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Construction of Six-Quark States from Parity Eigenfunctions for N–N Processes

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The study of the nucleon-nucleon interaction as a system of six quarks requires two important ingredients: the basis states and the quark-quark interaction usually taken to be the one gluon exchange.

The present work is devoted to the classification and construction of six-quark states as totally antisymmetric states of six fermions, each described by orbital, spin, isospin and color degrees of freedom. The classification scheme we propose can be viewed as an extension of the work done by Harvey (1981a) within the cluster model. A common feature with Harvey's work is the group theoretic classification scheme. Accordingly the orbital function $\psi_6(f)$ of partition f is coupled to the $SU_c(3)$ singlet color function $C_6(f')$ having $[f'] = [222]$ to give a state belonging to the representation $[\tilde{f}'']$ of the permutation group S_6. Together with the isospin-spin state $\Gamma_6(f'')$ of dual symmetry $[f'']$ it forms a totally antisymmetric state of six particles. The isospin-spin state Γ_6 is further subclassified by the chain $SU(4) \supset SU_T(2) \times SU_S(2)$ where T, S stands for isospin, spin. The wave function reads

$$\Psi_6 = \left\{ \{\psi_6(f) C_6(f')\}_{\tilde{f}''} \, \Gamma_6(f''TS) \right\}_{[1^6]} \tag{1}$$

The major difference with respect to Harvey (1981a) and—to our knowledge—all previous work on the nucleon-nucleon interaction (Harvey, 1981b; Harvey, 1984; Oka and Yazaki, 1984; Faessler, 1983; Maltman and Isgur, 1984; Suzuki and Hecht, 1984, 1986; DeTar, 1978; Myhrer and Wroldsen, 1987) is related to the spatial part of the wave function. Instead of the cluster model used previously, here we propose a classification scheme based on parity eigenfunctions. This means that the quarks, instead of occupying states R (right) or L (left) of a well located either at $+\frac{1}{2}Z$ or $-\frac{1}{2}Z$ on the Z-axis, now occupy single particle orbits in a deformed common well. The single particle hamiltonian is assumed to be reflectionally and axially symmetric and can be obtained, for example, from constrained Hartree-Fock or soliton mean field theories. Therefore the scheme can be used in either non-relativistic or relativistic independent particle models and, furthermore, can be applied to cluster models where it both simplifies and generalizes the states. The eigenstates are classified by parity π, projection of the angular momentum along the symmetry axis m, flavor f (u or d), color c and some other quantum number n which may designate the ordering of states of the same π, m, f, c at $Z = 0$. A Schematic representation of the single particle energies associated with these molecular type orbits is given in Fig. 1. In our study we limit ourselves to the first two orbits σ and π, each with projection $m = \pm\frac{1}{2}$ and of parity + and - respectively.

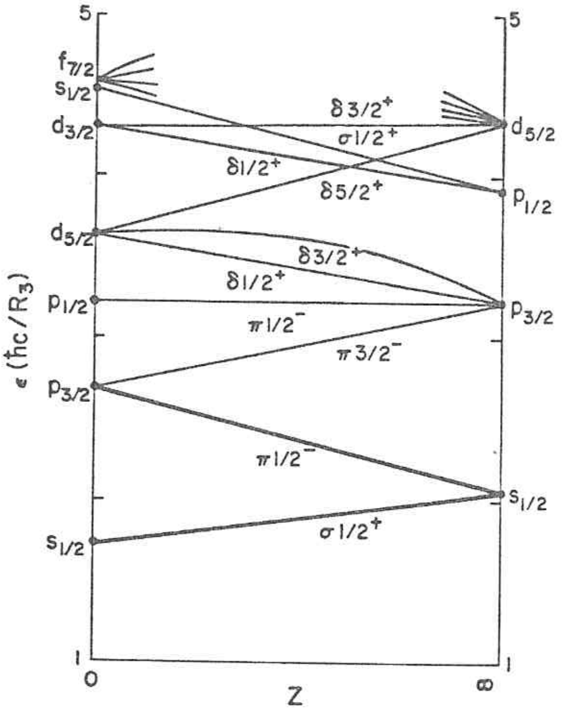

Fig. 1. *Schematic* representation of the eigenenergies of molecular-type orbitals for $0 \leq Z < \infty$. the $Z = \infty$ (separated bags) values are MIT eigenvalues in a bag of radius R_3; the $Z = 0$ (united bag) values are for an MIT bag of radius $R_6 = R_3/2^{1/4}$. For intermediate Z, straight lines (except for one case) are used to connect the limiting values. The lines are labelled by $|m|^{parity}$; the levels are degenerate with respect to the sign of m. The Greek letter is to be identified with the Roman label at $Z = 0$. Note that each state at $Z = \infty$ is degenerate with respect to parity as well as m. The heavy lines (lowest σ and π) are the only states considered here.

To make contact with the cluster model, it is useful to introduce pseudo-right r and pseudo-left ℓ states:

$$\begin{pmatrix} r \\ \ell \end{pmatrix} = 2^{-\frac{1}{2}}(\sigma \pm \pi) \quad \text{for all } Z. \tag{2}$$

At infinitely large separation distances, it is clear that $r \to R$ and $\ell \to L$, where R and L are cluster model states. A detailed comparison between (r, ℓ) and (R, L) states has been given by Stancu and Wilets (1987). An essential distinction between the two sets is that r and ℓ are orthogonal at any separation distance Z

while the cluster model states R and L become orthogonal only as $Z \to \infty$. The orthogonality property of r and ℓ makes calculations for many body states much simpler.

In the orbital space we can work either with (σ, π) states or with (r, ℓ) states. The allowed permutation symmetries for six quarks are therefore [6], [51], [42] and [33]. For each symmetry $[f]$ we give in Table 1 all the available positive and negative parity states with a specified structure $r^n \ell^{6-n}$ and its corresponding $\sigma^n \pi^{6-n}$ configurations. This basis is larger than the cluster model basis in two ways. First, the configurations of type $r^3\ell^3$ have a richer single particle composition. Let us consider for example the $r^3\ell^3[42]$ state. Bearing in mind that $\sigma \to s$ and $\pi - p$ when $Z \to 0$ one can see that this state contains both the s^4p^2 and the s^2p^4 limiting configurations while in the $R^3L^3[42]$ cluster model state the complementary s^2p^4 configuration is missing (Harvey, 1981a). Moreover, the function $r^3\ell^3$ [51] has two extra limiting configurations sp^5 and s^3p^3 and the function $r^3\ell^3$ [6] has three extra configurations p^6, s^2p^4 and s^4p^2, all missing in the cluster model. They will certainly play a role in the calculation of the nucleon-nucleon interaction at short separation distance.

Second, the configurations $r^n\ell^{6-n}$ with $m \neq 3$ are entirely absent in the cluster model approach. These states act as hidden color states. Their contribution to the energy expectation value is finite at zero or short separation distances and goes to infinity as $Z \to \infty$. The one gluon exchange produces a coupling between these channels and the physical N-N, N-Δ or Δ-Δ channels. Through this coupling they will have an influence on the physical channels at short distances similar to the effect of the hidden-color states described explicitly in (Harvey, 1981).

Table 1 Transformation from the r-ℓ to the σ-π representations.

Even parity		
Name	$r^n\ell^{6-n}$ configuration	$\sigma^n\pi^{6-n}$ configuration
$60^+[6]$	$\frac{1}{\sqrt{2}}[(r^6[6]) + (\ell^6[6])]$	$\frac{1}{4\sqrt{2}}[(\sigma^6[6]) + (\pi^6[6]) + \sqrt{15}(\sigma^4\pi^2[6]) + \sqrt{15}(\sigma^2\pi^4[6])]$
$51^+[6]$	$\frac{1}{\sqrt{2}}[(r^5\ell[6]) + (r\ell^5[6])]$	$\frac{1}{4}[\sqrt{3}(\sigma^6[6]) - \sqrt{3}(\pi^6[6]) + \sqrt{5}(\sigma^4\pi^2[6]) - \sqrt{5}(\sigma^2\pi^4[6])]$
$42^+[6]$	$\frac{1}{\sqrt{2}}[(r^4\ell^2[6]) + (r^2\ell^4[6])]$	$\frac{1}{4\sqrt{2}}[\sqrt{15}(\sigma^6[6]) + \sqrt{15}(\pi^6[6]) - (\sigma^4\pi^2[6]) - (\sigma^2\pi^4[6])]$
$33[6]$	$(r^3\ell^3[6])$	$\frac{1}{4}[\sqrt{5}(\sigma^6[6]) - \sqrt{5}(\pi^6[6]) - \sqrt{3}(\sigma^4\pi^2[6]) + \sqrt{3}(\sigma^2\pi^4[6])]$
$51^+[51]$	$\frac{1}{\sqrt{2}}[(r^5\ell[51]) + (r\ell^5[51])]$	$-\frac{1}{\sqrt{2}}[(\sigma^4\pi^2[51]) - (\sigma^2\pi^4[51])]$
$42^+[51]$	$\frac{1}{\sqrt{2}}[(r^4\ell^2[51]) + (r^2\ell^4[51])]$	$-\frac{1}{\sqrt{2}}[(\sigma^4\pi^2[51]) + (\sigma^2\pi^4[51])]$
$42^+[42]$	$\frac{1}{\sqrt{2}}[(r^4\ell^2[42]) + (r^2\ell^4[42])]$	$\frac{1}{\sqrt{2}}[(\sigma^4\pi^2[42]) + (\sigma^2\pi^4[42])]$
$33[42]$	$(r^3\ell^3[42])$	$\frac{1}{\sqrt{2}}[(\sigma^4\pi^2[42] - (\sigma^2\pi^4[42])]$
Odd parity		
$51^-[51]$	$\frac{1}{\sqrt{2}}[(r^5\ell[51]) - (r\ell^5[51])]$	$-\frac{1}{4}[\sqrt{2}(\sigma^5\pi[51]) - \sqrt{2}(\sigma\pi^5[51]) + 2\sqrt{3}(\sigma^3\pi^3[51])]$
$42^-[51]$	$\frac{1}{\sqrt{2}}[(r^4\ell^2[51]) - (r^2\ell^4[51])]$	$\cdot\frac{1}{\sqrt{2}}[(\sigma^5\pi[51]) + (\sigma\pi^5[51])$
$33[51]$	$(r^3\ell^3[51])$	$-\frac{1}{4}[\sqrt{6}(\sigma^5\pi[51]) - \sqrt{6}(\sigma\pi^5[51]) - 2(\sigma^3\pi^3[51])]$
$42^-[42]$	$\frac{1}{\sqrt{2}}[(r^4\ell^2[42]) - (r^2\ell^4[42])]$	$(\sigma^3\pi^3[42])$
$33[33]$	$(r^3\ell^3[33])$	$-(\sigma^3\pi^3[33])$

Using the orbital functions of Table 1, one can build color singlet totally antisymmetric states. In Table 2 we list all possible combinations of orbital and associated SU(4) symmetries which lead to six-quark antisymmetric states for the sectors $(TS) = (10)$ or (01), $(TS) = (00)$ and $(TS) = (11)$. In the $(TS) = (10)$ or (01) sector there are 16 orthogonal color-singlet channels, in the $(TS) = (00)$ there are 7 and in the $(TS) = (11)$ 25 channels. This is to be compared to 3, 3 and 6 channels considered by Harvey (1981).

Table 2 The orbital and SU(4) content for the color-singlet six quark states in the sectors (TS) (01), (10), (00) and (11).

Sector	Orbital	SU(4)
T=0,S=1 T=1,S=0	$60^+[6], 51^+[6], 42^+[6], 33[6]$	{33}
	$51^+[51], 42^+[51]$	{321}
	$42^+[42], 33[42]$	{51} {33} {411} {2211} {321}
T=0,S=0	$51^-[51], 42^-[51], 33[51]$	{42}
	$33[33]$	{6} {42} {222} {3111}
T=1,S=1	$51^-[51], 42^-[51], 33[51]$	{42}×2 {321}×2
	$42^-[42], 33[42]$	{51} {411} {321}×2
	$33[33]$	{6} {42}×2 {222} {311}

From cluster model states R and L, one can construct good parity orthonormal states for all Z by setting

$$\begin{pmatrix} \sigma \\ \pi \end{pmatrix} = [2(1 - < R\, L >)]^{-\frac{1}{2}} (R \pm L) \tag{3}$$

where R and L may be gaussians with a properly chosen size parameter, located at $+\frac{Z}{2}$ and $-\frac{Z}{2}$ on the Z-axis.

Our ultimate aim is to study N-N processes in the context of the (relativistic) soliton bag model (Wilets, 1987). Single particle states of the deformed mean field are already available (Schuh, 1985; Schuh, Pirner and

Wilets, 1986). As an intermediate stage we plan to perform a systematic study of the effect of the enlarged basis at $Z = 0$. Two models are being considered: the MIT bag model and the cluster model based on harmonic oscillator states.

Calculation of the matrix elements of the 6-quark hamiltonian will be reduced to the calculation of 2-body matrix elements through the procedure indicated by Harvey (1981) which makes use of fractional parentage coefficients and Clebsch-Gordan coefficients (K-matrix) of inner products of the symmetric group representations. The tables given by Harvey are now being extended to other representations or configurations appearing in our enlarged base.

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REFERENCES

DeTar, C. (1978). *Phys. Rev.* D 17, 302 ~~and 303.~~
Faessler, A., F. Fernandez, G. Lübeck, and K. Shimizu (1983). *Nucl. Phys.* A402, 555.
Harvey, M. (1981a). *Nucl. Phys.* A352, 301.
Harvey, M. (1981b). *Nucl. Phys.* A352, 326.
Harvey, M., J. Letourneux, and B. Lorazo (1984). *Nucl. Phys.* A424, 428.
Maltman, K. and N. Isgur (1984). *Phys. Rev.* D 29, 952.
For a review of the N-N interaction and the quark degrees of freedom, see Myhrer, F. and J. Wroldsen. CERN preprint 87-570.
Oka, M. and K. Yazaki (1984). *Quarks in Nuclei*, ed. W. Weise, World Scientific (Singapore), p. 489, and references therein.
Schuh, A. (1985). Doctoral Dissertation, University of Heidelberg.
Schuh, A., H. J. Pirner, and L. Wilets (1986). *Phys. Lett.* B174, 10.
Stancu, Fl. and L. Wilets (1987). *Phys. Rev.* C 36, 726.
Suzuki, Y. and K. T. Hecht (1984). *Nucl. Phys.* A420, 525.
Suzuki, Y. and K. T. Hecht (1986). *Nucl. Phys.* A446, 625.
Wilets, L. (1987). Present Proceedings.