BLED WORKSHOPS IN PHYSICS VOL. 2, NO. 1
ISSN 1580–4992

Proceedings of the Mini-Workshop

Selected Few-Body Problems in Hadronic and Atomic Physics

Bled, Slovenia, July 7–14, 2001

DMFA – ZALOŽNIŠTVO
LJUBLJANA, DECEMBER 2001

A three-body confinement force in constituent quark models *

Fl. Stancu[a]** and S. Pepin[b]

[a] Institute of Physics B.5, University of Liège, Belgium
[b] Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany

Abstract. We discuss the role of a three-body colour confinement interaction introduced on algebraic grounds and present some of its implications for the spectra of baryons, tetraquarks and six-quark systems within a simple quark model.

In quark models two distinct types of three-body forces have been introduced so far, namely long-range, confinement forces, as e.g. in [1] or short-range forces, associated to the instanton 't Hooft interaction, as e.g. in [2].

Here we shall discuss the implications on baryon spectroscopy of the long-range confinement forces only. We are not presently concerned with the origin of these forces which is still controversial, as lattice calculations suggest [3]. We shall use an algebraic approach inspired by Ref. [4], based on the invariant operators of SU(3).

Let us consider the Hamiltonian

$$H = T + V_{2b} + V_{3b}\,, \tag{1}$$

where T is the kinetic energy and V_{2b} a 2-body confinement interaction of the form

$$V_{2b} = \sum_{i<j} V_{ij}\,(c_1 + \frac{4}{3} + F_i^a F_j^a)\,, \tag{2}$$

where $F_i^a = \frac{1}{2}\lambda_i^a$ is the colour charge operator of the quark i and c_1 an arbitrary constant which we set equal to 1 as in Ref. [4]. For simplicity we take

$$V_{ij} = \frac{1}{2}\, m\omega^2\, (r_i - r_j)^2\,. \tag{3}$$

V_{3b} is the 3-body confinement interaction

$$V_{3b} = V_{ijk} = \mathcal{V}_{ijk}\mathcal{C}_{ijk}\,, \tag{4}$$

with

$$\mathcal{V}_{ijk} = \frac{1}{2}\, c\, m\omega^2\, [(r_i - r_j)^2 + (r_j - r_k)^2 + (r_k - r_i)^2]\,, \tag{5}$$

* Talk delivered by Fl. Stancu.

where c is a strength parameter and $\mathcal{C}_{ijk}$ a colour operator of type

$$\mathcal{C}_{ijk} = d^{abc}\, F_i^a\, F_j^b\, F_k^c \,. \tag{6}$$

The coefficients d^{abc} are some real constants, symmetric under any permutation of indices (see e.g. Ref. [5], chapter 8). Performing integration in the colour space and expressing H in terms of the internal coordinates $\rho = (r_1 - r_2)/\sqrt{2}$ and $\lambda = (r_1 + r_2 - 2r_3)/\sqrt{6}$, we have:

$$H = 3m - \frac{\hbar^2}{2m}(\nabla_\rho^2 + \nabla_\lambda^2) + \frac{3}{2}m\omega^2\chi_i(\rho^2 + \lambda^2)\,, \tag{7}$$

with

$$\chi_i = \begin{cases} \frac{5}{3} + \frac{10}{9}c & \text{i=1 (singlet)} \\ \frac{13}{6} - \frac{5}{36}c & \text{i=8 (octet)} \\ \frac{8}{3} + \frac{1}{9}c & \text{i=10 (decuplet)} \end{cases} \tag{8}$$

In the expressions of χ_i ($i = 1, 8$ or 10), the first and second terms stem from the colour part of V_{2b} and V_{3b} respectively. The energies E_1, E_8 and E_{10} of the singlet, octet and decuplet states in a $3q$ system are

$$E_1 = 3m + 3\hbar\omega\sqrt{3\chi_1}\,, \tag{9}$$

$$E_8 = 3m + 4\hbar\omega\sqrt{3\chi_8}\,, \tag{10}$$

$$E_{10} = 3m + 5\hbar\omega\sqrt{3\chi_{10}}\,. \tag{11}$$

The baryon spectrum is correctly described provided [4]

$$-\frac{3}{2} < c < \frac{2}{5}\,. \tag{12}$$

The lower limit comes from the stability constraint $\chi_1 > 0$ and the upper limit from imposing the right sequence in the spectrum, i.e. from the requirement $\chi_8 > \chi_1$. The closer c is to the lower limit, the larger is the gap between the colour octet and singlet states. To see this, let us take c = -1.43. There is of course some arbitrariness in choosing m and $\hbar\omega$. As typical values for quark models we take $m = 0.340$ GeV and $\hbar\omega = 0.6$ GeV [6]. This implies an octet-singlet gap $\Delta E = E_8 - E_1 \approx 5.5$ GeV. For $c = 0$ (no three-body force) one would have $\Delta E = 3.5\hbar\omega \approx 2.1$ GeV. Therefore the gap is increased considerably by a three-body force with a strong negative strength. This is a desired feature for quark models with three valence quarks (no gluons). In the same way one can show that the decuplet state is located above the octet with quite a large gap for a limiting value of c.

Let us now consider tetraquarks, i.e. $q^2\bar{q}^2$ systems and denote the quarks by 1 and 2 and the antiquarks by 3 and 4. One can first form $q\bar{q}$ pairs which are next coupled to colour singlets. These states are either singlet-singlet states $|1_{13}1_{24}\rangle$ or octet-octet states $|8_{13}8_{24}\rangle$ (see Ref. [5], chapter 10). One can have a three-body interaction acting in a $q^2\bar{q}$ subsystem as or a three-body interaction acting in a $q\overline{q}^2$ subsystem as

$$\overline{\mathcal{C}}_{ijk} = d^{abc}\, F_i^a\, \overline{F}_j^b\, \overline{F}_k^c\,, \tag{14}$$

where

$$\overline{F}_i^a = -\frac{1}{2}\lambda_i^{a*}\,, \tag{15}$$

is the charge operator of an antiquark. The operators (13) or (14) between these tetraquark states have the same eigenvalue, which is calculated in two steps. First, one evaluates the eigenvalue between the states $|\overline{3}_{12}3_{34}\rangle$ and $|6_{12}\overline{6}_{34}\rangle$, where the two quarks couple either to a $\overline{3}$ or a 6 state and the antiquarks to a 3 or a $\overline{6}$ state. One obtains $-5/9$ for the $|\overline{3}_{12}3_{34}\rangle$ state and $5/18$ for $|6_{12}\overline{6}_{34}\rangle$. The physically relevant states $|1_{13}1_{24}>$ and $|8_{13}8_{24}>$ are then defined by the transformations (see e.g. Ref. [7]):

$$|1_{13}1_{24}> = \sqrt{\frac{1}{3}}|\overline{3}_{12}3_{34}> + \sqrt{\frac{2}{3}}|6_{12}\overline{6}_{34}>\,, \tag{16}$$

$$|8_{13}8_{24}> = -\sqrt{\frac{2}{3}}|\overline{3}_{12}3_{34}> + \sqrt{\frac{1}{3}}|6_{12}\overline{6}_{34}>\,. \tag{17}$$

Thus one gets:

$$\langle 1_{13}1_{24}|\overline{\mathcal{C}}_{123}|1_{13}1_{24}\rangle \propto [\frac{1}{3}(-\frac{5}{9}) + \frac{2}{3}\frac{5}{18}]\, c = 0\,, \tag{18}$$

and

$$\langle 8_{13}8_{24}|\overline{\mathcal{C}}_{123}|8_{13}8_{24}\rangle \propto [\frac{2}{3}(-\frac{5}{9}) + \frac{1}{3}\frac{5}{18}]\, c = -\frac{5}{18}\, c\,, \tag{19}$$

which shows that with a negative c one raises the expectation value of the octet-octet above the singlet-singlet state more than with $c = 0$. This implies that in the presence of a 3-body confinement interaction with $c < 0$ the coupling between octet-octet and singlet-singlet states due to a hyperfine splitting will be diminished, which amounts to make a ground state tetraquark less stable. This seems to be consistent with the experimental observation that no stable tetraquark system has been seen so far.

The q^6 systems are important for the NN problem. Here we discuss the sector IS = (01) or (10). It is well known that the physical NN state is a combination of three symmetry states containing the orbital configurations $[6]_O$ and $[42]_O$, as shown for example in [8]. In fact the three symmetry states allowed by the Pauli principle can be combined into the NN and ΔΔ states and the unphysical hidden-colour CC state. The latter has an important role at short separation between the 3q clusters. Using 3-body fractional parentage coefficients, one can calculate the matrix elements of the three-body force (4)-(6) in the basis of the states $|NN>$,

$|\Delta\Delta>$ and $|CC>$. This gives rise to the following matrix:

	NN	ΔΔ	CC
NN	$\frac{28}{81}c$	$\frac{38\sqrt{5}}{405}c$	$\frac{38\sqrt{5}}{135}c$
ΔΔ	$\frac{38\sqrt{5}}{405}c$	$\frac{121}{405}c$	$\frac{76}{135}c$
CC	$\frac{38\sqrt{5}}{135}c$	$\frac{76}{135}c$	$\frac{9}{5}c$

(20)

The eigenvalues of this matrix are $E_1 = c/9, E_2 = c/9$ and $E_3 = 20c/9$. This shows that the effect of the 3-body colour confinement on NN and ΔΔ is identical and rather small as compared to that on CC. In particular for a negative value of c, the spectrum of NN, ΔΔ and CC lowers and shrinks. For a positive c, the situation is the other way round. This means that, for $c < 0$, V_{3b} itself brings some attraction and implies a stronger coupling of CC to NN and ΔΔ due to a hyperfine interaction. This will lead to a reduced hard core repulsion in the NN potential.

In conclusion, a three-body confinement force can affect the spectrum of multiquark systems in a positive or negative way, depending on the strength c. In particular, if c is negative, the unphysical octet and decuplet states of a system of three quarks become well separated from the colour singlet states, which is a desired feature for models with three valence quarks only. In tetraquarks its role is also positive because it decouples the colour octet-octet state from the singlet-singlet one, the first being unphysical. For six-quark systems the role of the three-body confinement force with a negative strength is controversial. It increases the coupling of the hidden-colour CC states to the physical NN and ΔΔ states. On one hand, this brings more attraction into the NN potential, which is useful to lower too high hard core potentials, but on the other hand this implies stronger Van der Waals forces. The latter is in contrast with the hopes of Ref. [4].

Details of the calculations can be found in Ref. [9]. One should notice that the present study is based on a simple harmonic oscillator confinement. It would be useful to extend it to a more realistic confinement. Also, for tetraquarks and six-quark systems the results are derived for compact configurations, i.e. for zero separation between the hadronic clusters, here of type $q\overline{q}$ or q^3. It would certainly be interesting to study non-zero separation distances (molecular type configurations).

Acknowledgements: We are most grateful for the useful comments made by Mitja Rosina regarding the role of the three-body forces in tetraquarks and six-quark systems.

References

2. A. E. Dorokhov and N. I. Kochelev, preprint JINR-E2-86-847, 1986; M. Oka and S. Takeuchi, Nucl. Phys. **A524** (1991) 649; S. Takeuchi and M. Oka, Phys. Rev. Lett. **66** (1991) 1271.
3. T. T. Takahashi, H. Matsufuru, Y. Nemoto and H. Suganuma, Phys. Rev. Lett. **86** (2001) 18; C. Alexandrou, Ph. de Forcrand and A. Tsapalis, hep-lat/0107006.
4. V. Dmitrasinovic, Phys. Lett **B499** (2001) 135.
5. Fl. Stancu, *Group Theory in Subnuclear Physics*, Oxford University Press, 1996.
6. Fl. Stancu, S. Pepin and L. Ya. Glozman, Phys. Rev. **C56** (1997) 2779; **C59** (1999) 1219 (erratum).
7. D. M. Brink and Fl. Stancu, Phys. Rev. **D49**, 4665 (1994)
8. M. Harvey, Nucl. Phys. **A352** (1981) 301; ibid. **A481** (1988) 834 (erratum).
9. S. Pepin and Fl. Stancu, Phys. Rev. **D**, to be published.