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SYMMETRIES OF SIX-QUARK STATES RELATED TO THE NUCLEON-NUCLEON PROBLEM

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ABSTRACT

We discuss two group theoretical aspects of the classification and construction of six-quark states related to the NN problem. One concerns the coupling schemes available in the literature and a unitary transformation is derived between the TS and CS coupling schemes. The other gives a prescription for using molecular orbitals as single particle quark states.

1. Introduction

Since the nucleon is now treated as a composite object one expects that its structure will influence the nucleon-nucleon interaction at short separation distances, when the two interacting nucleons overlap very strongly. If the quark core radius is of the order of 0.5 fm or 0.7 fm for an equivalent radius one expects the quark structure to be effective in a range up to about 1 fm. In this range the nucleon-nucleon system can be viewed as a six-quark system confined in a region of space and interacting via gluon exchange. In nonrelativistic quark models the confinement is achieved by the spin-independent interaction[1-4], in the MIT bag[5] by appropriate boundary conditions and in the soliton bag[6] by an additional scalar field or through a chromo-dielectric function[7] mediated by a scalar field. A description of the nucleon-nucleon interaction within the frame of quark models has been the subject of much research and we refer the reader to reviews written, for example, by Oka and Yazaki[8] or Myhrer and Wroldsen.[9]

The purpose of the present contribution is to discuss some group theoretical aspects of the six-quark system, inasmuch as the classification and construction of six-quark states is a central problem in any derivation of the nucleon-nucleon interaction. In principle the choice of a basis is irrelevant if a very large number of states is included but in practice one has to use a finite set which has to contain the most important ones. The discussion we give here is valid for any nonrelativistic or relativistic model which provides the single particle states necessary to build six-quark states.

The symmetries of the six-quark states are related to the degrees of freedom characterizing the quarks : orbital (O), colour (C), flavour or isospin (T) and spin (S). There are two distinct aspects related to the construction of six-quark states which we wish to present here. One is concerning the orbital part of the wavefunction and the other to the coupling schemes.

2. Coupling Schemes

Having four distinct degrees of freedom O, C, T and S in the N-N problem, several coupling schemes are possible in constructing totally antisymmetric six-quark states. There are two classification schemes available. One is based on the intermediate CS coupling and the other on the TS coupling. The TS coupling scheme allows to naturally construct the physical NN and $\Delta\Delta$ states[10] from symmetry states. The CS coupling scheme favours symmetry states for which the expectation value of the spin-spin interaction is lowest. These are not necessarily physical states. A unitary transformation has been derived[11] to relate states in the two coupling schemes. The chosen orbital symmetries are $[42]_O$ in the TS = (01) and $[33]_O$ in the TS = (00) sector. This allows to determine the correct CS composition of the NN state. For example the (TS) = (01) or (10) result is shown in Table 1.

TABLE 1. The content of NN in the CS scheme for TS = (01) or (10) .

	$[6]_O\{222\}_{CS}$	$[42]_O\{42\}_{CS}$	$[42]_O\{321\}_{CS}$	$[42]_O\{3111\}_{CS}$	$[42]_O\{222\}_{CS}$	$[42]_O\{21111\}_{CS}$
NN	$\frac{1}{3}$	$\frac{1}{6\sqrt{5}}$	$-\frac{4}{27\sqrt{5}}$	$-\frac{1}{27\sqrt{2}}$	$\frac{31}{54}$	$-\frac{20}{27}$

One can see that among the $[42]_O$ states the largest contribution comes from $[21111]_{CS}$ (54.9 %) and $[222]_{CS}$ (33 %) symmetries. On the other hand the

symmetry $[42]_{CS}$ gives the lowest (negative) contribution to the expectation value. As this state contributes to less than 1 % the dominant states should first of all be taken into account.

3. Molecular Orbitals

Here we use the TS coupling scheme, where one first couples the colour singlet $[222]_C$ state to a specific orbital symmetry $[f]_O$ to give a state of symmetry $[\tilde{f}']$ which has to be coupled to the dual $[f']_{TS}$ state in order to give an antisymmetric $[1^6]$ state which reads

$$\Psi_6 = \left(\left([f]_O [222]_C \right)_{[\tilde{f}']} [f']_{TS} \right)_{[1^6]} . \qquad (1)$$

The difference with respect to Harvey's scheme or other studies is that we use molecular single-particle states, σ and π, instead of cluster model states R (right) and L (left). The molecular orbitals are states of definite parity in contrast to cluster model states. Here σ has positive and π negative parity. They are eigenstates of the same hamiltonian which incorporates a mean field. Hence they are orthogonal at any separation. For the scattering problem, from σ and π it is convenient to construct orthogonal pseudo-right and pseudo-left orbitals, r and ℓ as

$$\begin{pmatrix} r \\ \ell \end{pmatrix} = 2^{-\frac{1}{2}} (\sigma \pm \pi). \qquad (2)$$

The r,ℓ states recover the R,L cluster model states at large separation. The algebra with r,ℓ states is much simpler than that with R,L states because of their orthogonality. The transformation from the r,ℓ to the σ,π representation of the relevant six-quark states was given in Table 1 of Ref. 12. In practical calculations it is simpler to work with r,ℓ instead of σ,π states. One can return to the σ,π states after the six-body matrix elements have been reduced to linear combinations of one- and two-body matrix elements by using the fractional parentage technique. In Refs. 13 and 14 we performed and compared calculations based on a cluster model basis on the one hand and a molecular basis on the other hand. Reference 13 refers to a constituent nonrelativistic quark model and Ref. 14 to a current (relativistic) quark model, the prototypical MIT model. In both cases we found that the

ground state energy was substantially lowered through the use of molecular orbitals, by making an appropriate choice of six-quark basis states. We found that at zero separation configurations of type $s^n p^{6-n}$ ($n \neq 3$), which are missing in the cluster model basis, are very important. As an example in Table 2 we reproduce results for the TS = (01) or (10) sectors in the MIT case. The first three basis states form the "physical" basis in Harvey's transformation[10] and contain configurations of type $r^3 \ell^3$. The other molecular configurations are of type $r^4 \ell^2 + r^2 \ell^4$ or $r^5 \ell + r \ell^5$, denoted by 42^+ and 51^+, respectively.

TABLE 2. Eigenvalues (MeV) of the MIT hamiltonian (bag energy plus quark single-particle energies plus one gluon exchange) for TS = (01) or (10) in the cluster R,L basis and molecular r,ℓ basis. In each case n×n is the matrix to be diagonalized ; 1×1 represents the diagonal matrix elements before diagonalization.

	R,L (cluster)		r, ℓ (molecular)		
channel	1×1	3×3	1×1	7×7	amplitude
NN	71	10	0.02	- 82	- 0.911
ΔΔ	354	237	418	242	– 0.035
CC	543	721	1010	436	- 0.060
(42+ [6] {33})			763	581	- 0.274
(42+ [42] {33})			968	772	- 0.188
(42+ [42] {51})			586	968	0.186
(51+ [6] {33})			578	1403	0.140

In the calculations mentioned above the results are for a united bag (zero separation) which is the extreme situation relevant for short distance interactions. At present we consider applications of the molecular orbital basis at finite separations (deformed bags) within a more realistic model, the chromo-dielectric model. Partial results can be found in L. Wilets' invited talk at this conference.[15]

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