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ON THE ROLE OF HIDDEN-COLOUR STATES IN $2q - 2\bar{q}$ SYSTEMS

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ABSTRACT

We study a system of two quarks and two antiquarks of equal masses in the framework of a nonrelativistic potential model with colour confinement and hyperfine spin-spin interaction. We propose a simple variational solution and conclude that the hyperfine interaction is needed to bind the system in agreement with previous studies. Our results explicitly show the important role played by the hidden-colour states in lowering the variational energy.

The diquonia or $2q - 2\bar{q}$ systems are a class of exotics. They provide an interpretation of some resonances whose properties cannot be understood with a $q\bar{q}$ model.[1-4] They also serve at modelling the meson-meson interaction.[5]

The resonances under discussion are also called hadronic molecules. There are several candidates in the literature. For example the well established $J^{PC} = 0^{++}$ states $a_0\,(980)$ and $f_0\,(975)$ are interpreted as $K\bar{K}$ molecules.[1-3] Their properties are such that they cannot be accommodated as members of the $^3P_0$ $q\bar{q}$ nonet. The $J^{PC} = 1^{++}$ state $f_1\,(1420)$ was long considered to be a member of the $^3P_1$ multiplet but now it is interpreted[5] as a quasi-bound state of $K^*\bar{K}$. Recently, proposals[5-7] were made to interpret the $f_0\,(1720)$ state as a vector meson molecule $K^*\bar{K}^*$ or $K^*\bar{K}^* + \omega\,\phi$. These interpretations are somewhat controversial[4] and more theoretical and experimental work is required.

A crucial problem related to the study of hadronic molecules as $2q-2\bar{q}$ systems is whether they possess bound states. Here we present results obtained from a rather rudimentary model in order to get answers to the following questions : 1) the role of spin-spin (one gluon exchange) ; 2) the role of hidden-colour (octet-octet) states.

Our framework is a nonrelativistic potential model with harmonic colour confinement and hyperfine spin-spin interaction and we choose the same hamiltonian as in Ref. 2. We discuss identical quarks and are concerned with the $T = 0$, $S = 0$ sector only. In the colour-spin space we introduce four independent channel wave functions : the pseudoscalar meson-pseudoscalar meson PP , the vector meson - vector meson VV and two hidden-colour C_0C_0 and C_1C_1 channels. This is in contradistinction to Refs. 2 and 3 where only the physical channels PP and VV have been considered.

The model hamiltonian is

$$H = \sum_{i=1}^{4} \left(m_i + \frac{p_i^2}{2m_i} \right) + \sum_{i<j} \left(V_{ij}^{conf} + V_{ij}^{hyp} \right), \tag{1}$$

with

$$V_{ij}^{conf} = -\left(e_0 + \frac{1}{2} k r_{ij}^2 \right) \frac{\hat{\lambda}_i}{2} \cdot \frac{\hat{\lambda}_j}{2} \tag{2}$$

and

$$V_{ij}^{hyp} = V_{ij}^{SS} = -\frac{8\pi\alpha_s}{3 m_i m_j} f(r_{ij})\, \vec{S}_i \cdot \vec{S}_j \, \frac{\hat{\lambda}_i}{2} \cdot \frac{\hat{\lambda}_j}{2}, \tag{3}$$

where $r_{ij} = |\vec{r}_i - \vec{r}_j|$ and m_i, $\vec{r}_i$, $\vec{p}_i$, $\vec{S}_i$ and $\hat{\lambda}_i/2$ are the mass, position, momentum, spin and colour operators of the i^{th} particle. The coordinate space part of the spin-spin interaction has the regularized form

$$f(r_{ij}) = \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2 r_{ij}^2} \tag{4}$$

where σ is a parameter. The confinement potential (2) has a harmonic form for simplicity. In order to allow comparison with Ref. 2 we choose one set of their parameters. These are :

$$\begin{aligned} m &= m_u = m_d = 330 \text{ MeV} \\ \omega_0 &= 200 \text{ MeV} \\ e_0 &= -352 \text{ MeV} \\ \alpha_s &= 2.7 \\ \sigma &= 2 \text{ fm}^{-1}, \end{aligned} \tag{5}$$

where the parameter $\omega_0 = (k/m)^{1/2}$ gives $k = 339 \text{ MeV fm}^{-2}$.

We deal with u and d quarks only. For the $q\bar{q}$ problem we choose a wave function of type

$$\phi_0 = \frac{a^{3/2}}{\pi^{3/4}} e^{-\frac{a^2}{2} r_{ij}^2}. \tag{6}$$

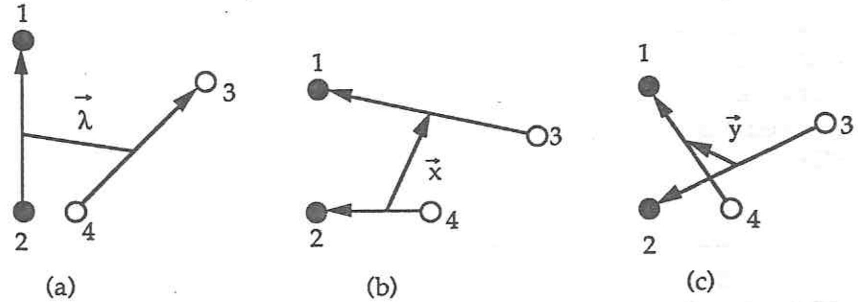
The parameter a is fixed by minimizing the two-body hamiltonian ground state energy. This corresponds to a pseudoscalar meson mass

$$m_\pi = 172.7 \text{ MeV at } a = 2 \text{ fm}^{-1}. \tag{7}$$

The value $a = 2 \text{ fm}^{-1}$ gives a radius of $\langle r^2 \rangle_\pi^{1/2} = 0.30$ fm .

For describing the $2q-2\bar{q}$ system one can introduce three alternative coordinate systems as shown in Fig. 1. They can be used at various stages in order to simplify calculations.

FIGURE 1. Three possible ways to define relative coordinates for a $2q-2\bar{q}$ system. Darkened and open circles represent quark and antiquarks respectively.



Including spin and colour degrees of freedom one can define four channel wave functions with total spin $S = 0$ as follows :

$$\Psi_{PP} = \frac{N_{PP}}{\sqrt{2}}\left[R_P(x)\,|1_{13}1_{24}\rangle\,|P_{13}P_{24}\rangle + R_P(y)\,|1_{14}1_{23}\rangle\,|P_{14}P_{23}\rangle\right] \tag{8}$$

$$\Psi_{VV} = \frac{N_{VV}}{\sqrt{2}}\left[R_V(x)\,|1_{13}1_{24}\rangle\,|V_{13}V_{24}\rangle + R_V(y)\,|1_{14}1_{23}\rangle\,|V_{14}V_{23}\rangle\right] \tag{9}$$

$$\Psi_{C_0C_0} = \frac{N_{C_0C_0}}{\sqrt{2}}\left[R_{C_0}(x)\,|8_{13}8_{24}\rangle\,|P_{13}P_{24}\rangle + R_{C_0}(y)\,|8_{14}8_{23}\rangle\,|P_{14}P_{23}\rangle\right] \tag{10}$$

$$\Psi_{C_1C_1} = \frac{N_{C_1C_1}}{\sqrt{2}}\left[R_{C_1}(x)\,|8_{13}8_{24}\rangle\,|V_{13}V_{24}\rangle + R_{C_1}(y)\,|8_{14}8_{23}\rangle\,|V_{14}V_{23}\rangle\right]. \tag{11}$$

Here $|1_{13}1_{24}\rangle$ and $|1_{14}1_{23}\rangle$ are singlet-singlet colour space states, and $|8_{13}8_{24}\rangle$ and $|8_{14}8_{23}\rangle$ are the octet-octet counterparts. In the spin space $|P_{13}P_{24}\rangle$ and $|P_{14}P_{23}\rangle$ are the pseudoscalar-pseudoscalar states and $|V_{13}V_{24}\rangle$ and $|V_{14}V_{23}\rangle$ are the vector-vector states. Then one can see that each of the channel wave functions has a direct and an exchange term and that Ψ_{PP} corresponds to the pseudoscalar-pseudoscalar channel and Ψ_{VV} to the vector-vector channel. The functions (10) and (11) represent closed (hidden-colour) channels. They are formed by coupling two colour octet $q\bar{q}$ pairs to a single $2q-2\bar{q}$ state. When the spin of a $q\bar{q}$ pair couples to zero the channel is denoted by C_0C_0 and when it couples to one the notation is C_1C_1. In this paper we study the explicit influence of the hidden-colour states.

The orbital wave functions $R_i(x)$ $(i = P, V, C_0, C_1)$ are expressed in terms of the coordinates defined in Fig. 1b and the exchange wave functions $R_i(y)$ in terms of the coordinates in Fig. 1c. For identical quarks the wave functions (8)-(11) are symmetric under permutations (12) and (34). The antisymmetry is re-established through the flavour part of the total wave function

$$\frac{1}{2}(ud - du)(\bar{u}\bar{d} - \bar{d}\bar{u}) \tag{12}$$

The wave functions (8)-(11) are not orthogonal to each other. This

raises the question as to whether they are linearly independent. It can be shown that the functions (10) and (11) are linearly dependent on (8) and (9) if the orbital wave functions are expresses in terms of a complete set of functions. In that case the hidden-colour wave functions are redundant. On the other hand the set (8)-(11) is not normally linearly dependent if the orbital functions are linear combinations of a finite set of functions with a restricted form. In that case the hidden-colour channels can introduce new components into the wave function and lower the variational energy.

In our calculations the orbital parts $R_i(x)$ and $R_i(y)$ are treated variationally in a Gaussian basis.[8] For example

$$R_i(x) = \sum_b a_i(b)\,R(x,b)\,, \tag{13}$$

where

$$R(x,b) = e^{-a^2\rho^2}\,e^{-a^2\rho'^2}\,e^{-b^2x^2}\,. \tag{14}$$

Some of our results are shown in Table 1. The first two rows show the effect of hidden-colour states when the orbital wave function is a simple Gaussian. The parameter a has been fixed by the two-body $q\bar{q}$ problem (eq. (7)). The parameter b was determined variationally in the $2q-2\bar{q}$ problem. By comparing rows 1 and 2 we see that including hidden-colour channels almost doubles the binding. Rows 3 and 4 are calculated with a wave function which is a linear combination of two Gaussians with range parameters $b = 1.1\ \mathrm{fm}^{-1}$ and $b' = 2.2\ \mathrm{fm}^{-1}$. Row 4 contains hidden-colour channels while row 3 does not. One can see again the role of the hidden colours in increasing the binding.

TABLE 1. The minimum in the interaction energy of a $2q-2\bar{q}$ system showing the effect of adding hidden-colour channels.

Channels	b (fm^{-1})	b' (fm^{-1})	E_{min} (MeV)
PP, VV	1.2		-28.9
PP, VV, C_0C_0, C_1C_1	1.1		-51.9
PP, VV, P'P', V'V'	1.1	2.2	-84.6
PP, VV, C_0C_0, C_1C_1, P'P', V'V', $C'_0C'_0$, $C'_1C'_1$	1.1	2.2	-102

Calculations of Table 1 include the hyperfine interaction in the hamiltonian. We found that without hyperfine interaction the system doesn't bind. This is in agreement with previous work.[2,3]

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