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THE NUCLEON-NUCLEON POTENTIAL IN THE CHROMODIELECTRIC SOLITON MODEL

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ABSTRACT

The short- and medium-range parts of the nucleon-nucleon interaction are being studied in the framework of the chromodielectric soliton model. The model consists of current quarks, gluons in the abelian approximation, and a scalar σ field which simulates the nonabelian interactions of the gluons and governs the medium through the dielectric function $\kappa(\sigma)$. Absolute color confinement is effected by the vanishing of the dielectric in vacuum; this also removes the troublesome van der Waals problem. We distinguish between *spatial* confinement, which arises from the self-energy of the quarks *in medium* (excluding MFA contributions), and *color* confinement which is effected through OGE in the MFA (including the corresponding self-energy contributions). The *static* (adiabatic) energies are computed as a function of deformation (generalized bag separation) in a constrained MFA. Six quark molecular-type wave functions in all important space-spin-isospin-color configurations are included. The gluon propagator is solved in the deformed dielectric medium. The resultant Hamiltonian matrix is diagonalized. Dynamics are handled in the Generator Coordinate Method, which leads to the Hill-Wheeler integral equation. In the present case, this yields a set of coupled equations corresponding to the various configurations. Although this can be approximated by a set of differential equations, we propose to solve the integral equations with some regularization scheme.

1. Introduction

In N-N phenomenology, relativistic or non-relativistic, one describes the nucleons as elementary particles interacting through a two-body potential which is either local or includes some nonlocality through momentum-dependence in the interaction.

Meson field-theoretic models usually treat the nucleons as elementary particles with an empirical form factor, and their interactions are carried through one-boson exchange (OBE) plus two pion exchange (TPE) where the TPE is frequently simulated by a (fictitious) sigma meson.

Within these descriptions, the long range (1.5 fm $\lesssim r$) part of the N-N interaction is controlled by one pion exchange, while the intermediate range (0.5 fm $\lesssim r \lesssim$ 1.5 fm) is dominated by OBE and TPE. Short range ($r \lesssim$ 0.5 fm) is the "mystery" region in such descriptions. Hard or soft cores in potential models, or *ad hoc* cutoffs in field theories, have been invoked.

The advent of QCD and quark models has lifted the veil of mystery from the short range N-N problem exposing a new level of simplicity. However, the system is no longer just a two-body system, but is at least a six-body problem and more properly a field theory problem. The quark core of nucleons is of the order of 0.7 fm (an rms radius of 0.5 fm) and one expects that the quark substructure to be effective within a range of N-N separations up to about 1 fm.

A description of the nucleon-nucleon interaction within the framework of quark models has been the subject of much research. The ideal calculation would be a lattice gauge theory calculation, but we are not yet at that stage. We mention, non-exhaustively, several categories of models which have been employed:

- Non-relativistic constituent (potential) quark models,
- Relativistic current quark models, including various soliton models,
- String models,
- Skyrme models.

There are many varieties under each category, and we will not attempt to review them all here, but recommend the reader to Oka and Yazaki,[1] and Myhrer and Wroldsen.[2]

The quark models hold promise to give a good description for short range and into the intermediate range, but beyond about 1 fm the interaction, although in principle describable in terms of quark degrees of freedom, is more easily represented by meson models, with nucleon substructure giving rise to form factors for the nucleon-meson couplings.

The ultimate object of our study is not only to reproduce two-body data, such as phase shifts and bound state properties, but also to quantify the quark substructure of nuclei. With respect to the latter, the collision process can be described as an act of fusion followed by separation into three-quark clusters, and this can be used in conjunction with the Independent Pair Model of nuclei (which incorporates two-body correlations).

The main aspects of our project are described as follows:

(1) We employ the chromodielectric soliton model, which has the following features:

- Covariance.
- Absolute color confinement.
- One gluon exchange with a confined gluon propagator.
- No color van der Waals problem.

(2) "Molecular" basis states for the six quark system are used including all spin-isospin-color configurations based on the two lowest spatial configurations.[3] This allows for basis states normally omitted in calculations based on cluster models. These states have been demonstrated to be important in lowering the upper bound of variational solutions.[4,5]

(3) Dynamics will be handled through the generator coordinate method which leads to a set of coupled integral equations. No statements concerning a potential are meaningful without dynamics. It has been shown that a significant part of short-range repulsion is due to dynamics.[6] The effective interaction is very non-local in terms of N-N separation, and even the meaning of N-N separation must be reinterpreted for overlapping clusters.

(4) In order to reproduce two-body properties, we attach the interaction to a phenomenological local OBE potential beyond about 0.5 or 0.7 fm. We could consider extending the model calculations more deeply into the OBE intermediate range region by including configurations of the form $q^7\bar{q}$ along with surface oscillations of the σ-field.

2. The Model

The chromodielectric model[7] is an evolution of the Friedberg-Lee nontopological soliton model.[8,9] Its Lagrangian is the same as the fundamental QCD Lagrangian, supplemented by a scalar field, σ, which simulates the gluon condensate and other scalar structures which inhabit the complicated physical vacuum. The model Lagrangian is covariant and, for massless quarks, satisfies chiral symmetry. The extra degrees of freedom introduced by the sigma field are redundant, but we are dealing with an effective theory. In order to avoid double counting we do not include structures (diagrams) of scalar objects already represented by the σ field. There are parameters in the model which are adjusted at every level of approximation to fit key physical data. One might hope that as the sophistication of the calculations is increased, one would find a decoupling of the σ degrees of freedom and one should be left with pure QCD and a phantom scalar field. We are far from that stage of sophistication. Currently we treat the gluons in the Abelian approximation with the σ field simulating non-linear and non-perturbative effects. Although the model has a basis in QCD, we regard it as phenomenological.

2.1 The Lagrangian

The primary role of the σ field is to mediate the chromodielectric function $\kappa(\sigma)$. The model Lagrangian density is given by

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_\sigma + \mathcal{L}_G , \tag{1}$$

with

$$\begin{aligned} \mathcal{L}_q &= \bar{\psi}\ (i\gamma^\mu D_\mu - m_q)\ \psi \\ \mathcal{L}_\sigma &= \frac{1}{2}\ \partial_\mu\sigma\partial^\mu\sigma - U(\sigma) \\ \mathcal{L}_G &= -\frac{1}{4}\ \kappa(\sigma)\ F^c_{\mu\nu}\ F^{\mu\nu}_c . \end{aligned}$$

Note that there is no direct coupling between the quarks and the σ field. Here m_q is the quark mass matrix, and for massless quarks the Lagrangian is chirally invariant. We consider only massless quarks in this paper.

The gauge field tensor is given

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^a_{\ bc} A^b_\mu A^c_\nu\,, \tag{2}$$

where the $f^a_{\ bc}$ are the SU(3) structure constants.

$U(\sigma)$ is the self-interaction energy of the σ field and is taken to be of the form

$$U(\sigma) \;=\; \frac{a}{2!}\sigma^2 \;+\; \frac{b}{3!}\sigma^3 \;+\; \frac{c}{4!}\sigma^4 \;+\; B\,, \tag{3}$$

which has a minimum at a value $\sigma = \sigma_v$. B, which has the same meaning as the bag constant in the MIT model, is a function of a, b, c, such that $U(\sigma_v) \;=\; U'(\sigma_v) \;=\; 0$. Then

$$U''(\sigma_v) \;\equiv\; m^2_{GB}, \tag{4}$$

were m_{GB} is identified with the mass of the lowest 0^{++} glueball state.

The quartic form of $U(\sigma)$ assures renormalizability *if κ were a constant.* Although this is convenient, it is not demanded because we already have an effective theory.

The dielectric function must satisfy

$$\kappa(\sigma_v) \;=\; \kappa'(\sigma_v) \;=\; \kappa'(0) \;= 0\,, \qquad \kappa(0) \;=\; 1\,. \tag{5}$$

We choose the form (with $x = \sigma/\sigma_v$)

$$\kappa(\sigma) = 1 \;+ \theta(x)x^n(nx - (n+1))\,. \tag{6}$$

In our present calculations, we choose $n = 2$.

2.2 Quark Dressing by Gluons, and Confinement

Although the quarks are massless and there is no direct quark-sigma coupling, the quarks acquire a self-energy and hence an effective mass through interaction with the gluon field. The quark self-energy has been studied in two cases: (1) a uniform dielectric function[10] κ, and (2) a cavity model[11] in which the function is unity at the center and goes to zero outside the bag. In both cases the self-energy is nonlocal (or momentum-dependent). We distinguish below between *spatial* confinement and *color* confinement.

The uniform κ study exhibits spatial confinement: the quark acquires a "dynamic" mass below some critical κ_c and becomes infinite as $\kappa \to 0$. But this mass is "color blind."

The cavity study exhibits color confinement. Indeed any cavity surrounded by vacuum ($\kappa \to 0$) exhibits absolute color confinement at the mean field level.

We utilize the results of these studies to introduce an effective quark mass which simulates spatial confinement by adding a term to the Lagrangian density,

$$g_{eff}(\sigma) \;=\; g_o\, \sigma_v \left(\frac{1}{\kappa(\sigma)} \;-\; 1 \right)\,. \tag{7}$$

The exact form does not seem to be important.

Color confinement is effected by including one gluon exchange (OGE) for the valence quarks both in their self energies and in their mutual interactions. We work in Coulomb gauge, and it is the A^0 component which yields the color confinement.

2.3 The Gluon Fields

We solve the field theory problem by variation of the energy with respect to various parameters. The gluon potentials, $(A^\mu) = (A^0, \mathbf{A})$, are constructed in the Abelian approximation, consistent with OGE. In the Coulomb, or transverse, gauge we have $\boldsymbol{\nabla}\cdot(\kappa\mathbf{A}) = 0$. The time component A^0 is instantaneous and frequency-independent. We can write the effective Hamiltonian as

$$H = \int d^3\mathbf{r}\, \mathcal{H}(\mathbf{r}), \tag{8}$$

where

$$\mathcal{H}(\mathbf{r}) = \psi^\dagger(\mathbf{r})\Big\{\, \vec{\alpha}\cdot[\mathbf{p} - g_s\mathbf{A}] + \beta\; g_{eff}(\sigma(\mathbf{r})) \;+\; g_s A^0(\mathbf{r}) \Big\}\psi(\mathbf{r})$$
$$+\tfrac{1}{2}\pi(\mathbf{r})^2 + \tfrac{1}{2}|\vec{\nabla}\sigma(\mathbf{r})|^2 + U(\sigma), \tag{9}$$

with $\pi = \dot{\sigma}$. This must be supplemented by the gluon field equations,

$$\partial^\mu \kappa\big[\partial_\mu A_\nu - \partial_\nu A_\mu\big] = J_\nu\,, \tag{10}$$

which are solved in the transverse gauge.

3. Constrained Mean Field and Coherent State Approximations

The input to our generator coordinate calculations[12,9] is a wave function which is characterized by one or a set of deformation parameters, which we will denote collectively as α, and a set of state vectors $|\alpha, n>$.

We determine the state vector basis set $|\alpha, n>$ by extremizing the expectation value of the total Hamiltonian (neglecting OGE), $<\alpha|H|\alpha>$ with respect to a variational mean field wave function for the quarks and a mean field function for the soliton field $\sigma(\mathbf{r})$ subject to a constraint

$$<\alpha|Q|\alpha> = Q_0\,, \tag{11}$$

where Q is some moment of the quark distribution

$$Q = \int \overline{\psi}\, q(\mathbf{r})\, \psi\, d^3r. \tag{12}$$

The constrained mean field equations now assume the form

$$\left\{ \vec{\alpha} \cdot \mathbf{p} + \beta \left[g_{eff}(\sigma(\mathbf{r})) - \lambda\, q(\mathbf{r}) \right] - \epsilon_n \right\} \psi_n = 0, \tag{13}$$

$$-\nabla^2 \sigma + U'(\sigma) + g'_{eff}(\sigma(\mathbf{r})) \sum \overline{\psi}_n \psi_n = 0. \tag{14}$$

where λ is a Lagrange multiplier. Instead of specifying the constraint function $q(\mathbf{r})$ explicitly and solving the pair of equations (13) and (14) simultaneously and self-consistently, it is actually more physical and simpler to specify the function in square brackets in (13)

$$g_{eff}(\sigma(\mathbf{r})) - \lambda\, q(\mathbf{r}) \equiv \mathcal{V}(\mathbf{r}). \tag{15}$$

This plays the role[6] of a scalar generating potential for the quarks wave functions. We choose

$$\mathcal{V}(\mathbf{r}) = g_{eff}(\sigma_\alpha(\mathbf{r})), \tag{16}$$

[see Eq. (7)] with σ_α determined by folding the volume formed by the union (for $\alpha > 0$) or the intersection (for $\alpha < 0$) of two spheres, whose centers are separated by a distance $|\alpha|$, with a Yukawa form factor. $\alpha > 0$ corresponds to prolate deformations and $\alpha < 0$ to oblate deformations. See Fig. 1. Note that these geometric configurations are folded with a form factor to yield a smooth $\sigma_\alpha(\mathbf{r})$ which agrees with the self-consistent calculations for separated and united clusters. *Without iteration for self-consistency,* the σ field is now calculated from Eq. (17).

One-gluon exchange is calculated perturbatively.

The model parameters a, b, c and g_0 are fitted at the nucleon level.

4. Status

The first part of the program has been completed, through the calculation of the diagonal energies $< \alpha|H|\alpha >$ including diagonalization with respect to the relevant six-quark basis states. This yields the adiabatic energies $E(\alpha)$. A repulsive core obtains due to color electrostatic repulsion.Note, however, that in three quark states with identical spatial configurations, electrostatic energy vanishes (the mutual and valence self-energy terms cancel exactly).

The next phase, already in progress, involves the employment of generator coordinates and the solution of the Hill-Wheeler integral equation. Finally, there will be the inclusion of intermediate phenomenology to fit two-body data. As stated, one of our goals is the calculation of the quark structure function in nuclei.

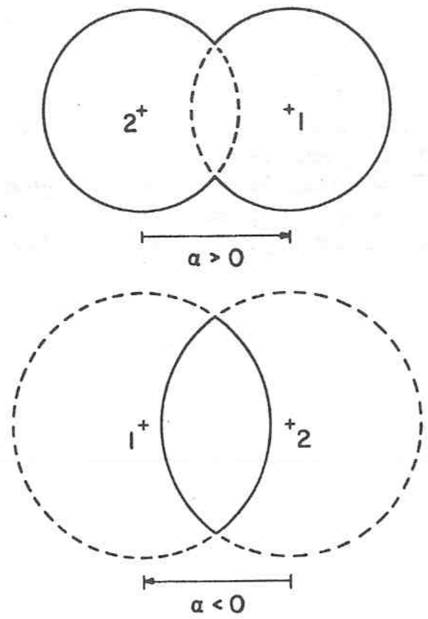

Figure 1. Geometric configurations used to construct $\sigma_\alpha(r)$ for insertion into Eq. (6). The geometric shape, enclosed by the solid arcs, is folded with a Yukawa form factor.

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6. References

1. M.Oka and K.Yazaki, "Quarks and nuclei", *International Review of Nuclear Physics*, vol.1, ed. W. Weise(World Scientific, Singapore, 1984), p. 489.
2. F.Myhrer and J. Wroldsen, *Rev. Mod. Phys.* **60** (1988) 629.
3. Fl. Stancu and L. Wilets, *Phys. Rev.* **C36** (1987) 726.
4. Fl. Stancu and L. Wilets, *Phys. Rev.* **C38** (1988) 1145; **C40** (1989) 1901.
5. Fl. Stancu and L. Wilets, Proceedings of this Conference.
6. A. Schuh, Doctoral Dissertation, University of Heidelberg (1985); A. Schuh, H. J. Pirner and L. Wilets, *Physics Letters* **B 174** (1986) 10.
7. G. Fai, R.J. Perry and L. Wilets, *Phys. Lett.* **B 208** (1988) 1.

8. R. Friedberg, T. D. Lee and A. Sirlin, *Phys. Rev.* **D 13** (1976) 2739; *Nucl. Phys.* **B115** (1976) 1 and 32; R. Friedberg and T. D. Lee, *Phys. Rev.* **D15** (1977) 1694; **D18** (1978) 2623.
9. L. Wilets, *Nontopological Soliton Models,* World Scientific, 1989.
10. G. Krein, P. Tang, L. Wilets and A. G. Williams, *Phys. Lett.* **B 212** (1988) 362; *Nucl. Phys.* **A 523** (1991) 548.
11. P. Tang, Doctoral Dissertation, University of Washington (1992); L. Wilets, S. Hartmann and P. Tang, in preparation.
12. D. L. Hill and J. A. Wheeler *Phys. Rev.* **89** (1953) 1109.