

# Identification method of an advanced constitutive law for nickel-based alloy Haynes 230 used in solar receivers

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**Abstract.** A model to study panels of thin tubes of Haynes 230 nickel alloy used in solar receivers has been developed. The thermo-mechanical behavior of these tubes is simulated using an advanced model capable of representing specific characteristics such as viscosity, kinematic and cyclic hardening, static recovery, or dynamic recovery. The constitutive law implemented is a finite-element visco-plastic model based on the work of Chaboche. Due to its complexity, the model uses a significant number of parameters that need to be identified at several temperatures.

The aim of this article is to define an efficient method for the identification of the parameters of this Chaboche model adapted to cyclic thermo-mechanical loading.

## 1. Introduction

Solar receivers are a key component in Concentrated Solar Power (CSP) systems. Such systems produce renewable energy by using mirrors to concentrate sunlight radiation onto a solar receiver. The solar receiver consists in panels of adjacent tubes in which circulates a working fluid. These thin tubes are manufactured by welding.

The solar receiver is exposed to complex thermo-mechanical loadings. During daytime, the front of the tube, exposed to concentrated sunlight, reaches temperatures up to 700°C, while the back of the tube stays at a temperature around 560°C. This temperature gradient generates stresses in the tube. However during nighttime, the tubes are at ambient temperature and are not subjected to thermo-mechanical stress. These tubes are therefore submitted to cyclic loading with high temperatures.

A visco-plastic Chaboche model such as the one developed in [1] is adapted for the simulation of the behavior of the tubes, as it contains all the necessary features to model the complex behavior of the material. In particular, an innovative aspect of this model is the representation of cyclic hardening. However, the accuracy of this type of model largely depends on the quality of the material parameters identified from experimental tests. Due to the complexity of the model and the large number of parameters it contains, the use of a general purpose automatic optimization procedure should be restrained to the fine tuning of a set of parameters initially determined by a specific methodology dedicated to the present model.

The present article establishes such a methodology for the initial parameter identification based on the physical meaning of these parameters. A new implementation of the model within non-linear finite-element software Lagamine has been developed in MSM team. Experimental data for Haynes 230 are available by literature survey and ongoing research project.

## 2. Description of the model

The model is a visco-plastic Chaboche model with multiple features. Details about the model can be found in [1, 2]. A brief summary of the main equations is given hereafter. For simplification, the model is presented with one-dimensional equations.

### 2.1. Visco-plasticity

The mechanical strain is decomposed into an elastic contribution and a visco-plastic contribution as in equation (1). The elastic strain and the stress are related through Hooke's law as in equation (2).

$$\varepsilon = \varepsilon^e + \varepsilon^{vp} \quad (1)$$

$$\sigma = E\varepsilon^e \quad (2)$$

The yield locus is defined by the von-Mises criterion, shown in equation (3), where  $X$  is the back-stress and  $\sigma_0$  the initial yield limit.

$$f = |\sigma - X| - \sigma_0 \begin{cases} < 0 & \text{in the elastic regime} \\ \geq 0 & \text{in the visco-plastic regime} \end{cases} \quad (3)$$

The viscosity is modeled through Norton's equation, with viscous parameters  $n$  and  $K$  (equation (4)). For temperatures below 500°C where Haynes 230 shows no viscosity, the value of parameter  $K$  is close to zero.

$$\dot{p} = |\dot{\varepsilon}^{vp}| = \left\langle \frac{f}{K} \right\rangle^n \quad (4)$$

$$\text{where } \langle x \rangle = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

### 2.2. Hardening equations

Hardening of Haynes 230 is controlled by the back-stress  $X$ . It is considered for this material that no isotropic hardening occurs. The global back-stress  $X$  is the sum of several back-stresses  $X_i$ , as shown in equation (5). The number of back-stresses  $N_X$  must be determined considering the cyclic hardening behavior of the material. To model Haynes 230, the value of  $N_X$  was set to 3. Equation (6) describing the evolution of one back-stress  $X_i$  consists of 4 terms:

- Strain hardening, controlled by parameter  $C_i$ ;
- Dynamic recovery, controlled by parameter  $\gamma_i$ ;
- Static recovery, controlled by parameters  $b_i$  and  $r_i$ ;
- Temperature rate dependence, with the influence of parameter  $C_i$ .

$$X = \sum_{i=1}^{N_X} X_i \quad (5)$$

$$\dot{X}_i = C_i \dot{\varepsilon}^{vp} - \gamma_i X_i \dot{p} - b_i |X_i|^{r_i} X_i + \frac{1}{C_i} \frac{\partial C_i}{\partial T} \dot{T} X_i \quad (6)$$

Cyclic hardening is represented via the evolution of the parameter  $\gamma_i$  from its initial value  $\gamma_i^{init}$  to its final value  $\gamma_i^F$ . The evolution of this parameter depends on the plastic strain norm  $p$  as shown in equation (7).  $\gamma_i^{init}$ ,  $\gamma_i^F$  and  $D_{\gamma_i}$  are material parameters.

$$\gamma_i = \gamma_i^F - (\gamma_i^F - \gamma_i^{init}) e^{-D_{\gamma_i} p} \quad (7)$$

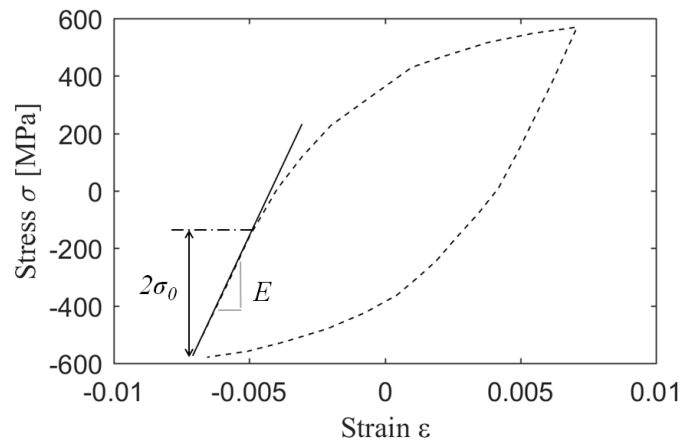
In total, 22 parameters have to be identified:

- 2 elastic parameters  $E$  and  $\sigma_0$ ;
- 2 viscosity parameters  $K$  and  $n$ ;

- 2 kinematic hardening parameters per back-stress  $C_i$  and  $\gamma_i^{Init}$  ;
- 2 static recovery parameters per back-stress  $b_i$  and  $r_i$  ;
- 2 cyclic hardening parameters per back-stress  $\gamma_i^F$  and  $D_{\gamma_i}$  ;

### 3. Method for the identification of parameters

The identification of the elastic parameters  $E$  and  $\sigma_0$  is trivial, although it can be noted that to model cyclic behavior, these parameters should be measured from cyclic tests and not from monotonic tests, as shown in Figure 1. The material parameters vary with the temperature and must be identified for several temperature within the application range. The model can be simplified for temperatures where the material shows no rate-dependence (absence of viscosity). Therefore the number of parameters to be identified is smaller.



**Figure 1.** Identification of the elastic parameters on the stress-strain hysteresis loop of a cyclic test taken from [2]

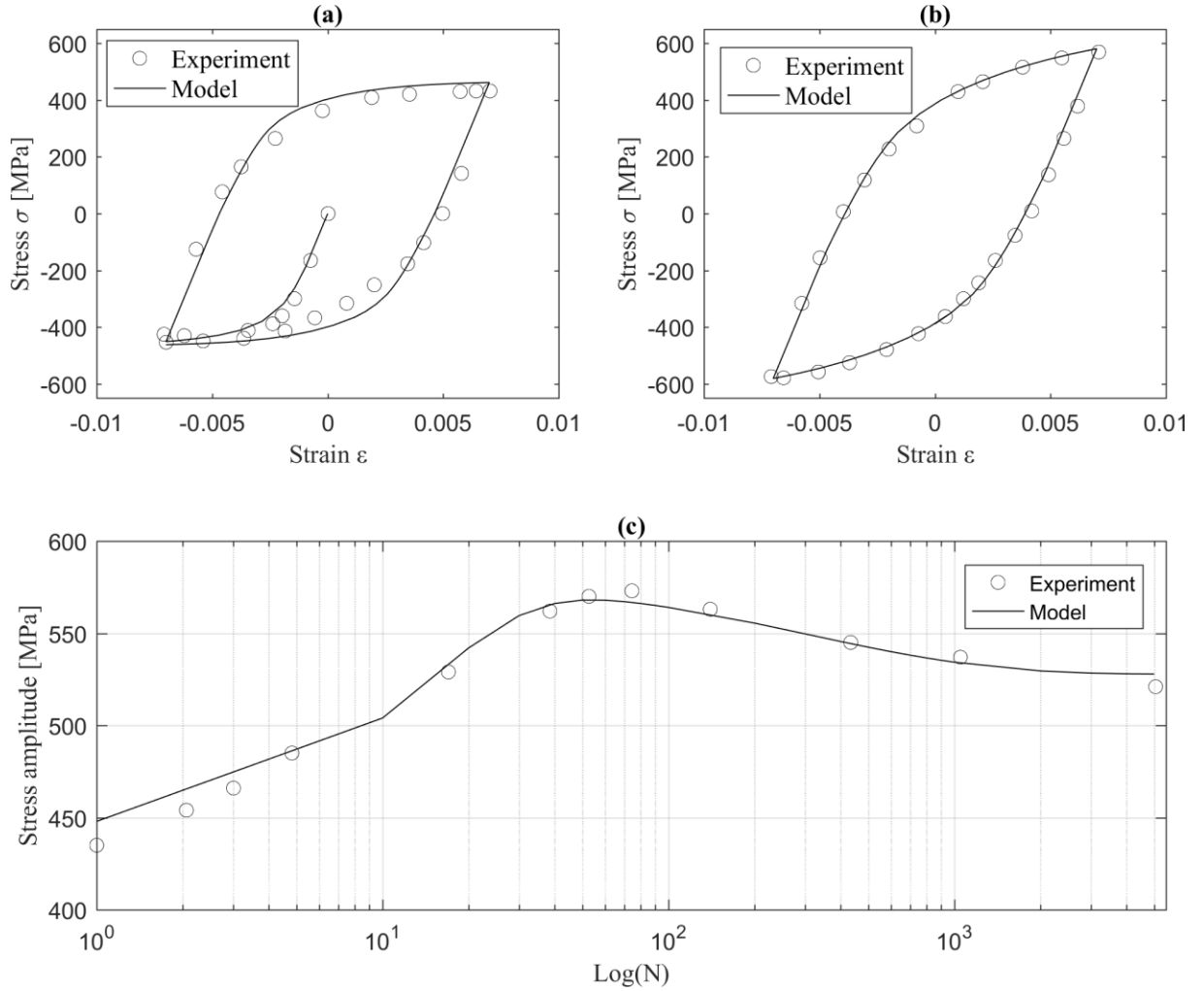
#### 3.1. Identification in the absence of viscosity

For temperatures below 500°C, the behaviour of Haynes 230 alloy is elasto-plastic, which means the yield surface  $f$  cannot be strictly positive and  $f = 0$  corresponds to plasticity. In addition, further simplifications can be made in the expression of the back-stresses, as static recovery is a temperature-activated phenomenon that only appears in the viscosity range. For isothermal loading conditions, the evolution of one back-stress is therefore determined by equation (8).

$$\dot{X}_i = C_i \dot{\varepsilon}^{vp} - \gamma_i X_i \dot{p} \quad (8)$$

The first step for the identification of the parameters is to determine the number of back-stresses to be used in the model. It depends on the cyclic hardening behaviour of the material. If no cyclic hardening is observed, two back-stresses are sufficient to model the hardening behaviour of most materials, as shown in [3] and the cyclic hardening parameters are set to zero. In the case where the material shows cyclic hardening, the number of back-stresses should be derived from the evolution of the stress amplitude observed for a cyclic loading.

Figure 2. (c) shows the evolution of the stress amplitude for a strain-controlled low-cycle-fatigue test on alloy Haynes 230 taken from [4]. Two different phases can be observed: a rapid hardening at the beginning (cycle 1 to 75), followed by a slow softening from cycle 75 to 5000. A back stress  $X_1$  is used to describe the hardening phase and a back stress  $X_2$  is used for the cyclic softening. These two back-stresses control the stress amplitude. A third back-stress  $X_3$  models the shape of the hysteresis loop (see Figure 2 (a-b)), and more particularly the slope of the stress-strain curve at relatively high values of the inelastic strain.



**Figure 2.** Low cycle fatigue response of Haynes 230 at 24°C, (a) hysteresis loop at the 1<sup>st</sup> cycle, (b) hysteresis loop at the 75<sup>th</sup> cycle, (c) stress amplitude as a function of cycles - experiment from [2]

Once the number of back-stresses is set, the kinematic hardening parameters  $C_i$  and  $\gamma_i^{init}$  can be identified. On the first cycle, it is difficult to differentiate the role of back-stresses  $X_1$  and  $X_2$ . Therefore, to simplify the identification process, they are assembled as one single back-stress  $X_{1/2}$  following equation (9). The identification of parameters  $C_{1/2}$ ,  $C_3$ ,  $\gamma_{1/2}^{init}$  and  $\gamma_3^{init}$  on the first cycle can be achieved using the method proposed in [3].

$$\begin{aligned} \dot{X}_{1/2} &= C_{1/2} \dot{\epsilon}^{vp} - \gamma_{1/2}^{init} X_{1/2} \dot{p} \\ C_1 &= C_2 = \frac{C_{1/2}}{2} \\ \gamma_1^{init} &= \gamma_2^{init} = \gamma_{1/2}^{init} \end{aligned} \quad (9)$$

The cyclic hardening parameter  $\gamma_3^F$  can be identified by trial and error on the stabilised hysteresis loop, or on the cycle where the stress amplitude is maximal. The value  $\gamma_3^F$  should correspond to the right fitting of the slope of the stress-strain curve in the range of high inelastic deformations.

Let  $n$  be the cycle number corresponding to the turning point of the stress amplitude curve and  $r$  the cycle number corresponding to the rupture of the sample. Parameters  $\gamma_1^F$  and  $\gamma_2^F$  can be identified

using the value of the stress amplitude respectively at cycle  $n$  and  $r$  to solve equation (10). At cycle  $n$ , the value of  $\gamma_3$  is  $\gamma_3^F$  and the value of  $\gamma_2$  is approximately equal to  $\gamma_2^{Init}$ .

$$\sigma_{amp} = \sigma_0 + \sum_i \frac{C_i}{\gamma_i} (1 - \exp(-\gamma_i \varepsilon_{max}^{vp})) \quad (10)$$

Finally, the rates of cyclic hardening  $D_{\gamma_i}$  can be identified using the stress amplitude curve. Considering equation (7) and equation (10), one can plot the theoretical stress amplitude curve and fit the values of  $D_{\gamma_i}$  by trial and error. A good approximation for the plastic strain norm  $p$  is given by equation (11).

$$p \simeq 2N\Delta\varepsilon^{vp} \quad (11)$$

The parameters identified through this procedure are summarized in Table 1 and the response of the model using these parameters is shown in Figure 2.

**Table 1.** Hardening parameters identified for Haynes 230 at 24°C

$i$	$C_i$	$\gamma_i^{Init}$	$\gamma_i^F$	$D_{\gamma_i}$
1	306157	4164	2093	5
2	306157	4164	13007	0.06
3	49069	468	200	5

### 3.2. Identification in the visco-plastic domain

The identification of the parameters at temperatures where the material exhibits a viscous behavior can be similarly achieved. First, parameters  $C_i$  and  $\gamma_i^{Init}$  can be identified using the method from [3]. Then, viscous parameters  $K$  and  $n$  can be identified from a relaxation test or multiple creep tests. An iterative process is then necessary to adapt  $C_i$ ,  $\gamma_i^{Init}$ ,  $K$  and  $n$  as the addition of the viscosity parameters has an influence both on the stress amplitude and on the shape of the hysteresis loop.

For the identification of cyclic hardening parameters  $\gamma_i^F$ , viscosity must also be taken into account. In the visco-plastic domain, equation (10) is replaced by equation (12). The values of parameters  $\gamma_i^F$  can be computed using equation (12) or by trial and error. The process for the identification of the rate parameters  $D_{\gamma_i}$  is the same as described above.

$$\sigma_{amp} = \sigma_0 + \sum_i \frac{C_i}{\gamma_i} (1 - \exp(-\gamma_i \varepsilon_{max}^{vp})) + \langle f \rangle \quad (12)$$

As mentioned in [5], static recovery is a thermally activated phenomenon that takes place in the regime of very low velocities. The static recovery parameters  $b_i$  and  $r_i$  can be identified using long duration creep tests or relaxation tests. The parameter  $b_i$  should be small enough to have a negligible influence on simulations of low-cycle fatigue or short-term relaxation.

## 4. Conclusions

The use of advanced numerical models is necessary to simulate the behaviour of materials under complex loadings, such as thermo-mechanical cyclic loading. Models such as the Chaboche model are well adapted to this type of problem as they can contain multiple features to represent various aspects of the material behaviour. As a result, the identification of the numerous material parameters can become arduous and the incorrect use of an optimization technique is likely to lead to an erroneous set of parameters. Therefore, it is important to base this identification process on physical considerations.

A procedure for the identification of hardening parameters with and without consideration of viscosity was established and tested for the nickel-based alloy Haynes 230. The parameters obtained from this method give good results, even without optimization, as can be seen in Figure 2.

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