Computational & Multiscale
Mechanics of Materials

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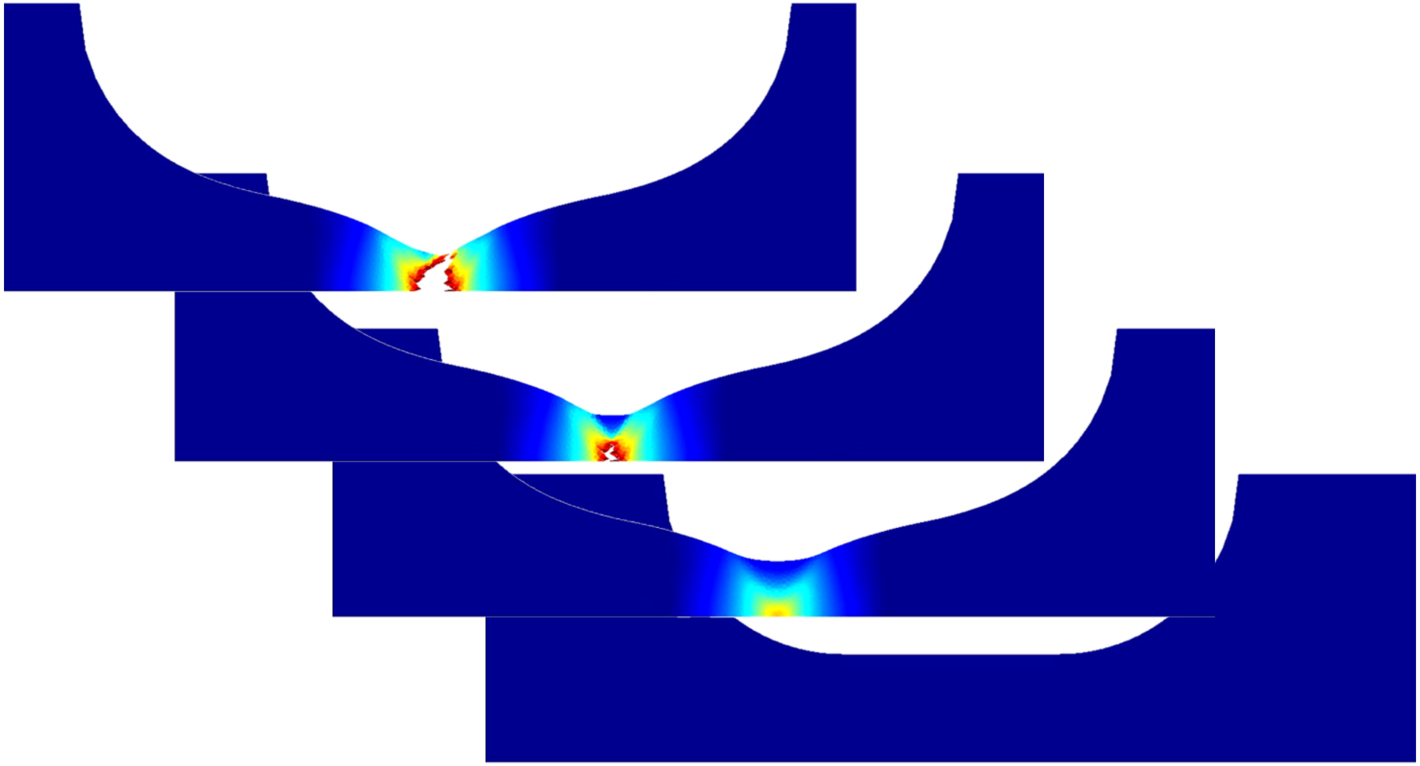
A Damage to Crack Transition Framework for Ductile Materials Accounting for Stress Triaxiality

Julien Leclerc, Ling Wu, Van-Dung Nguyen, Ludovic Noels



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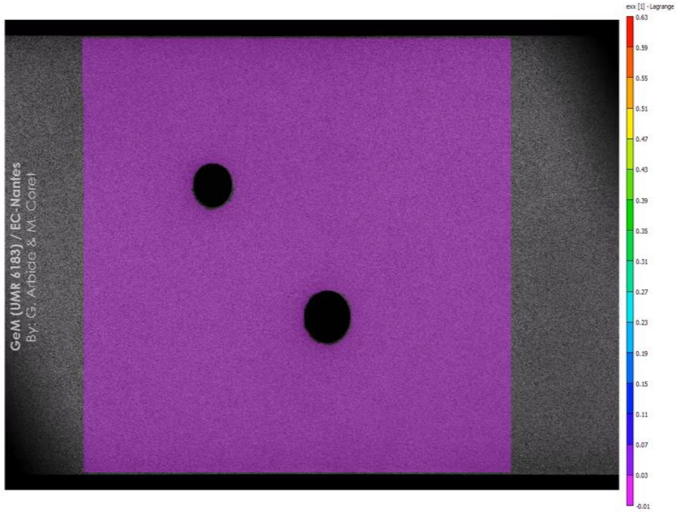
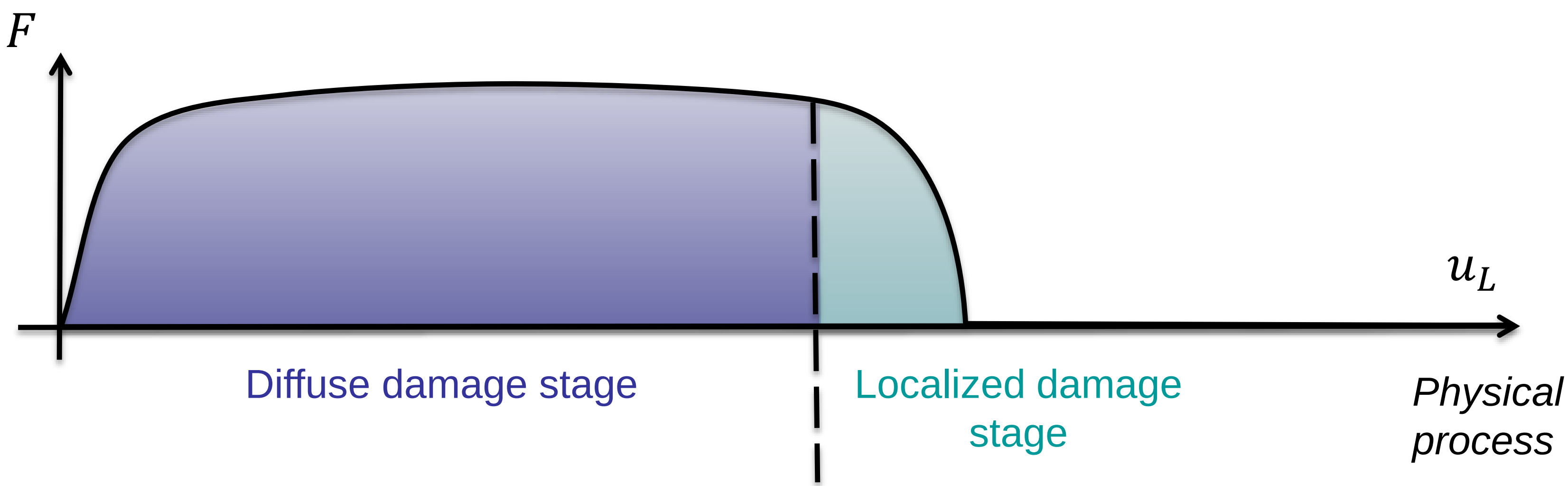
Introduction

- Goal: To capture the whole ductile failure process
  - A diffuse stage
    - Damage onset / nucleation, growth…
  - Followed by a localized stage
    - Damage coalescence
    - Crack initiation and propagation
    - …



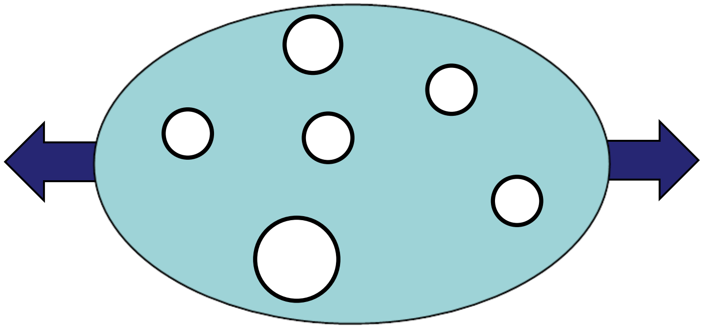
[http://radome.ec-nantes.fr/]



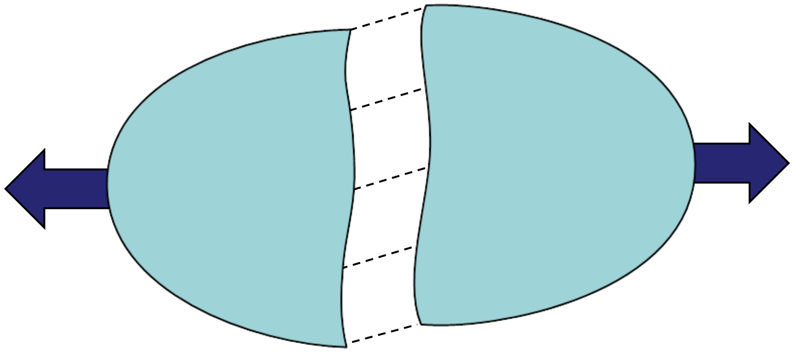
State of art: Modeling approaches

- State-of-the-art
  - 2 approaches modeling material failure:

    - Continuum Damage Models (CDM)
      - Lemaitre-Chaboche,
      - Gurson,
      - …

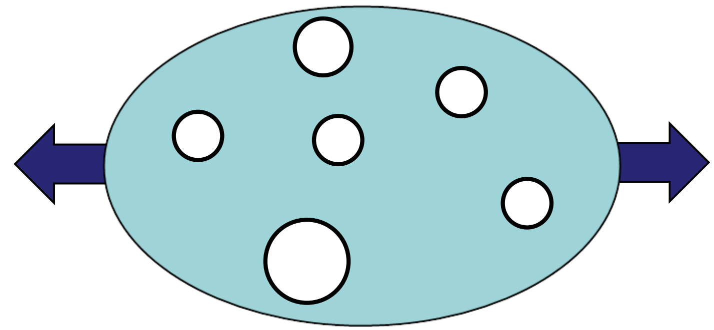


    - Discontinuous: Fracture mechanics
      - Cohesive zone,
      - XFEM
      - …

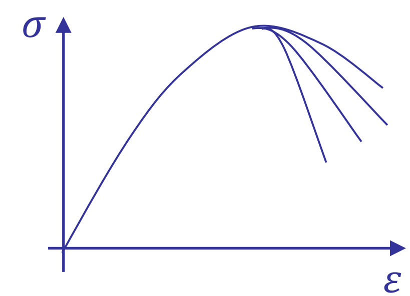


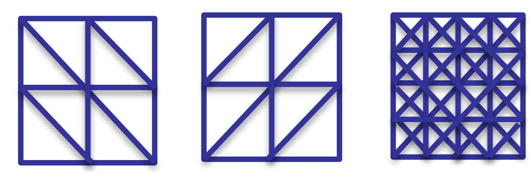
State of art: Continuous approaches

- Material properties degradation modelled through internal variables evolution
  - Form $\boldsymbol{\sigma}(\boldsymbol{\varepsilon}; Z(t'))$
    - With internal variables history $Z(t')$
  - Lemaitre-Chaboche model,
  - Gurson model,
    - Porosity evolution f_V



- Continuous Damage Model (CDM) implementation:
  - Local form
    - Mesh-dependency
  - Non-local form needed [Bažant 1988]
    - Introduction of a characteristic length l_c
    - In terms of Weight functions: $\tilde{f}_V(\boldsymbol{x}) = \int_{V_C} W(\boldsymbol{y}; \boldsymbol{x}, l_c) f_V(\mathbf{y}) d\boldsymbol{y}$
  - Implicit formulation [Peerlings et al. 1998]
    - New non-local degrees of freedom $\tilde{f}_V$
    - New Helmholtz type equation to be solved $\tilde{f}_V - l_c^2 \Delta \tilde{f}_V = f_V$



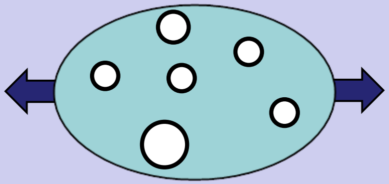


The numerical results change without convergence

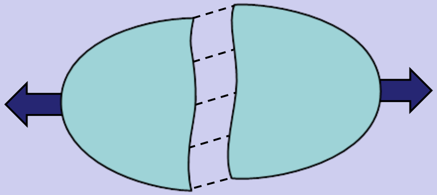
State of art: Comparison (1)

Continuous:

Continuous Damage Model (CDM)



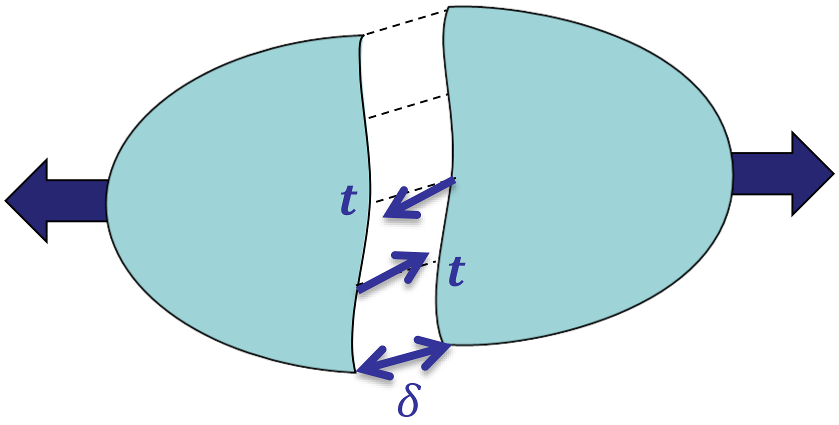
Discontinuous:



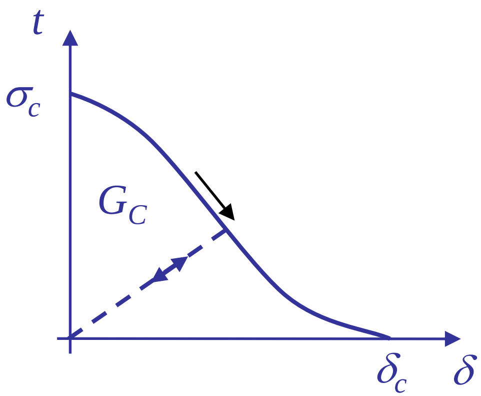
+ Capture the **diffuse damage stage**
+ Capture stress **triaxiality** and **Lode** variable effects

- **Mesh dependency without (implicit) non-local form**
- **Numerical problems** with highly damaged elements
- **Cannot represent cracks** without remeshing / element deletion at $D \rightarrow 1$ (loss of accuracy, mesh modification ...)
- Crack initiation observed for lower damage values

- Based on fracture mechanics concepts
  - Characterized by
    - Strength σ_c &
    - Critical energy release rate G_C



- One of the most used methods:
  - Cohesive Zone Model (CZM) modelling the crack tip behavior
  - Integrate a Traction Separation Law (TSL):
    - At interface elements between two elements
    - Using element enrichment (EFEM) [Armero et al. 2009]
    - Using mesh enrichment (xFEM) [Moes et al. 2002]
    - …



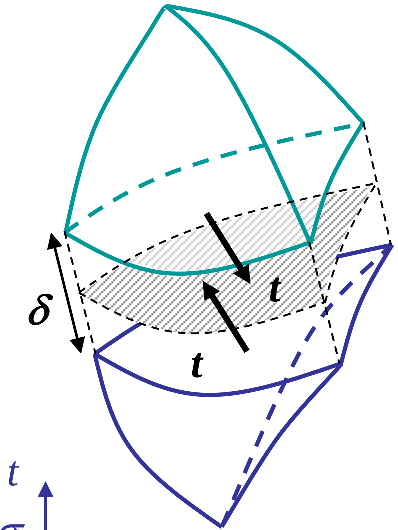
State of art: Discontinuous approaches

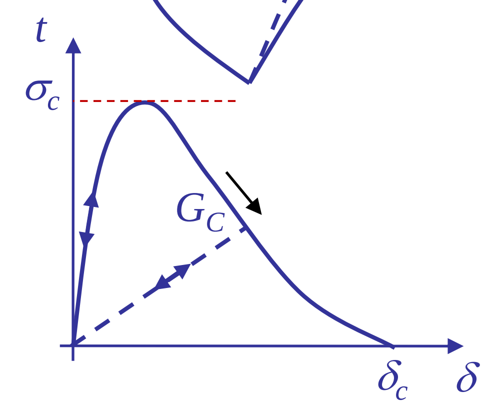
- Cohesive elements
  - Inserted between volume elements

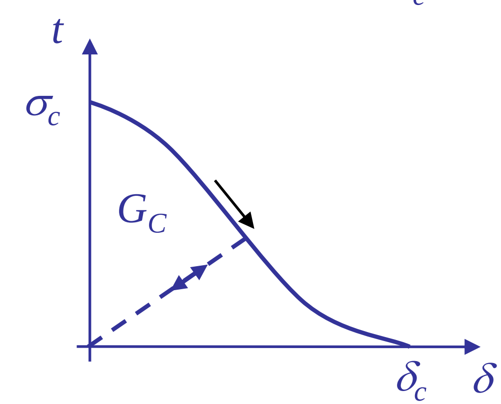
    - Zero-thickness ⇒ no triaxiality accounted for

  - Intrinsic Cohesive Law (ICL)
    - Cohesive elements inserted from the beginning
    - Efficient if a priori knowledge of the crack path
    - Mesh dependency [Xu & Needelman, 1994]
    - Initial slope modifies the effective elastic modulus
    - This slope should tend to infinity [Klein et al. 2001]:
      - Alteration of a wave propagation
      - Critical time step is reduced

  - Extrinsic Cohesive Law (ECL)
    - Cohesive elements inserted on the fly when the failure criterion is verified [Ortiz & Pandolfi 1999]
    - Complex implementation in 3D (parallelization)







State of art: Discontinuous approaches

- Hybrid framework [Radovitzky et al. 2011]
  - Discontinuous Galerkin (DG) framework
    - Test and shape functions discontinuous
    - **Consistency**, **convergence rate**, **uniqueness** recovered though interface terms

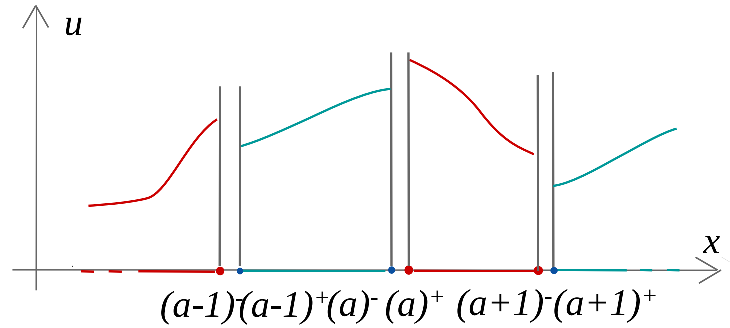
$$\int_{\Omega_0} \mathbf{P} : \nabla_0 \boldsymbol{\delta u} d\Omega +$$

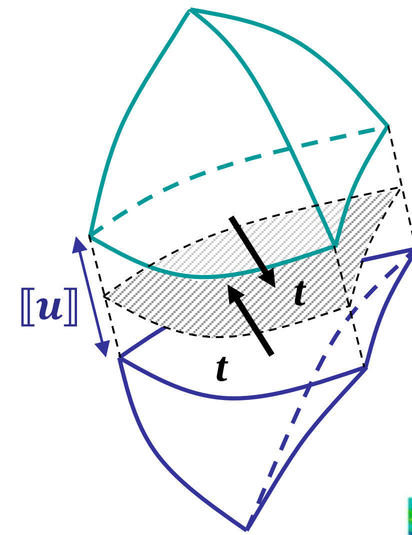
$$\int_{\partial_I \Omega_0} [\![\boldsymbol{\delta u}]\!] \cdot \langle \mathbf{P} \rangle \cdot \boldsymbol{N}^- d\partial\Omega +$$

$$\int_{\partial_I \Omega_0} [\![\boldsymbol{u}]\!] \cdot \langle \boldsymbol{C}^{\mathrm{el}} : \nabla_0 \boldsymbol{\delta u} \rangle \cdot \boldsymbol{N}^- d\partial\Omega +$$

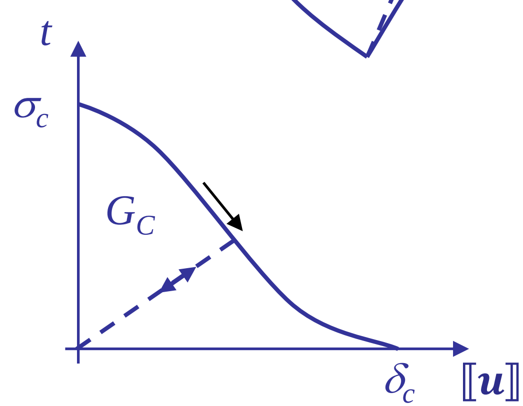
$$\int_{\partial_I \Omega_0} [\![\boldsymbol{u}]\!] \otimes \boldsymbol{N}^- : \langle \frac{\beta_s \boldsymbol{C}^{\mathrm{el}}}{h^s} \rangle : [\![\boldsymbol{\delta u}]\!] \otimes \boldsymbol{N}^- d\partial\Omega = 0$$

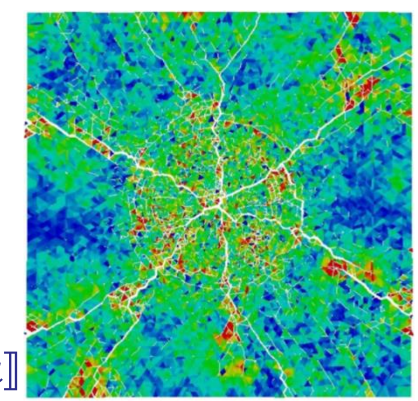
    - Interface terms integrated on interface elements





  - Combination with extrinsic cohesive laws
    - Interface elements already there
    - Switch to traction separation law
    - Efficient for fragmentation simulations

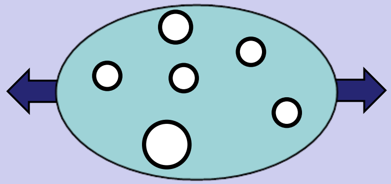




State of art: Comparison (2)

Continuous:

Continuous Damage Model (CDM)



- \+ Capture the **diffuse damage stage**
- \+ Capture stress **triaxiality** and **Lode** variable effects

- **Mesh dependency without (implicit) non-local form**
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- **Cannot represent cracks** without remeshing / element deletion at $D \rightarrow 1$ (loss of accuracy, mesh modification ...)
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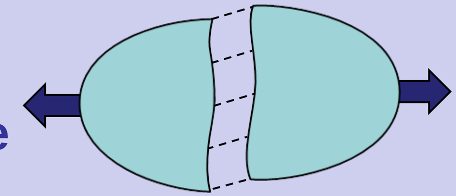
Discontinuous:

Extrinsic Cohesive Zone Model (CZM)



- \+ **Multiple crack initiation** and propagation naturally managed

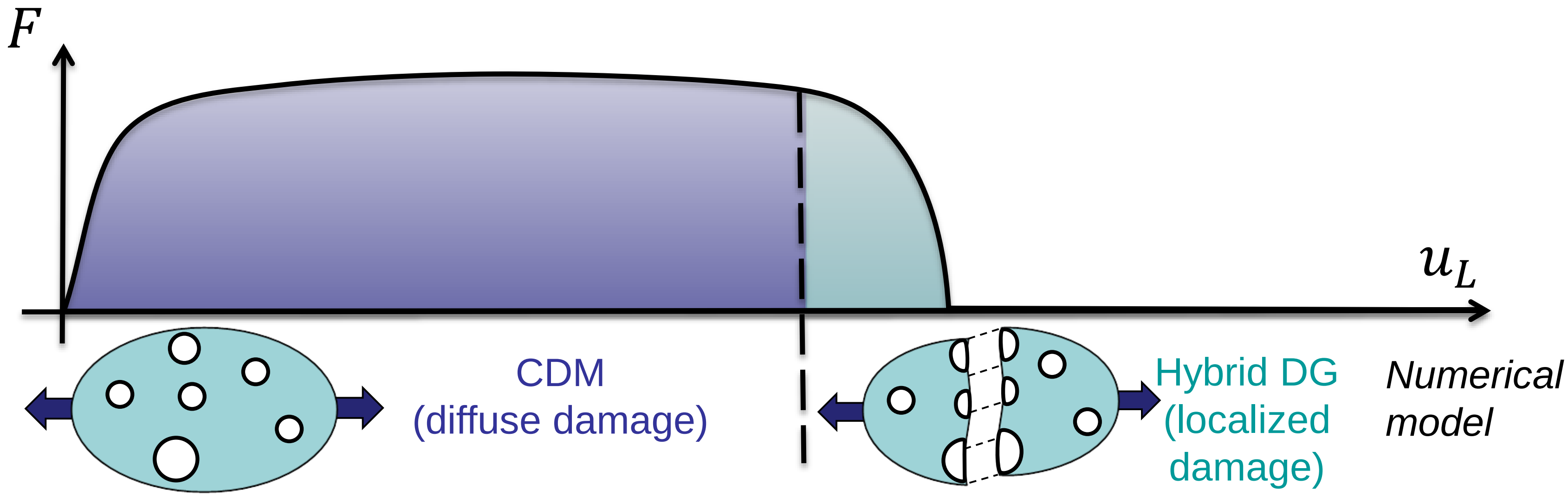
- **Cannot capture diffuse damage**
- **No triaxiality** effect
- Currently valid for brittle / small scale yielding elasto-plastic materials

Goals of research

- Goal:
  - Simulation of the whole ductile failure process with accuracy
- Main idea:
  - Combination of 2 complementary methods in a single finite element framework:
    - Continuous (non-local damage model)

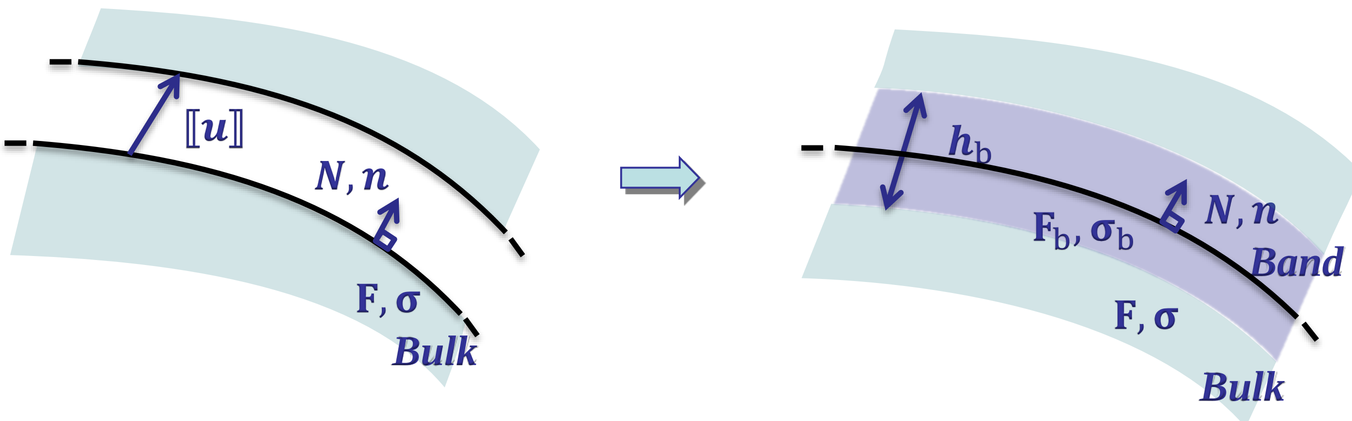
      \+ transition to
    - Discontinuous (Hybrid DG model)

    → Damage to crack transition
  - However triaxiality effects are important → Cohesive zone model not adequate!!!



Damage to crack transition – Principles

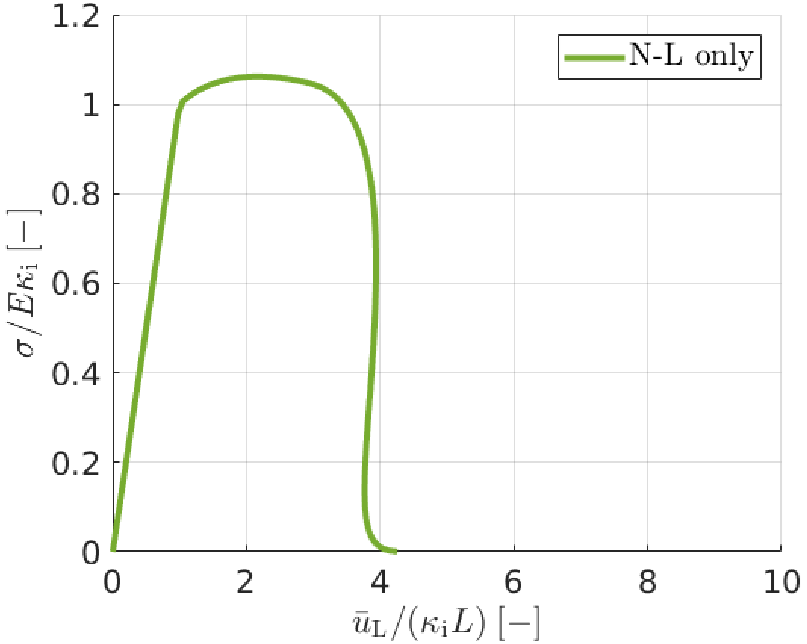
- Hybrid DG model: use of a Cohesive Band Model (CBM)
  - Hypothesis
    - In the last stage of failure, all damaging process occurs in an uniform thin band
  - Principles
    - Substitute TSL of CZM by the behavior of a uniform band of thickness h_{b} [Remmers et al. 2013]



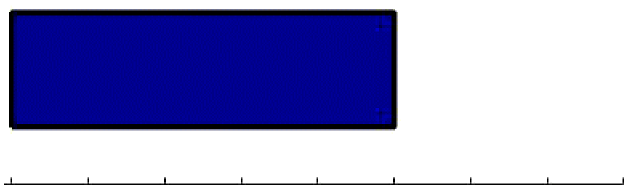
  - Methodology [Leclerc et al. 2018]
    1. Compute a band strain tensor $\mathbf{F}_{\mathrm{b}} = \mathbf{F} + \frac{[\![\boldsymbol{u}]\!] \otimes \boldsymbol{N}}{h_b} + \frac{1}{2} \boldsymbol{\nabla}_T [\![\boldsymbol{u}]\!]$
    2. Compute a band stress tensor $\boldsymbol{\sigma}_{\mathrm{b}}\left(\mathbf{F}_{\mathrm{b}}; \mathrm{Z}(\tau)\right)$ using the same CDM as bulk elements
    3. Recover a surface traction $\boldsymbol{t}([\![\boldsymbol{u}]\!], \mathbf{F}) = \boldsymbol{\sigma}_{\mathrm{b}} . \boldsymbol{n}$
  - What is the effect of h_{b} (band thickness)
    - A priori determined with underlying non-local damage model to ensure energy consistency

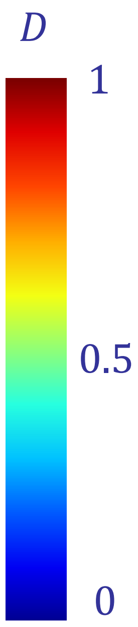
Damage to crack transition for elastic damage – Proof of concept

- Elastic damage material model
  - Constitutive equations
    - Helmholtz energy: $\rho\psi(\boldsymbol{\varepsilon}, D) = \frac{1}{2}(1-D)\boldsymbol{\varepsilon}: H: \boldsymbol{\varepsilon}$
    - Non-local maximum principal strain: $\tilde{e} - l_c^2 \Delta\tilde{e} = e$
    - Damage evolution $\dot{D}(\kappa) = (1-D)\left(\frac{\beta}{\kappa} + \frac{\alpha}{\kappa_c - \kappa}\right)\dot{\kappa}$ with $\kappa = \max_{t'} \tilde{e}(t')$
  - 1D non-local test



Non-local only





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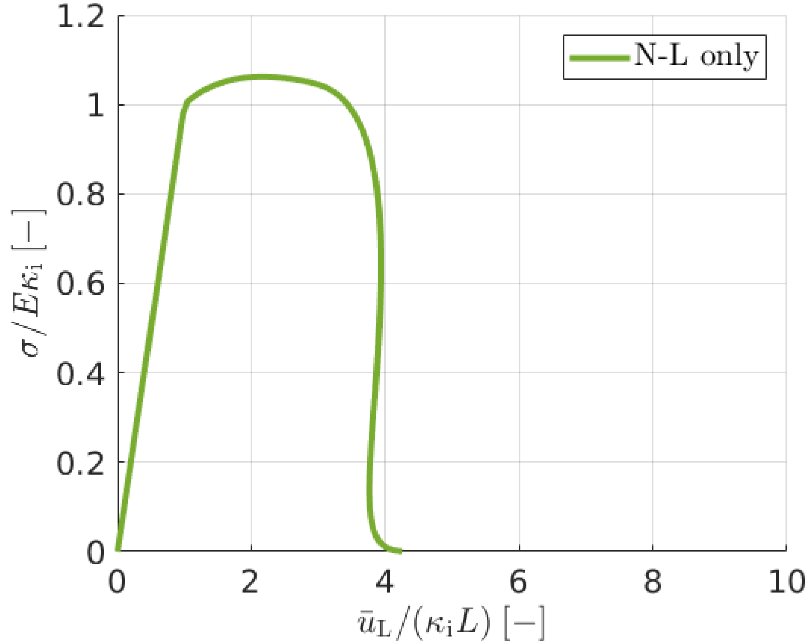

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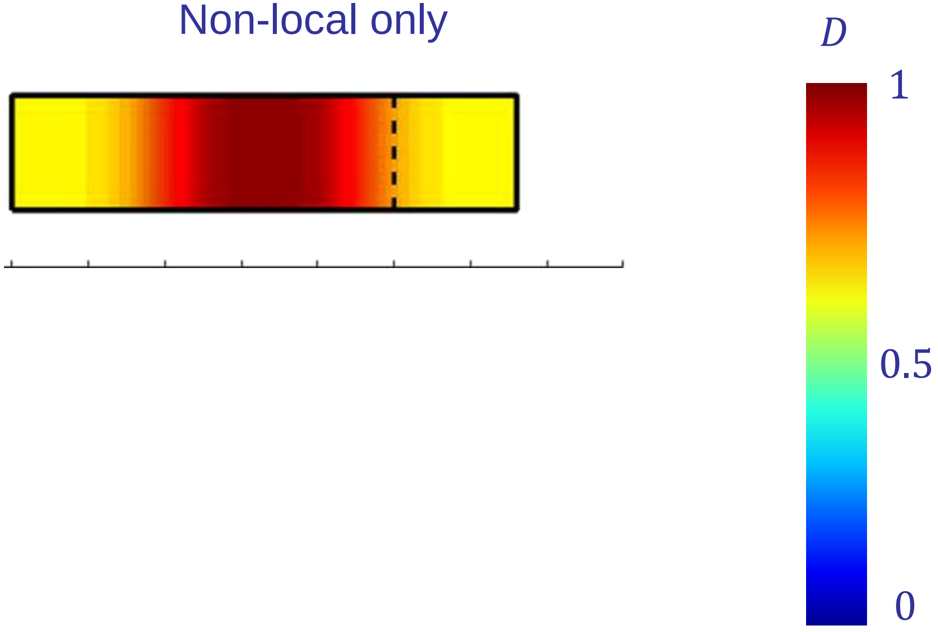
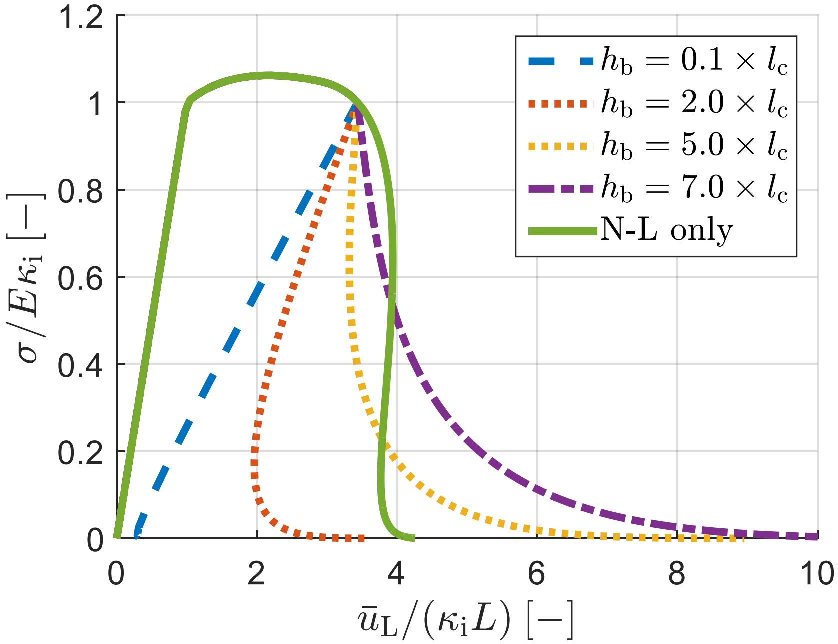
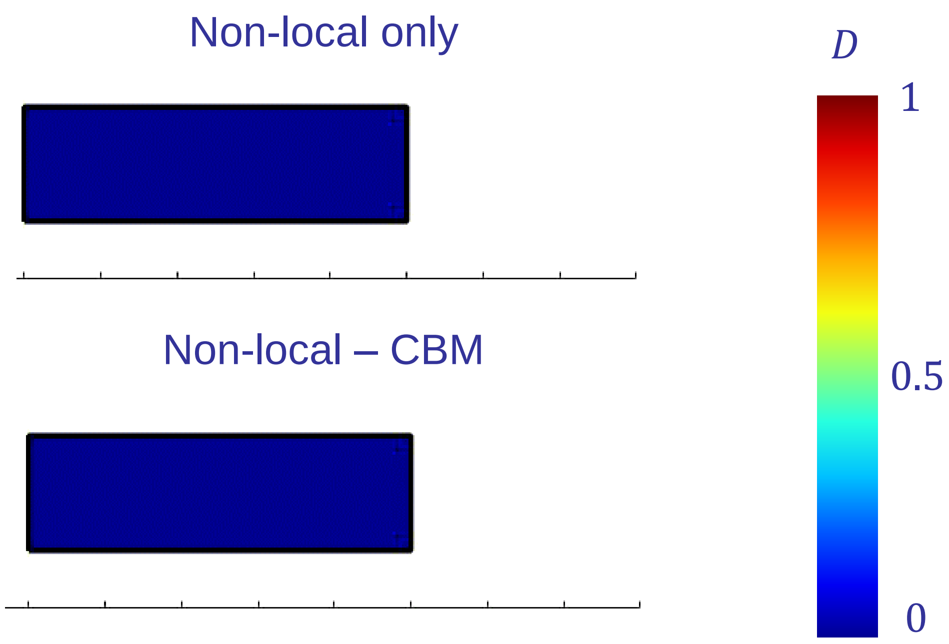


Damage to crack transition for elastic damage – Proof of concept

- Non-local elastic damage to Cohesive Band Model transition
  - Influence of h_b (for a given l_c) on response in a 1D elastic case [Leclerc et al. 2018]
    - Has effect on the totally dissipated energy Φ
    - Could be chosen to conserve energy dissipation (physically based)





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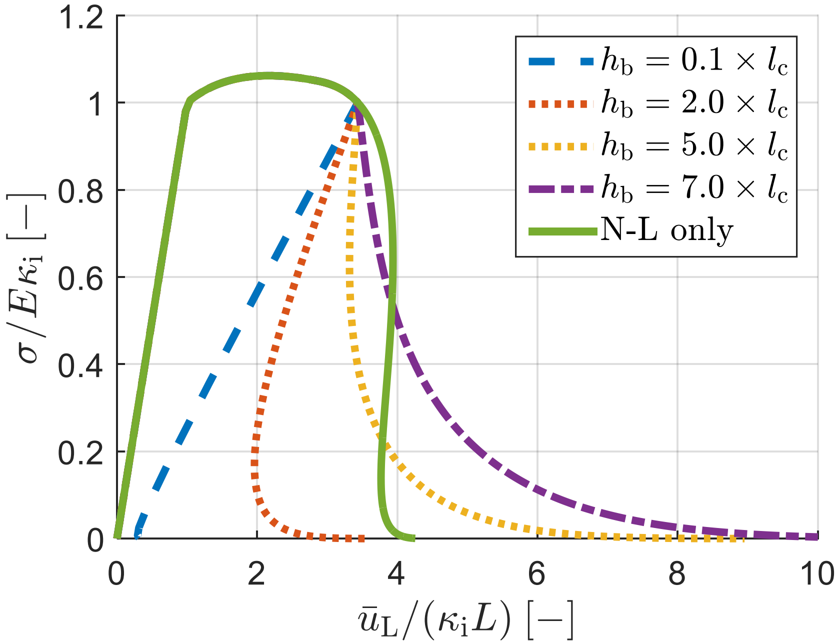
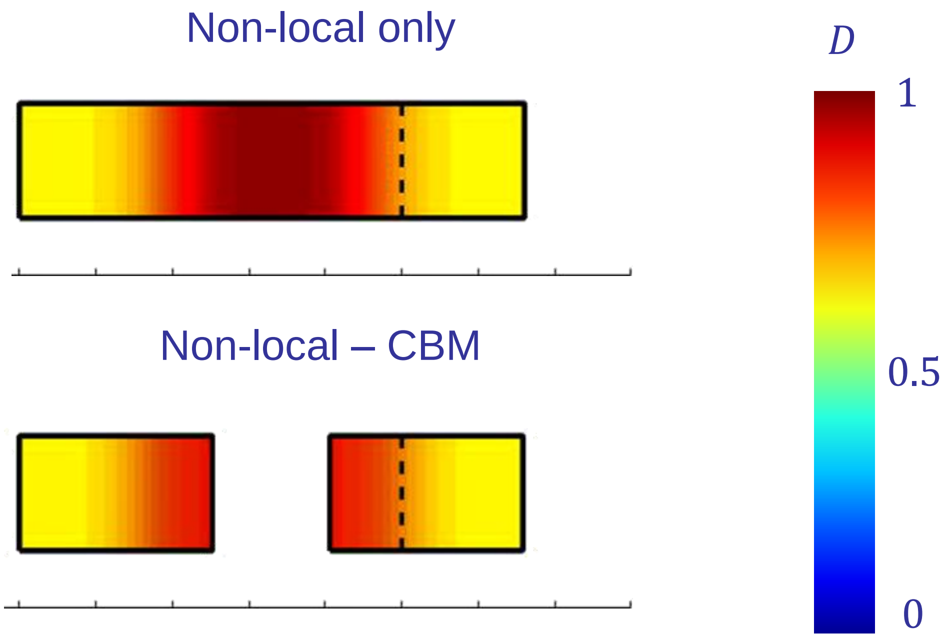
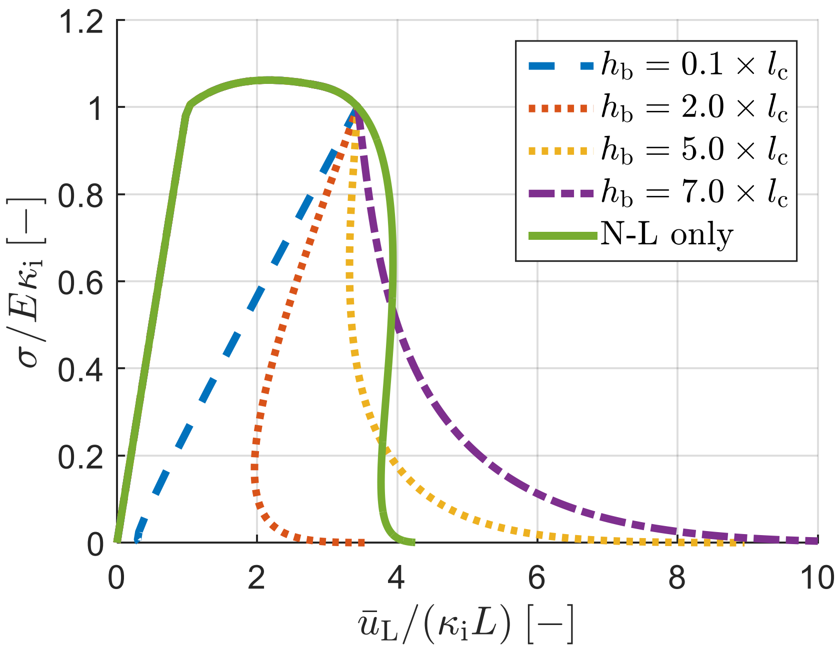
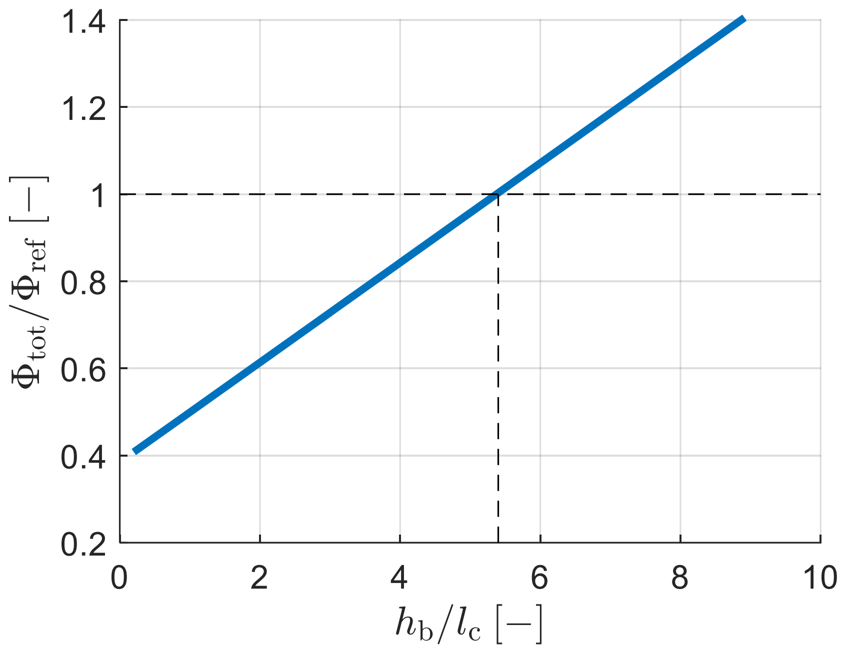




Damage to crack transition for elastic damage – Proof of concept

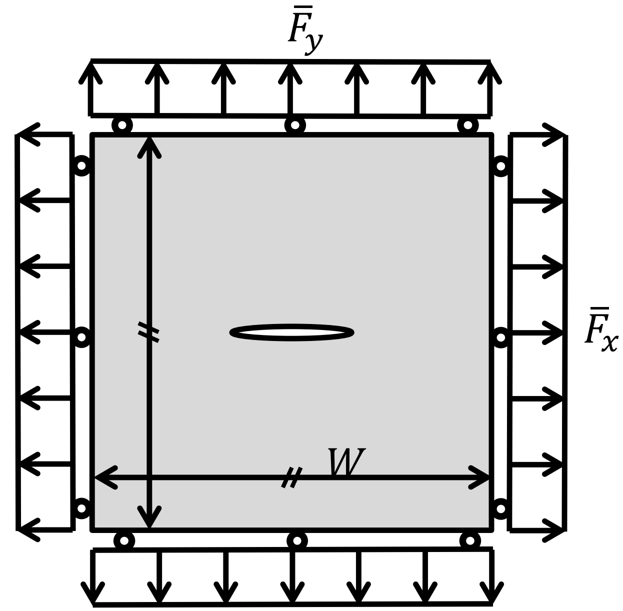
- Non-local elastic damage to Cohesive Band Model transition
  - Influence of h_{b} (for a given l_{c}) on response in a 1D elastic case [Leclerc et al. 2018]
    - Has effect on the totally dissipated energy Φ
    - Could be chosen to conserve energy dissipation (physically based)
    - For elastic damage: $h_{\mathrm{b}} \simeq 5.4\ l_c$

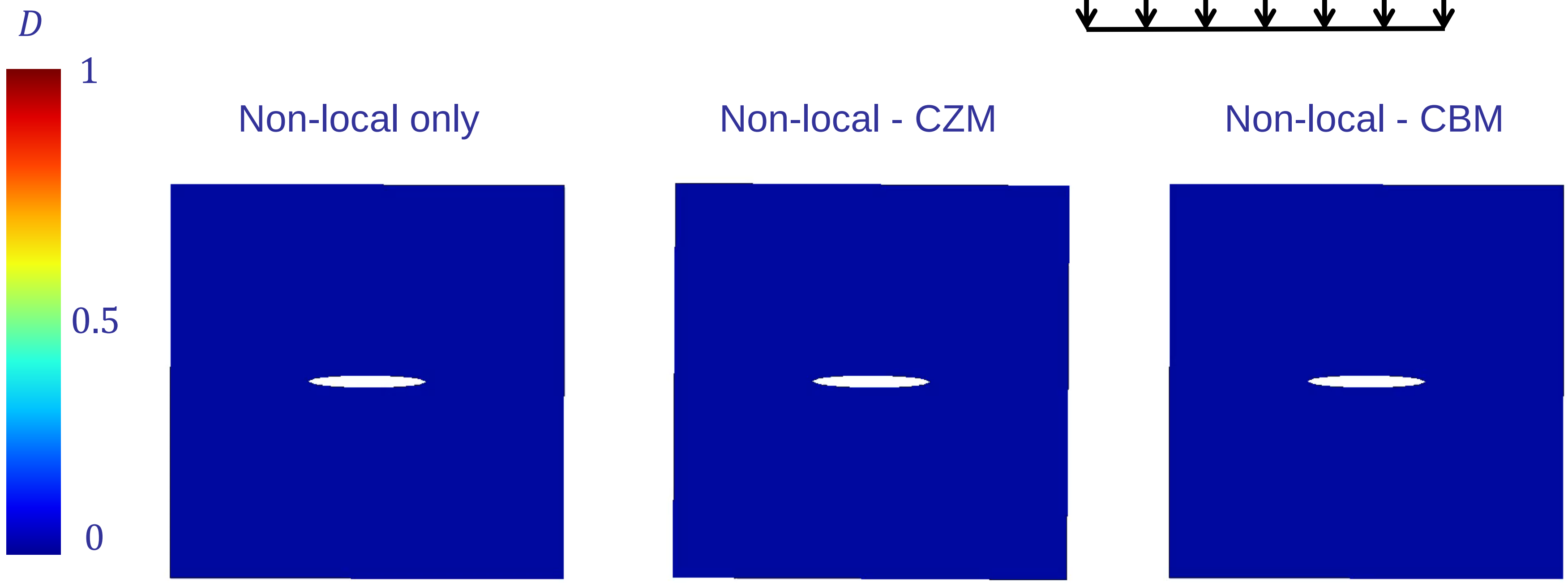




Damage to crack transition for elastic damage – Proof of concept

- Study of triaxiality effect on a slit-plate
  - Biaxial loading
    - Loading at constant $\bar{F}_x/\bar{F}_y$ ratio
    - Plane-strain state
  - Comparison between:
    - Pure non-local
    - Non-local + cohesive zone (CZM)
    - Non-local + cohesive band (CBM)





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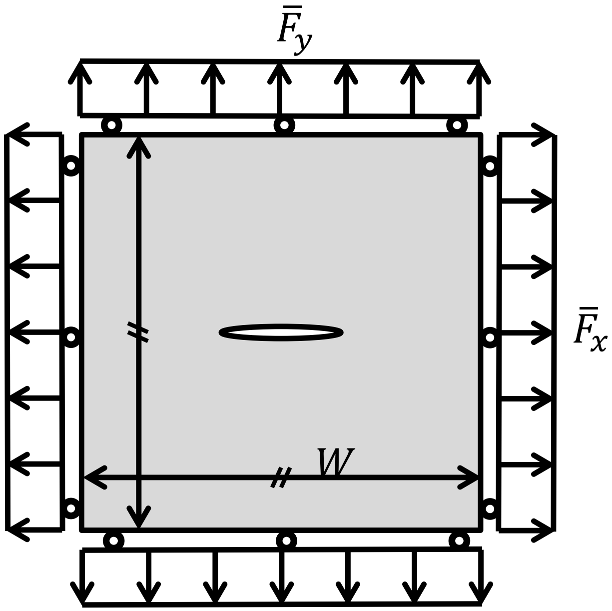
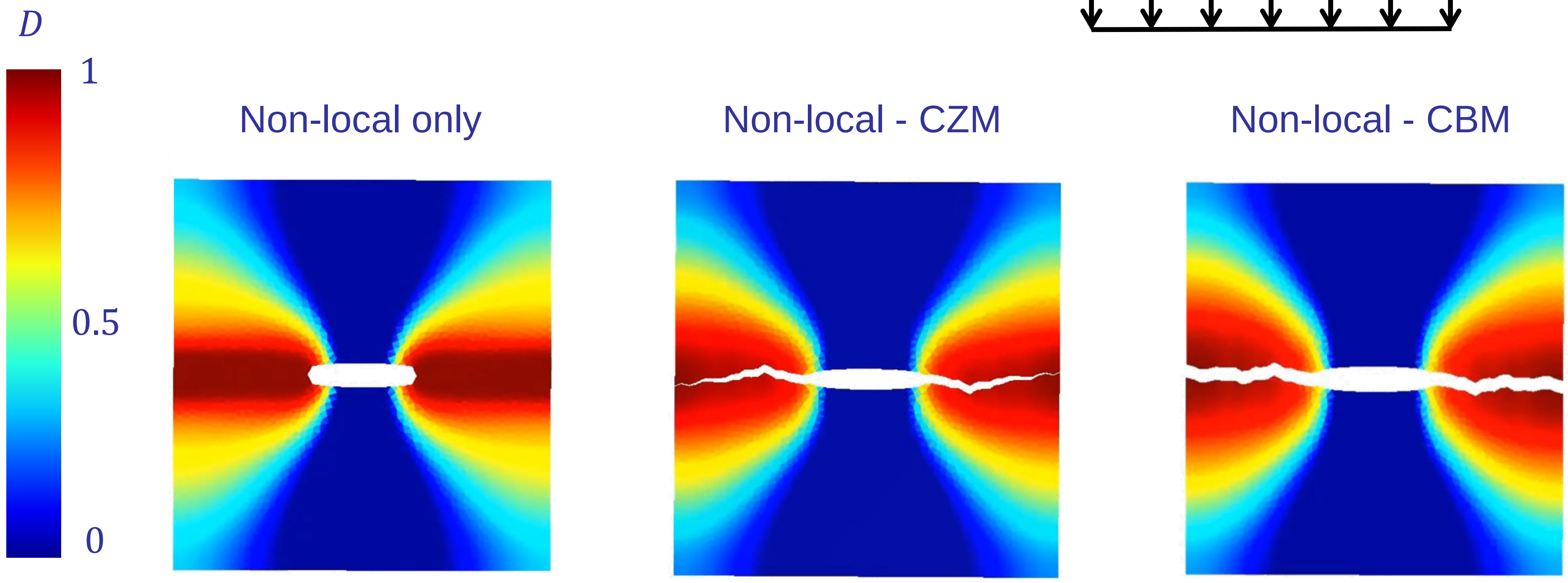


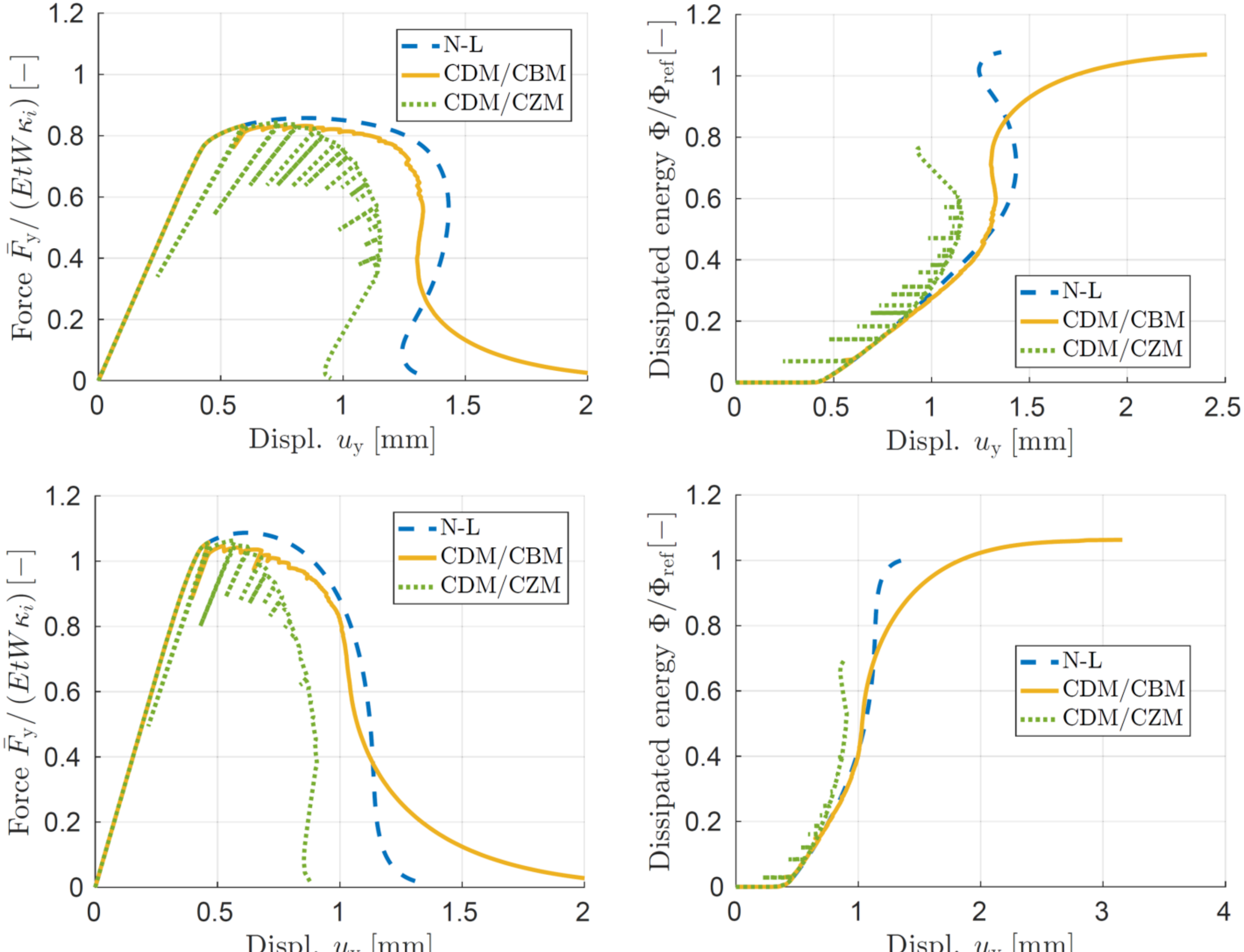


Damage to crack transition for elastic damage – Proof of concept

- Study of triaxiality effect on a slit-plate
  - Reference dissipated energy Φ_{ref} for non-local with $\bar{F}_x/\bar{F}_y$ =0

$\frac{\bar{F}_x}{\bar{F}_y} = -\frac{1}{2}$

$\frac{\bar{F}_x}{\bar{F}_y} = \frac{1}{2}$



Non-Local only

Non-Local - CZM

Non-Local - CBM

Error on total diss. energy

CZM: ~30%

CBM: ~3%

Non-local porous plasticity model

- Hyperelastic-based formulation
  - Multiplicative decomposition

    $\mathbf{F} = \mathbf{F}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{p}}, \quad \mathbf{C}^{\mathrm{e}} = \mathbf{F}^{\mathrm{e}T} \cdot \mathbf{F}^{\mathrm{e}}, \quad J^{e} = \det(\mathbf{F}^{\mathrm{e}})$
  - Stress tensor definition
    - Elastic potential $\psi(\mathbf{C}^{\mathrm{e}})$
    - First Piola-Kirchhoff stress tensor

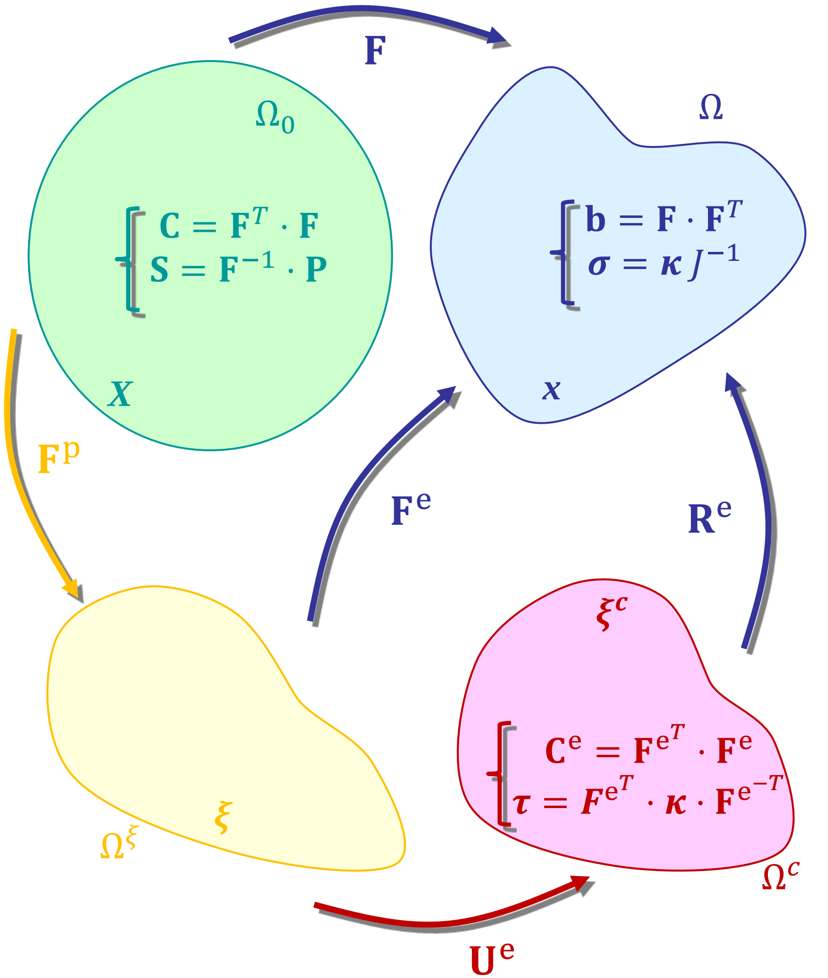
      $$\mathbf{P} = 2\mathbf{F}^{\mathrm{e}} \cdot \frac{\partial \psi(\mathbf{C}^{\mathrm{e}})}{\partial \mathbf{C}^{\mathrm{e}}} \cdot \mathbf{F}^{\mathrm{p}^{-T}}$$
    - Kirchhoff stress tensors
      - In current configuration

        $$\boldsymbol{\kappa} = \mathbf{P} \cdot \mathbf{F}^{T} = 2\mathbf{F}^{\mathrm{e}} \cdot \frac{\partial \psi(\mathbf{C}^{\mathrm{e}})}{\partial \mathbf{C}^{\mathrm{e}}} \cdot \mathbf{F}^{\mathrm{e}T}$$
      - In co-rotational space

        $$\boldsymbol{\tau} = \mathbf{C}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{e}^{-1}} \cdot \boldsymbol{\kappa} \cdot \mathbf{F}^{\mathrm{e}^{-T}} = 2\mathbf{C}^{\mathrm{e}} \cdot \frac{\partial \psi(\mathbf{C}^{\mathrm{e}})}{\partial \mathbf{C}^{\mathrm{e}}}$$
- Logarithmic deformation
  - Elastic potential ψ:

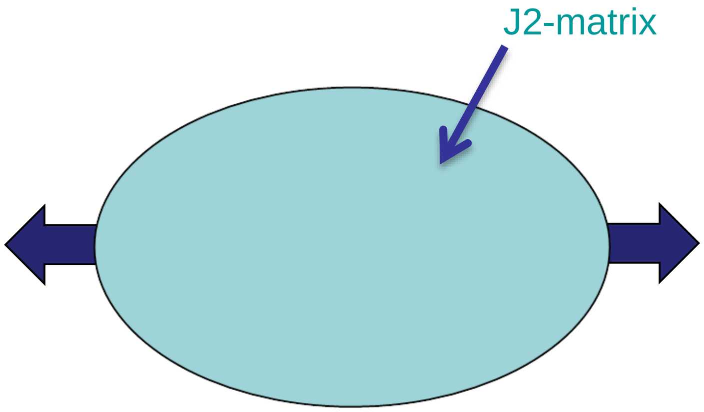
    $$\psi(\mathbf{C}^{\mathrm{e}}) = \frac{K}{2}\ln^2(J^e) + \frac{G}{4}(\ln(\mathbf{C}^{\mathrm{e}}))^{\mathrm{dev}} : (\ln(\mathbf{C}^{\mathrm{e}}))^{\mathrm{dev}}$$
  - Stress tensor in co-rotational space

    $$\boldsymbol{\tau} = \underbrace{K\ln(J^e)}_{p}\,\mathbf{I} + G(\ln(\mathbf{C}^{\mathrm{e}}))^{\mathrm{dev}}$$



Damage to crack transition in porous elasto-plasticity

- Porous plasticity (or Gurson) approach
  - Assuming a J2-(visco-)plastic matrix



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- Porous plasticity (or Gurson) approach
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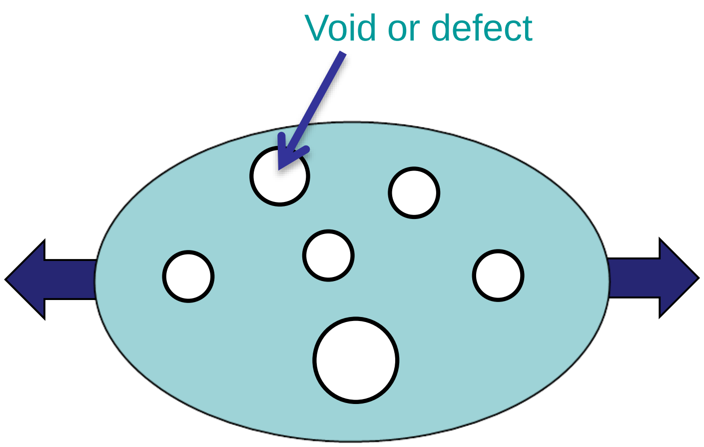
  - Including effects of void/defect or porosity on plastic behavior
    - Apparent macroscopic yield surface $f(\tau_{\text{eq}}, p) \leq 0$ due to microstructural state:



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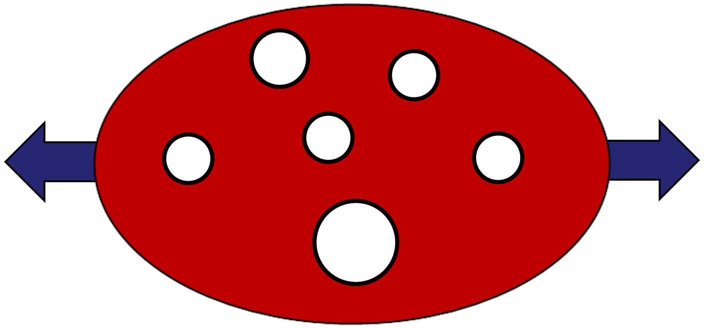
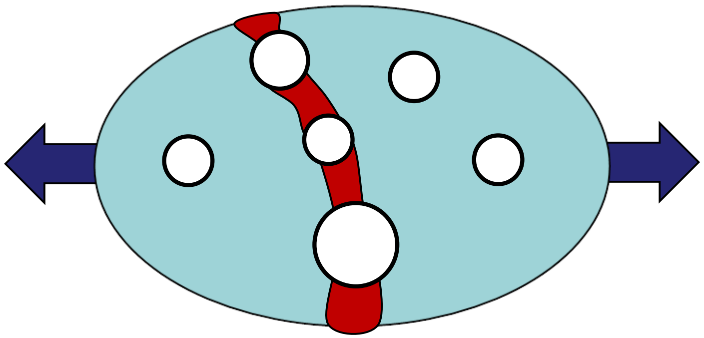
      » Diffuse plastic flow spreads in the matrix

      » Gurson model



Damage to crack transition in porous elasto-plasticity

- Porous plasticity (or Gurson) approach
  - Assuming a J2-(visco-)plastic matrix
  - Including effects of void/defect or porosity on plastic behavior
    - Apparent macroscopic yield surface $f(\tau_{\mathrm{eq}}, p) \leq 0$ due to microstructural state:
      - Competition between two deformation modes:
        - » Diffuse plastic flow spreads in the matrix
          - » Gurson model
        - » Before failure: coalescence or localized plastic flow between voids
          - » GTN or Thomason models



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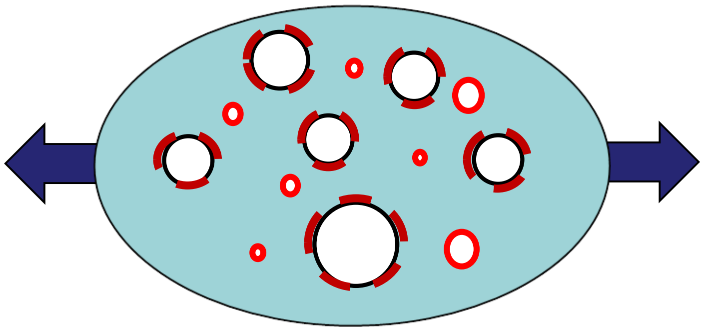
      - Competition between two deformation modes:
        - » Diffuse plastic flow spreads in the matrix
          - » Gurson model
        - » Before failure: coalescence or localized plastic flow between voids
          - » GTN or Thomason models
  - Including evolution of microstructure during failure process
    - Void growth by diffuse plastic flow
    - Apparent growth by shearing
    - Nucleation / appearance of new voids
    - Void coalescence until failure



Non-local porous plasticity model

- Yield surface is considered in the co-rotational space
  - Non-local form $f\left(\tau_{\mathrm{eq}}, p; \tau_{\mathrm{Y}}, \boldsymbol{Z}(t'), \tilde{f}_V(t')\right) \leq 0$
    - τ_{eq} is the von Mises equivalent Kirchhoff stress and p the pressure
    - $\tau_{\mathrm{Y}} = \tau_{\mathrm{Y}}(\hat{p}, \dot{\hat{p}})$ is the viscoplastic yield stress in terms of equivalent matrix plastic strain $\hat{p}$
    - f_{V} is the porosity and $\tilde{f}_{\mathrm{V}}$, its non-local counterpart with $\tilde{f}_V - l_c^2 \Delta \tilde{f}_V = f_V$
    - $\boldsymbol{Z}$ is the vector of internal variables
    - l_{c} is the non-local length
  - Normal plastic flow $\mathbf{D}^{\mathrm{p}}$

$$\mathbf{D}^{\mathrm{p}} = \dot{\mathbf{F}}^{\mathrm{p}} \cdot \mathbf{F}^{\mathrm{p}^{-1}} = \dot{\gamma}\frac{\partial f}{\partial \boldsymbol{\tau}} = \dot{\hat{d}}\frac{\partial \tau_{\mathrm{eq}}}{\partial \boldsymbol{\tau}} + \dot{\hat{q}}\frac{\partial p}{\partial \boldsymbol{\tau}}$$

  - Microstructure evolution (spherical voids):
    - Equivalent matrix plastic strain rate

$$\dot{\hat{p}} = \frac{\boldsymbol{\tau}:\mathbf{D}^{\mathrm{p}}}{(1-f_{V_0})\tau_{\mathrm{Y}}}$$

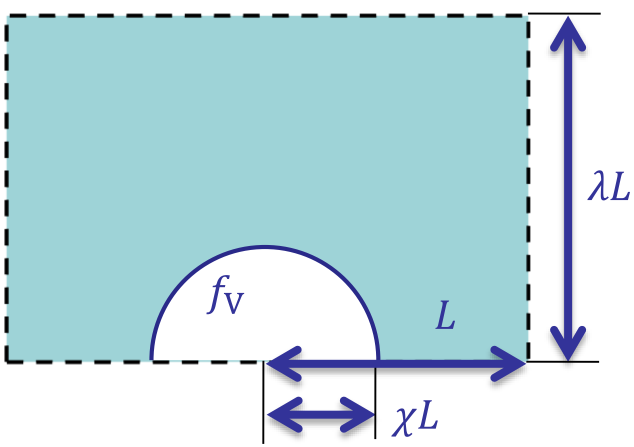
    - Porosity:

$$\dot{f}_V = (1 - f_V)\mathrm{tr}(\mathbf{D}^{\mathrm{p}}) + \dot{f}_{\mathrm{nucl}} + \dot{f}_{\mathrm{shear}}$$

    - Ligament ratio:

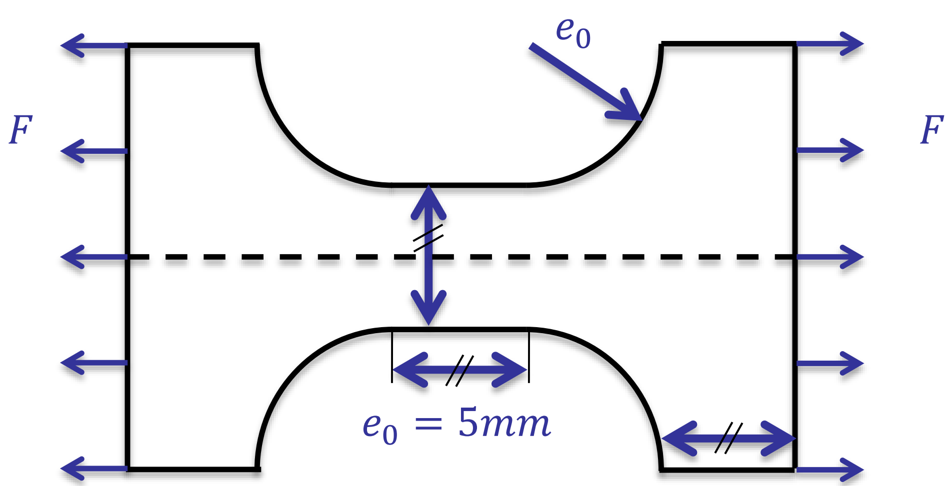
$$\dot{\chi} = \dot{\chi}(\chi, \tilde{f}_V, \kappa, \lambda, \boldsymbol{Z})$$

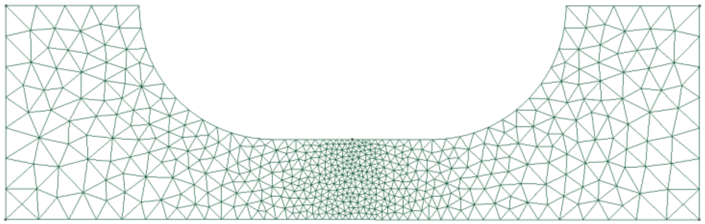
Microstructure parameters



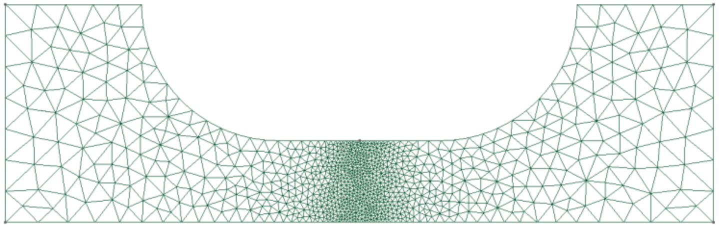
Non-local porous plasticity – Comparison with literature results

- Plane strain specimen [Besson et al. 2003]
  - Only half specimen is modelled
  - Three ≠ mesh sizes

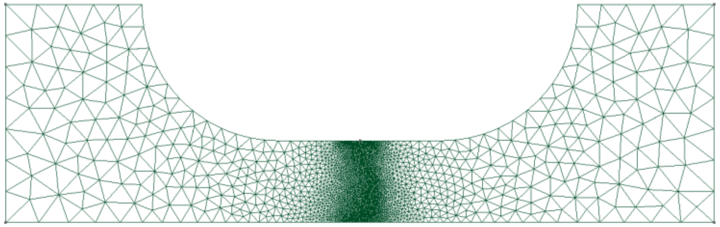




Coarse mesh
(~4600 elements, $l_m \cong 1.12\ l_c$)



Medium mesh
(~8100 elements, $l_m \cong 0.75\ l_c$)

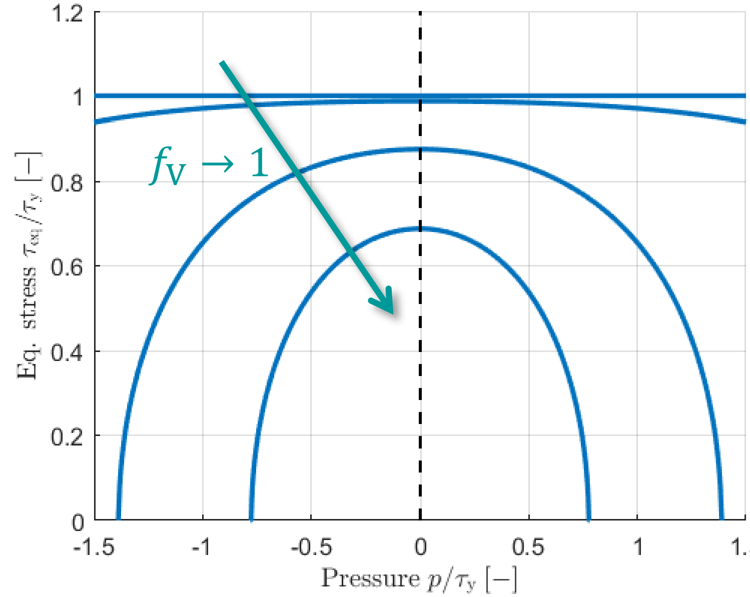


Fine mesh
(~15500 elements, $l_m \cong 0.5\ l_c$)

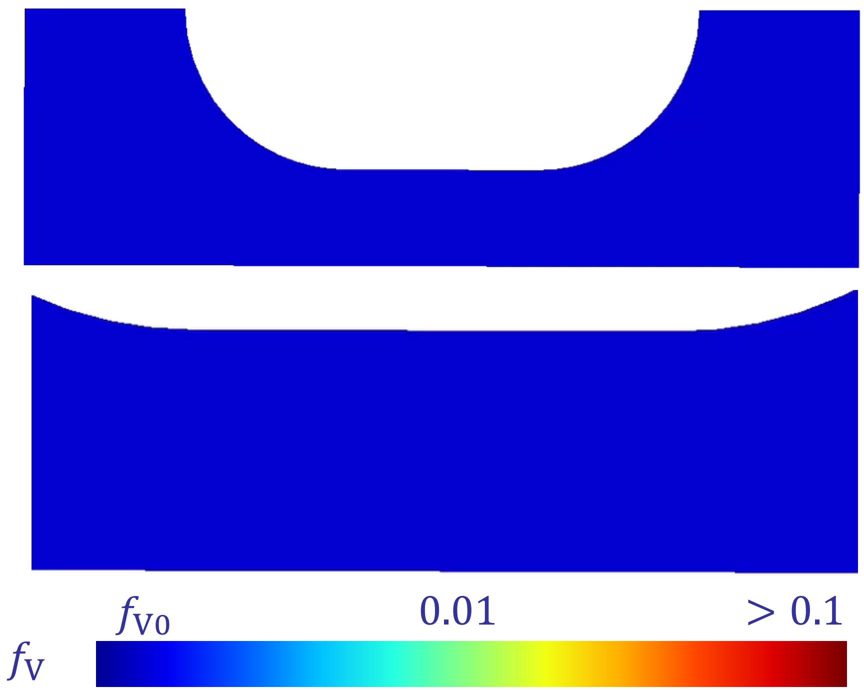
Non-local porous plasticity – void growth

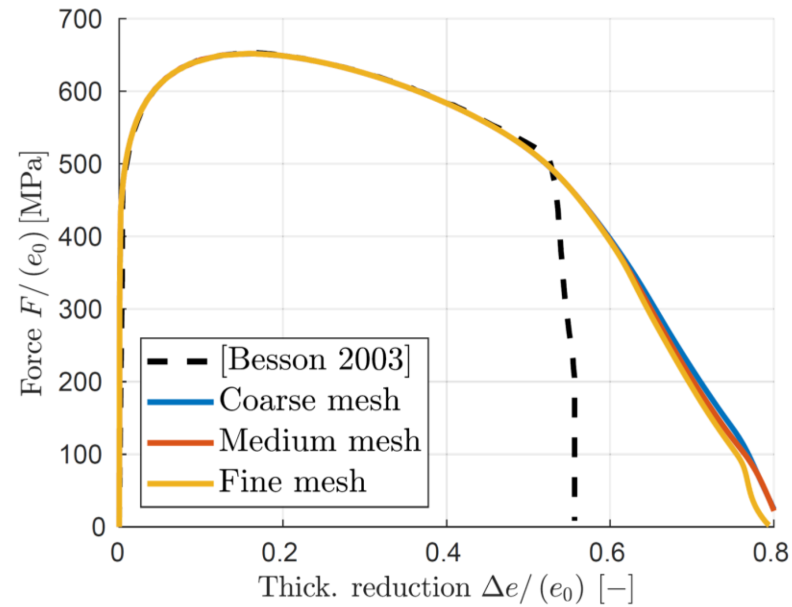
- Gurson model [Reush et al. 2003]
  - Particularized yield surface

$$f_{\mathrm{G}} = \frac{\tau_{\mathrm{eq}}^2}{\tau_{\mathrm{Y}}^2} + 2q_1\tilde{f}_V \cosh\left(\frac{q_2 p}{2\tau_{\mathrm{Y}}}\right) - 1 - q_3^2\tilde{f}_V^2 \leq 0$$



- Verification of non-local model





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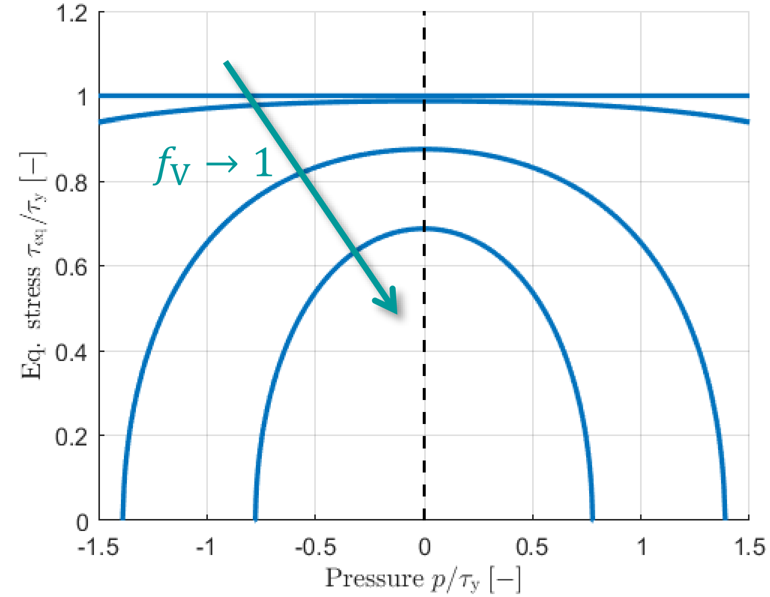

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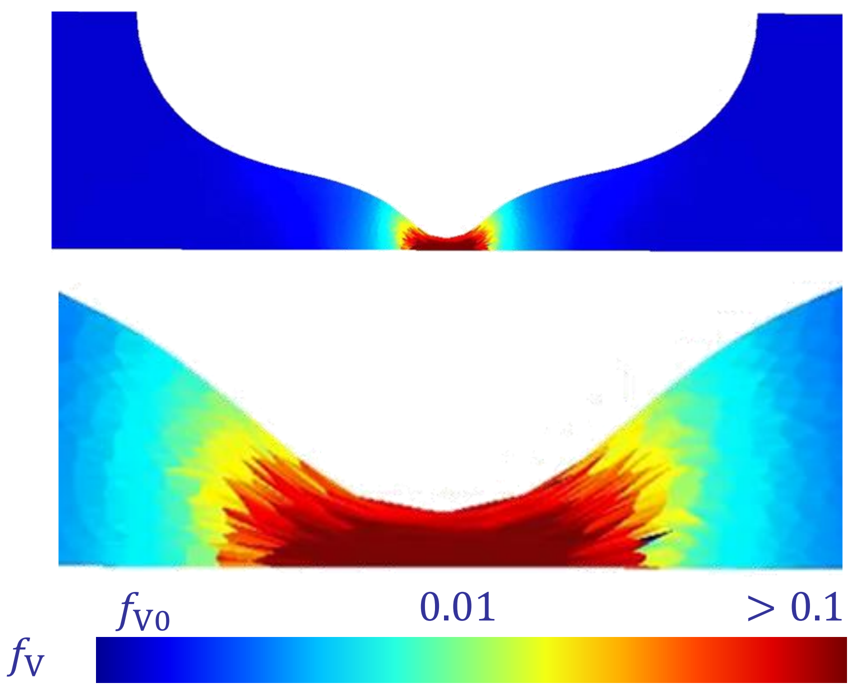
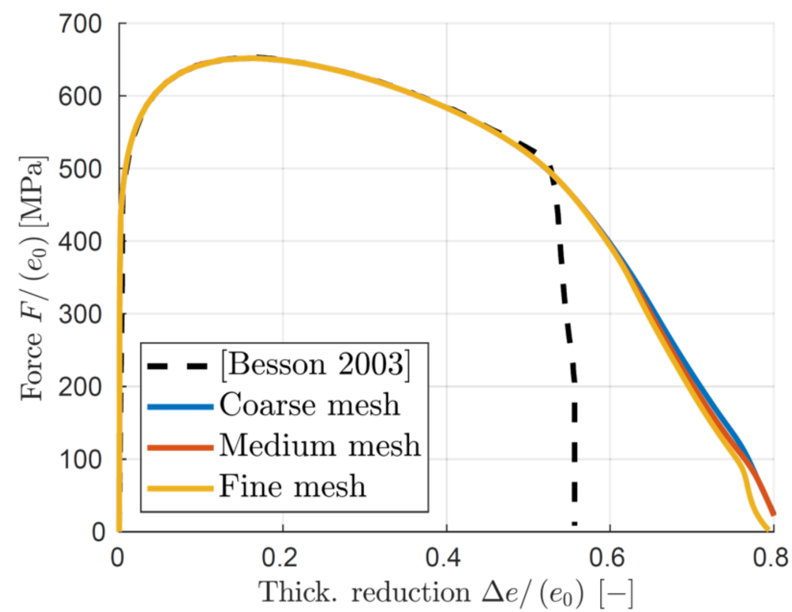


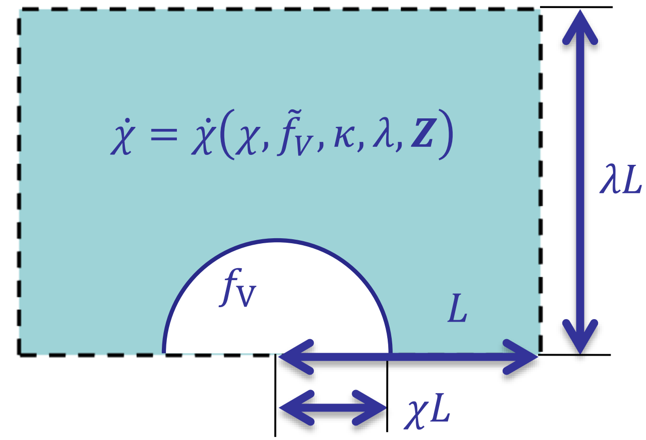


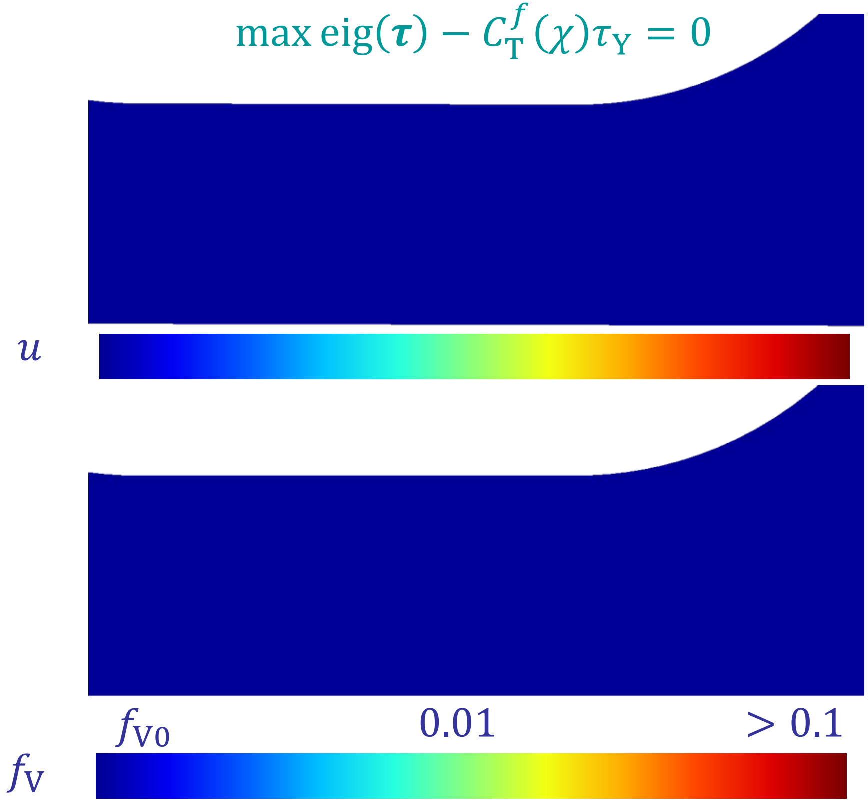
- Gurson model [Reush et al. 2003]
  - Phenomenological coalescence model:
    - Replace $\tilde{f}_{\mathrm{V}}$ by an effective value $\tilde{f}_{\mathrm{V}}^{*}$:

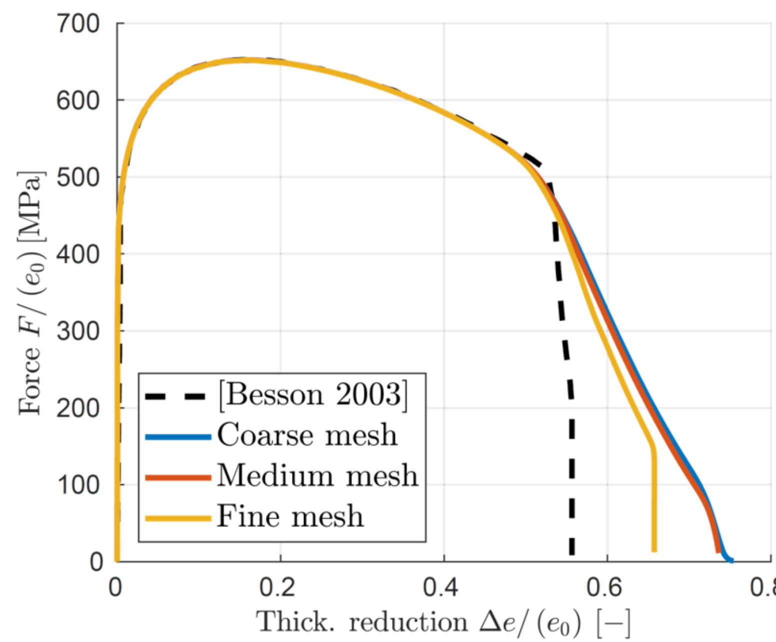
$$\tilde{f}_V^* = \begin{cases} \tilde{f}_V & \text{if } \tilde{f}_V \leq f_C \\ f_C + R\left(\tilde{f}_V - f_C\right) & \text{if } \tilde{f}_V > f_C \end{cases}$$

    - f_C from concentration factor $C_{\mathrm{T}}^{f}(\chi)$ [Benzerga2014]

$$\max \operatorname{eig}(\boldsymbol{\tau}) - C_{\mathrm{T}}^{f}(\chi)\tau_{\mathrm{Y}} = 0$$







Non-local porous plasticity – void growth and coalescence

- Gurson model [Reush et al. 2003]
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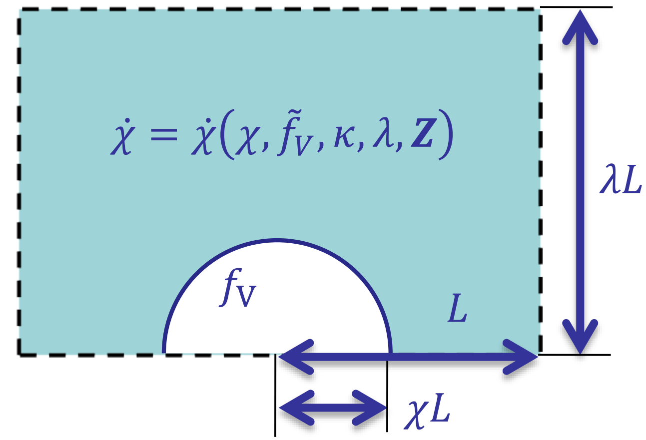
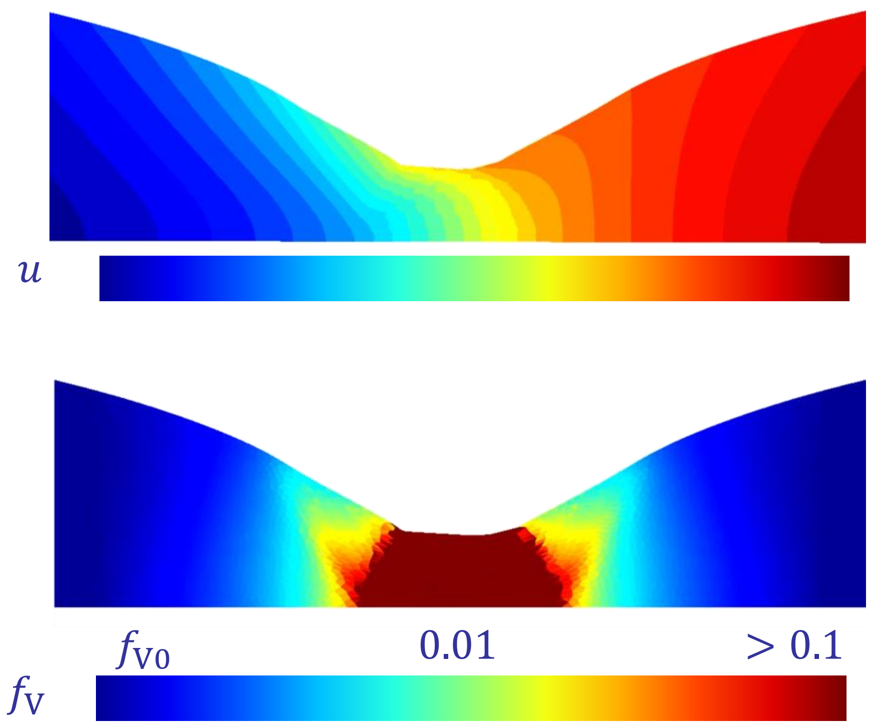
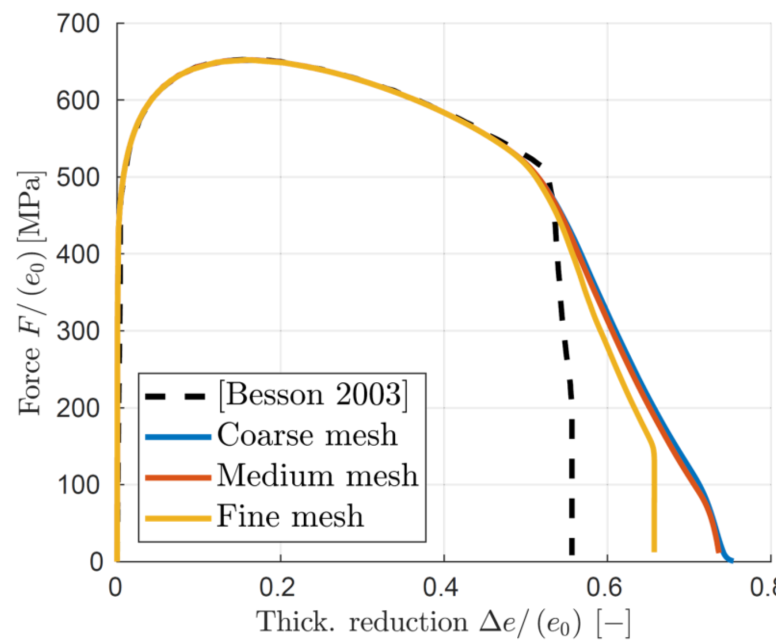




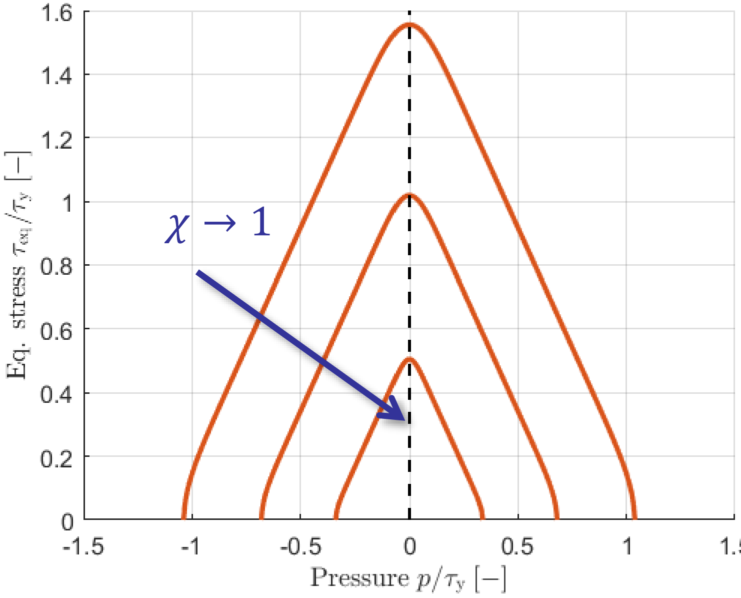


Non-local porous plasticity – void coalescence

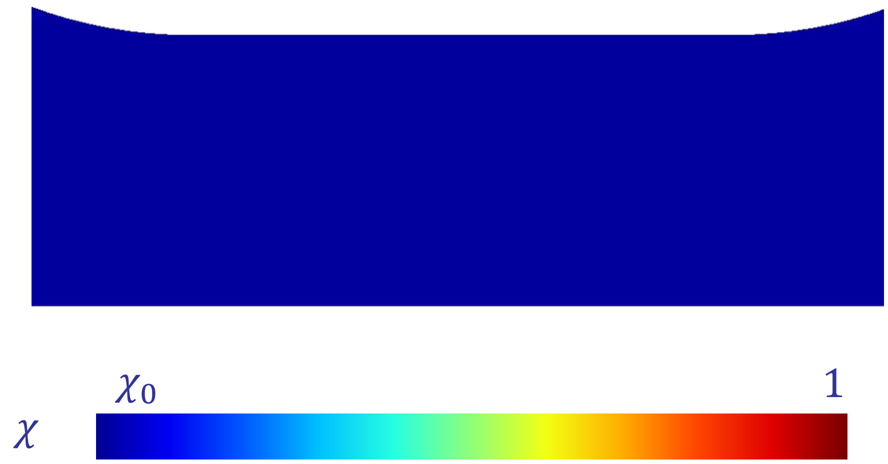
- Thomason model [Benzerga 2014, Besson 2009]
  - Particularized yield surface

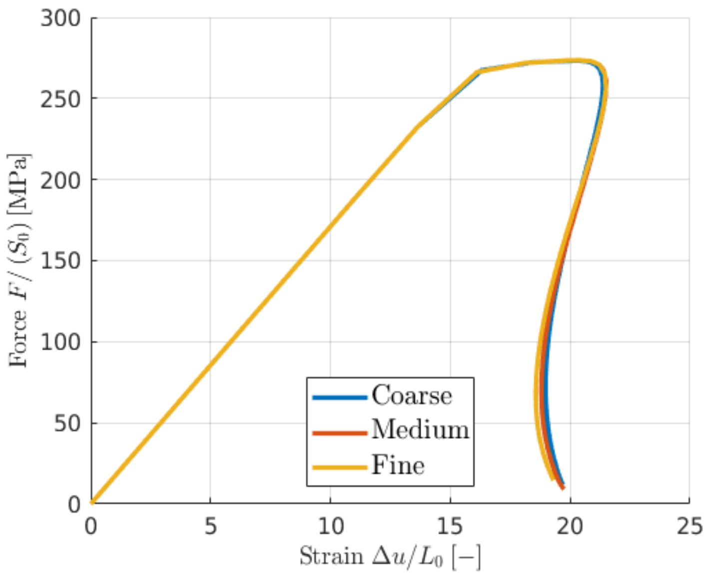
$$f_T = \frac{2}{3}\tau_{eq} + |p| - C_T^f(\chi)\tau_Y \leq 0$$

  - Higher porosity to trigger coalescence
  - No lateral contraction due to plasticity



- Verification of non-local model
  - For $\kappa = 0.5; \lambda = 0.5; l_c = 50\ \mu m$





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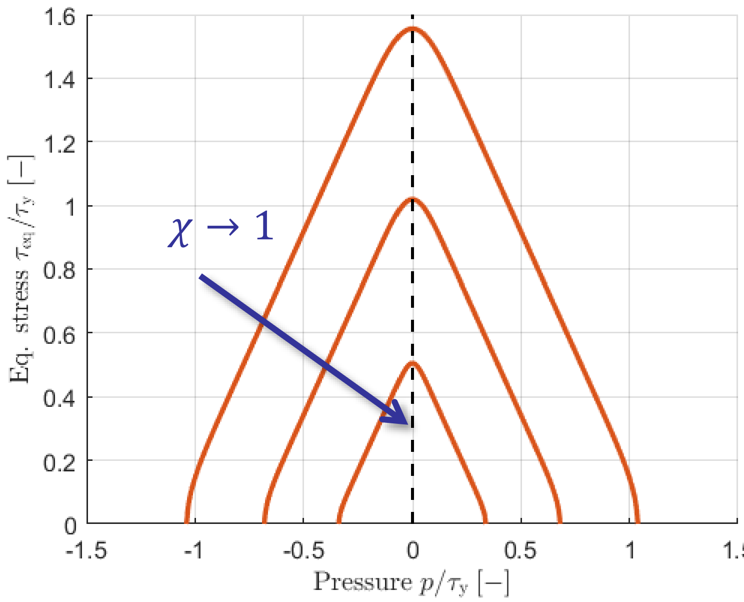

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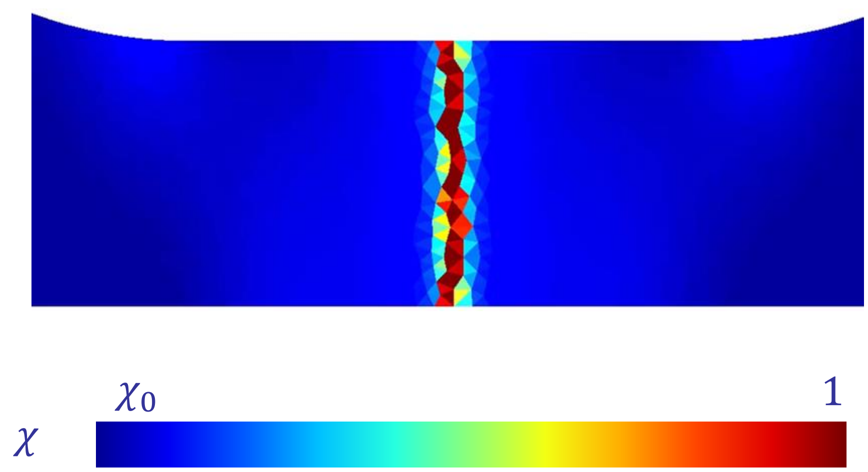
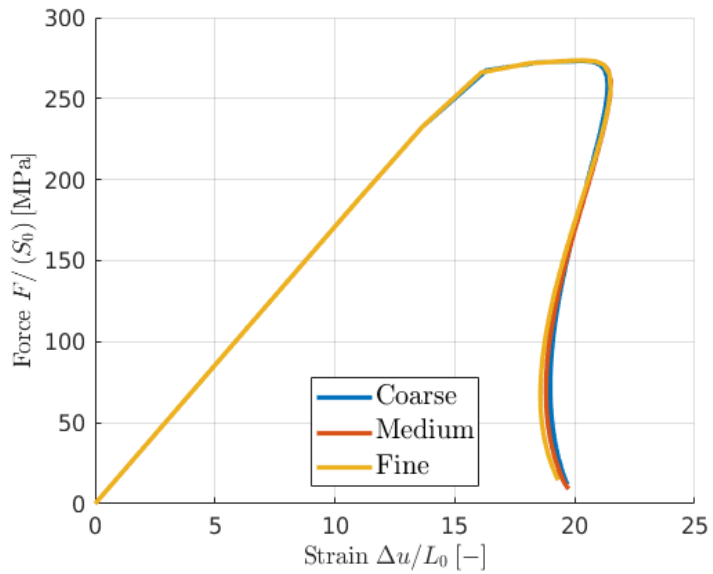
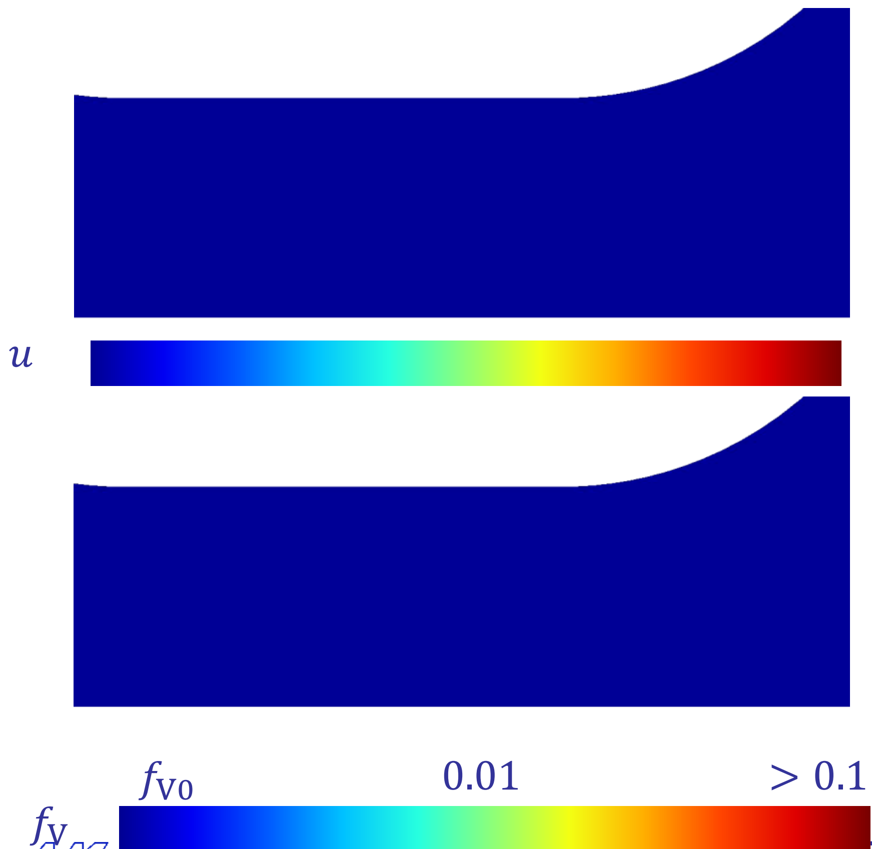


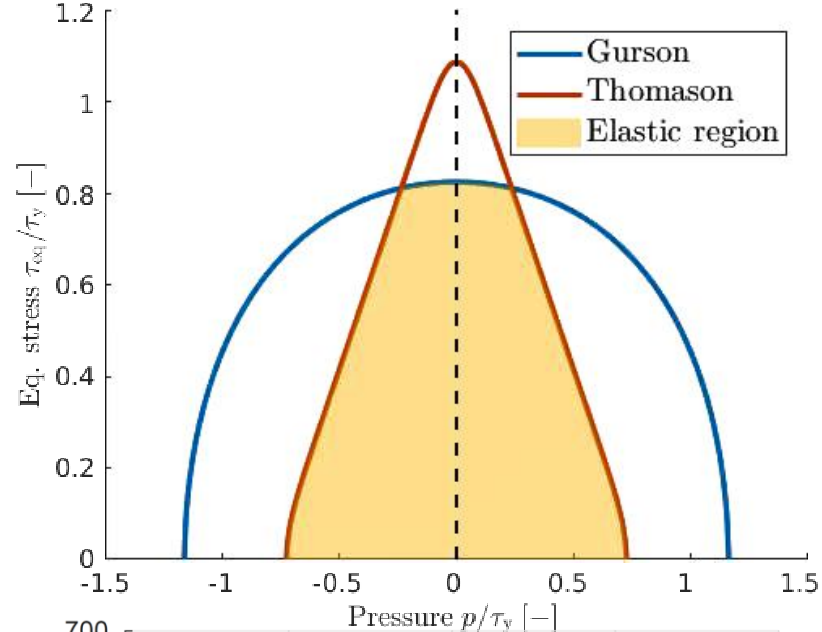


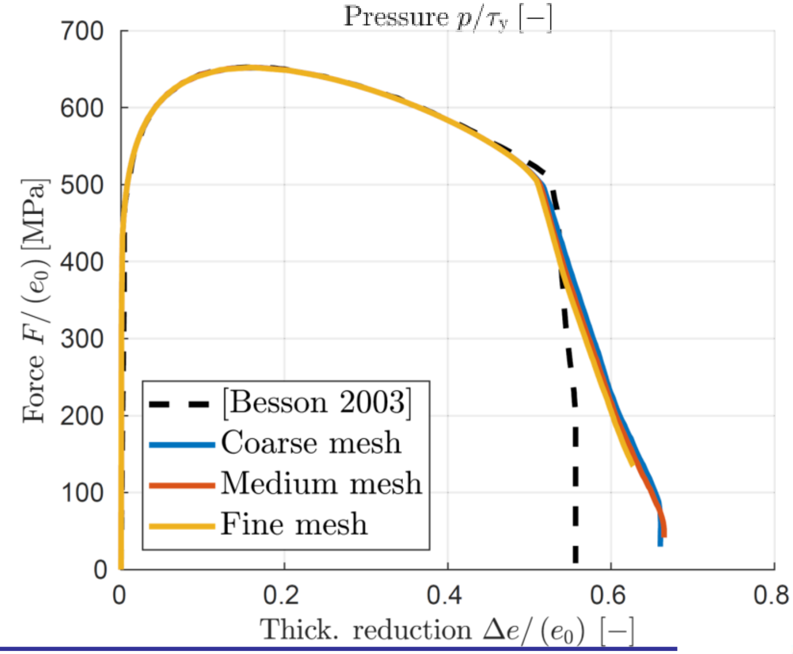
- Coupled non-local Gurson-Thomason
  - Competition between f_G and f_T

$$\begin{cases} f_\mathrm{G} = \frac{\tau_\mathrm{eq}^2}{\tau_\mathrm{Y}^2} + 2q_1 \tilde{f}_V \cosh\left(\frac{q_2 p}{2\tau_\mathrm{Y}}\right) - 1 - q_3^2 \tilde{f}_V^2 \leq 0 \\ f_\mathrm{T} = \frac{2}{3}\tau_\mathrm{eq} + |p| - C_\mathrm{T}^f(\chi)\tau_\mathrm{Y} \leq 0 \end{cases}$$

- For $\kappa = 0.5; \lambda = 0.5; l_c = 50\ \mu\mathrm{m}$





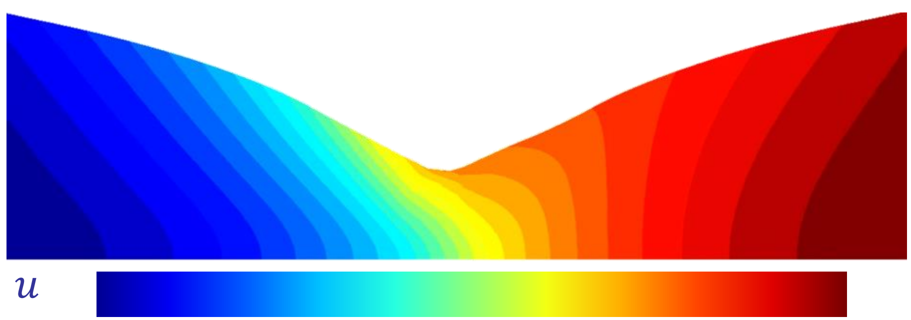


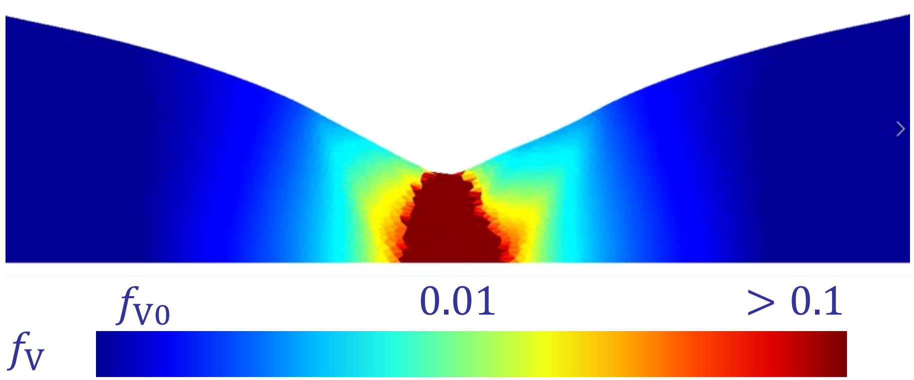
Non-local porous plasticity – void growth and coalescence

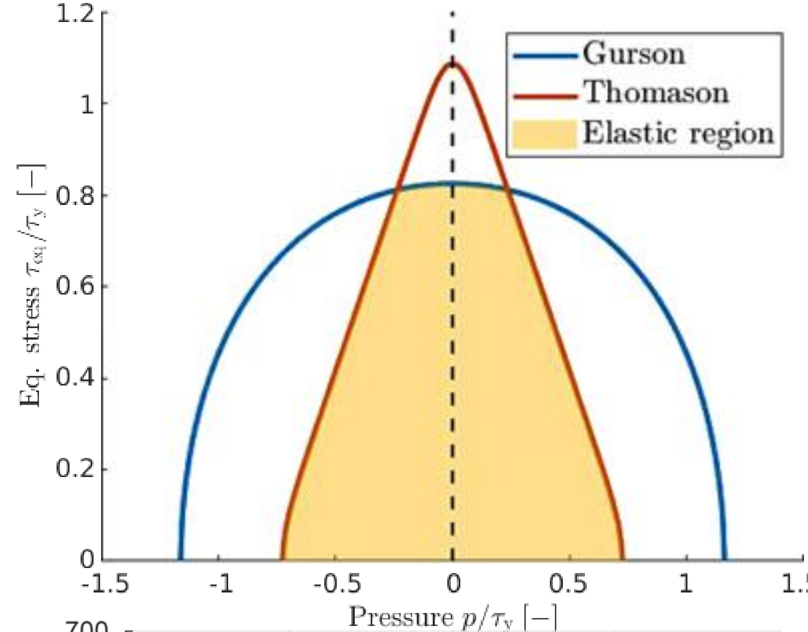
- Coupled non-local Gurson-Thomason
  - Competition between f_{G} and f_{T}

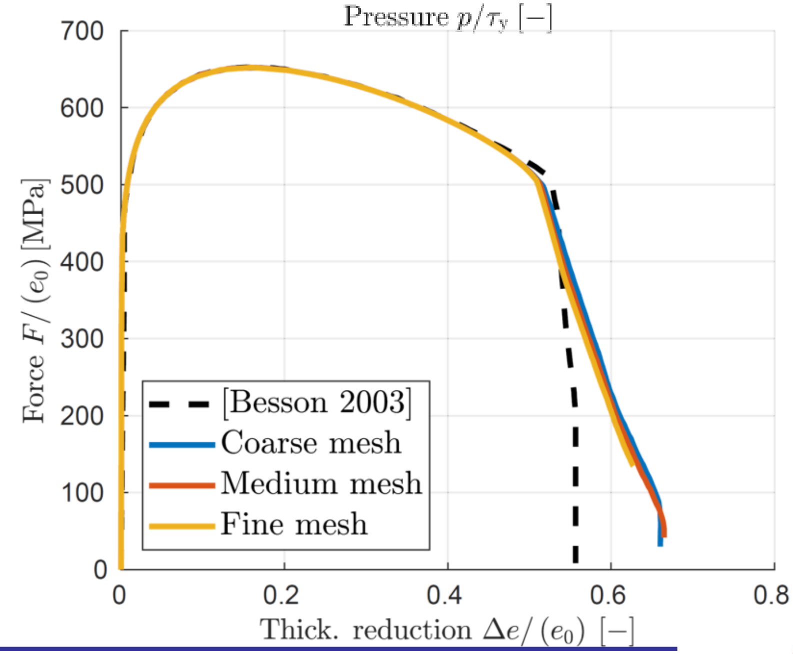
$$\begin{cases} f_{\text{G}} = \frac{\tau_{\text{eq}}^2}{\tau_{\text{Y}}^2} + 2q_1\tilde{f}_V \cosh\left(\frac{q_2 p}{2\tau_{\text{Y}}}\right) - 1 - q_3^2\tilde{f}_V^2 \leq 0 \\ f_{\text{T}} = \frac{2}{3}\tau_{\text{eq}} + |p| - C_{\text{T}}^f(\chi)\tau_{\text{Y}} \leq 0 \end{cases}$$

- For $\kappa = 0.5; \lambda = 0.5; l_c = 50\ \mu m$



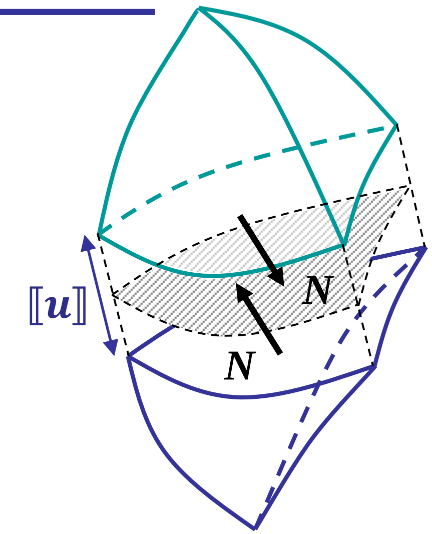


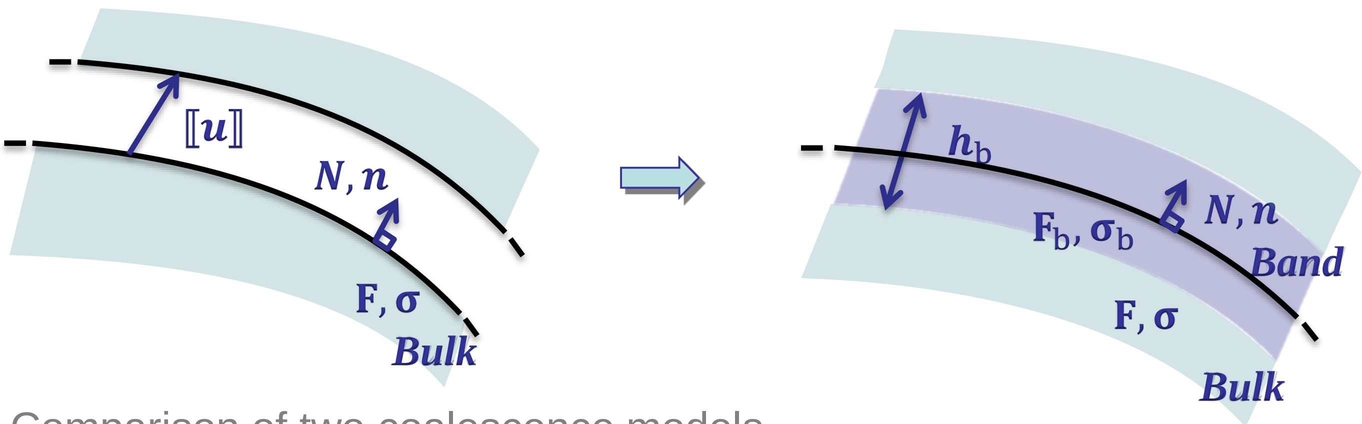




Damage to crack transition for porous plasticity

- Non-local Gurson model – CBM (arbitrary crack paths)
  - Gurson material model $f_{\mathrm{G}} = \frac{\tau_{\mathrm{eq}}^2}{\tau_{\mathrm{Y}}^2} + 2q_1\tilde{f}_V \cosh\left(\frac{q_2 p}{2\tau_{\mathrm{Y}}}\right) - 1 - q_3^2\tilde{f}_V^2 \leq 0$
  - Crack insertion at Thomasson criterion $\boldsymbol{N}\cdot\boldsymbol{\tau}\cdot\boldsymbol{N} - C_{\mathrm{T}}^f(\chi)\tau_{\mathrm{Y}} = 0$
  - At crack insertion: Cohesive Band Model



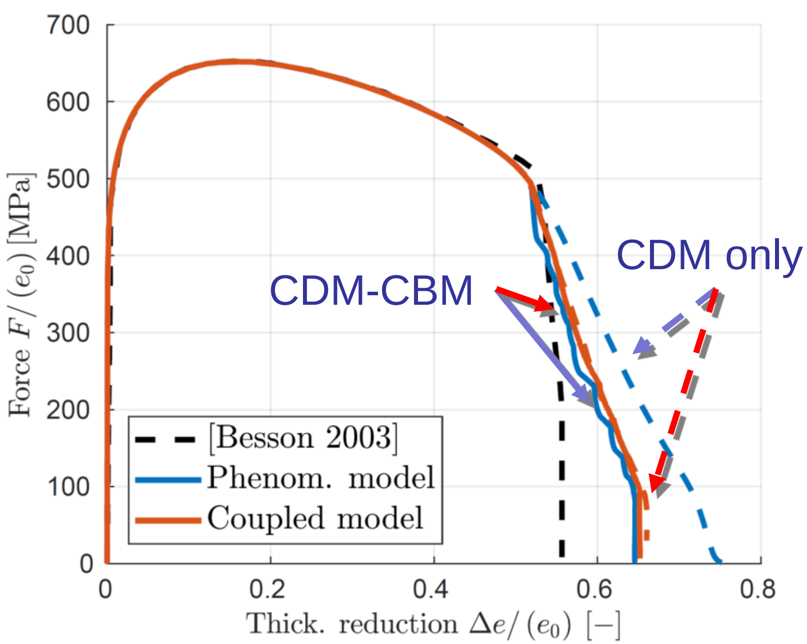


  - Comparison of two coalescence models
    - Phenomenological approach: $\tilde{f}_V^* = \begin{cases} \tilde{f}_V & \text{if } \tilde{f}_V \leq f_C \\ f_C + R(\tilde{f}_V - f_C) & \text{if } \tilde{f}_V > f_C \end{cases}$
    - Thomason model: $f_{\mathrm{T}} = \frac{2}{3}\tau_{\mathrm{eq}} + |p| - C_{\mathrm{T}}^f(\chi)\tau_{\mathrm{Y}} \leq 0$

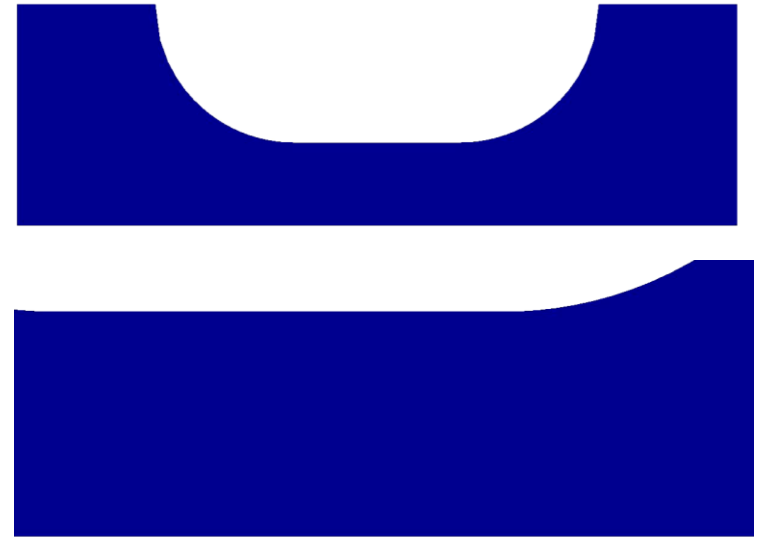
Damage to crack transition for porous plasticity

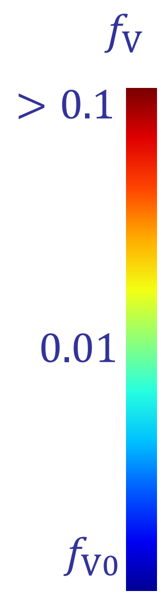
- Non-local Gurson model – CBM
  - CBM insertion at Thomason criterion
  - CBM with coalescence model
    - Comparison of 2 coalescence models
    - For $\kappa = 0.5; \lambda = 0.5; l_c = 50\ \mu m$
  - Crack path in cup-cone shape

[Besson et al. 2001]

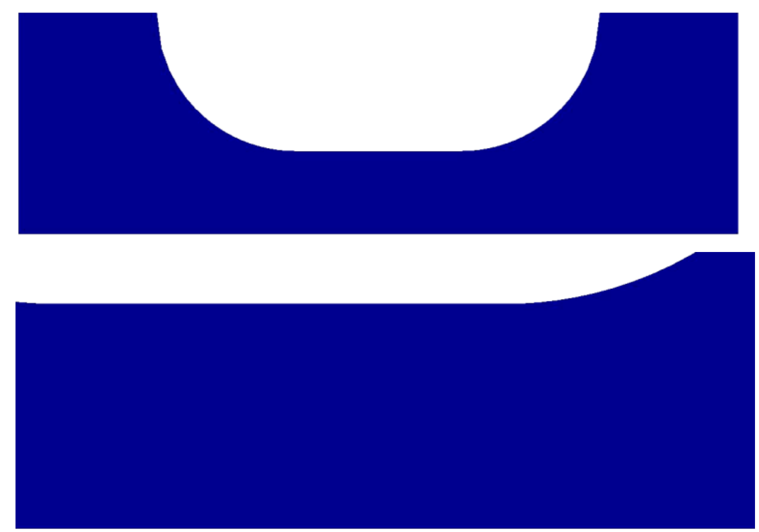


Thomason coalescence





Phenomenological coalescence



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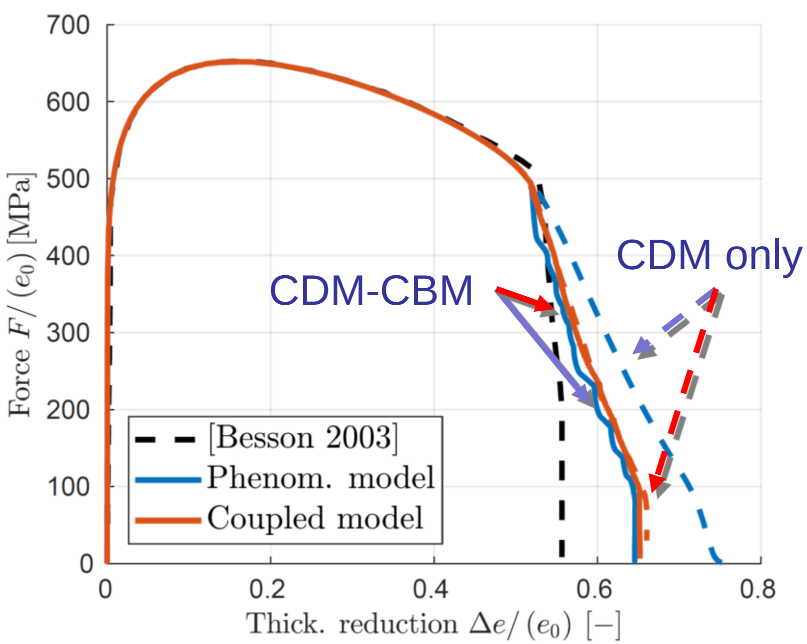
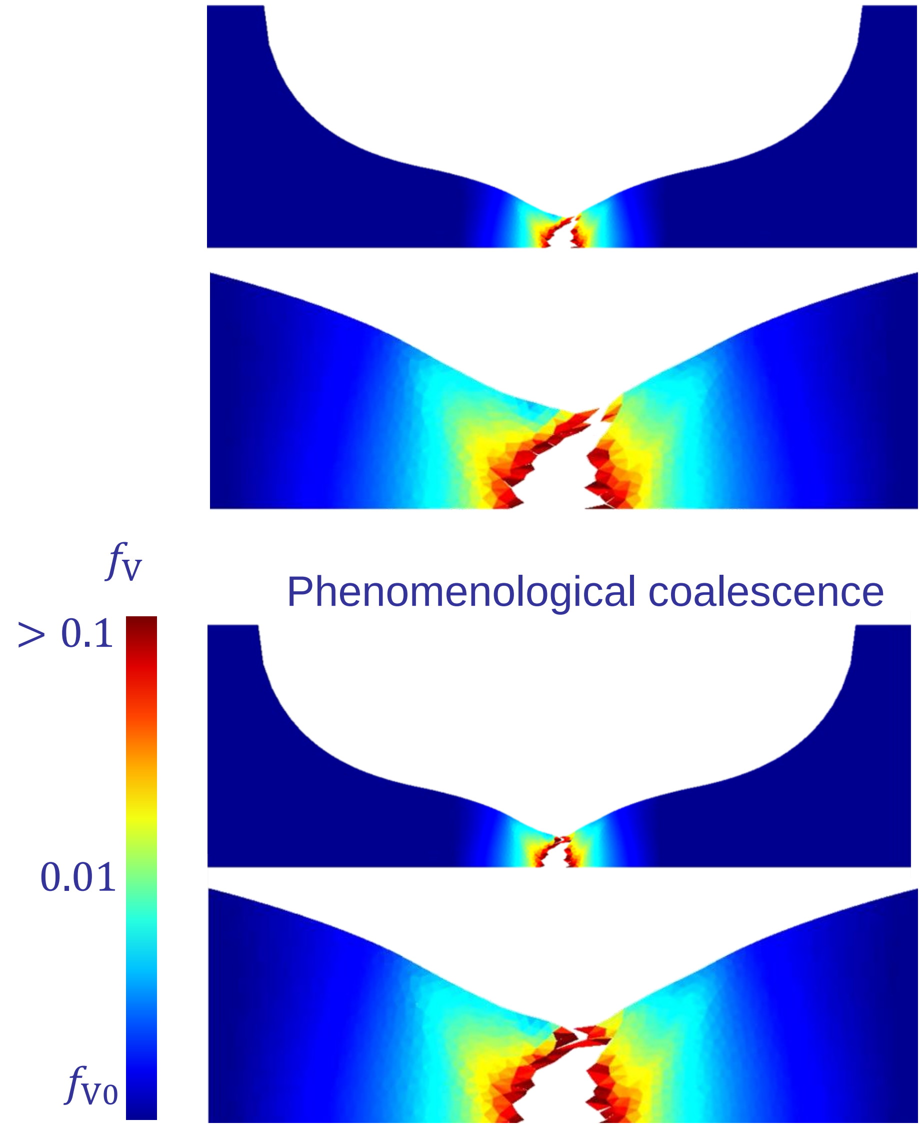


Thomason coalescence



Conclusions

- Objective:
  - Simulation of material degradation and crack initiation / propagation
- Methodology
  - Combination of non-local Continuum Damage Model (CDM)
  - Cohesive Band Model (CBM)
  - Integrated in a Hybrid Discontinuous Galerkin framework
- Proof of concept
  - Elastic damage material model
  - Cohesive band thickness controls the failure energy dissipation
- Ductile materials
  - Implementation of hyperelastic non-local porous-plastic model
    - Coupled Gurson-Thomason model
  - First results of the CDM-CBM transition
  - Upcoming tasks:
    - Enrichment of nucleation model and coalescence model
    - Calibration of the band thickness
    - Validation/Calibration with literature/experimental tests

Thank you for your attention

Computational & Multiscale Mechanics of Materials – CM3
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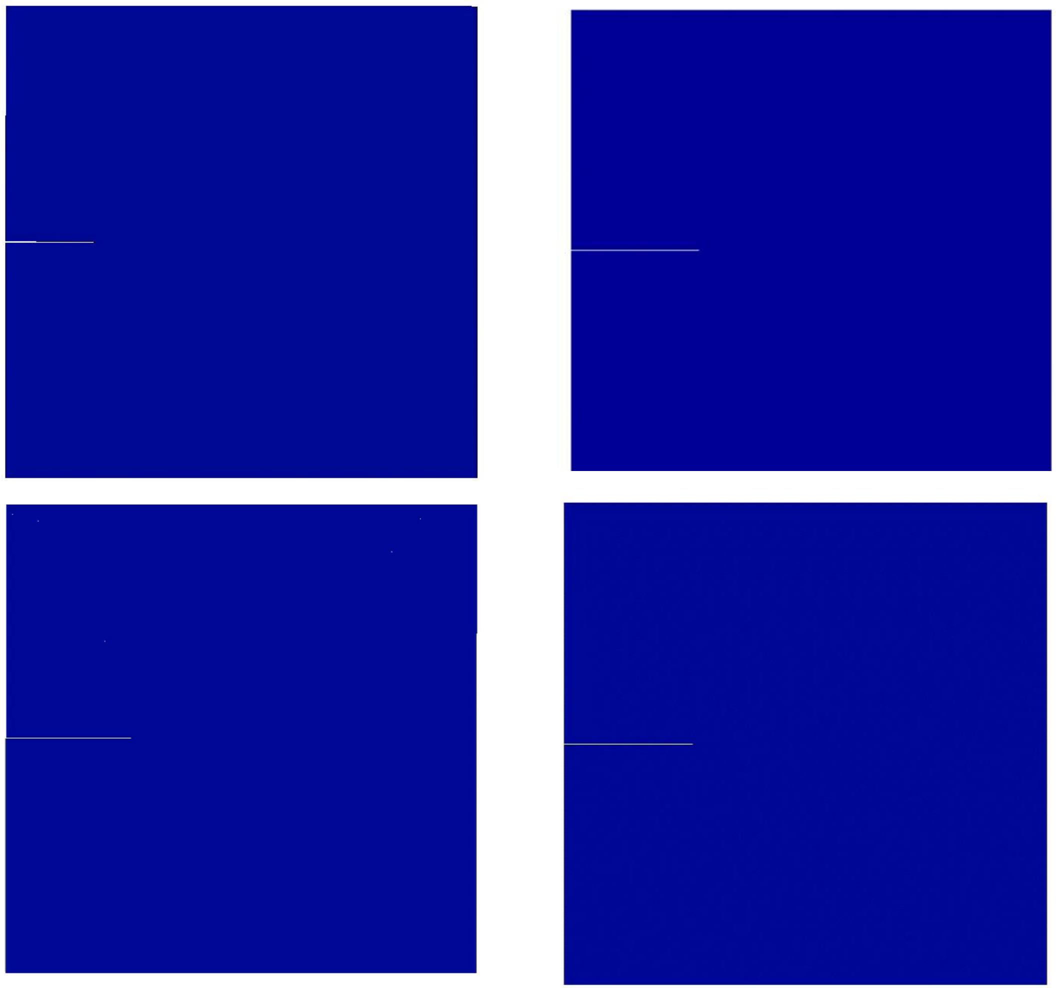
Allée de la découverte 9, B4000 Liège
Julien.Leclerc@ulg.ac.be

Damage to crack transition for elastic damage – Proof of concept

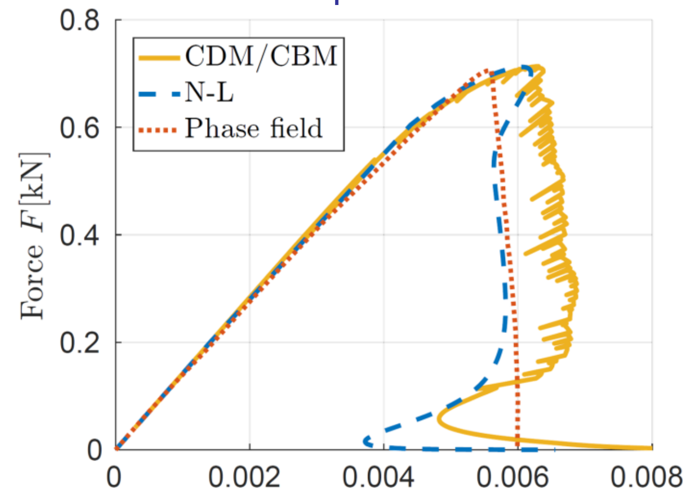
- Comparison with phase field
  - Single edge notched specimen [Miehe et al. 2010]
    - Calibration of damage and CBM parameters with 1D case [Leclerc et al. 2018]

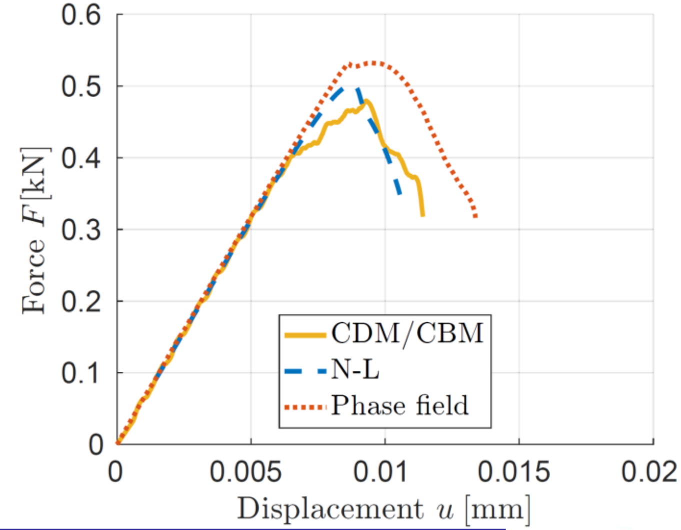
Non-local model | Cohesive band model

Tension test / Shearing test



Force-displacement curve





- Comparison with phase field
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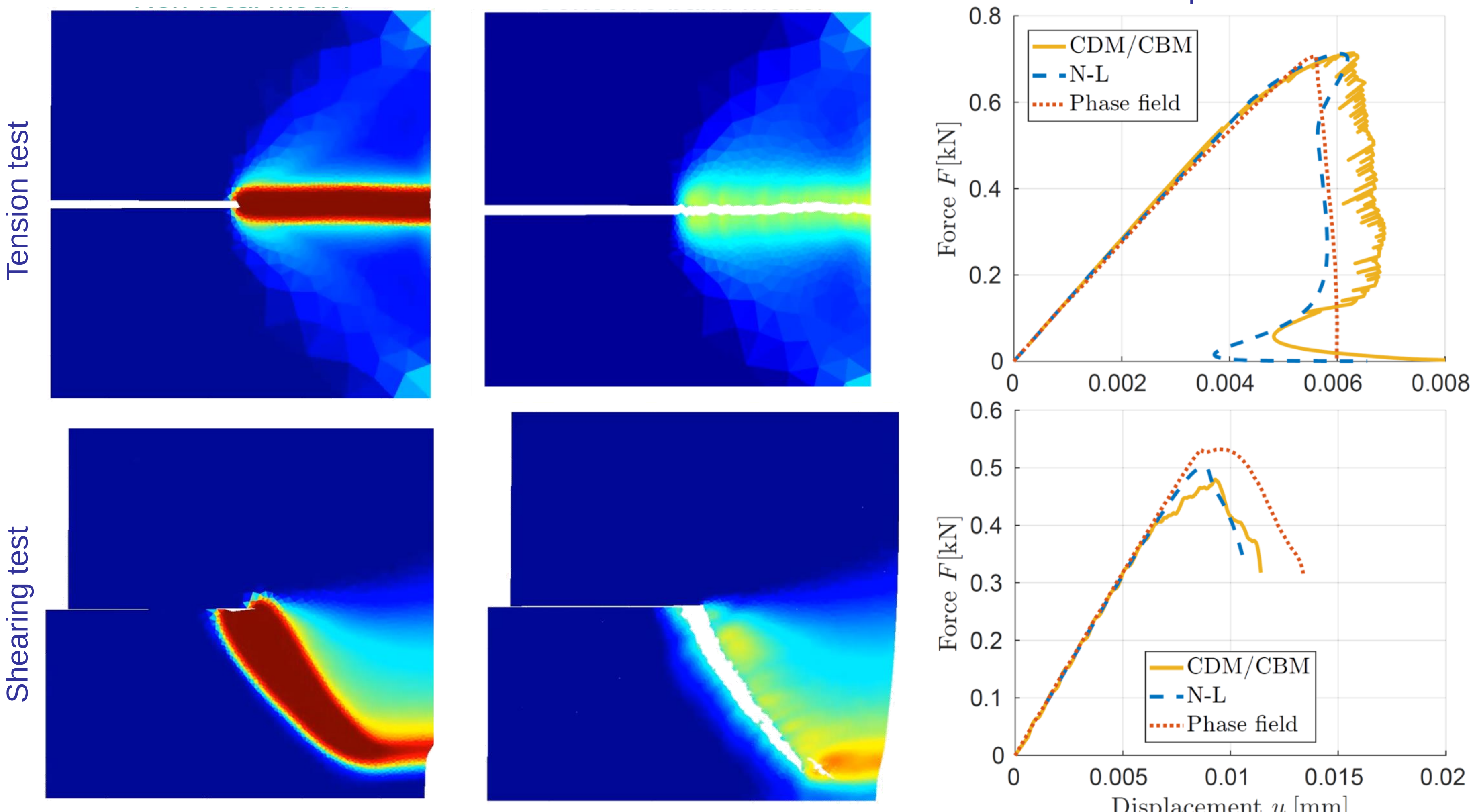
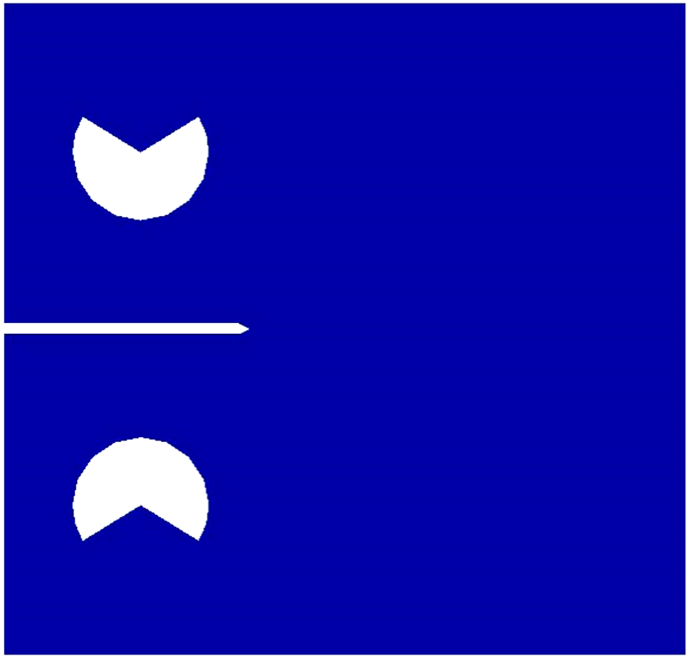
    - Calibration of damage and CBM parameters with 1D case [Leclerc et al. 2018]

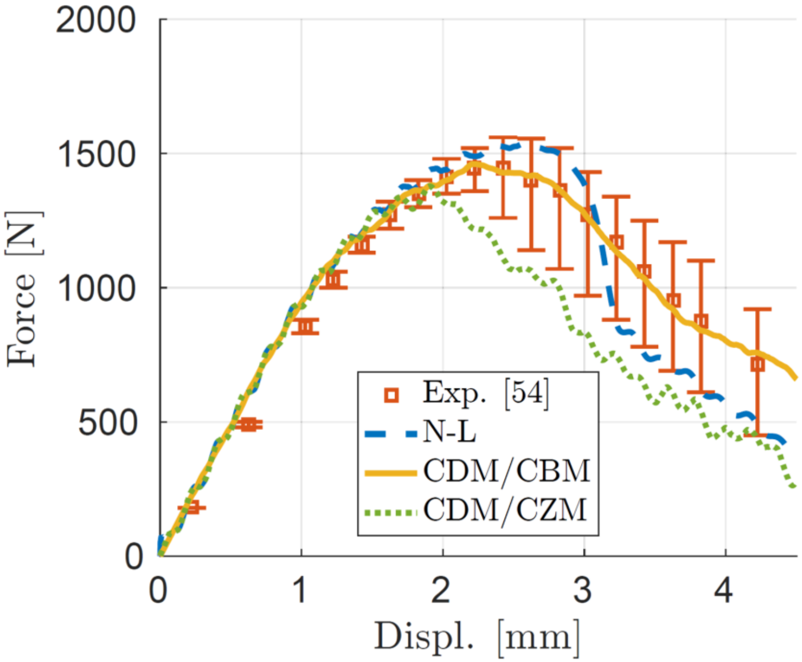
Non-local model | Cohesive band model | Force-displacement curve



Damage to crack transition for elastic damage – Proof of concept

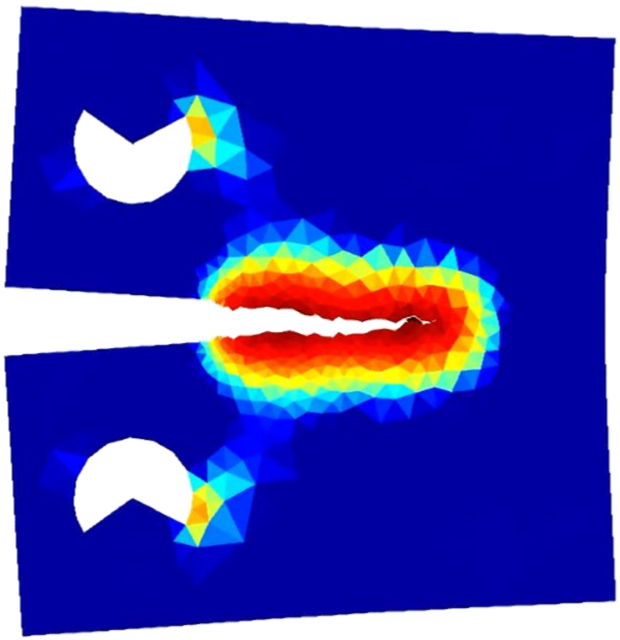
- Validation with Compact Tension Specimen [Geers 1997]
  - Better agreement with the cohesive band model than the cohesive zone model or the non-local model alone [Leclerc et al. 2018]

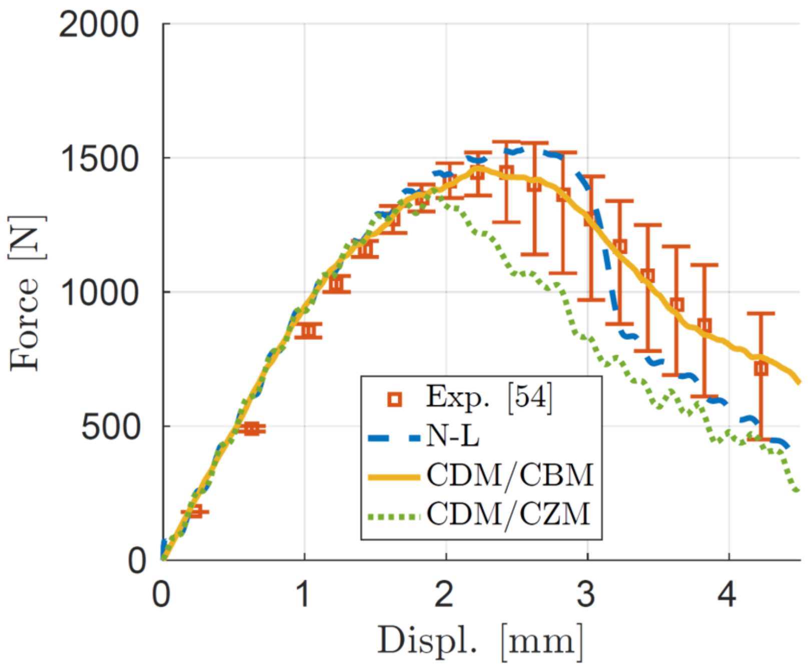


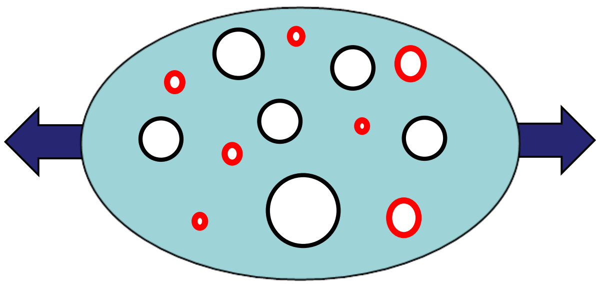


Damage to crack transition for elastic damage – Proof of concept

- Validation with Compact Tension Specimen [Geers 1997]
  - Better agreement with the cohesive band model than the cohesive zone model or the non-local model alone [Leclerc et al. 2018]







- Evolution of local porosity

$$\dot{f}_V = (1 - f_V)\mathrm{tr}(\mathbf{D}^{\mathrm{p}}) + \dot{f}_{\mathrm{nucl}} + \dot{f}_{\mathrm{shear}}$$

  – Voids nucleation $\dot{f}_{\mathrm{nucl}}$ modifies porosity growth rate
    - Linear strain-controlled growth

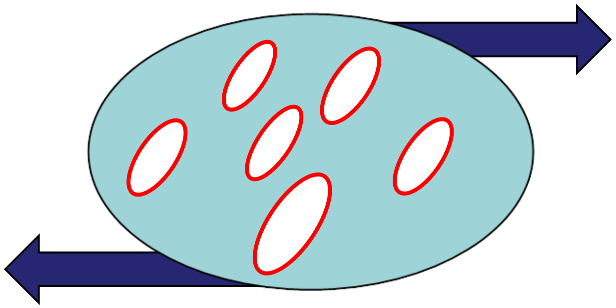
$$\dot{f}_{\mathrm{nucl}} = A_{\mathrm{N}}\dot{\hat{p}} \quad \text{with} \begin{cases} A_{\mathrm{N}} \neq 0 & \text{if } f_V > f_{\mathrm{N}} \\ A_{\mathrm{N}} = 0 & \text{if } f_V \leq f_{\mathrm{N}} \end{cases}$$

    - Gaussian strain-controlled growth

$$\dot{f}_{\mathrm{nucl}} = \frac{f_{\mathrm{N}}}{\sqrt{\{2\pi s_{\mathrm{N}}^2\}}} \exp\left(-\frac{(\hat{p} - \epsilon_{\mathrm{N}})^2}{2 s_{\mathrm{N}}^2}\right)\dot{\hat{p}}$$

    - where A_{N}, f_{N}, ϵ_{N}, s_{N} are material parameters

Porous plasticity – Voids nucleation



- Evolution of local porosity

$$\dot{f}_V = (1 - f_V)\mathrm{tr}(\mathbf{D}^\mathrm{p}) + \dot{f}_\mathrm{nucl} + \dot{f}_\mathrm{shear}$$

  – Shearing affect voids nucleation: $\dot{f}_\mathrm{shear}$

    - Includes Lode variable effect $\zeta(\boldsymbol{\tau}) = -\frac{27\,\det(\boldsymbol{\tau}^\mathrm{dev})}{2\,\tau_\mathrm{eq}^3}$

$$\dot{f}_\mathrm{shear} = f_V k_\mathrm{w}\left(1 - \zeta^2(\boldsymbol{\tau})\right)\frac{\boldsymbol{\tau}^\mathrm{dev} : \mathbf{D}^\mathrm{p}}{\tau_\mathrm{eq}}$$

    - where k_w is a material parameter

Integration algorithm

- Predictor-corrector procedure
  - Elastic predictor

$$\mathbf{F}^{\mathrm{e}^{\mathbf{pr}}} = \mathbf{F}\cdot\mathbf{F}_{\boldsymbol{n}}^{\mathrm{p}^{-1}}$$

  - Plastic corrector (radial return-like algorithm)
    - 3 equations
      - Consistency equation: $f\left(\tau_{\mathrm{eq}}, p; \tau_{\mathrm{Y}}, \boldsymbol{Z}(t'), \tilde{f}_V(t')\right) = 0$
      - Plastic flow rule: $\mathbf{D}^{\mathrm{p}} = \dot{\mathbf{F}}^{\mathrm{p}}\cdot\mathbf{F}^{\mathrm{p}^{-1}} = \dot{\gamma}\frac{\partial f}{\partial \boldsymbol{\tau}} = \dot{\hat{d}}\frac{\partial \tau_{\mathrm{eq}}}{\partial \boldsymbol{\tau}} + \dot{\hat{q}}\frac{\partial p}{\partial \boldsymbol{\tau}}$
      - Matrix plastic strain evolution: $\dot{\hat{p}} = \frac{\boldsymbol{\tau}:\mathbf{D}^{\mathrm{p}}}{(1-f_{V_0})\tau_{\mathrm{Y}}}$
    - 3 Unknowns $\Delta\hat{d}$, $\Delta\hat{q}$, $\Delta\hat{p}$
    - 3 linearized equations
      - Consistency equation: $f\left(\tau_{\mathrm{eq}}\left(\Delta\hat{d}\right), p(\Delta\hat{q}); \tau_{\mathrm{Y}}(\Delta\hat{p}), \boldsymbol{Z}\left(\Delta\hat{d}, \Delta\hat{q}, \Delta\hat{p}\right), \tilde{f}_V\right) = 0$
      - Plastic flow rule: $\Delta\hat{d}\frac{\partial f}{\partial p} - \Delta\hat{q}\frac{\partial f}{\partial \tau_{\mathrm{eq}}} = 0$
      - Matrix plastic strain evolution: $\left(1 - f_{V_0}\right)\tau_{\mathrm{Y}}\Delta\hat{p} = \tau_{\mathrm{eq}}\Delta\hat{d} + p\Delta\hat{q}$