University of Liège
Aerospace & Mechanical Engineering

Non-local damage to crack transition framework for ductile failure based on a cohesive band model

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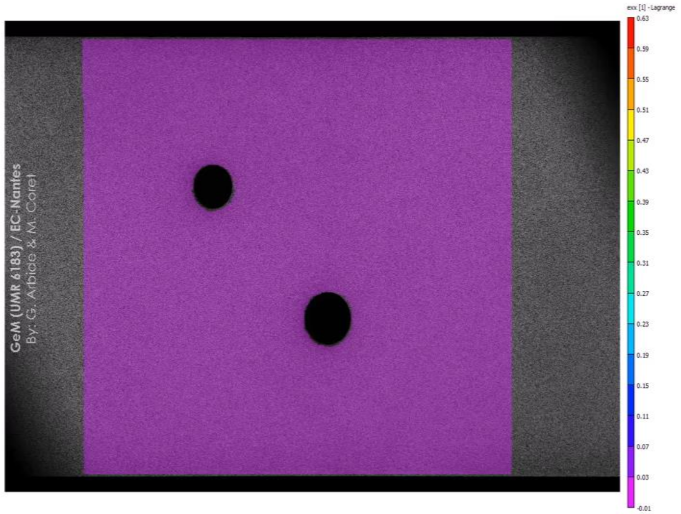
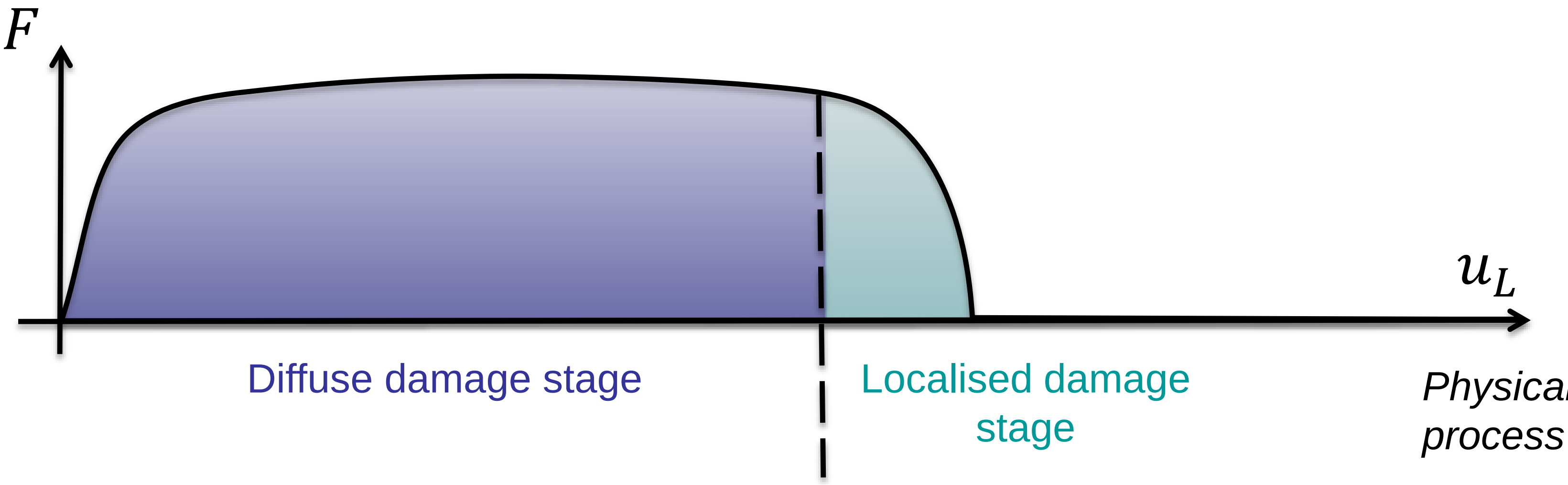
- Goal:
  - To capture the whole ductile failure process made of:
    - A diffuse stage
      - damage onset / nucleation, growth…

    followed by
    - A localised stage
      - damage coalescence
      - crack initiation and propagation
      - …

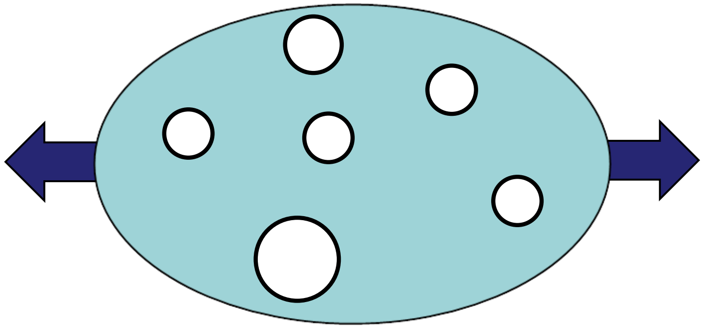


[http://radome.ec-nantes.fr/]

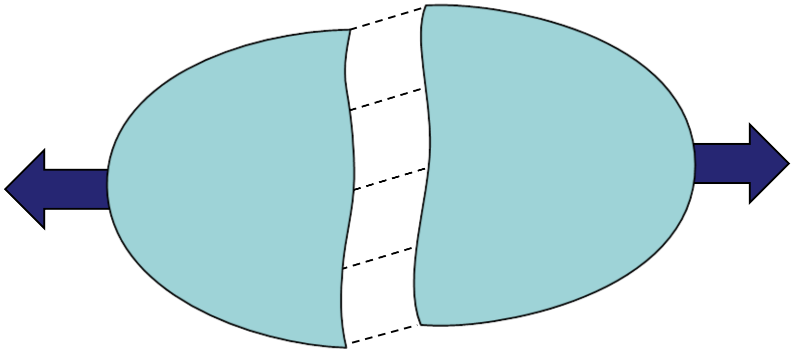


Modeling strategy

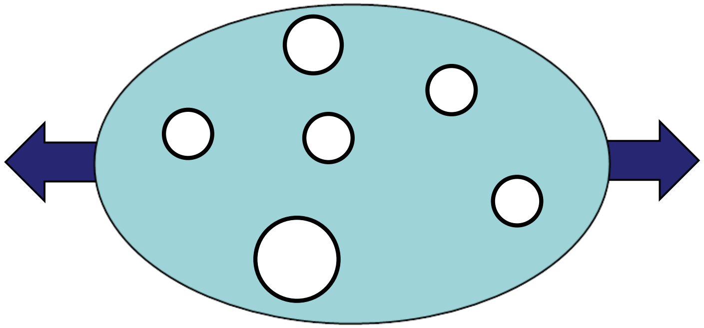
- Two principal approaches to describe material failure:
  - Continuous:
    - Damage models
      - Lemaitre-Chaboche,
      - Gurson,
      - …



  - Discontinuous:
    - Fracture mechanics
      - Cohesive zone,
      - XFEM
      - …

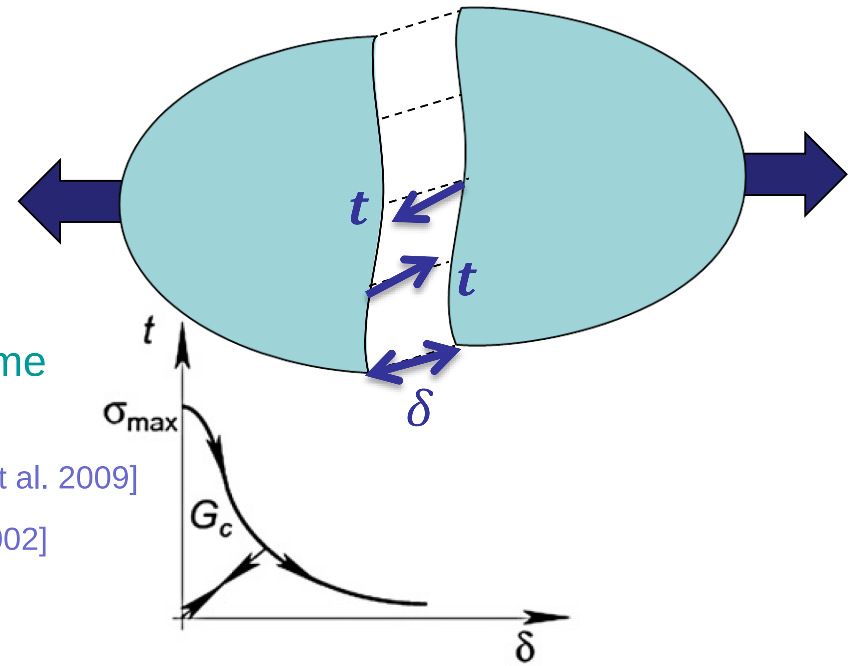


- Continuous approaches
  - Material properties degradation modelled by internal variables (= damage):
    - Lemaitre-Chaboche models,
    - Gurson-based models,
      - Porosity evolution
    - …



  - Continuous Damage Model (CDM) implementation:
    - Local form
      - Mesh-dependent
    - Non-local form needed [Peerlings et al. 1998]

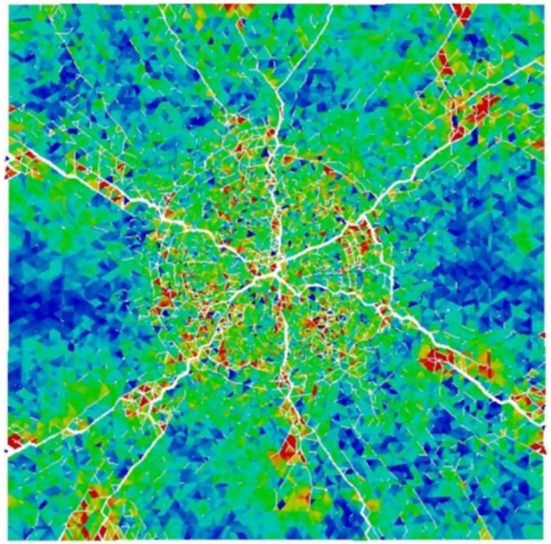
- Discontinuous approaches
  - Similar to fracture mechanics
  - One of the most used methods:
    - Cohesive Zone Model (CZM) modelling the crack tip behaviour inserted by:
      - Interface elements between two volume elements
      - Element enrichment (EFEM) [Armero et al. 2009]
      - Mesh enrichment (XFEM) [Moes et al. 2002]
    - …



  - Consistent and efficient hybrid framework for brittle fragmentation: [Radovitzky et al. 2011]
    - Extrinsic cohesive interface elements

      +

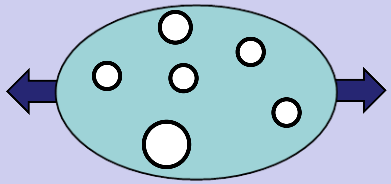
    - Discontinuous Galerkin (DG) framework (enables inter-elements discontinuities)



Modeling strategy

Continuous:

Continuous Damage Model (CDM)

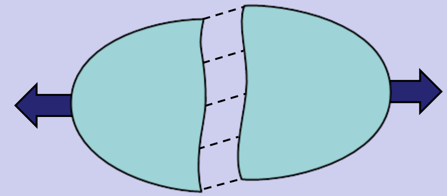


- + Capture the **diffuse damage stage**
- + Capture stress **triaxiality** and **Lode** variable effects

- − **Mesh dependency** without implicit non-local
- − **Numerical problems** with highly damaged elements
- − **Cannot represent cracks** without remeshing / element deletion at $D \rightarrow 1$ (loss of accuracy, mesh modification ...)
- − Crack initiation observed for lower damage values

Discontinuous:

Extrinsic Cohesive Zone Model (CZM)

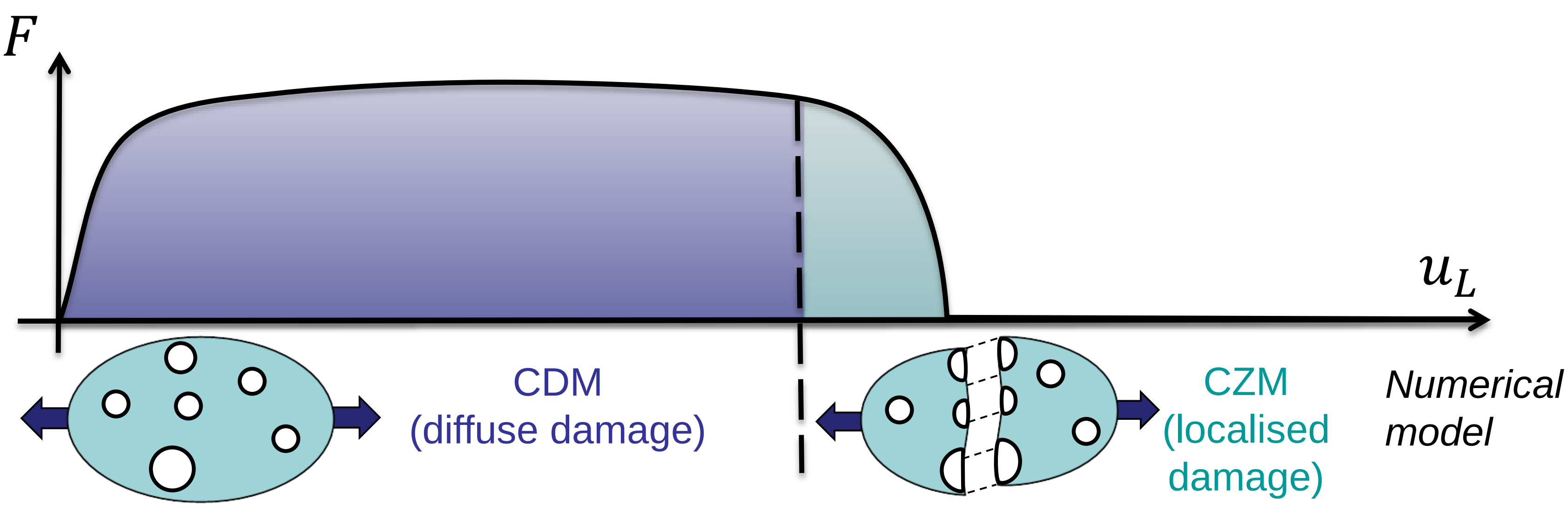


- + **Multiple crack initiation** and propagation naturally managed

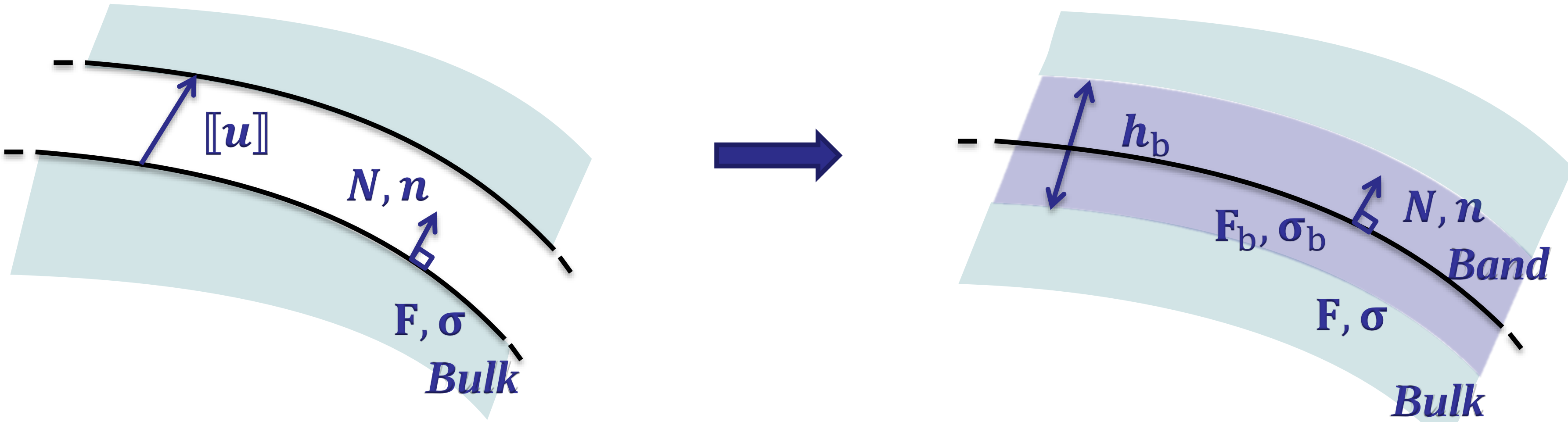
- − **Cannot capture diffuse damage**
- − **No triaxiality** effect
- − Currently valid for brittle / small scale yielding elasto-plastic materials

- Goal:
  - Simulation of the whole ductile failure process with accuracy
- Main idea:
  - Combination of 2 complementary methods in a single finite element framework:
    - continuous (non-local damage model)
      + transition to
    - discontinuous (cohesive model)

    → Damage to crack transition



- Discontinuous model here = Cohesive Band Model (CBM):
  - Hypothesis
    - In the last stage of failure, all damaging process occurs in an uniform thin band
  - Principles
    - Replacing the traction-separation law of a cohesive zone by the behaviour of a uniform band of given thickness h_{b} [Remmers et al. 2013]
  - Methodology [Leclerc et al. 2017]
    1. Compute a band strain tensor $\mathbf{F}_{\mathrm{b}} = \mathbf{F} + \frac{[\![\boldsymbol{u}]\!] \times \boldsymbol{N}}{h_{\mathrm{b}}} + \frac{1}{2}\nabla_T [\![\boldsymbol{u}]\!]$
    2. Compute then a band stress tensor $\boldsymbol{\sigma}_{\mathrm{b}}$
    3. Recover traction forces $\boldsymbol{t}([\![\boldsymbol{u}]\!], \mathbf{F}) = \boldsymbol{\sigma}_{\mathrm{b}}.\boldsymbol{n}$



Damage to crack transition – Principles

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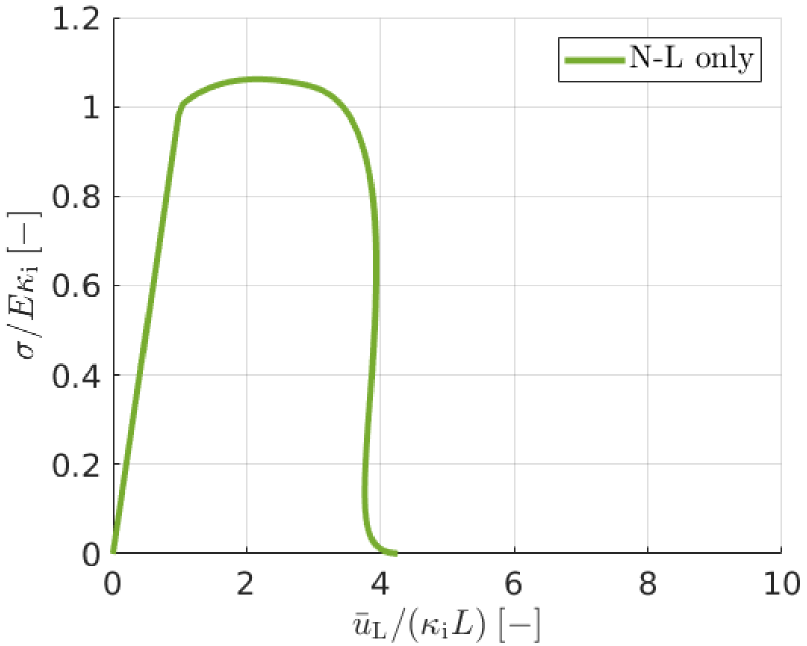
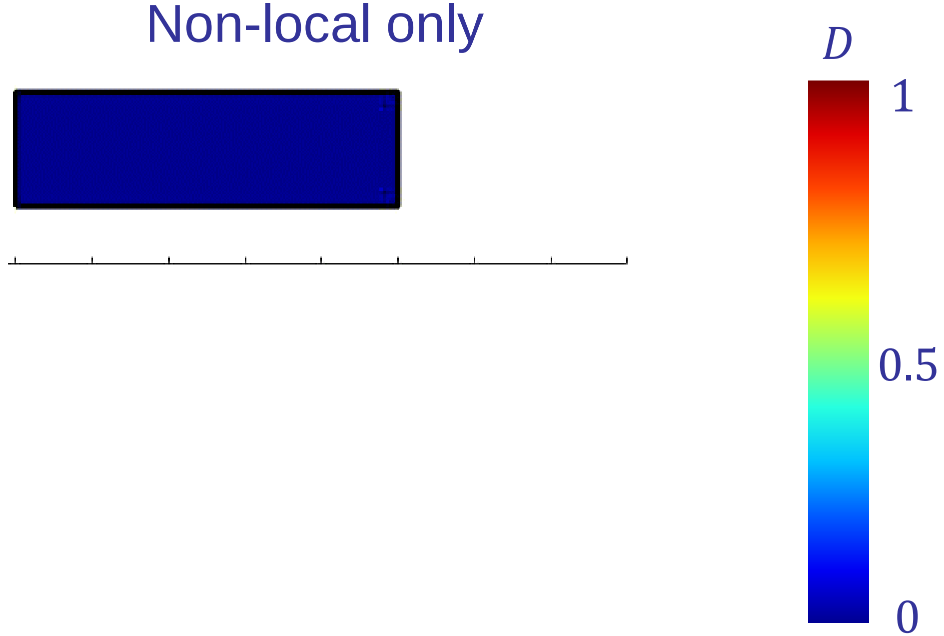
  - At crack insertion, framework only dependent on h_{b} (band thickness)
    - $h_{\mathrm{b}} \neq$ new material parameter
    - A priori determined with underlying non-local damage model to ensure energy consistency

- Influence of h_b (for a given l_c) on response in a 1D elastic case [Leclerc et al. 2017]:
  - Total dissipated energy Φ
    - Has to be chosen to conserve energy dissipation (physically based)



Non-local only



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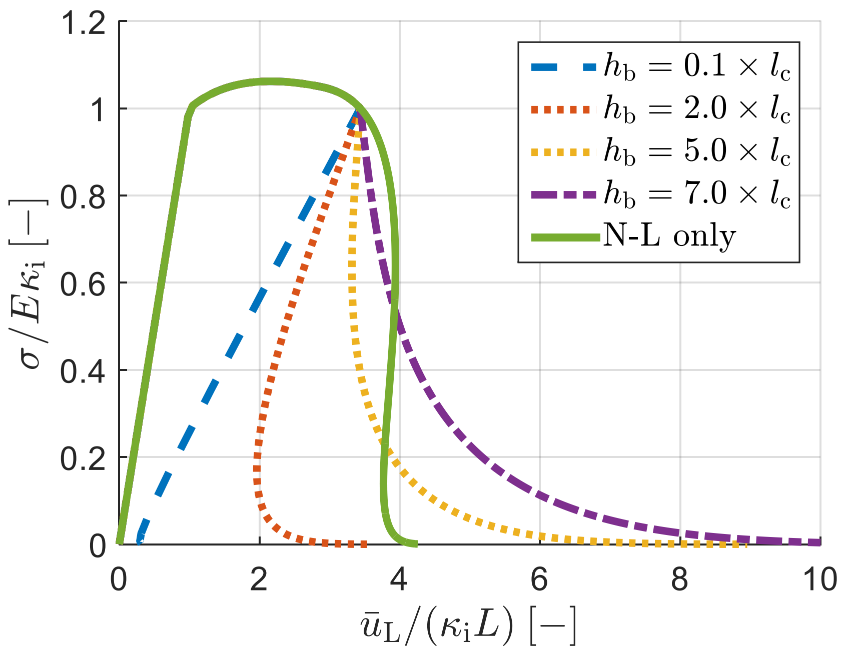
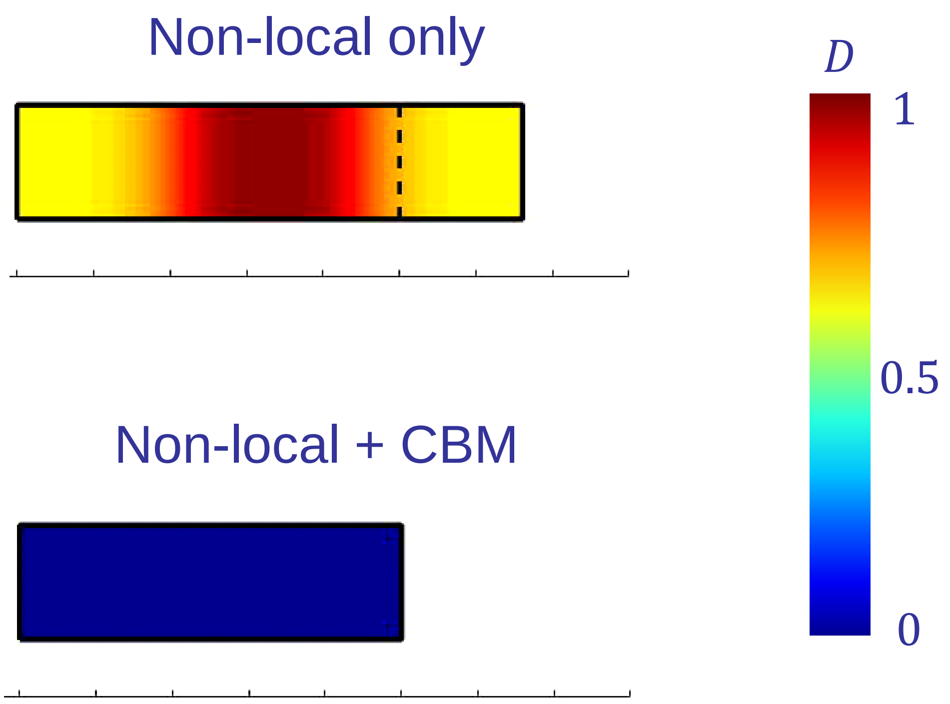
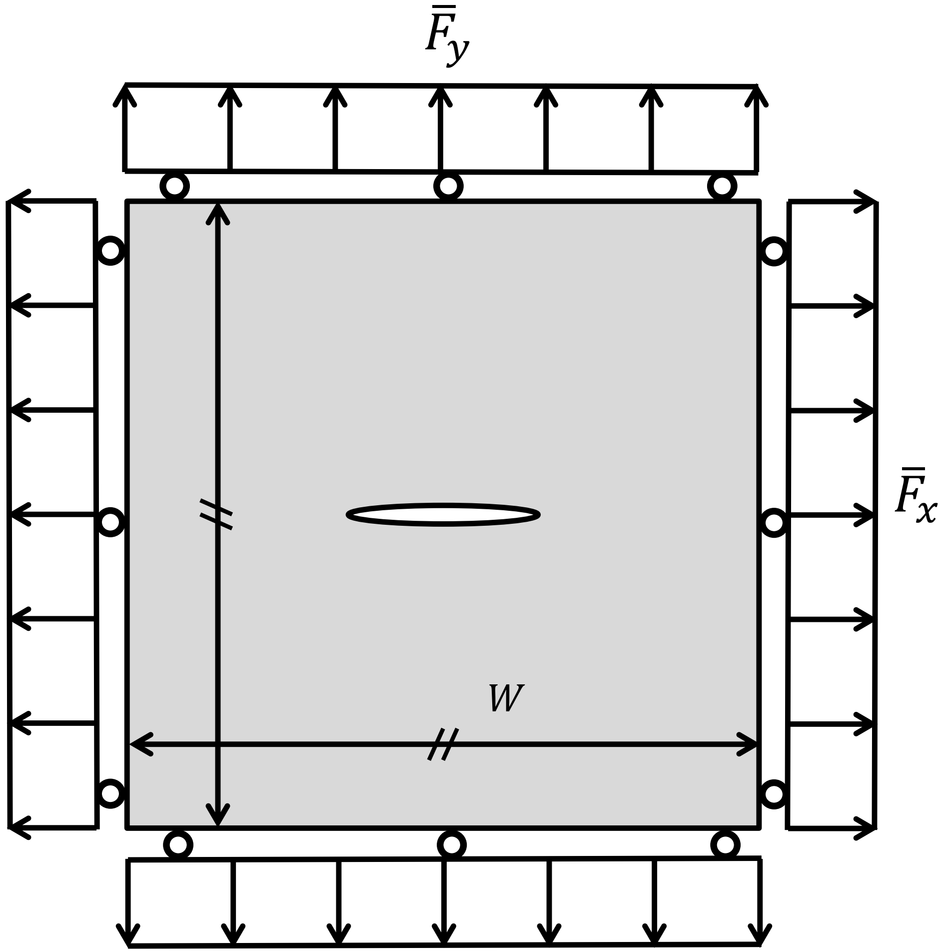


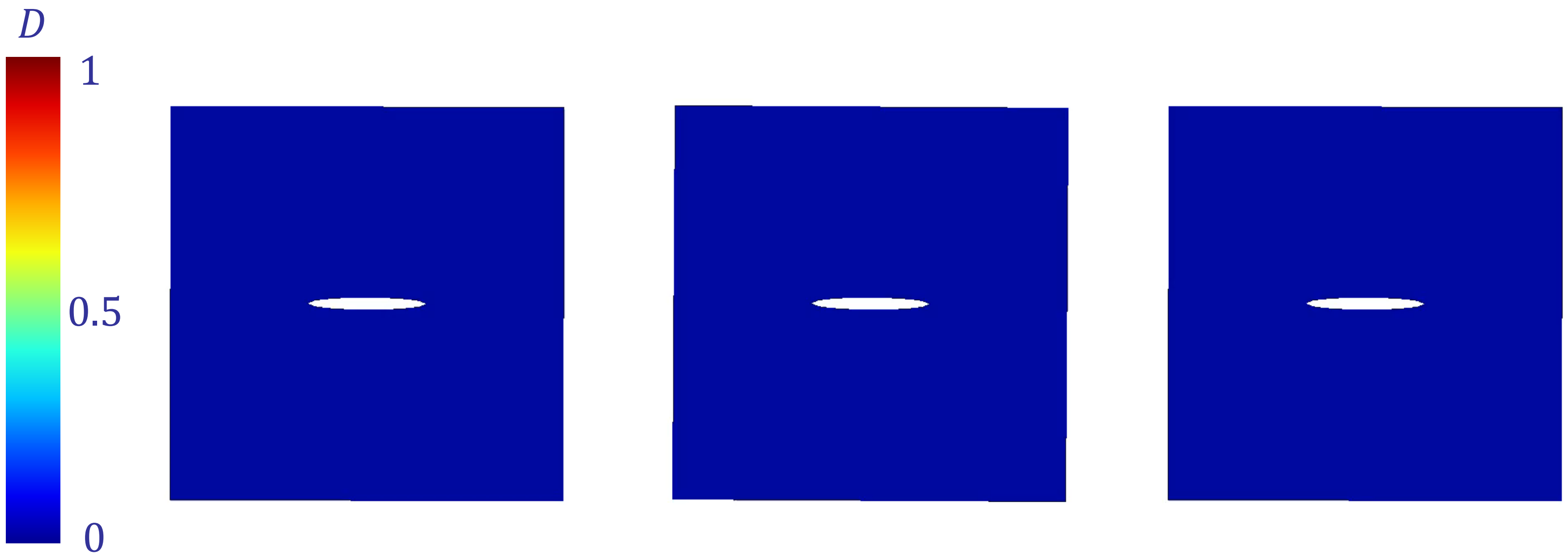


- 2D elastic plate with a defect
  - Biaxial loading
    - Ratio $\bar{F}_x / \bar{F}_y$ constant during a test
  - In plane strain
  - Path following method
  - Comparison between:
    - Pure non-local
    - Non-local + cohesive zone (CZM)
    - Non-local + cohesive band (CBM)



Damage to crack transition for elasticity – Proof of concept

- 2D plate in plane strain: $\bar{F}_x/\bar{F}_y$ = 0



Non-local only — Non-local + CZM — Non-local + CBM

no crack insertion

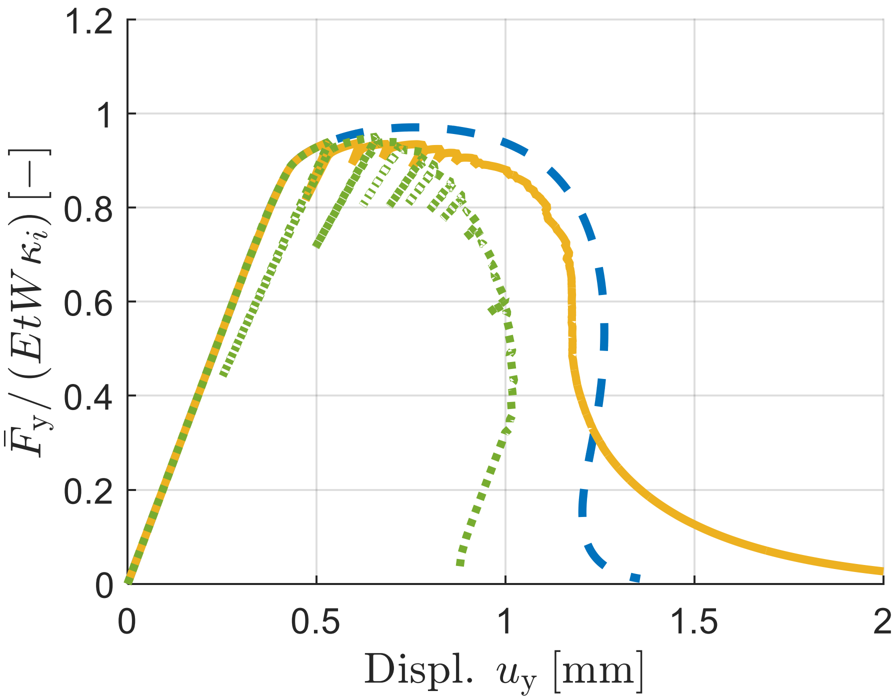
cohesive models calibrated on 1D bar under uniaxial stress state

Damage to crack transition for elasticity – Proof of concept

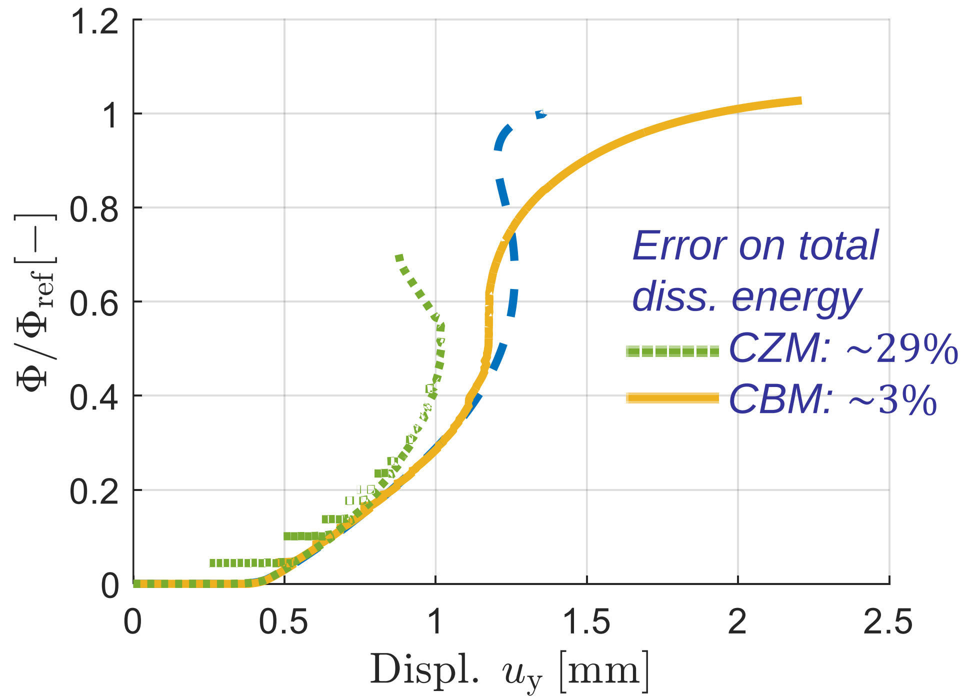
- 2D plate in plane strain: $\bar{F}_x / \bar{F}_y = 0$

Non-Local only
Non-Local + CZM
Non-Local + CBM

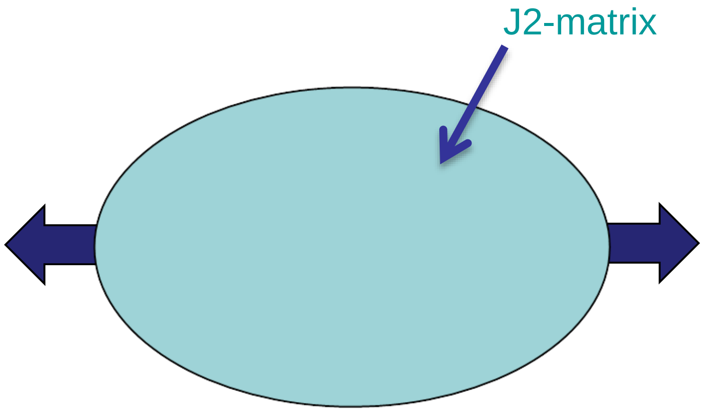
- Force evolution



- Dissipated energy evolution

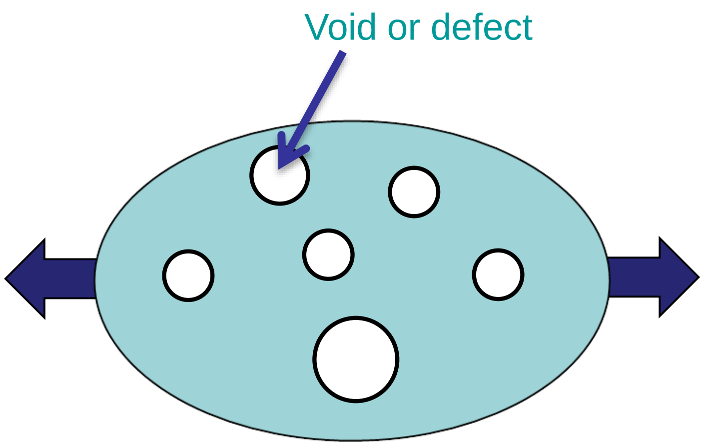


- Porous plasticity (or Gurson) approach
  - Assuming a J2-(visco-)plastic matrix



Application of the transition to plasticity

- Porous plasticity (or Gurson) approach
  - Assuming a J2-(visco-)plastic matrix
  - Including effects of void/defect or porosity on plastic behavior
    - Apparent macroscopic yield surface $f(\tau_{\text{eq}}, p, \tau_{\text{y}}, \boldsymbol{Z}) \leq 0$ due to microstructural state:



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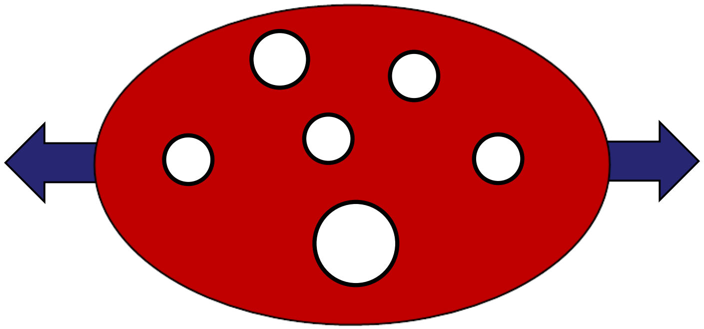
      - Competition between two deformation modes:
        - » Diffuse plastic flow spreads in the matrix
          - » Gurson-Tvergaard-Needleman (GTN) model



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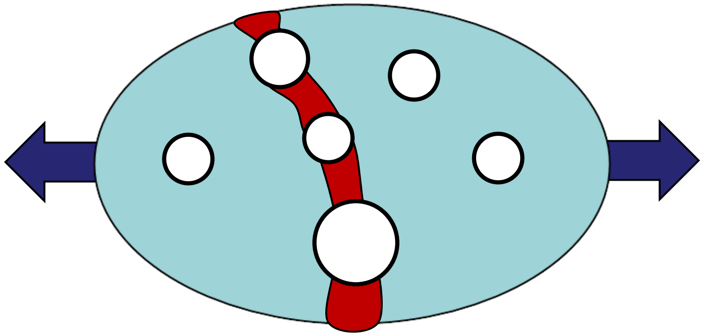
        - » Before failure: coalescence or localized plastic flow between voids
          - » GTN or Thomason models



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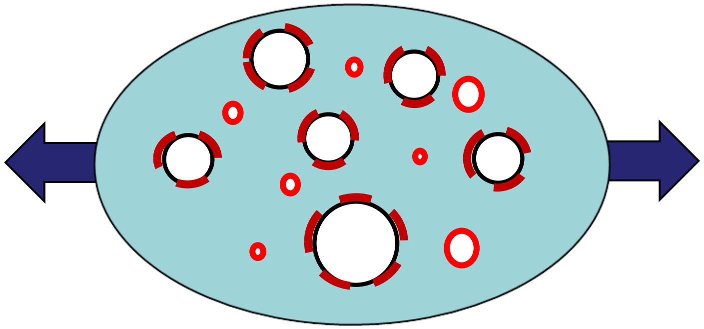
        - » Before failure: coalescence or localized plastic flow between voids
          - » GTN or Thomason models
  - Including evolution of microstructure during failure process
    - Void growth by diffuse plastic flow
    - Apparent growth by shearing
    - Nucleation / appearance of new voids
    - Void coalescence until failure



Non-local porous plasticity model

- Yield surface is considered in the co-rotational space
  - Non-local form: $f\left(\tau_{\text{eq}}, p, \tau_{\text{y}}, \boldsymbol{Z}, \tilde{\boldsymbol{Z}}\right) \leqslant 0$ with $\tilde{f}_{\text{V}} - l_{\text{c}}^2 \Delta \tilde{f}_{\text{V}} = f_{\text{V}}$
    - τ^{eq} is the von Mises equivalent Kirchhoff stress and p the pressure
    - $\tau_{\text{Y}} = \tau_{\text{Y}}(\hat{p}, \dot{\hat{p}})$ is the viscoplastic yield stress
    - f_{V} is the porosity and $\tilde{f}_{\text{V}}$, its non-local counterpart
    - $\boldsymbol{Z}$ is the vector of internal variables
    - l_{c} is the non-local length
  - Normal plastic flow $\mathbf{D}^{\text{p}}$
  - Microstructure evolution (spherical voids):
    - Eq. plastic strain of the matrix:

$$\dot{\hat{p}} = \frac{\boldsymbol{\tau} : \mathbf{D}^{\text{p}}}{(1 - f_{\text{V}0})\tau_{\text{Y}}}$$

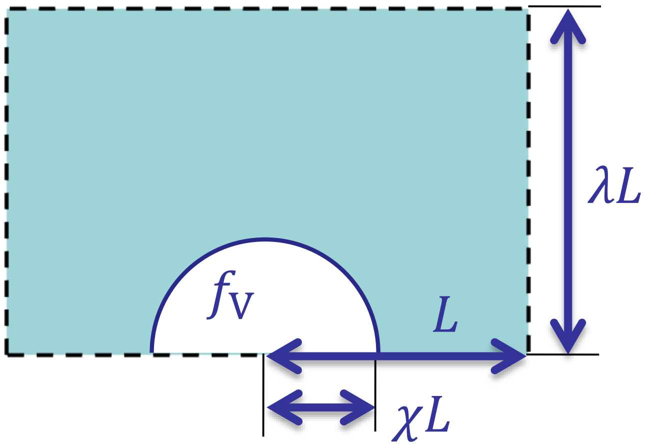
    - Porosity:

$$\dot{f}_{\text{V}} = (1 - f_{\text{V}})\text{tr}\,\mathbf{D}^{\text{p}} + \dot{f}_{\text{nucl}} + \dot{f}_{\text{shear}}$$

    - Ligament ratio:

$$\dot{\chi} = \dot{\chi}\left(\chi, \tilde{f}_V, \underbrace{\kappa, \lambda,}_{} \mathbf{Z}\right)$$

Microstructure parameters



Non-local porous plasticity – void growth and coalescence

- Gurson–Tvergaard–Needleman (GTN) model:

$$f = \frac{\tau_{\mathrm{eq}}^2}{\tau_{\mathrm{Y}}^2} + 2q_1 \tilde{f}_{\mathrm{V}} \cosh\left(\frac{q_2 p}{2\tau_{\mathrm{Y}}}\right) - 1 - q_3^2 \tilde{f}_{\mathrm{V}}^2 \leqslant 0$$

$$\tilde{f}_{\mathrm{V}} - l_{\mathrm{c}}^2 \Delta \tilde{f}_{\mathrm{V}} = f_{\mathrm{V}}$$

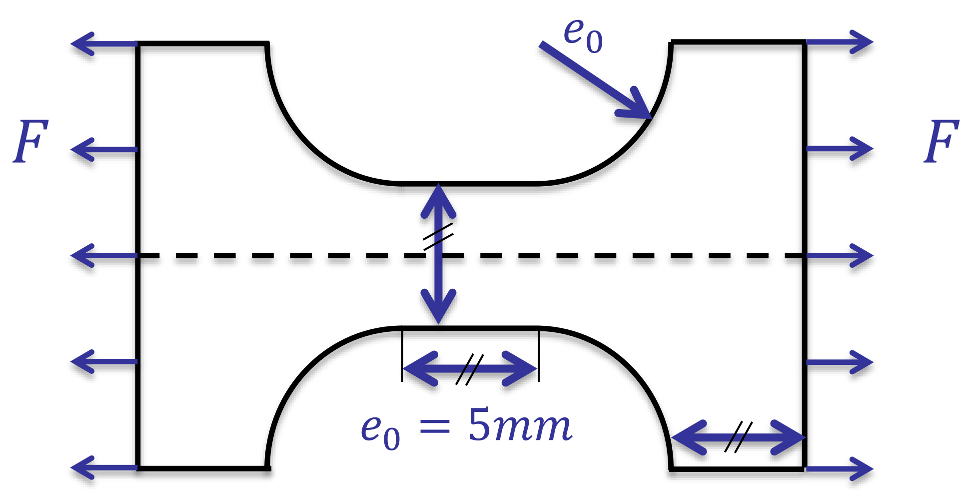
  – Phenomenological coalescence model:
    - replace $\tilde{f}_{\mathrm{V}}$ by an effective value $\tilde{f}_{\mathrm{V}}^*$:

$$\tilde{f}_{\mathrm{V}}^{\star} = \begin{cases} \tilde{f}_{\mathrm{V}} & \text{if } \tilde{f}_{\mathrm{V}} \leq f_{\mathrm{C}} \\ f_{\mathrm{C}} + R\left(\tilde{f}_{\mathrm{V}} - f_{\mathrm{C}}\right) & \text{if } \tilde{f}_{\mathrm{V}} > f_{\mathrm{C}} \end{cases}$$

    - f_C is determined by Thomason criterion [Benzerga2014]:

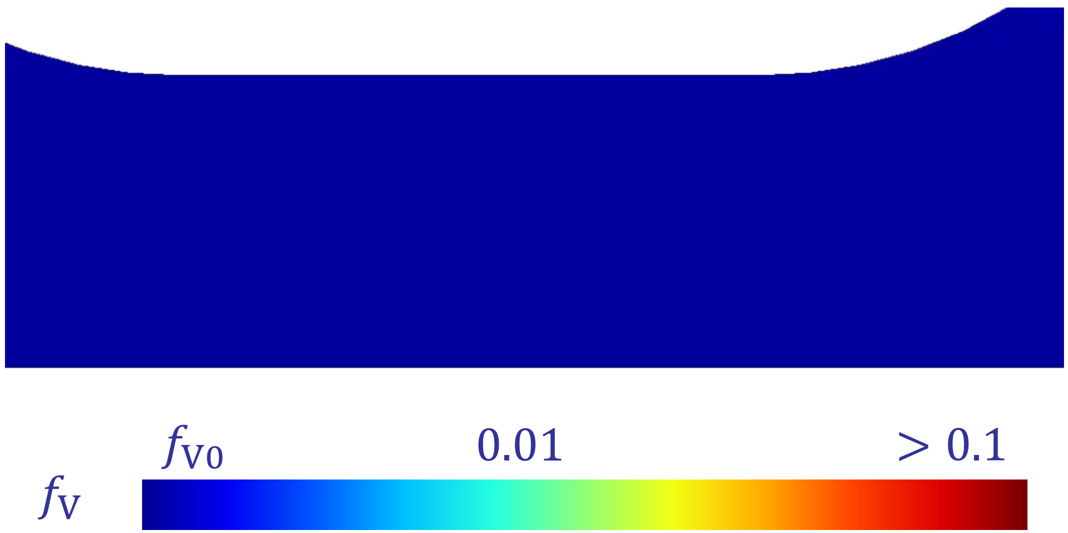
$$\max \mathrm{eig}\left(\boldsymbol{\tau}\right) - C_{\mathrm{T}}^{\mathrm{f}}\left(\chi\right) \tau_{\mathrm{Y}} > 0$$

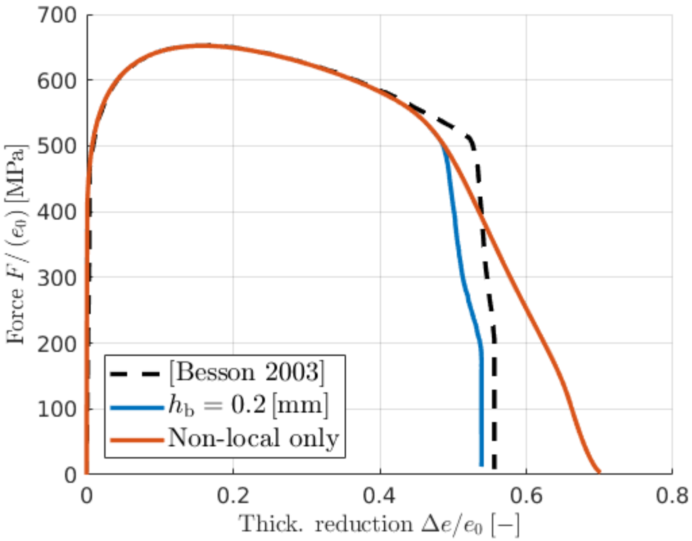
- Damage to crack transition for porous plasticity
  - Plane strain specimen [Besson et al. 2003]
    - Only an half is modelled



- Damage to crack transition for porous plasticity
  - Discontinuous Galerkin formulation + cohesive band model [Leclerc et al. 2017]
  - Coalescence is detected at interfaces of elements:

$$\max \mathrm{eig}\left(\boldsymbol{\tau}\right) - C_{\mathrm{T}}^{\mathrm{f}}\left(\chi\right)\tau_{\mathrm{Y}} > 0 \quad \Rightarrow \quad \boldsymbol{n}\cdot\boldsymbol{\tau}\cdot\boldsymbol{n} - C_{\mathrm{T}}^{\mathrm{f}}\left(\chi\right)\tau_{\mathrm{Y}} > 0$$



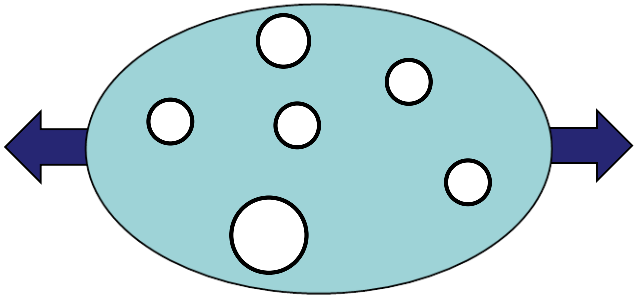


Conclusion

- Objective:
  - Simulation of material degradation and crack initiation / propagation during the ductile failure process

- Upcoming tasks:
  - Enrichment of nucleation model and coalescence model
  - Calibration of the band thickness
  - Validation/Calibration with literature/experimental tests

Thank you for your attention



- Non-local model
  - Principles
    - variable ξ ➔ non-local / “averaged” counterpart $\tilde{\xi}$
  - Formulation
    - Integral form [Bažant 1988]

$$\tilde{\xi}(\boldsymbol{x}) = \frac{1}{V}\int_V W(\boldsymbol{x}-\boldsymbol{y})\,\xi(\boldsymbol{y})\,dV$$

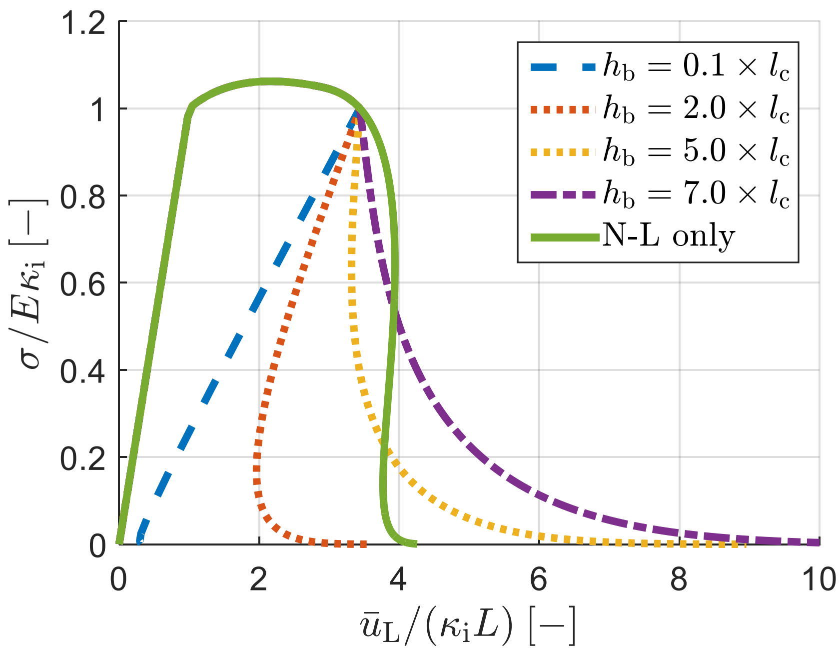
      » not practical for complex geometries

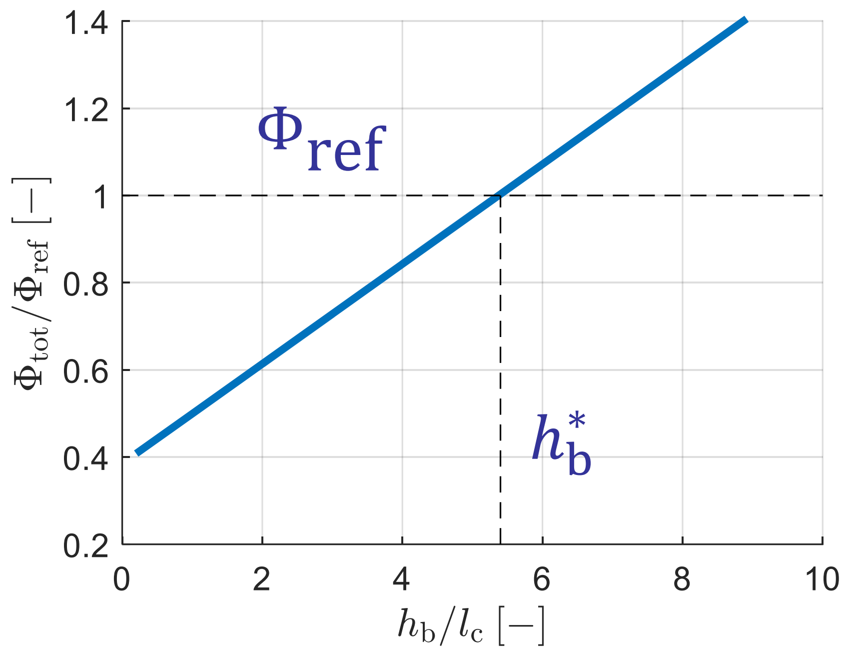
    - Differential forms [Peerlings et al. 2001]
      - Explicit formulation / gradient-enhanced formulation: $\tilde{\xi}(\boldsymbol{x}) = f\left(\xi, \nabla\xi, \nabla^2\xi, ...\right)$
        » does not remove mesh-dependency
      - Implicit formulation: $\tilde{\xi}(\boldsymbol{x}) = f\left(\xi, \nabla\tilde{\xi}, \nabla^2\tilde{\xi}, ...\right)$

$$\tilde{\xi}(\boldsymbol{x}) - l_c^2\Delta\tilde{\xi}(\boldsymbol{x}) = \xi(\boldsymbol{x})$$

        » removes mesh-dependency but one added unknown field

- Influence of h_b (for a given l_c) on response in a 1D elastic case [Leclerc et al. 2017]:
  - Total dissipated energy Φ = linear with h_b:
    - Has to be chosen to conserve energy dissipation (physically based)





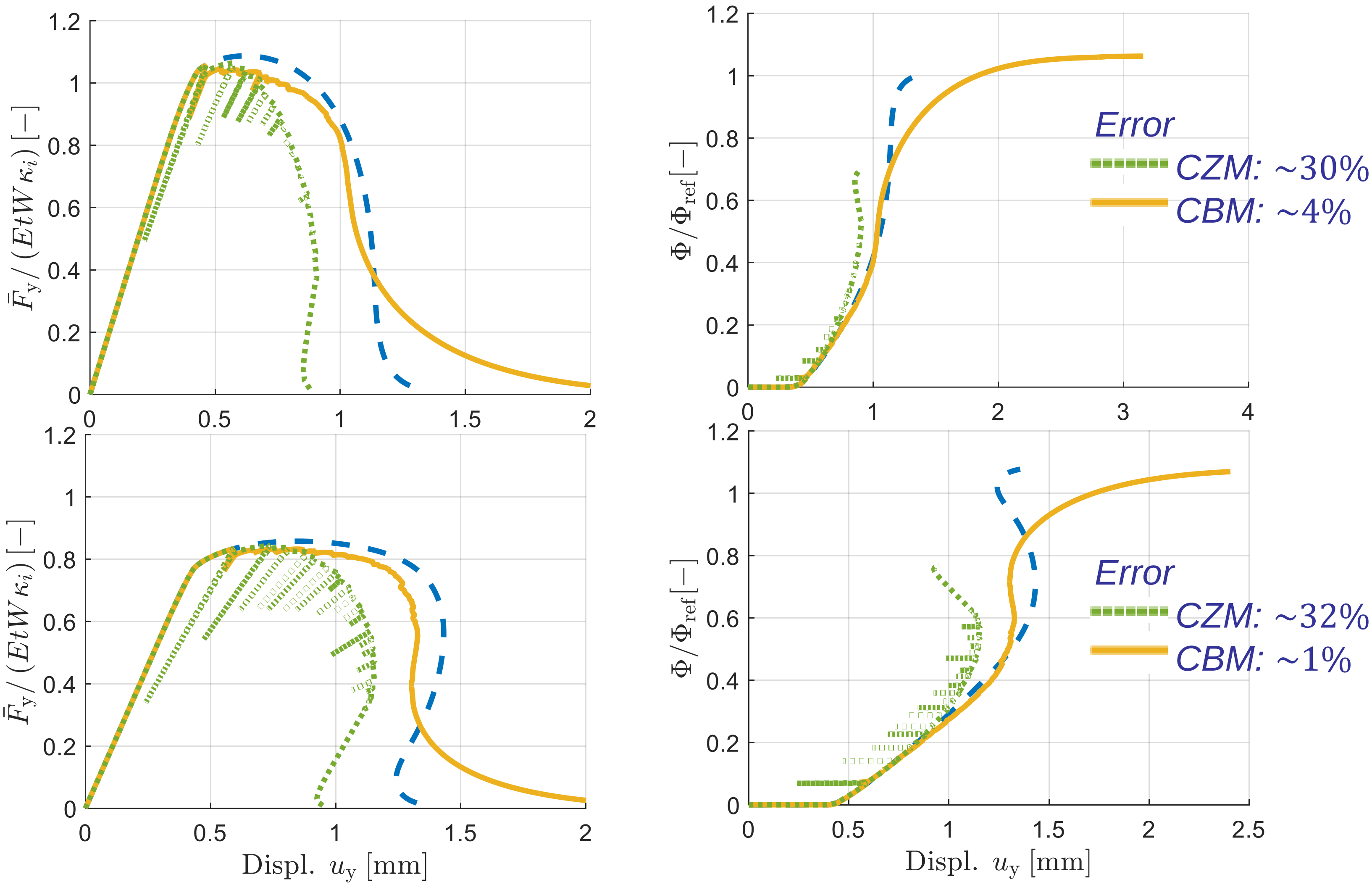
Damage to crack transition for elasticity – Proof of concept

- 2D plate in plane strain:
  - Same trends with ≠ force ratio

Non-Local only
Non-Local + CZM
Non-Local + CBM

$$\frac{\bar{F}_x}{\bar{F}_y} = +0.5$$

$$\frac{\bar{F}_x}{\bar{F}_y} = -0.5$$

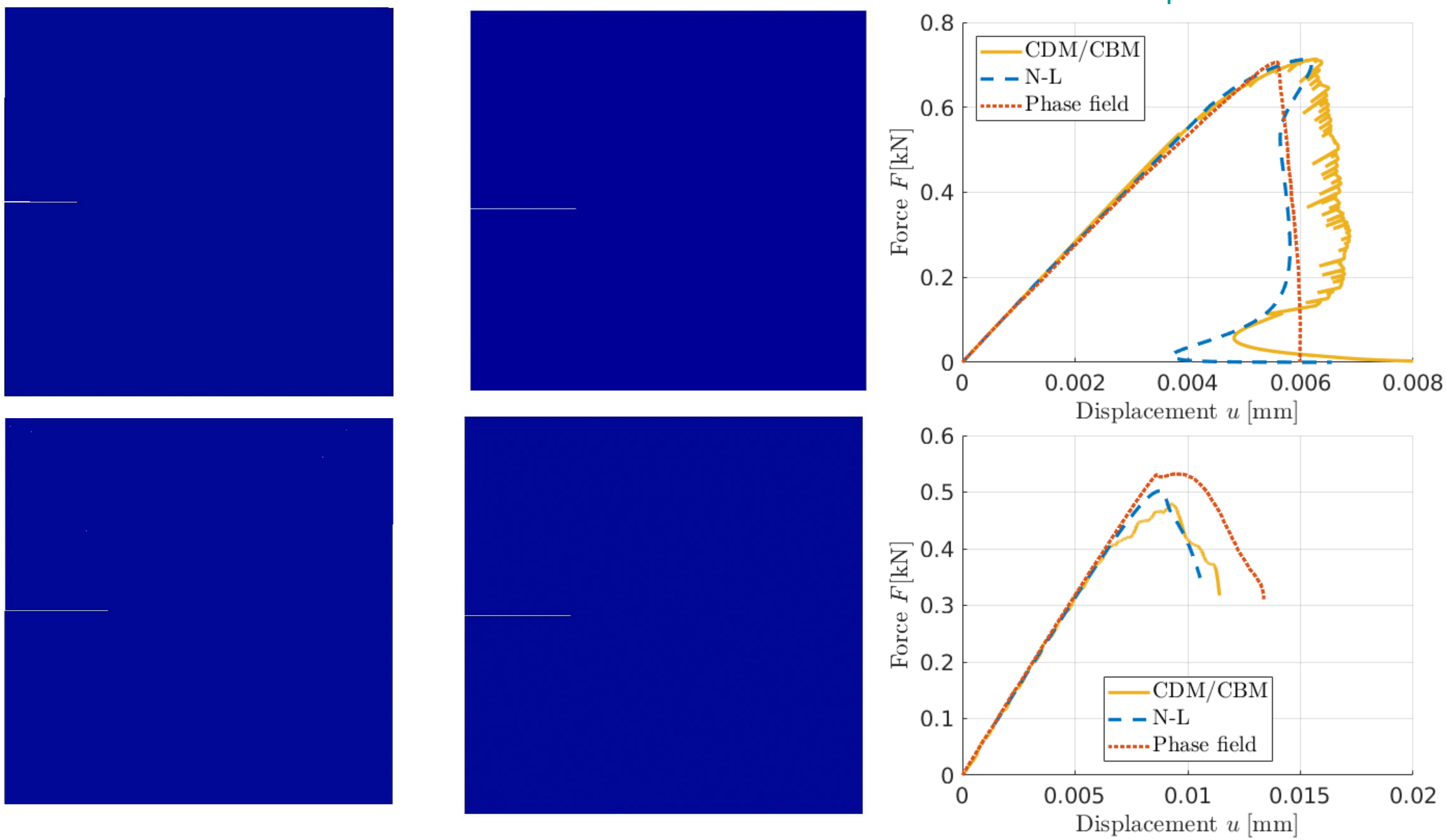


- Comparison with phase field
  - Single edge notched specimen [Miehe et al. 2010]:
    - Calibration of damage and CBM parameters with 1D case [Leclerc et al. 2017]:

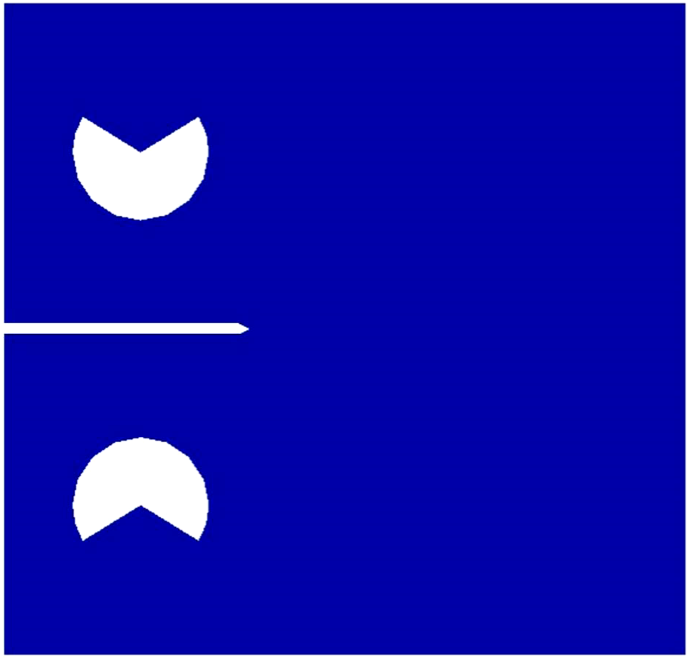
Non-local model | Cohesive band model | Force-displacement curve

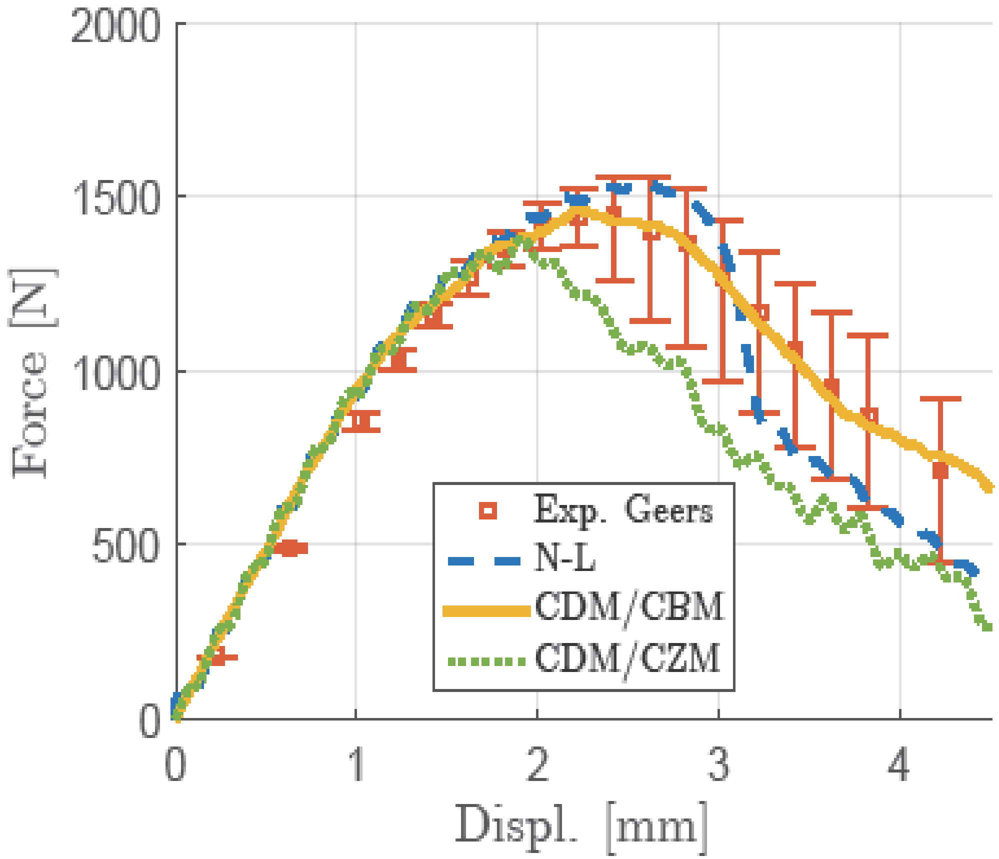
Tension test

Shearing test



- Validation with Compact Tension Specimen [Geers 1997]:
  - Better agreement with the cohesive band model than the cohesive zone model or the non-local model alone [Leclerc et al. 2017]





Porous plasticity – principles (2)

- Yield surface is considered in the co-rotational space
  - Local form: $f\left(\tau_{\mathrm{eq}}, p, \tau_{\mathrm{y}}, \boldsymbol{Z}\right) \leqslant 0$

    - τ^{eq} is the von Mises equivalent Kirchhoff stress and $p,$ the pressure
    - $\tau_{\mathrm{Y}} = \tau_{\mathrm{Y}}(\hat{p}, \dot{\hat{p}})$ is the viscoplastic yield stress
    - $\boldsymbol{Z}$ is the vector of internal variables

    - Normal plastic flow decomposition:

$$\mathbf{D}^{\mathrm{p}} = \dot{\mathbf{F}}^{\mathrm{p}} \cdot \mathbf{F}^{\mathrm{p}-1} = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\tau}} = \dot{\hat{d}}\frac{\partial \tau_{\mathrm{eq}}}{\partial \boldsymbol{\tau}} + \dot{\hat{q}}\frac{\partial p}{\partial \boldsymbol{\tau}}$$

    - Plastic deformation of the matrix from the equivalence of plastic energy:

$$(1 - f_{\mathrm{V0}})\tau_{\mathrm{Y}}\dot{\hat{p}} = \boldsymbol{\tau} : \mathbf{D}^{\mathrm{p}}$$

    - Microstructure evolution (porosity f_V and ligament ratio χ):

$$\dot{f}_{\mathrm{V}} = (1 - f_{\mathrm{V}})\operatorname{tr}\mathbf{D}^{\mathrm{p}} + \dot{f}_{\mathrm{nucl}} + \dot{f}_{\mathrm{shear}}$$

$$\dot{\chi} = \dot{\chi}\left(\chi, f_V, \mathbf{Z}\right)$$

    - Drawbacks
      - The numerical results change with the size and the direction of mesh

- Evolution of local porosity

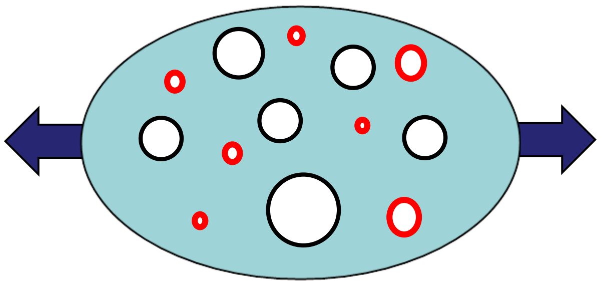
$$\dot{f}_\mathrm{V} = (1 - f_\mathrm{V})\mathrm{tr}\,\mathbf{D}^\mathrm{p} + \dot{f}_\mathrm{nucl} + \dot{f}_\mathrm{shear}$$

  – Void nucleation $\dot{f}_\mathrm{nucl}$
    - Modify porosity growth rate (where A_N, f_N, ϵ_N, s_N are material parameters)
      – Linear strain-controlled growth

$$\dot{f}_\mathrm{nucl} = A_\mathrm{N}\dot{\hat{p}} \quad \text{with} \quad A_\mathrm{N}\begin{cases} \neq 0 & \text{if } f_\mathrm{V} > f_\mathrm{N}, \\ = 0 & \text{otherwise.} \end{cases}$$

      – Gaussian strain-controlled growth

$$\dot{f}_\mathrm{nucl} = \frac{f_\mathrm{N}}{\sqrt{2\pi s_\mathrm{N}^2}} \exp\left(-\frac{(\hat{p} - \epsilon_\mathrm{N})^2}{2 s_\mathrm{N}^2}\right)\dot{\hat{p}}$$

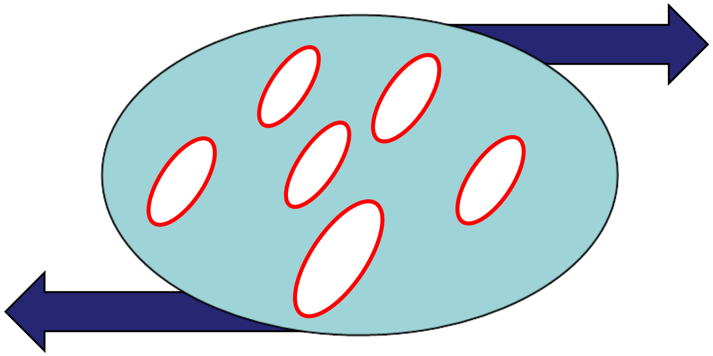


- Evolution of local porosity

$$\dot{f}_{\mathrm{V}} = (1 - f_{\mathrm{V}})\mathrm{tr}\,\mathbf{D}^{\mathrm{p}} + \dot{f}_{\mathrm{nucl}} + \dot{f}_{\mathrm{shear}}$$

  - Shear-induced voids growth $\dot{f}_{\mathrm{shear}}$
    - Includes Lode variable effect (where k_{w} is a material parameter)

$$\dot{f}_{\mathrm{shear}} = f_{\mathrm{V}} k_{\mathrm{w}} \omega\left(\boldsymbol{\tau}\right) \frac{\boldsymbol{\tau}^{\mathrm{dev}} : \mathbf{D}^{\mathrm{p}}}{\tau^{\mathrm{eq}}}$$



Ductile non-local damage model

- Hyperelastic formulation:
  - Multiplicative decomposition of deformation gradient in elastic and plastic parts: $\mathbf{F} = \mathbf{F}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{p}}$
  - Logarithmic elastic potential ψ:

$$\psi(\mathbf{C}^{\mathrm{e}}) = \frac{K}{2} \ln^2 J^{\mathrm{e}} + \frac{G}{4} (\ln \mathbf{C}^{\mathrm{e}})^{\mathrm{dev}} : (\ln \mathbf{C}^{\mathrm{e}})^{\mathrm{dev}}$$

with $\mathbf{C}^{\mathrm{e}} = \mathbf{F}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{e}\,\mathrm{T}}$ and $J^{\mathrm{e}} = \det \mathbf{F}^{\mathrm{e}}$

  - Stress tensor definition
    - PK1 stress: $\mathbf{P} = 2\mathbf{F} \cdot \dfrac{\partial \psi}{\partial \mathbf{C}}$
    - Kirchhoff stresses: $\boldsymbol{\kappa} = \mathbf{P} \cdot \mathbf{F}^{\mathrm{T}}$ or again:

$$\boldsymbol{\kappa} = p\mathbf{I} + (\boldsymbol{\kappa})^{\mathrm{dev}} = p\mathbf{I} + \mathbf{F}^{\mathrm{e}} \cdot \left[\mathbf{C}^{\mathrm{e}-1} \cdot (\boldsymbol{\tau})^{\mathrm{dev}} \right] \cdot \mathbf{F}^{\mathrm{e}T}$$

$$\boldsymbol{\tau} = p\mathbf{I} + (\boldsymbol{\tau})^{\mathrm{dev}} = p\mathbf{I} + 2G \left(\ln \sqrt{\mathbf{C}^{\mathrm{e}}} \right)^{\mathrm{dev}}$$

- Predictor-corrector procedure
  - Elastic predictor
  - Plastic corrector (radial return-like algorithm)
    - 3 Unknowns $\Delta\hat{d}$, $\Delta\hat{q}$, $\Delta\hat{p}$
    - 3 Equations
      - Consistency equation:

$$f\left(\tau_{\mathrm{eq}}(\Delta\hat{d}), p(\Delta\hat{q}), \tau_{\mathrm{Y}}(\Delta\hat{p}), \boldsymbol{Z}(\Delta\hat{d}, \Delta\hat{q}, \Delta\hat{p}), \tilde{\boldsymbol{Z}}\right) = 0$$

      - Plastic flow rule:

$$\Delta\hat{d}\frac{\partial f}{\partial p} - \Delta\hat{q}\frac{\partial f}{\partial \tau_{\mathrm{eq}}} = 0$$

      - Matrix plastic strain evolution:

$$(1 - f_{\mathrm{V0}})\,\tau_{\mathrm{Y}}\Delta\hat{p} = \tau_{\mathrm{eq}}\Delta\hat{d} + p\Delta\hat{q}$$