Non-local damage to crack transition framework for ductile failure based on a cohesive band model

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• **Goal:**
  - To capture the whole ductile failure process made of:
    • A diffuse stage
      - damage onset / nucleation, growth… followed by
    • A localised stage
      - damage coalescence
      - crack initiation and propagation
      - …
Two principal approaches to describe material failure:

- Continuous:
  - Damage models
    - Lemaitre-Chaboche,
    - Gurson,
    - ...

- Discontinuous:
  - Fracture mechanics
    - Cohesive zone,
    - XFEM
    - ...

Modeling strategy
• **Continuous approaches**
  – Material properties degradation modelled by internal variables ($\varepsilon = \text{damage}$):
    • Lemaitre-Chaboche models,
    • Gurson-based models,
      – Porosity evolution
    • ...

  – **Continuous Damage Model (CDM) implementation:**
    • Local form
      – Mesh-dependent
    • Non-local form needed [Peerlings et al. 1998]
• **Discontinuous approaches**
  
  – Similar to fracture mechanics
  – One of the most used methods:
    • Cohesive Zone Model (CZM) modelling the crack tip behaviour inserted by:
      – Interface elements between two volume elements
      – Element enrichment (EFEM) \cite{Armero} [Armero et al. 2009]
      – Mesh enrichment (XFEM) \cite{Moes} [Moes et al. 2002]
    • …

  – Consistent and efficient hybrid framework for brittle fragmentation: \cite{Radovitzky}
    • Extrinsic cohesive interface elements
    +
    • Discontinuous Galerkin (DG) framework (enables inter-elements discontinuities)
### Modeling strategy

<table>
<thead>
<tr>
<th>Continuous: Continuous Damage Model (CDM)</th>
<th>Discontinuous: Extrinsic Cohesive Zone Model (CZM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ Capture the <strong>diffuse damage stage</strong></td>
<td>+ <strong>Multiple crack initiation</strong> and propagation naturally managed</td>
</tr>
<tr>
<td>+ Capture stress <strong>triaxiality</strong> and <strong>Lode variable effects</strong></td>
<td></td>
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<tr>
<td>- Mesh dependency without implicit non-local</td>
<td>- <strong>Cannot capture diffuse damage</strong></td>
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<tr>
<td>- <strong>Numerical problems</strong> with highly damaged elements</td>
<td>- <strong>No triaxiality</strong> effect</td>
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<tr>
<td>- <strong>Cannot represent cracks</strong> without remeshing / element deletion at $D \rightarrow 1$ (loss of accuracy, mesh modification ...)</td>
<td>- Currently valid for brittle / small scale yielding elasto-plastic materials</td>
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<td>- Crack initiation observed for lower damage values</td>
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**Continuous Damage Model (CDM)**

**Discontinuous: Extrinsic Cohesive Zone Model (CZM)**
Goals of research

- **Goal:**
  - Simulation of the whole ductile failure process with accuracy

- **Main idea:**
  - Combination of 2 complementary methods in a single finite element framework:
    - continuous (non-local damage model)
      + transition to
    - discontinuous (cohesive model)

![Diagram showing the transition between CDM and CZM models](Image)
Discontinuous model here = Cohesive Band Model (CBM):

- **Hypothesis**
  - In the last stage of failure, all damaging process occurs in an uniform thin band

- **Principles**
  - Replacing the traction-separation law of a cohesive zone by the behaviour of a uniform band of given thickness $h_b$ [Remmers et al. 2013]

- **Methodology [Leclerc et al. 2017]**
  1. Compute a band strain tensor $F_b = F + \frac{[u] \times N}{h_b} + \frac{1}{2} \nabla T [u]$
  2. Compute then a band stress tensor $\sigma_b$
  3. Recover traction forces $t([u], F) = \sigma_b \cdot n$
• **Discontinuous model here = Cohesive Band Model (CBM):**
  
  – **Hypothesis**
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  – **Principles**
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  – **Methodology [Leclerc et al. 2017]**
    1. Compute a band strain tensor $F_b = F + \left[ \frac{u}{h_b} \right] \times \frac{N}{h_b} + \frac{1}{2} \nabla_T [u]$
    2. Compute then a band stress tensor $\sigma_b$
    3. Recover traction forces $t([u], F) = \sigma_b \cdot n$
  
  – At crack insertion, framework only dependent on $h_b$ (band thickness)
    - $h_b \neq$ new material parameter
    - A priori determined with underlying non-local damage model to ensure energy consistency
Influence of $h_b$ (for a given $l_c$) on response in a 1D elastic case [Leclerc et al. 2017]:
- Total dissipated energy $\Phi$
  - Has to be chosen to conserve energy dissipation (physically based)
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- Total dissipated energy $\Phi$:
  - Has to be chosen to conserve energy dissipation (physically based)
• 2D elastic plate with a defect
  – Biaxial loading
    • Ratio $\bar{F}_x/\bar{F}_y$ constant during a test
  – In plane strain
  – Path following method
  – Comparison between:
    • Pure non-local
    • Non-local + cohesive zone (CZM)
    • Non-local + cohesive band (CBM)
Damage to crack transition for elasticity – Proof of concept

- 2D plate in plane strain: $\frac{F_x}{F_y} = 0$

### Images:

- **Non-local only**: no crack insertion
- **Non-local + CZM**: cohesive models calibrated on 1D bar under uniaxial stress state
- **Non-local + CBM**:
Damage to crack transition for elasticity – Proof of concept

- 2D plate in plane strain: $F_x / F_y = 0$

- Force evolution

- Dissipated energy evolution

```
Non-Local only
Non-Local + CZM
Non-Local + CBM
```

```
Error on total diss. energy
CZM: ~29%
CBM: ~3%
```
Application of the transition to plasticity

- Porous plasticity (or Gurson) approach
  - Assuming a J2-(visco-)plastic matrix
Porous plasticity (or Gurson) approach
  
  - Assuming a J2-(visco-)plastic matrix

  - Including effects of void/defect or porosity on plastic behavior
    - Apparent macroscopic yield surface \( f(\tau_{eq}, p, \tau_y, Z) \leq 0 \) due to microstructural state:
• Porous plasticity (or Gurson) approach
  – Assuming a J2-(visco-)plastic matrix
  – Including effects of void/defect or porosity on plastic behavior
    • Apparent macroscopic yield surface $f(\tau_{eq}, p, \tau_y, Z) \leq 0$ due to microstructural state:
      – Competition between two deformation modes:
        » Diffuse plastic flow spreads in the matrix
        » Gurson-Tvergaard-Needleman (GTN) model
Porous plasticity (or Gurson) approach

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      » Before failure: coalescence or localized plastic flow between voids
        » GTN or Thomason models
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          » GTN or Thomason models

  – Including evolution of microstructure during failure process
    • Void growth by diffuse plastic flow
    • Apparent growth by shearing
    • Nucleation / appearance of new voids
    • Void coalescence until failure
Non-local porous plasticity model

- Yield surface is considered in the co-rotational space
  - Non-local form: \( f\left(\tau_{eq}, p, \tau_Y, Z, \tilde{Z}\right) \leq 0 \) with \( \tilde{f}_V - l_c^2 \Delta \tilde{f}_V = f_V \)

  * \( \tau^{\text{eq}} \) is the von Mises equivalent Kirchhoff stress and \( p \) the pressure
  * \( \tau_Y = \tau_Y(\dot{\rho}, \dot{\rho}) \) is the viscoplastic yield stress
  * \( f_V \) is the porosity and \( \tilde{f}_V \), its non-local counterpart
  * \( Z \) is the vector of internal variables
  * \( l_c \) is the non-local length

- Normal plastic flow \( D_p \)
- Microstructure evolution (spherical voids):
  - Eq. plastic strain of the matrix:
    \[ \dot{\rho} = \frac{\tau : D_p}{(1 - f_{V0}) \tau_Y} \]
  - Porosity:
    \[ \dot{f}_V = (1 - f_V) \text{tr} D_p + \dot{f}_{\text{nucl}} + \dot{f}_{\text{shear}} \]
  - Ligament ratio:
    \[ \dot{\chi} = \dot{\chi}\left(\chi, \tilde{f}_V, \kappa, \lambda, Z\right) \]

  Microstructure parameters
Non-local porous plasticity – void growth and coalescence

- Gurson–Tvergaard–Needleman (GTN) model:

\[ f = \frac{\tau_{eq}^2}{\tau_Y^2} + 2q_1 \tilde{f}_V \cosh \left( \frac{q_2 p}{2\tau_Y} \right) - 1 - q_3^2 \tilde{f}_V^2 \leq 0 \]

\[ \tilde{f}_V - l_c^2 \Delta \tilde{f}_V = f_V \]

- Phenomenological coalescence model:
  - replace \( \tilde{f}_V \) by an effective value \( \tilde{f}_V^* \):

\[ \tilde{f}_V^* = \begin{cases} 
\tilde{f}_V & \text{if } \tilde{f}_V \leq f_C \\
 f_C + R (\tilde{f}_V - f_C) & \text{if } \tilde{f}_V > f_C 
\end{cases} \]

- \( f_C \) is determined by Thomason criterion [Benzerga2014]:

\[ \max \text{eig} \left( \tau \right) - C_T^f (\chi) \tau_Y > 0 \]
Non-local porous plasticity – void growth and coalescence

• Damage to crack transition for porous plasticity
  – Plane strain specimen [Besson et al. 2003]
    • Only an half is modelled

\( e_0 = 5 \text{mm} \)
Non-local porous plasticity – void growth and coalescence

- Damage to crack transition for porous plasticity
  - Discontinuous Galerkin formulation + cohesive band model [Leclerc et al. 2017]
  - Coalescence is detected at interfaces of elements:

\[
\max \text{eig} (\tau) - C^f_T (\chi) \tau_Y > 0 \quad \Rightarrow \quad n \cdot \tau \cdot n - C^f_T (\chi) \tau_Y > 0
\]
Conclusion

- **Objective:**
  - Simulation of material degradation and crack initiation / propagation during the ductile failure process

- **Upcoming tasks:**
  - Enrichment of nucleation model and coalescence model
  - Calibration of the band thickness
  - Validation/Calibration with literature/experimental tests
Thank you for your attention
State of art: two main approaches – 1. Continuous approaches

• Non-local model
  – Principles
    • variable $\xi \Rightarrow$ non-local / “averaged” counterpart $\tilde{\xi}$
  – Formulation
    • Integral form [Bažant 1988]
      \[
      \tilde{\xi}(x) = \frac{1}{V} \int_V W(x - y) \xi(y) \, dV
      \]
      » not practical for complex geometries
    • Differential forms [Peerlings et al. 2001]
      – Explicit formulation / gradient-enhanced formulation: $\tilde{\xi}(x) = f(\xi, \nabla\xi, \nabla^2\xi, ...)$
        » does not remove mesh-dependency
    – Implicit formulation: $\tilde{\xi}(x) = f(\xi, \nabla\tilde{\xi}, \nabla^2\tilde{\xi}, ...)$
      \[
      \tilde{\xi}(x) - l_c^2 \Delta \tilde{\xi}(x) = \xi(x)
      \]
      » removes mesh-dependency but one added unknown field
• Influence of $h_b$ (for a given $l_c$) on response in a 1D elastic case [Leclerc et al. 2017]:
  – Total dissipated energy $\Phi$ = linear with $h_b$:
    • Has to be chosen to conserve energy dissipation (physically based)
2D plate in plane strain:
- Same trends with ≠ force ratio

\[
\frac{F_x}{F_y} = +0.5
\]

\[
\frac{F_x}{F_y} = -0.5
\]

Damage to crack transition for elasticity – Proof of concept

- Non-Local only
- Non-Local + CZM
- Non-Local + CBM
Damage to crack transition for elasticity – Proof of concept

- Comparison with phase field
  - Single edge notched specimen [Miehe et al. 2010]:
    - Calibration of damage and CBM parameters with 1D case [Leclerc et al. 2017]:

Non-local model

Cohesive band model

Force-displacement curve

Tension test

Shearing test
Damage to crack transition for elasticity – Proof of concept

- Validation with Compact Tension Specimen [Geers 1997]:
  - Better agreement with the cohesive band model than the cohesive zone model or the non-local model alone [Leclerc et al. 2017]
Porous plasticity – principles (2)

• Yield surface is considered in the co-rotational space
  – Local form: \( f(\tau_{eq}, p, \tau_Y, Z) \leq 0 \)

  • \( \tau^\text{eq} \) is the von Mises equivalent Kirchhoff stress and \( p \), the pressure
  • \( \tau_Y = \tau_Y(\dot{p}, \ddot{p}) \) is the viscoplastic yield stress
  • \( Z \) is the vector of internal variables

• Normal plastic flow decomposition:
  \[
  D^p = \dot{F}^p \cdot F^p^{-1} = \dot{\lambda} \frac{\partial f}{\partial \tau} = \dot{d} \frac{\partial \tau_{eq}}{\partial \tau} + \dot{q} \frac{\partial p}{\partial \tau}
  \]

• Plastic deformation of the matrix from the equivalence of plastic energy:
  \[
  (1 - f^0_V)\tau_Y \dot{p} = \tau : D^p
  \]

• Microstructure evolution (porosity \( f_V \) and ligament ratio \( \chi \)):
  \[
  \dot{f}_V = (1 - f_V) \text{tr} D^p + \dot{f}_\text{nucl} + \dot{f}_\text{shear}
  \]
  \[
  \dot{\chi} = \dot{\chi}(\chi, f_V, Z)
  \]

• Drawbacks
  – The numerical results change with the size and the direction of mesh
Porous plasticity – principles (3)

- Evolution of local porosity

\[ \dot{f}_V = (1 - f_V) \text{tr } D^p + \dot{f}_{\text{nucl}} + \dot{f}_{\text{shear}} \]

- Void nucleation \( \dot{f}_{\text{nucl}} \)
  - Modify porosity growth rate (where \( A_N, f_N, \epsilon_N, s_N \) are material parameters)
    - Linear strain-controlled growth
      \[ \dot{f}_{\text{nucl}} = A_N \dot{\hat{p}} \quad \text{with} \quad A_N \begin{cases} \neq 0 & \text{if } f_V > f_N, \\ = 0 & \text{otherwise.} \end{cases} \]
    - Gaussian strain-controlled growth
      \[ \dot{f}_{\text{nucl}} = \frac{f_N}{\sqrt{2\pi s_N^2}} \exp \left( -\frac{(\hat{p} - \epsilon_N)^2}{2s_N^2} \right) \dot{\hat{p}} \]
Porous plasticity – principles (3)

- Evolution of local porosity

\[ \dot{f}_V = (1 - f_V) \text{tr } D^p + \dot{f}_{\text{nucl}} + \dot{f}_{\text{shear}} \]

- Shear-induced voids growth \( \dot{f}_{\text{shear}} \)
  - Includes Lode variable effect (where \( k_w \) is a material parameter)

\[ \dot{f}_{\text{shear}} = f_V k_w \omega(\tau) \frac{\tau^{\text{dev}} : D^p}{\tau^{\text{eq}}} \]
Ductile non-local damage model

- **Hyperelastic formulation:**
  - Multiplicative decomposition of deformation gradient in elastic and plastic parts:
    \[ \mathbf{F} = \mathbf{F}^e \cdot \mathbf{F}^p \]
  - Logarithmic elastic potential \( \psi \):
    \[ \psi(\mathbf{C}^e) = \frac{K}{2} \ln^2 J^e + \frac{G}{4} (\ln \mathbf{C}^e)^{\text{dev}} : (\ln \mathbf{C}^e)^{\text{dev}} \]
    with \( \mathbf{C}^e = \mathbf{F}^e \cdot (\mathbf{F}^e)^T \) and \( J^e = \det \mathbf{F}^e \)
  - Stress tensor definition
    - PK1 stress: \( \mathbf{P} = 2\mathbf{F} \cdot \frac{\partial \psi}{\partial \mathbf{C}} \)
    - Kirchhoff stresses: \( \mathbf{\kappa} = \mathbf{P} \cdot (\mathbf{F}^e)^T \) or again:
      \[ \mathbf{\kappa} = p\mathbf{I} + (\mathbf{\kappa})^{\text{dev}} = p\mathbf{I} + \mathbf{F}^e \cdot \left[ \mathbf{C}^{e-1} \cdot (\mathbf{\tau})^{\text{dev}} \right] \cdot (\mathbf{F}^e)^T \]
      \[ \mathbf{\tau} = p\mathbf{I} + (\mathbf{\tau})^{\text{dev}} = p\mathbf{I} + 2G \left( \ln \sqrt{\mathbf{C}^e} \right)^{\text{dev}} \]
Integration algorithm

• **Predictor-corrector procedure**
  – Elastic predictor
  – Plastic corrector (radial return-like algorithm)
    • 3 Unknowns $\Delta \hat{d}$, $\Delta \hat{q}$, $\Delta \hat{p}$
    • 3 Equations
      – Consistency equation:
        \[ f \left( \tau_{eq}(\Delta \hat{d}), p(\Delta \hat{q}), \tau_Y(\Delta \hat{p}), Z(\Delta \hat{d}, \Delta \hat{q}, \Delta \hat{p}), \tilde{Z} \right) = 0 \]
      – Plastic flow rule:
        \[ \Delta \hat{d} \frac{\partial f}{\partial p} - \Delta \hat{q} \frac{\partial f}{\partial \tau_{eq}} = 0 \]
      – Matrix plastic strain evolution:
        \[ (1 - f_{V0}) \tau_Y \Delta \hat{p} = \tau_{eq} \Delta \hat{d} + p \Delta \hat{q} \]