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NEGATIVE PARITY NON-STRANGE BARYONS

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Our previous study is extended to negative parity baryon resonances up to $J = \frac{9}{2}^-$. The framework is a semi-relativistic constituent quark model. The quark-quark interaction contains a Coulomb plus linear confinement terms and a short distance spin-spin and tensor terms. It is emphasized that a linear confinement potential gives too large a mass to the D_{35} (1930) resonance.

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The purpose of the present work is to extend our previous analysis [1,2] of the non-strange baryon masses of the N = 0, 1 and 2 bands to the N = 3 band. These are negative parity resonances of masses around 2000 MeV or more. The interest in the N = 3 band has been raised about a decade ago by a search of assignment of the $D_{35}(1930)$ resonance. (For a review, see Ref. [3]). There are essentially two different kinds of approaches which have been considered until now in the treatment of the N = 3 band. One is based on a non-relativistic harmonic-oscillator quark model with anharmonicity corrections. These have been first treated by a spectrum generating algebra Sp (12,R) [4,5]. The result was a mass formula with four parameters. Three of these parameters also appear in studies of the N = 0, 1 and 2 bands [6-8]. The authors have claimed success in describing the $D_{35}(1930)$ resonance. But the harmonic potential is only a convenient first approximation to a realistic potential and its parameters need to be adjusted for each band in order to optimize the approximation. A similar parametrization has recently been obtained by Richard and Taxil [9] with a simpler method which relates the splitting pattern of the three-body problem to the properties of the two-body binding energies. Although the number of parameters is the same their meaning is different. The other kind of approach consists in solving for a three-body Hamiltonian incorporating a linear confinement potential inspired by QCD lattice calculations. This approach has been followed by Cutkoski and Hendrick [10] soon after a πN partial wave analysis [11] had shown evidence for the existence of a D_{35} resonance. An extended study has been recently performed by Capstick and Isgur [12] in a large harmonic oscillator basis up to N = 8. Both studies predict a D_{35} mass of about 200 MeV above the experimental value.

Our work follows this second approach. Our starting point is the semi relativistic flux tube model of Carlson, Kogut and Pandharipande [13] and their varational wave function for the ground state. Although their Hamiltonian has strong similarities to those of Refs. [10] and [12] their ground state wave function has the appropriate asymptotic behaviour for a linear potential. Let us recall that the Hamiltonian of Ref. [13] is :

$$H_0 = \sum_i \left(p_i^2 + m^2\right)^{\frac{1}{2}} + V(\vec{r}_1,\vec{r}_2,\vec{r}_3) + E_0^B \tag{1}$$

where the potential energy

$$V(\vec{r}_1,\vec{r}_2,\vec{r}_3) = \frac{1}{2}\sum_{i<j}\left(-\frac{4}{3}\frac{\alpha_s}{r_{ij}} + \sqrt{\sigma}\, r_{ij}\right) + \sqrt{\sigma}\left(\sum_i r_{i4} - \frac{1}{2}\sum_{i<j} r_{ij}\right) \tag{2}$$

has two-body and three-body terms. In the three-body term r_{i4} is the distance between the quark i and a point of equilibrium energy, r_4 . The values of the strong coupling constant and the string tension

$$\frac{4}{3}\alpha_s = 0.5 \qquad \sqrt{\sigma} = 1 \text{ GeV/fm} \tag{3}$$

are consistent with parameters used to fit the spectrum of charmonium [13]. The constant E_0^B is adjusted to the nucleon mass $m_N = 939$ MeV.

To H_0 we have added [1] a short range spin dependent interaction under the form of a hyperfine interaction $H^{hyp} = \sum_{i<j} H_{ij}^{hyp}$ having a spin-spin term V^{SS} and a tensor term V^T

$$H_{ij}^{hyp} = V_{ij}^{SS} + V_{ij}^{T} \tag{4}$$

with

$$V_{12}^{SS} = \frac{4\sqrt{2}\,\pi\,\alpha_s}{9\,m^2}\,\frac{1}{\left(2\pi\Lambda^2\right)^{\frac{3}{2}}}\, e^{-\frac{\rho^2}{2\Lambda^2}}\,\vec{S}_1\cdot\vec{S}_2 \tag{5}$$

$$V_{12}^{T} = \frac{\alpha_s}{\sqrt{2}\, m^2 \rho^3} \left\{ \mathrm{erf}\left(\frac{\rho}{2\Lambda}\right) - \frac{\sqrt{2}}{3\sqrt{\pi}} \frac{\rho^3}{\Lambda^3} \left(1 + 3\frac{\Lambda^2}{\rho^2}\right) e^{-\frac{\rho^2}{2\Lambda^2}} \right\}$$

$$\times \left(\frac{1}{\rho^2} \vec{S}_1 \cdot \vec{\rho}\ \vec{S}_2 \cdot \vec{\rho} - \frac{1}{3} \vec{S}_1 \cdot \vec{S}_2 \right) \qquad (6)$$

$\vec{\rho}$ being one of the internal (Jacobi) coordinates

$$\vec{\rho} = \frac{1}{\sqrt{2}}\left(\vec{r}_1 - \vec{r}_2\right) \quad , \quad \vec{\lambda} = \frac{1}{\sqrt{6}}\left(\vec{r}_1 + \vec{r}_2 - 2\,\vec{r}_1\right) \quad . \qquad (7)$$

The hyperfine interaction contains two parameters : the quark mass m and the finite size of the quark Λ, used to regularize the hyperfine interaction at the origin.

The N = 3 band introduces eight new SU(6) multiplets. The corresponding states ψ_{L0}^{μ}, where μ = S, A, ρ, λ stands for the S_3 symmetry, are introduced in Table 1 together with those with N = 0, 1 for completeness. F is the variational function of Ref. [13].

$$F = \left\{ 1 - \beta \sqrt{\sigma} \left[\sum_{i=1}^{3} \left| \vec{r}_i - \vec{r}_4 \right| - \frac{1}{2} \sum_{i<j} r_{ij} \right] \right\} \prod_{i<j} f\left(r_{ij}\right) \qquad (8)$$

where

$$f(r) = \exp\left\{ - \left(\gamma_1 r + \gamma_2 r^2\right) W(r) - \gamma_{1.5} r^{1.5} \left[1 - W(r)\right] \right\}$$

$$W(r) = \frac{1 + \exp\left[\left(r - r_0\right)/a\right]}{1 + \exp\left(r_0/a\right)} \qquad (9)$$

with the variational parameters :

$\gamma_1 = 0.3965\ \mathrm{fm}^{-1}$, $\gamma_2 = 0.637\ \mathrm{fm}^{-2}$, $\gamma_{1.5} = 1.40\ \mathrm{fm}^{-1.5}$,

$r_0 = 0.12$ fm , $a = 0.12$ fm , $\beta = 0.25\ \mathrm{GeV}^{-1}$.

The first factor in (8) is the effect of the three-body part of the interaction (2) taken into account perturbatively. The two body correlation function f has an Airy function asymptotic behaviour

$$f(r) \underset{a \to \infty}{\longrightarrow} \exp\left(- \gamma_{1.5}\, r^{1.5} \right) \tag{10}$$

appropriate for a linear confinement.

The polynomial structure of ψ^{μ}_{L0} has been derived by the method of Moshinsky [14]. For S_3 mixed symmetry representations only the state with $\mu = \rho$ is given. The associated $\mu = \lambda$ state can be obtained from the $\mu = \rho$ state by interchanging the ρ and λ coordinates. The coefficients c, α_0, α_1 and α_2 are such that the 70, 70' and 70" states are orthogonal to each other. The expression of the ($70',1^-$) states containing one unit of radial excitation has been built in analogy with the radial form we have recently proposed [2] for the description of the Roper resonance. The value k = 4 corresponds to the conventional harmonic excitation which gives too large a mass to the Roper resonance. We have shown in [2] that taking k = 1 the position of the Roper resonance is lowered by ~ 100 MeV and brought to around 1500 MeV. Here we found that the eigenstate having ($70'$, 1^-) as a main component is rather insensitive to the value of k. Below we reproduce results corresponding to k = 4. The compact form of states of Table 1 have an advantage over harmonic oscillator expansions (see e.g. Ref. [12]). They avoid problem of convergence and are practical in further calculations as e.g. decay widths. The third column of Table 1 gives the expectation value of H_0 for each multiplet. The level ordering has close similarities to that obtained in Ref.[9] for a linear potential. The first two levels are ($70'$, 1^-) and (56, 1^-) and (70, 2^-) is the last. For the other levels the ordering is different. This means that it depends on the detailed form of the hamiltonian. However if the pairs (56, L^-) and (20, L^-) with L = 1 and 3 are replaced by their centre of gravity one obtains the same level ordering as that of Fig. 4 of Ref. [9] or Fig. 2 of Ref. [10]. This is in contrast to the mass formula results of Refs. [4,5] where the level ordering is entirely different and (56, 1^-) has the lowest value.

In Table 2 we display the lowest eigenvalues of $H = H_0 + \sum_{i<j} H_{ij}^{hyp}$ likely to be compared to the available experimental data. This is a typical example corresponding to $\Lambda = 0.13$ fm. The mass spectrum dependence on m has the same type of behaviour for any Λ, the masses decrease as m increases (the statement beling valid for positive parity states as well). It is the combined effect of the kinetic energy and $\sum_{i<j} H_{ij}^{hyp}$. For a fixed value of m, masses decrease as Λ increases as a result of eqs. (4)-(6). The net result is that there are several sets of (Λ,m) which give very close eigenvalues, for example the sets (0.11 fm, 333 MeV), (0.13 fm, 313 MeV) and (0.15 fm, 293 MeV). The hyperfine interaction plays a role through the spin-spin term and the diagonal contributions of the tensor term. Non-diagonal tensor matrix elements can safely be neglected between N = 1 and N = 3 and inside the N = 3 band. They are typical of the order of ~ 10 MeV or less and the unperturbed expectation values are highly separated as seen from Table 1. The spin-spin term doesn't bring them close enough to allow the tensor term to mix them.

In Table 2 the best fit appears for m = 313 MeV. The discussion which follows refers to this value. The experimental intervals for four- and three-star resonances are those from the summary table of Particle Data Group [15]. For two- and one-star resonances we quote the results of Cutkoski et al. [16]. Our assignments are quite close to those made by Cutkoski and Forsyth [17] in their simultaneous fit of masses and πN decays. However they introduce a special parameter to adjust the position of the (56, 1^-) multiplet.

Most masses fall within or near the experimental interval with few exceptions. The lowest masses in the N $\frac{1}{2}^-$ and N $\frac{3}{2}^-$ sectors are about 50 MeV lower than the experimental values. A similar situation appeared in Refs. [12, 18]. However a discrepancy of a few tens of MeV should not be regarded as a failure at the present level of development, the coupling of baryons to baryon-meson channels being ignored. This coupling should be included as there are indications that it could produce important mass shifts [19-21].

The D_{35} (1930) resonance is predicted about 200 MeV above the experimental value, like in Refs. [10] and [12]. It is a consequence of the too high position of its main component, the multiplet $(56,1^-)$. This seems to be a typical property of a linear confinement potential. The spin-spin contribution to the $(56,1^-)$ multiplet is of the order of 75 MeV in the quark mass range considered here and the tensor contribution doesn't affect its position at all. In view of the conflict between practical calculations with reasonably realistic potentials and the simply derived parametric formulae of [9] on the one hand and the mass formula of [4] on the other hand, it would be useful to reconsider a derivation of a mass formulae for the N = 3 band through spectrum-generating algebra methods.

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Table Captions

Table 1

SU(6) multiplets for the N = 0, 1 and 3 bands (column 1) ; their wave functions ψ^{μ}_{L0} (column 2) in terms of the Jacobi coordinates ρ and λ ($\rho_{\pm} = \rho_x + i\,\rho_y$, $\lambda_{\pm} = \lambda_x \pm i\,\lambda_y$) and the variational function F of Ref. [13] ; the expectation value (column 3) of H_0 of eqs. (1)-(3) for each multiplet.

Table 2

Negative parity mass spectrum (MeV) for Λ = 0.13 fm at various quark masses. Column 1 : sectors. Column 2 : dimension of the matrix to be diagonalized in each sector. Column 3 : main component. Columns 4-8 : masses. Column 9 : resonance status. Column 10 : experimental range [15,16].

TABLE 1

Multiplet	ψ^{μ}_{L0}	$\langle H_0 \rangle$ (GeV)
$(56,0^+)$	$N^{S}_{00}\, F$	1.135
$(70,1^-)$	$N^{\rho}_{10}\, \rho_0\, F$	1.575
$(70',1^-)$	$N^{\rho'}_{10}\left[1 - c\left(\rho^2 + \lambda^2\right)^{k/4}\right]\rho_0\, F$	2.088
$(70'',1^-)$	$N^{\rho''}_{10}\left[\vec{\rho}\cdot\vec{\lambda}\,\lambda_0 + \left(\alpha_0 + \alpha_1\,\rho^2 + \alpha_2\,\lambda^2\right)\rho_0\right] F$	2.221
$(56,3^-)$	$N^{S}_{30}\left\{\left[5\left(\lambda_0^2 - 3\,\rho_0^2\right) + 3\left(\rho^2 - \lambda^2\right)\right]\lambda_0 + 6\,\vec{\rho}\cdot\vec{\lambda}\,\rho_0\right\} F$	2.253
$(20,3^-)$	$N^{A}_{30}\left\{\left[5\left(\rho_0^2 - 3\,\lambda_0^2\right) - 3\left(\rho^2 - \lambda^2\right)\right]\rho_0 + 6\,\vec{\rho}\cdot\vec{\lambda}\,\lambda_0\right\} F$	2.284
$(70,3^-)$	$N^{\rho}_{30}\left\{\left[5\left(\rho_0^2 + \lambda_0^2\right) - 3\,\rho^2 - \lambda^2\right]\rho_0 - 2\,\vec{\rho}\cdot\vec{\lambda}\,\lambda_0\right\} F$	2.214
$(70,2^-)$	$N^{\rho}_{20}\left(\rho_+\,\lambda_- - \rho_-\,\lambda_+\right)\lambda_0\, F$	2.487
$(56,1^-)$	$N^{S}_{10}\left[-\left(\rho^2 - \lambda^2\right)\lambda_0 - 2\,\vec{\rho}\cdot\vec{\lambda}\,\rho_0\right] F$	2.128
$(20,1^-)$	$N^{A}_{10}\left[\left(\rho^2 - \lambda^2\right)\rho_0 - 2\,\vec{\rho}\cdot\vec{\lambda}\,\lambda_0\right] F$	2.192

TABLE 2

Resonance	n	Main component	Quark mass (MeV) 280	300	313	330	350	Status	Experiment
$N\frac{9}{2}^-$	1	$^4N(70,3^-)$	2309	2262	2236	2206	2175	****	2130-2270
$\Delta\frac{9}{2}^-$	1	$^4\Delta(56,3^-)$	2331	2287	2261	2233	2204	**	2200-2400
$N\frac{7}{2}^-$	5	$^2N(70,3^-)$	2133	2110	2097	2083	2067	****	2120-2230
$\Delta\frac{7}{2}^-$	2	$^2\Delta(70,3^-)$	2305	2259	2232	2203	2173	*	2120-2280
$N\frac{5}{2}^-$	9	$^4N(70,1^-)$	1695	1647	1621	1592	1563	****	1660-1690
		$^2N(70,3^-)$	2132	2110	2097	2082	2067	**	2100-2260
$\Delta\frac{5}{2}^-$	4	$^4\Delta(56,1^-)$	2269	2216	2187	2154	2120	***	1890-1960
		$^2\Delta(70,3^-)$	2303	2257	2231	2202	2172		
		$^4\Delta(56,3^-)$	2354	2306	2279	2249	2218	*	2275-2525
		$^2\Delta(70,2^-)$	2496	2459	2437	2413	2388		
$N\frac{3}{2}^-$	11	$^2N(70,1^-)$	1484	1467	1457	1445	1434	****	1510-1530
		$^4N(70,1^-)$	1753	1699	1669	1635	1601	***	1670-1730
		$^2N(70',1^-)$	2036	2009	1994	1977	1959	**	1780-1980
		$^2N(56,1^-)$	2126	2101	2087	2072	2057	**	1980-2140
$\Delta\frac{3}{2}^-$	6	$^2\Delta(70,1^-)$	1648	1606	1583	1557	1532	****	1630-1740
		$^2\Delta(70',1^-)$	2106	2071	2051	2029	2006	*	1840-2040
$N\frac{1}{2}^-$	9	$^2N(70,1^-)$	1467	1452	1443	1433	1423	****	1520-1560
		$^4N(70,1^-)$	1663	1620	1596	1570	1543	****	1620-1680
		$^2N(70',1^-)$	2033	2007	1992	1975	1958	*	2000-2260
		$^2N(56,1^-)$	2068	2040	2025	2007	1990		
$\Delta\frac{1}{2}^-$	4	$^2\Delta(70,1^-)$	1647	1603	1580	1555	1530	****	1600-1650
		$^2\Delta(70',1^-)$	2106	2071	2052	2029	2006	***	1850-2000
		$^2\Delta(70'',1^-)$	2271	2218	2189	2155	2120	*	2050-2250