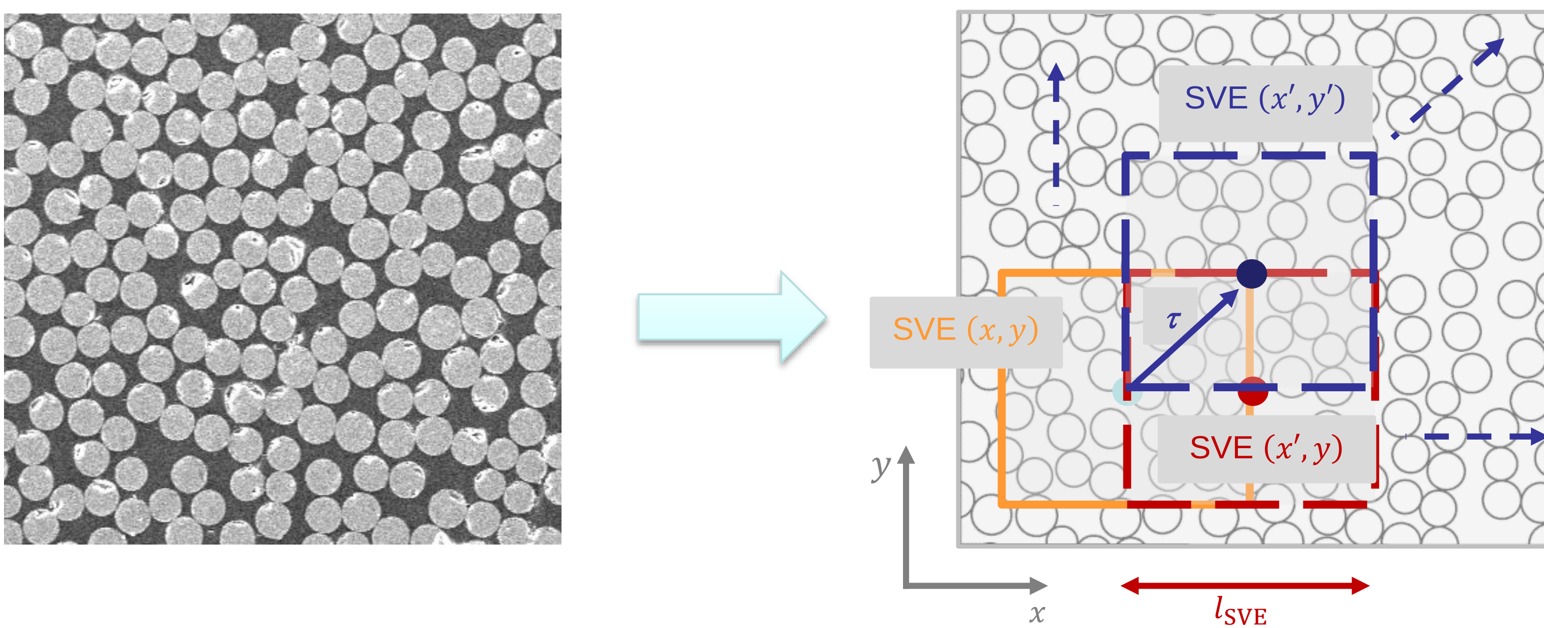
A Probabilistic Mean-Field-Homogenization Approach Applied to Study Unidirectional Composite Structures

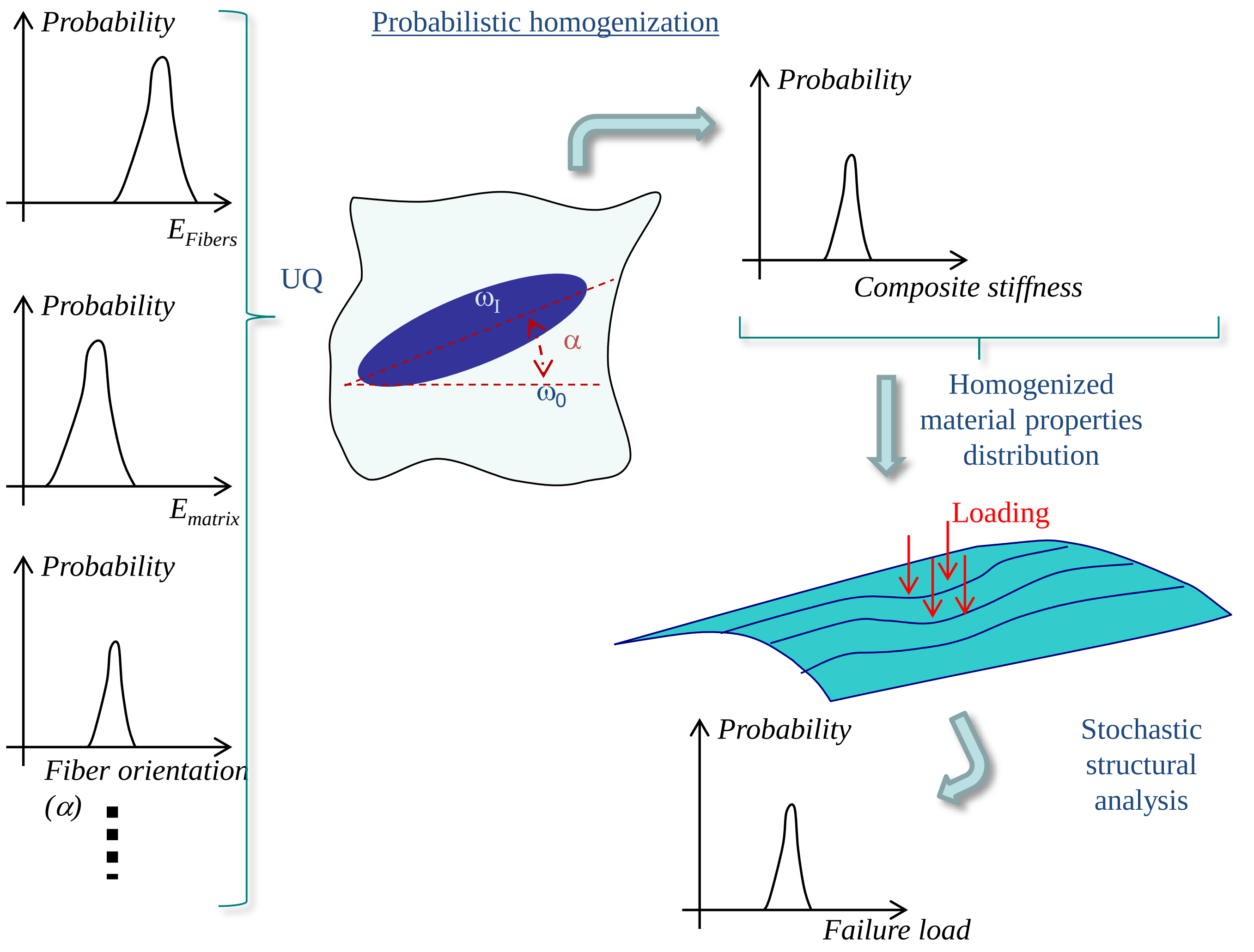
Wu Ling, Adam Laurent (e-Xstream), Noels Ludovic



The research has been funded by the Walloon Region under the agreement no 1410246-STOMMMAC (CT-INT 2013-03-28) in the context of the M-ERA.NET Joint Call 2014. SEM images by Major Zoltan, Nghia Chnug Chi, JKU, Austria

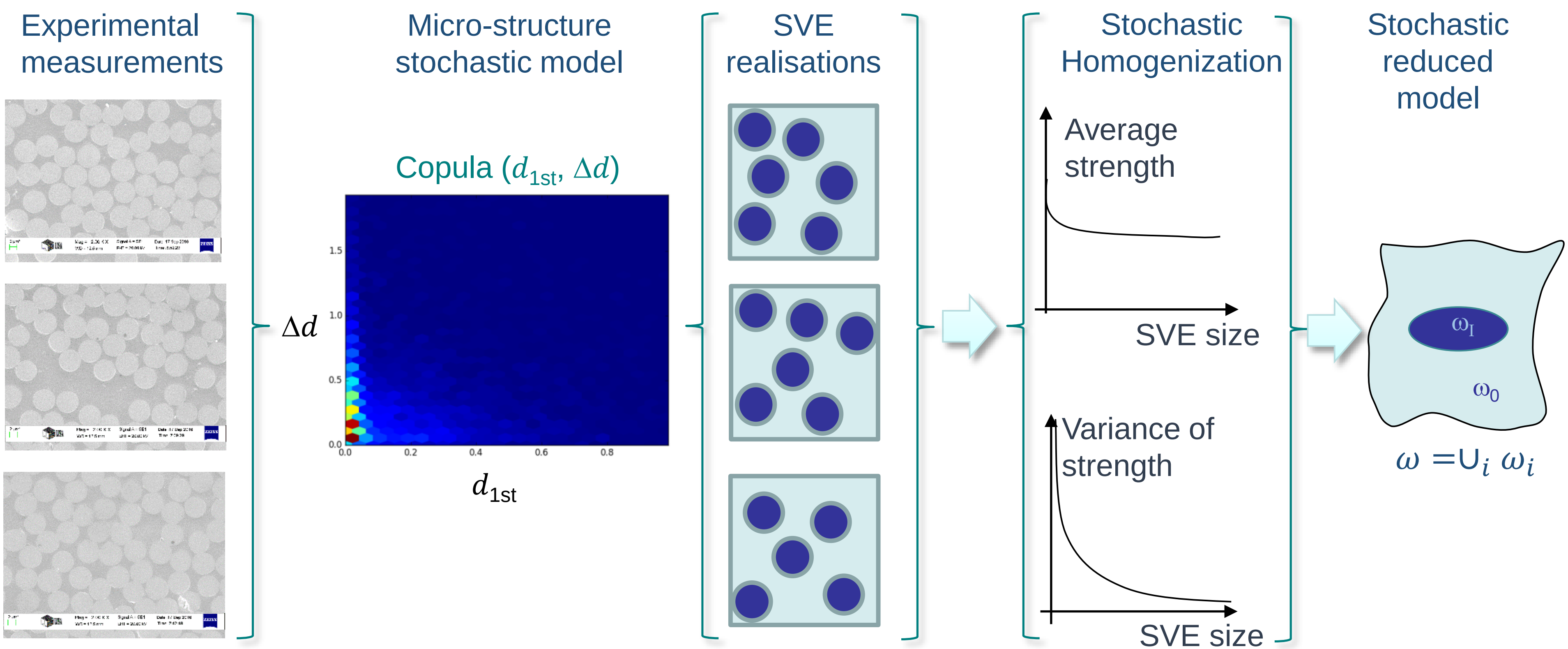
The problem

- Material uncertainties affect structural behaviors



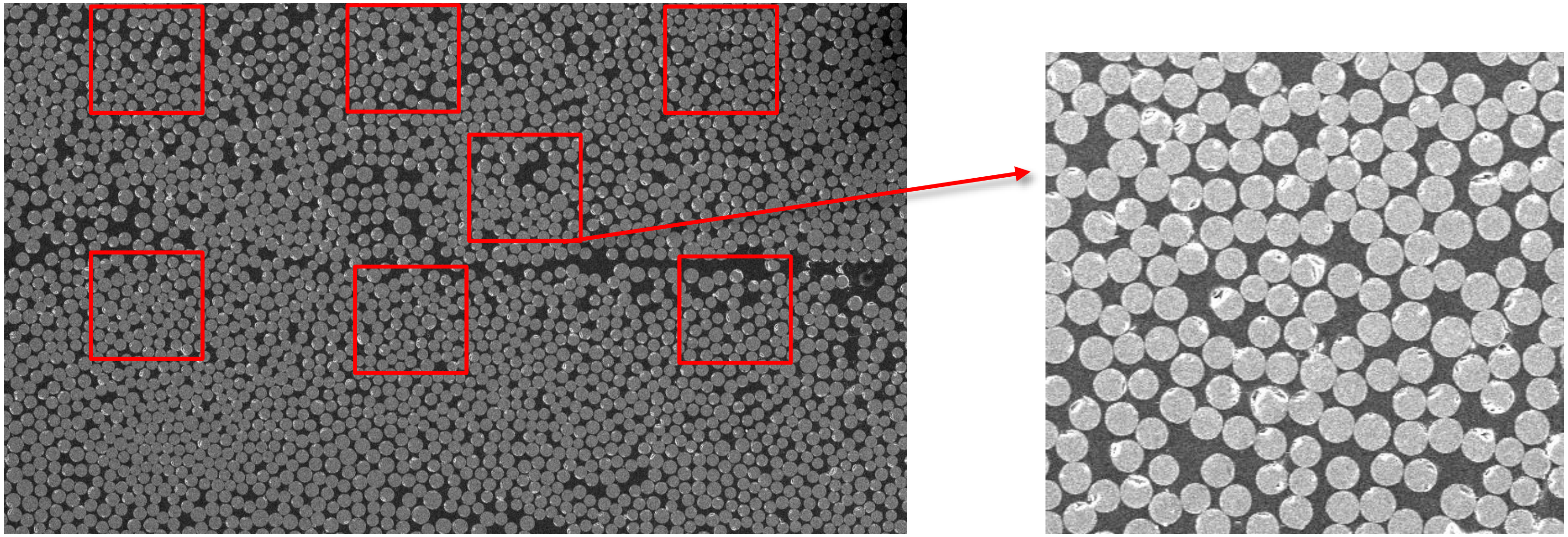
The problem

- Proposed methodology:

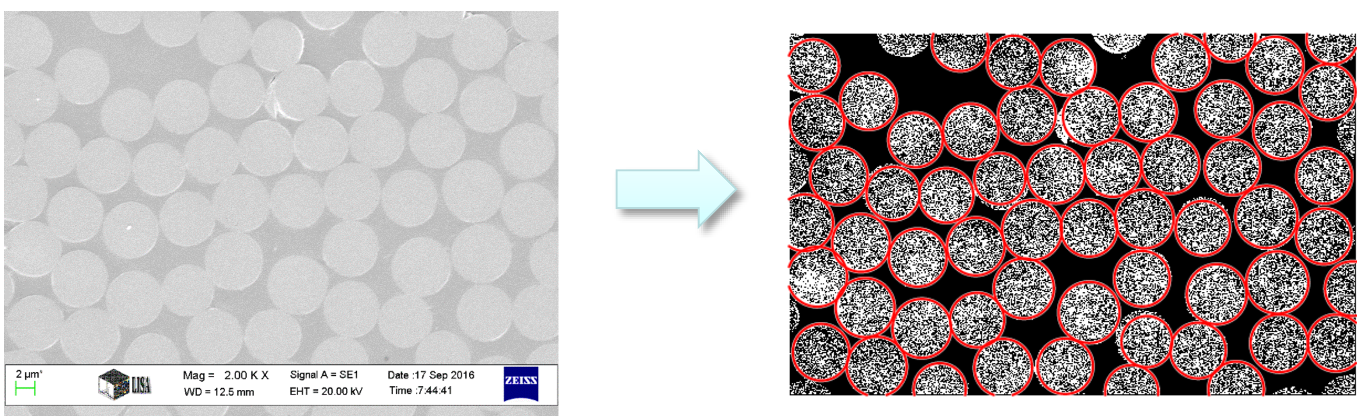


Experimental measurements

- 2000x and 3000x SEM images



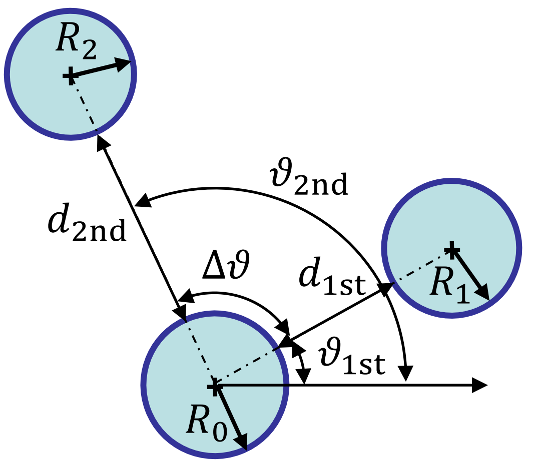
- Fibers detection



Micro-structure stochastic model

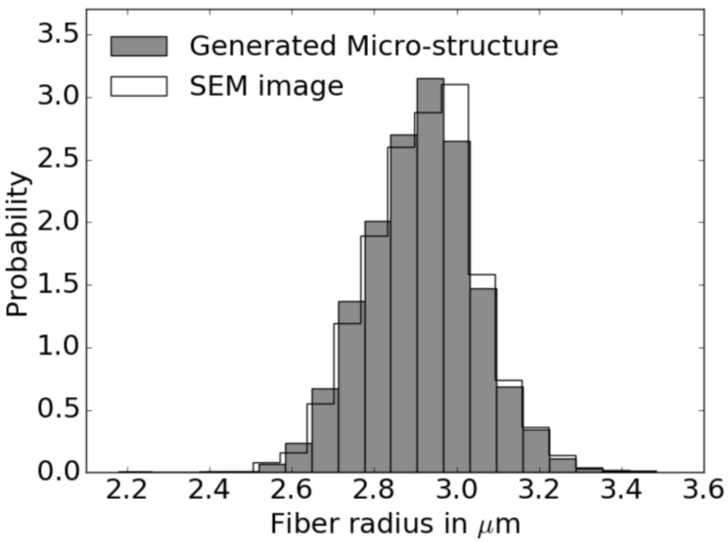
- Basic geometric information of fibers' cross sections
  - Fiber radius distribution $p_R(r)$

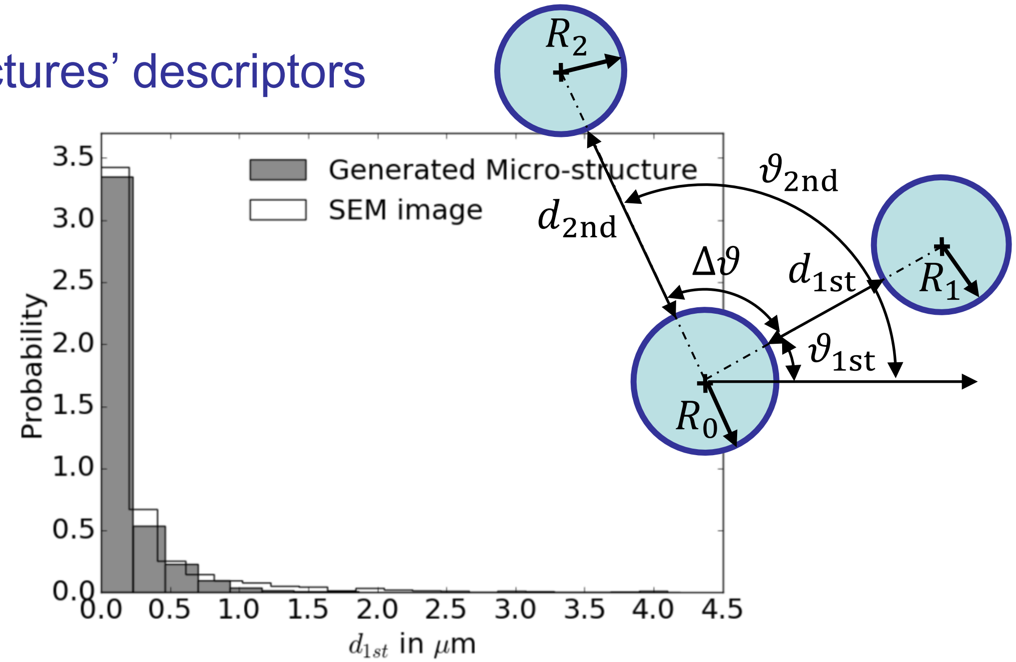
- Basic spatial information of fibers
  - The distribution of the nearest-neighbor net distance function $p_{d_{1\text{st}}}(d)$
  - The distribution of the orientation of the undirected line connecting the center points of a fiber to its nearest neighbor $p_{\vartheta_{1\text{st}}}(\theta)$
  - The distribution of the difference between the net distance to the second and the first nearest-neighbor $p_{\Delta d}(d)$ with $\Delta d = d_{2\text{nd}} - d_{1\text{st}}$
  - The distribution of the second nearest-neighbor's location referring to the first nearest-neighbor $p_{\Delta\vartheta}(\theta)$ with $\Delta\vartheta = \vartheta_{2\text{nd}} - \vartheta_{1\text{st}}$

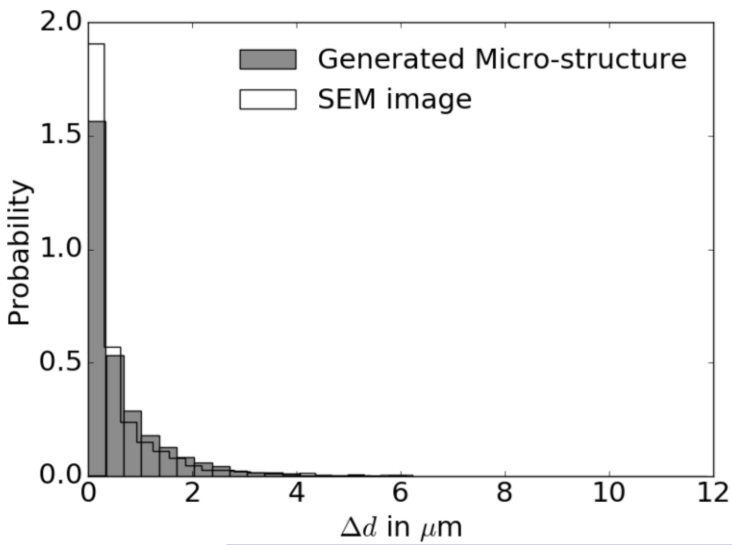


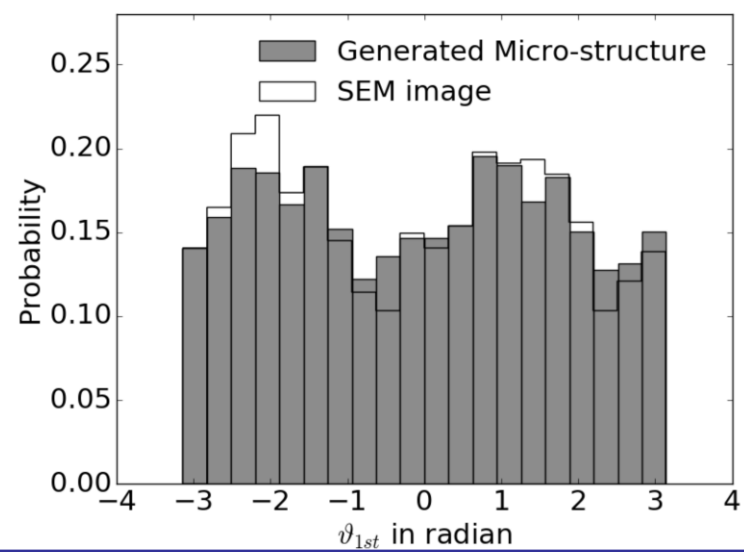
Micro-structure stochastic model

- Histograms of random micro-structures' descriptors



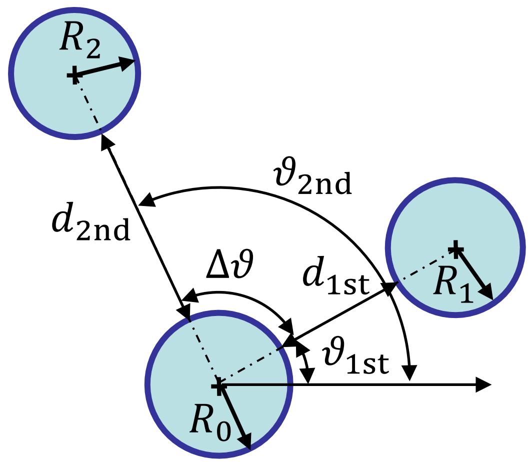






Micro-structure stochastic model

- Dependency of the four random variables $d_{1\text{st}}, \Delta d, \vartheta_{1\text{st}}, \Delta\vartheta$
- Correlation matrix



	$d_{1\text{st}}$	Δd	$\vartheta_{1\text{st}}$	$\Delta\vartheta$
$d_{1\text{st}}$	1.0	0.21	0.01	0.02
Δd		1.0	0.002	-0.005
$\vartheta_{1\text{st}}$			1.0	0.02
$\Delta\vartheta$				1.0

- Distances correlation matrix

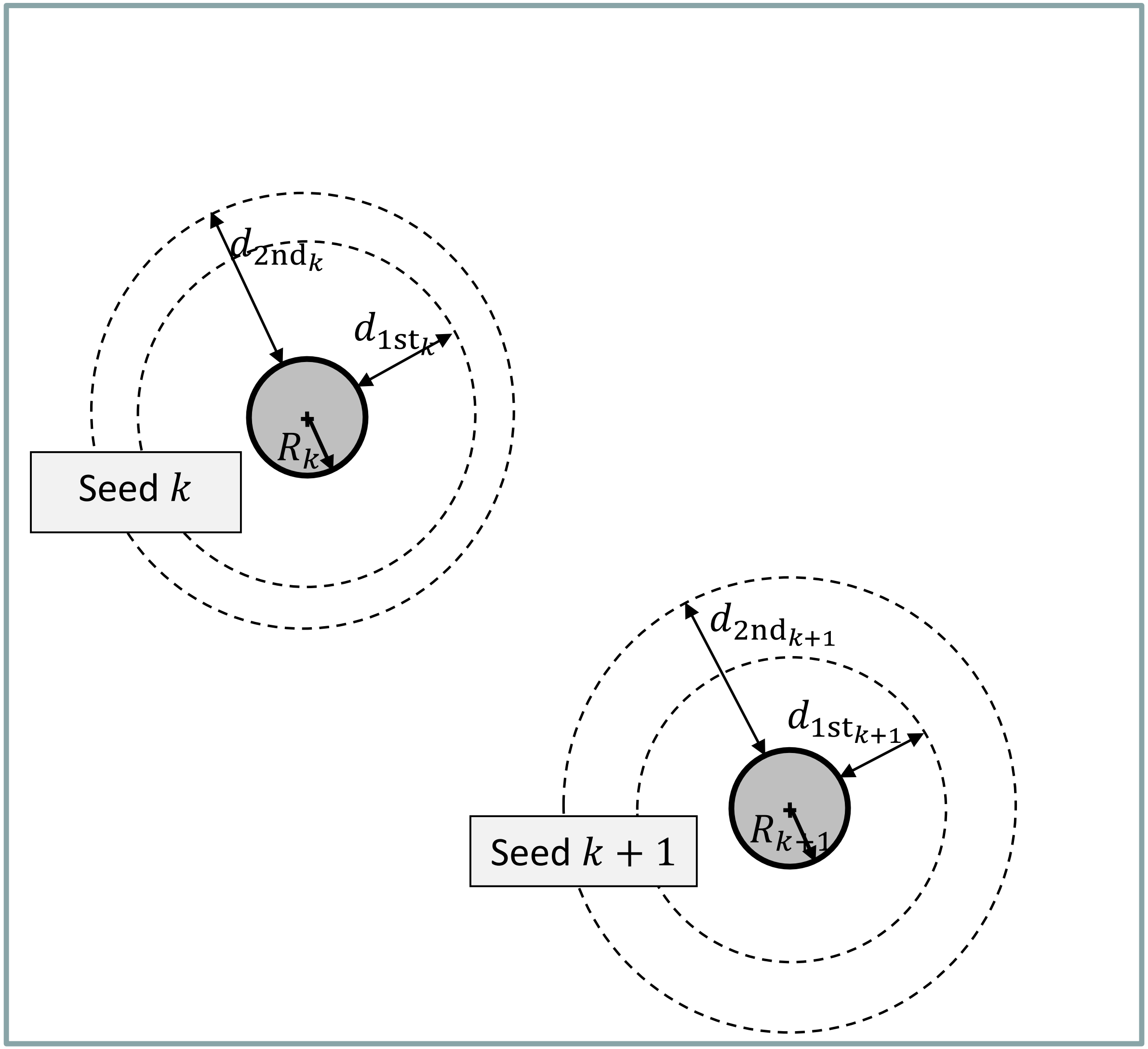
	$d_{1\text{st}}$	Δd	$\vartheta_{1\text{st}}$	$\Delta\vartheta$
$d_{1\text{st}}$	1.0	0.27	0.04	0.08
Δd		1.0	0.05	0.06
$\vartheta_{1\text{st}}$			1.0	0.05
$\Delta\vartheta$				1.0

$d_{1\text{st}}$ and Δd are dependent

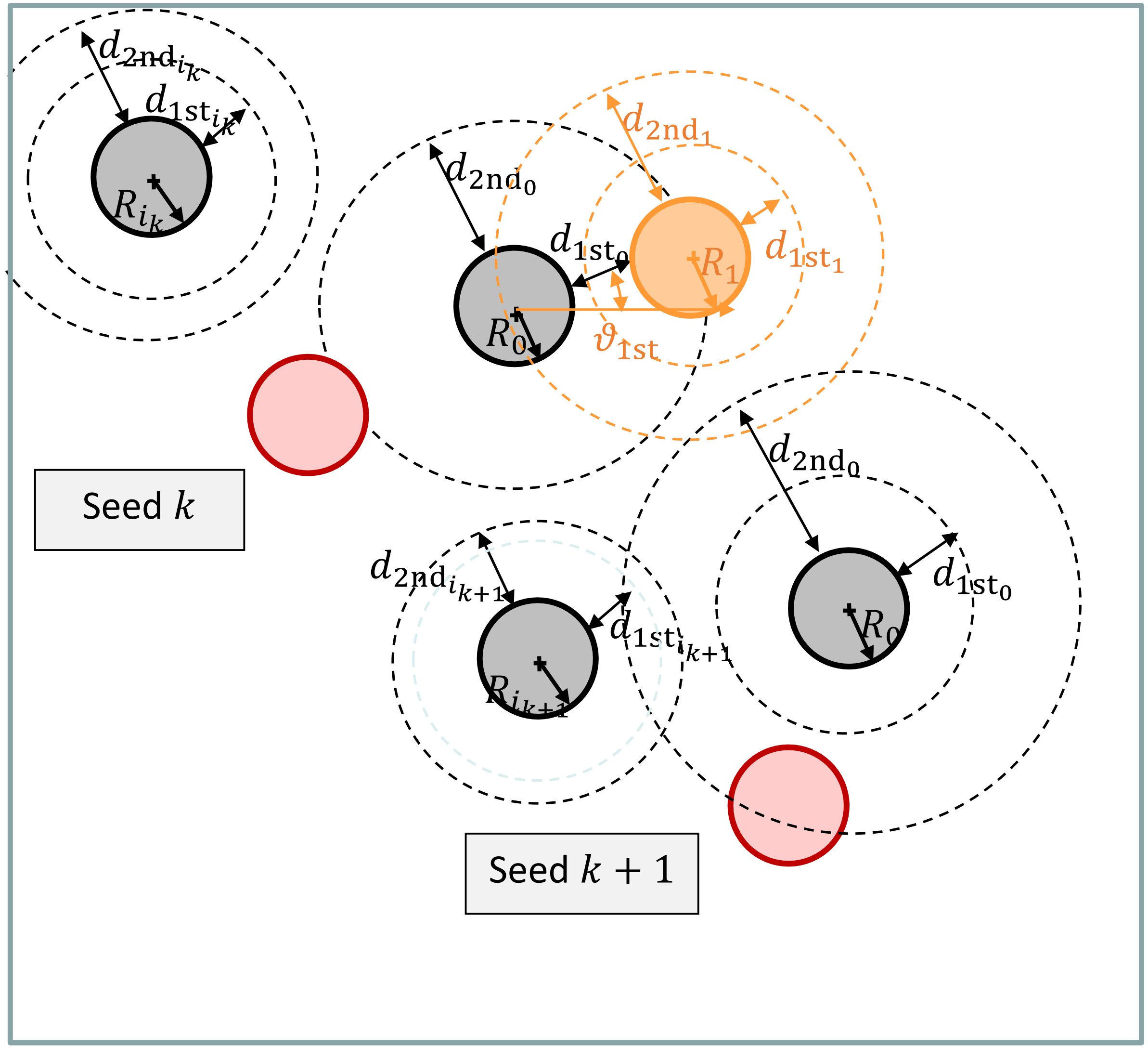
⇨ they will have to be generated from their empirical copula

Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process
  1) Define N seeds with first and second neighbors distances

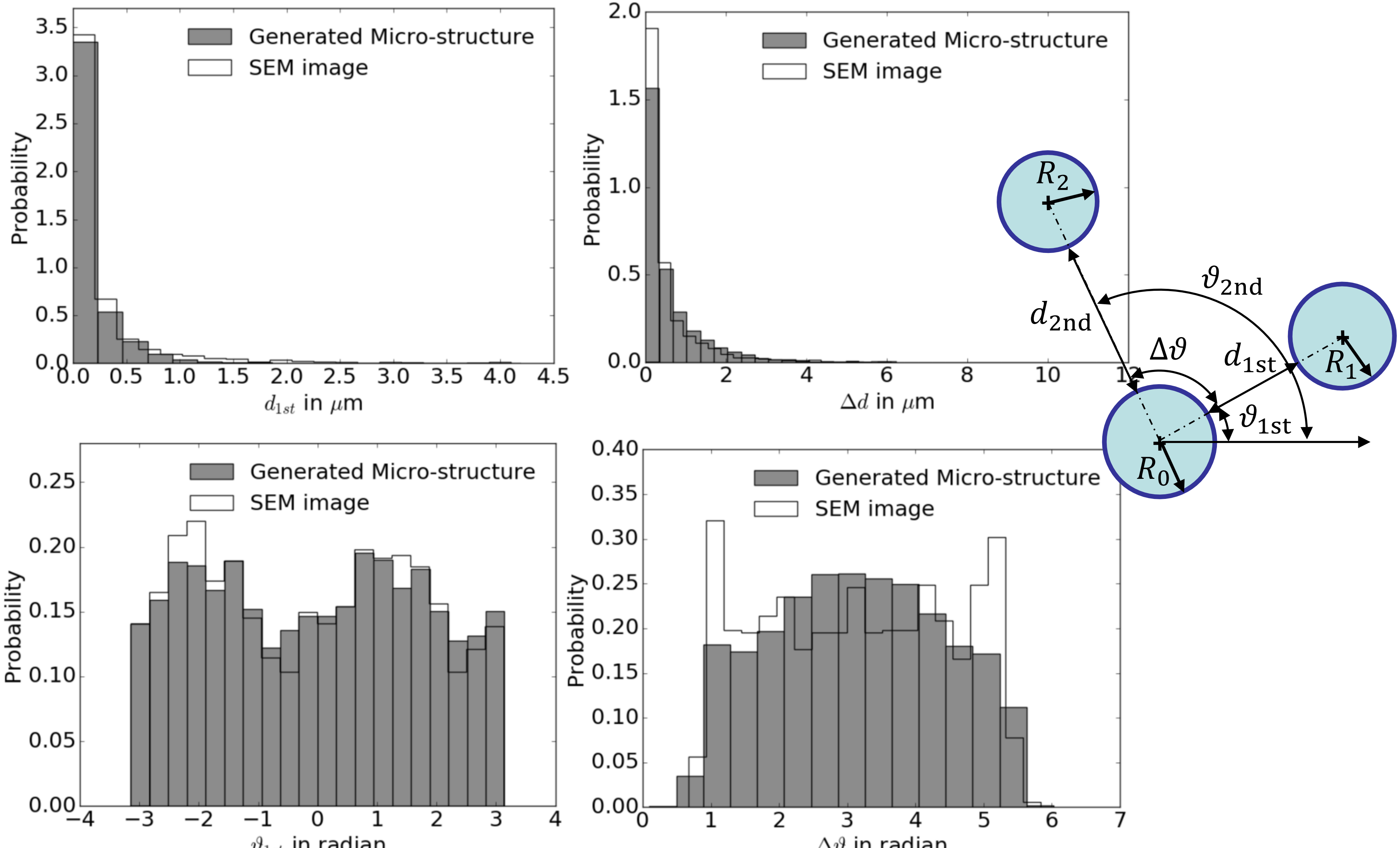


- The numerical micro-structure is generated by a fiber additive process
  1) Define N seeds with first and second neighbors distances
  2) Generate first neighbor with its own first and second neighbors distances
  3) Generate second neighbor with its own first and second neighbors distances
  4) Change seeds & then change central fiber of the seeds



Micro-structure stochastic model

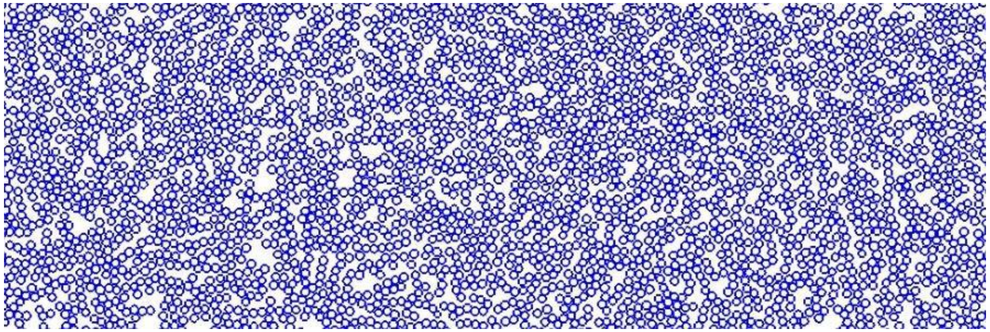
- Comparisons of fibers spatial information

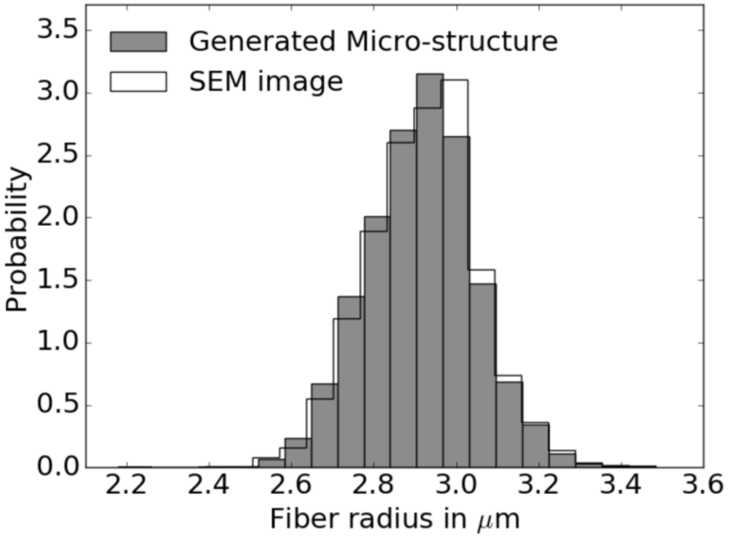


LIÈGE université

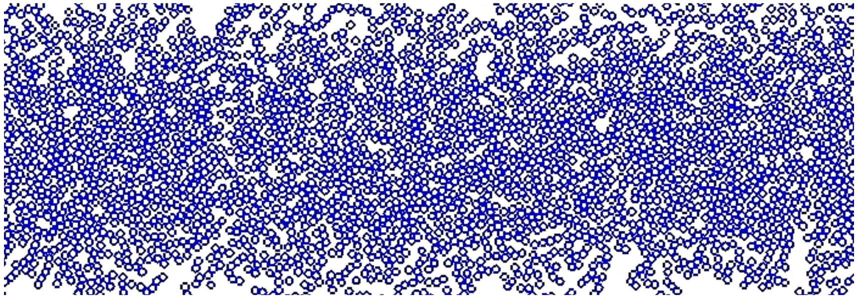
Micro-structure stochastic model

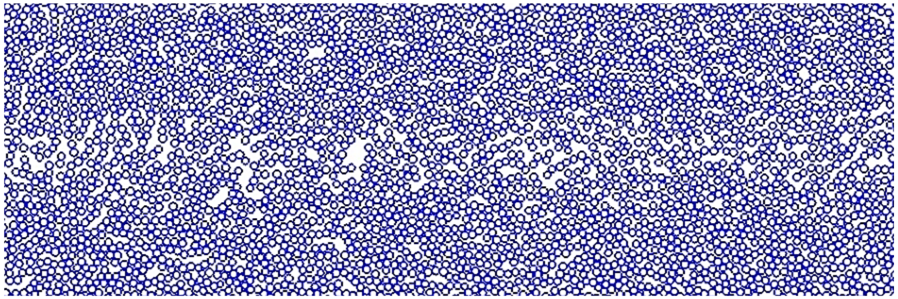
- Numerical micro-structures are generated by a fiber additive process
  - Arbitrary size
  - Arbitrary number





  - Possibility to generate non-homogenous distributions





Stochastic homogenization on the SVEs

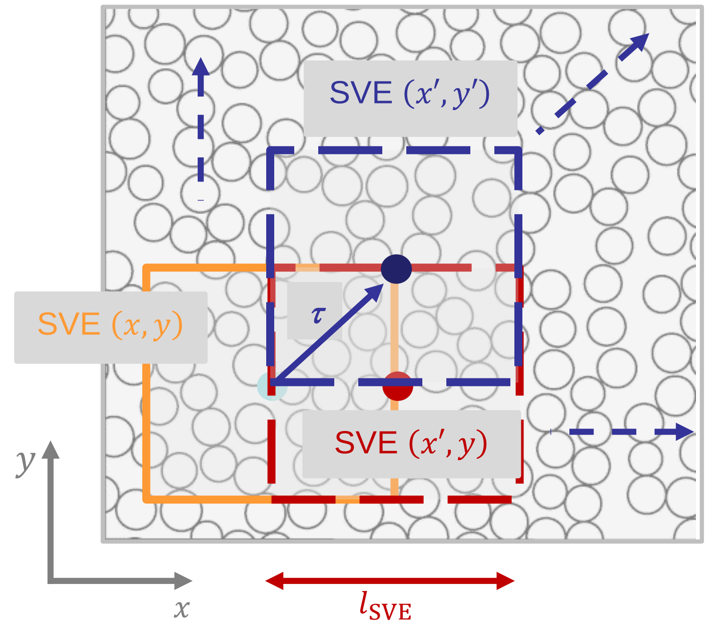
- Stochastic homogenization
  - Extraction of Stochastic Volume Elements
    - 2 sizes considered: $l_{\mathrm{SVE}} = 10\ \mu m$ & $l_{\mathrm{SVE}} = 25\ \mu m$
    - Window technique to capture correlation

$$R_{\boldsymbol{rs}}(\boldsymbol{\tau}) = \frac{\mathbb{E}\left[\left(r(\boldsymbol{x}) - \mathbb{E}(r)\right)\left(s(\boldsymbol{x}+\boldsymbol{\tau}) - \mathbb{E}(s)\right)\right]}{\sqrt{\mathbb{E}\left[\left(r - \mathbb{E}(r)\right)^2\right]}\sqrt{\mathbb{E}\left[\left(s - \mathbb{E}(s)\right)^2\right]}}$$

  - For each SVE
    - Extract apparent homogenized material tensor $\mathbb{C}_{\mathrm{M}}$

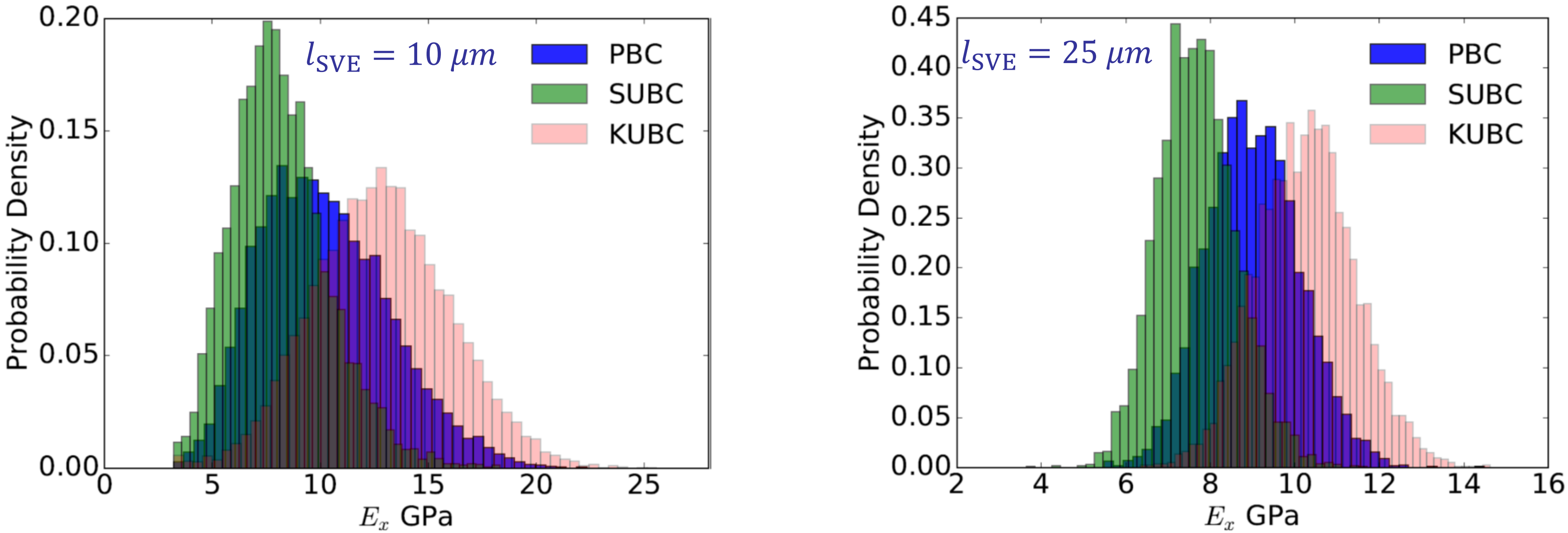
$$\begin{cases} \boldsymbol{\varepsilon}_{\mathrm{M}} = \dfrac{1}{V(\omega)} \displaystyle\int_{\omega} \boldsymbol{\varepsilon}_{\mathrm{m}} d\omega \\ \boldsymbol{\sigma}_{\mathrm{M}} = \dfrac{1}{V(\omega)} \displaystyle\int_{\omega} \boldsymbol{\sigma}_{\mathrm{m}} d\omega \\ \mathbb{C}_{\mathrm{M}} = \dfrac{\partial \boldsymbol{\sigma}_{\mathrm{M}}}{\partial \boldsymbol{u}_{\mathrm{M}} \otimes \boldsymbol{\nabla}_{\mathrm{M}}} \end{cases}$$

    - Consistent boundary conditions:
      - Periodic (PBC)
      - Minimum kinematics (SUBC)
      - Kinematic (KUBC)



Stochastic homogenization on the SVEs

- Apparent properties



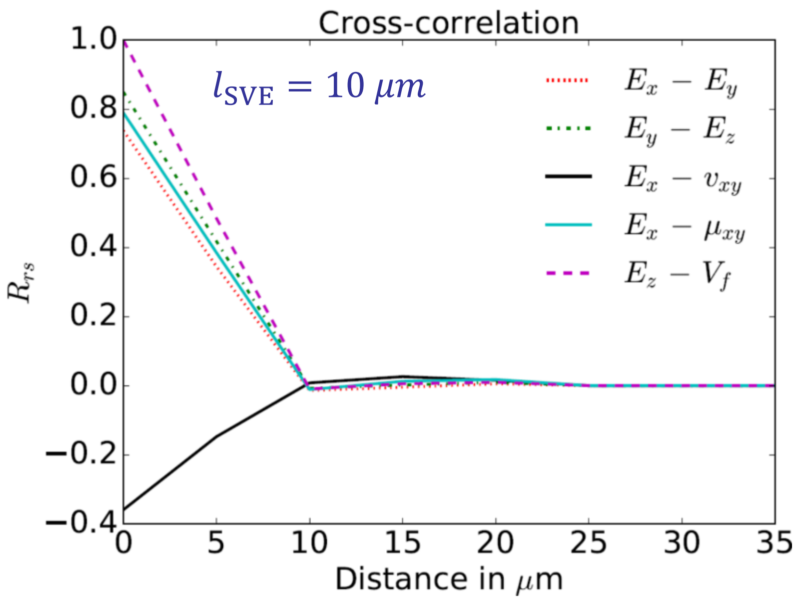
Increasing l_{SVE}

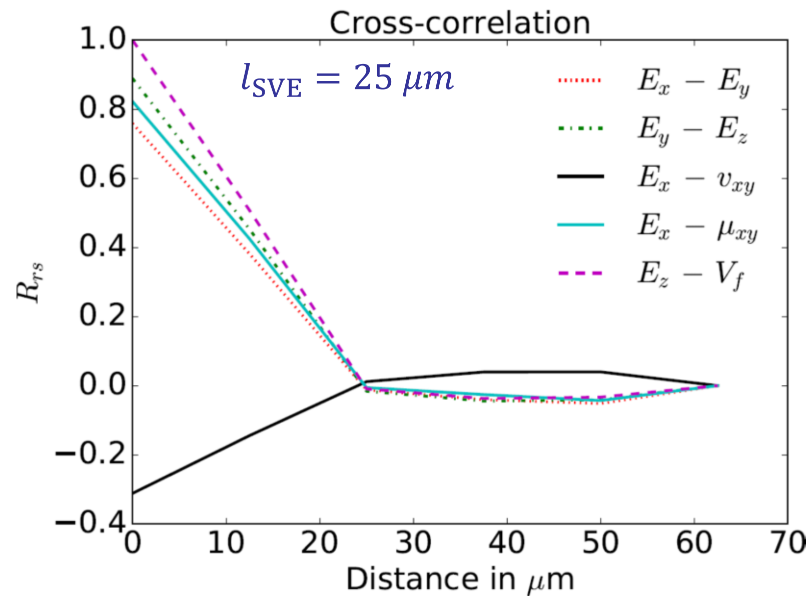
When l_{SVE} increases

- Average values for different BCs get closer (to PBC one)
- Distributions narrow
- Distributions get closer to normal

Stochastic homogenization on the SVEs

- Correlation





Increasing l_{SVE}

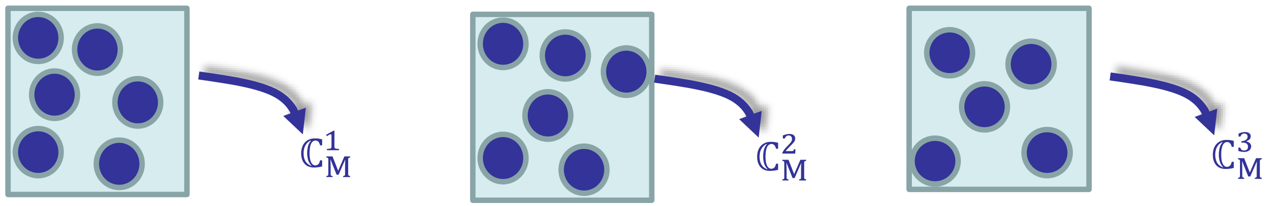
(1) Auto/cross correlation vanishes at $\tau = l_{\mathrm{SVE}}$
(2) When l_{SVE} increases, distributions get closer to normal

(1)+(2) Apparent properties are independent random variables
However the distribution depend on

- l_{SVE}
- The boundary conditions

Stochastic reduced order model

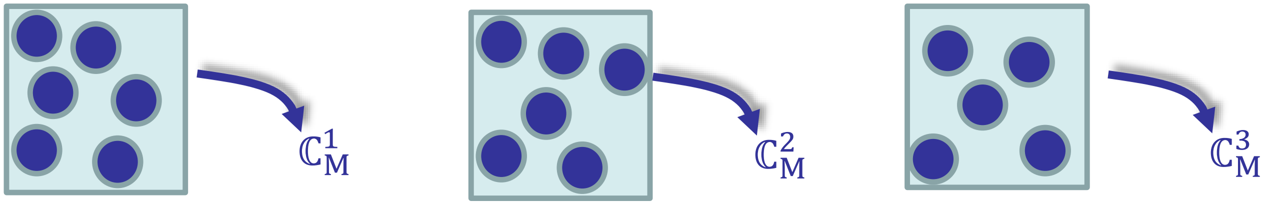
- Stochastic model of the anisotropic elasticity tensor
  - Extract (uncorrelated) tensor realizations $\mathbb{C}_{\mathrm{M}}^{i}$



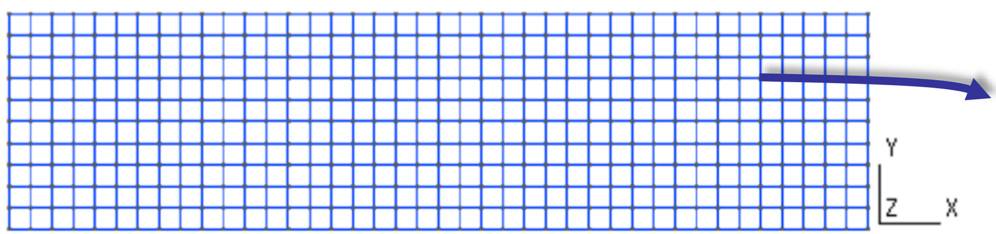
  - Represent each realization $\mathbb{C}_{\mathrm{M}}^{i}$ by a vector $\boldsymbol{\mathcal{V}}$ of 9 (dependent) $\boldsymbol{\mathcal{V}}^{(r)}$ variables
  - Generate random vectors $\boldsymbol{\mathcal{V}}$ using the Copula method

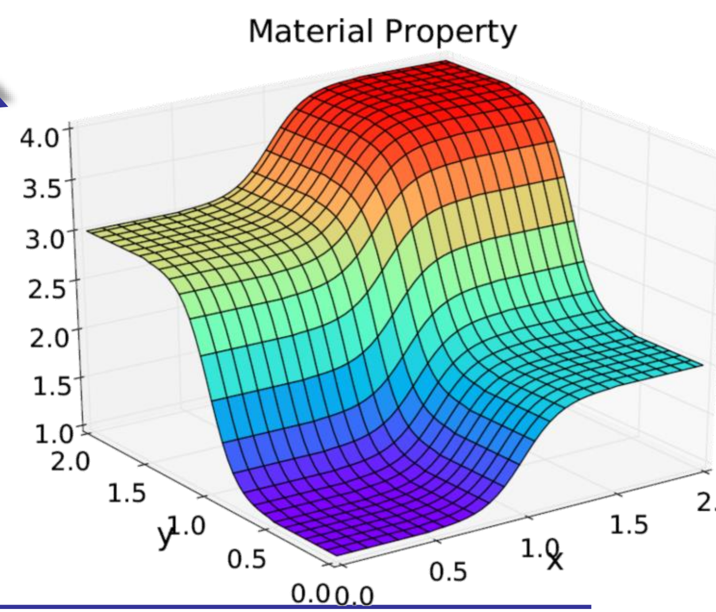
Stochastic reduced order model

- Stochastic model of the anisotropic elasticity tensor
  - Extract (uncorrelated) tensor realizations $\mathbb{C}_{\mathrm{M}}^{i}$

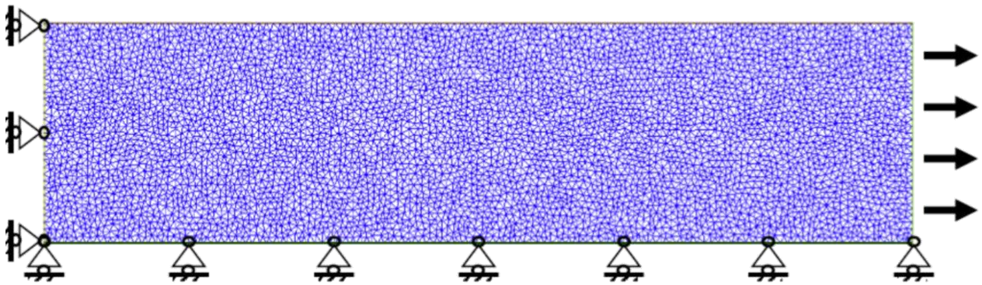


  - Represent each realization $\mathbb{C}_{\mathrm{M}}^{i}$ by a vector $\boldsymbol{\mathcal{V}}$ of 9 (dependent) $\boldsymbol{\mathcal{V}}^{(r)}$ variables
  - Generate random vectors $\boldsymbol{\mathcal{V}}$ using the Copula method
- Simulations require two discretizations
  - Random vector field discretization



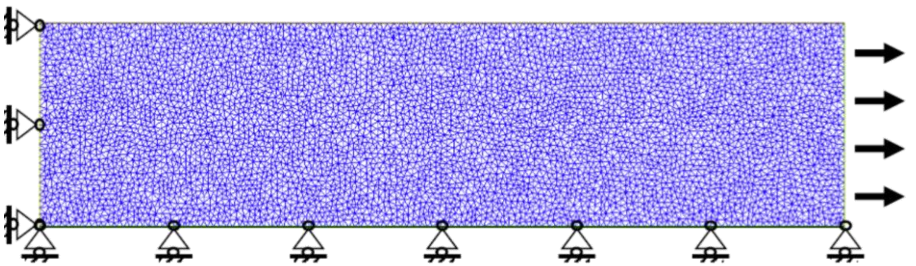


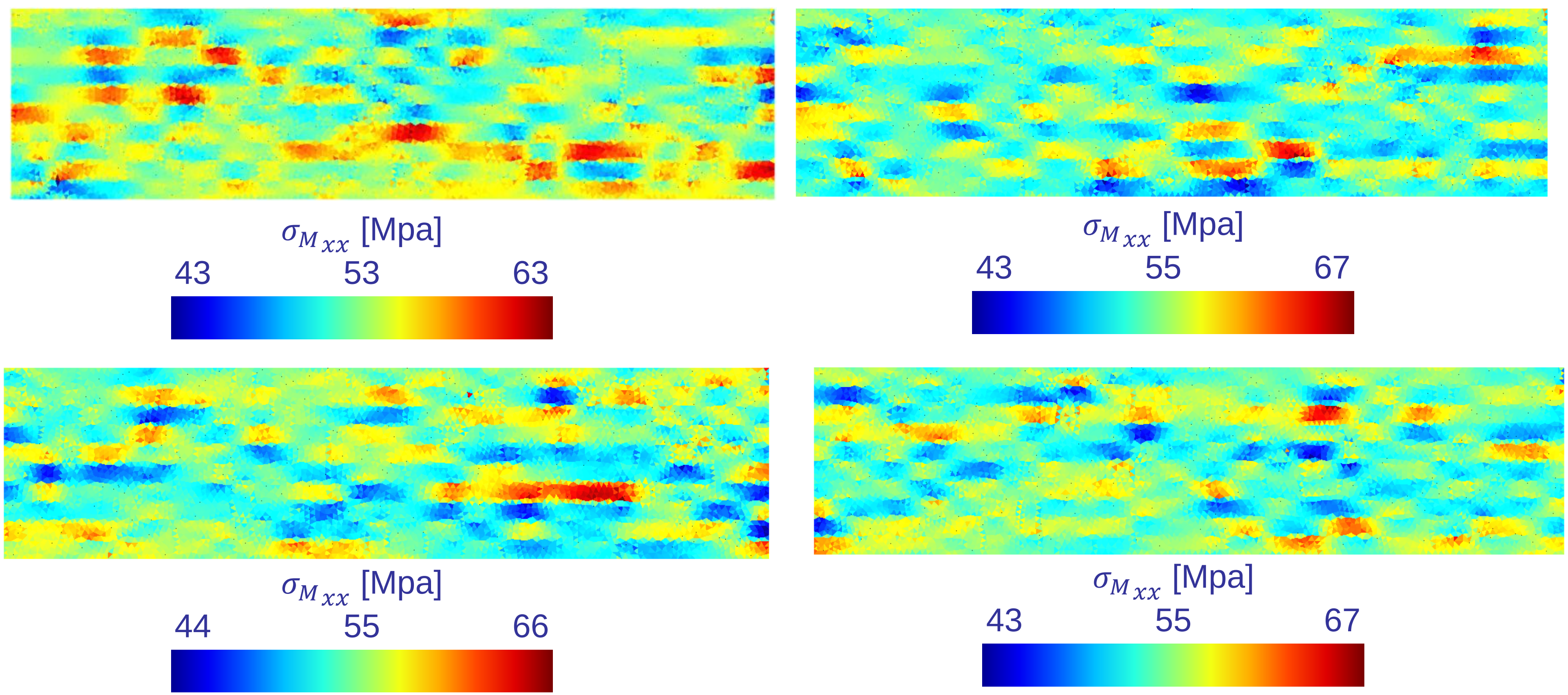
  - Finite element discretization



Stochastic reduced order model

- Ply loading realizations
  - Non-uniform homogenized stress distributions
  - Different realizations yield different solutions



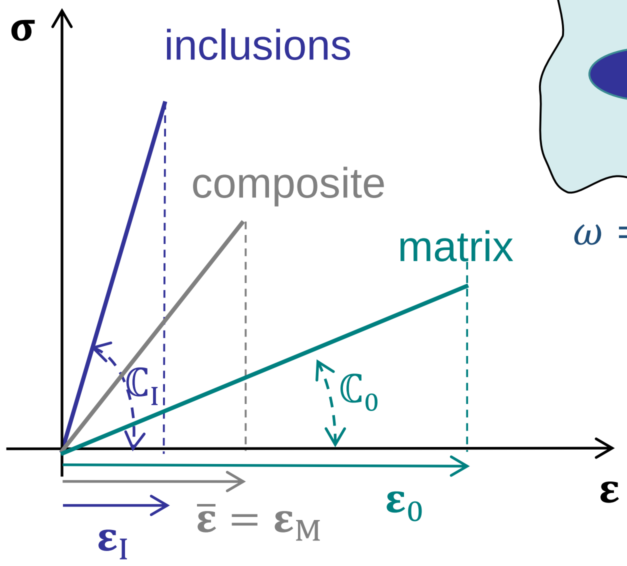


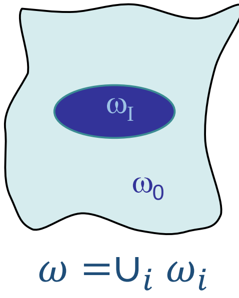
Stochastic Mean-Field Homogenization

- Mean-Field-homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbf{B}^{\varepsilon}\left(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}\right) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$$\Rightarrow \quad \hat{\mathbb{C}}_{\mathrm{M}} = \hat{\mathbb{C}}_{\mathrm{M}}\left(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}}\right)$$





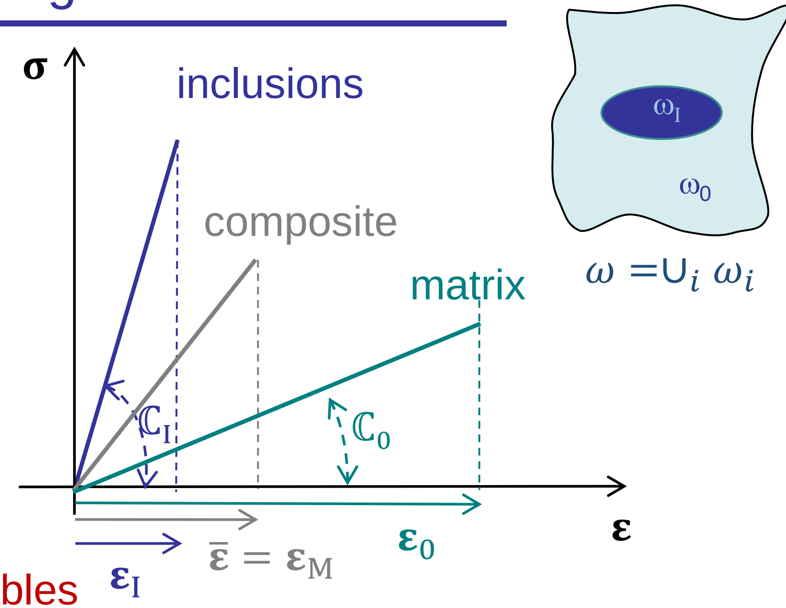
  - We use Mori-Tanaka assumption for $\mathbf{B}^{\varepsilon}\left(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}\right)$
- Stochastic MFH
  - How to define random vectors $\boldsymbol{\mathcal{V}}_{\mathrm{MT}}$ of $\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}}$?

Stochastic Mean-Field Homogenization

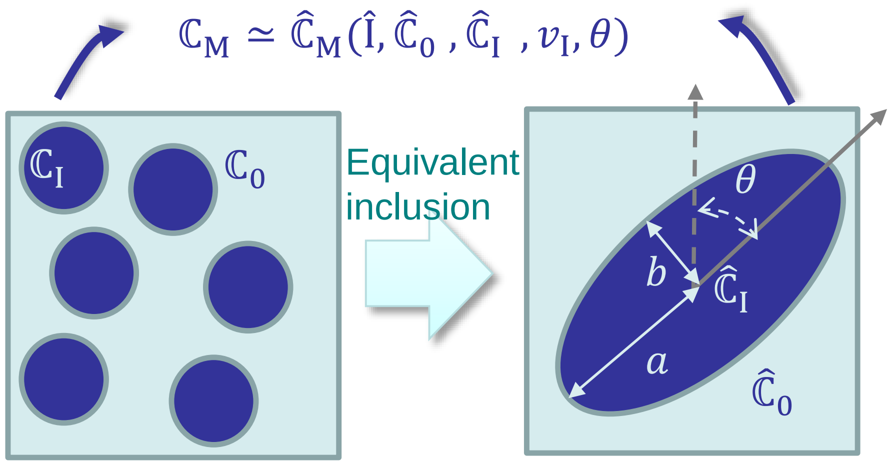
- Mean-Field-homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$\Rightarrow$ $\hat{\mathbb{C}}_{\mathrm{M}} = \hat{\mathbb{C}}_{\mathrm{M}}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}})$ → Defined as random variables



- Consider an equivalent system
  - For each SVE realization i:
    - $\Rightarrow$ $\mathbb{C}_{\mathrm{M}}$ and v_{I} known
  - Anisotropy from $\mathbb{C}_{\mathrm{M}}^i$
    - $\Rightarrow$ θ is evaluated
  - Fiber behavior uniform
    - $\Rightarrow$ $\hat{\mathbb{C}}_{\mathrm{I}}$ for one SVE

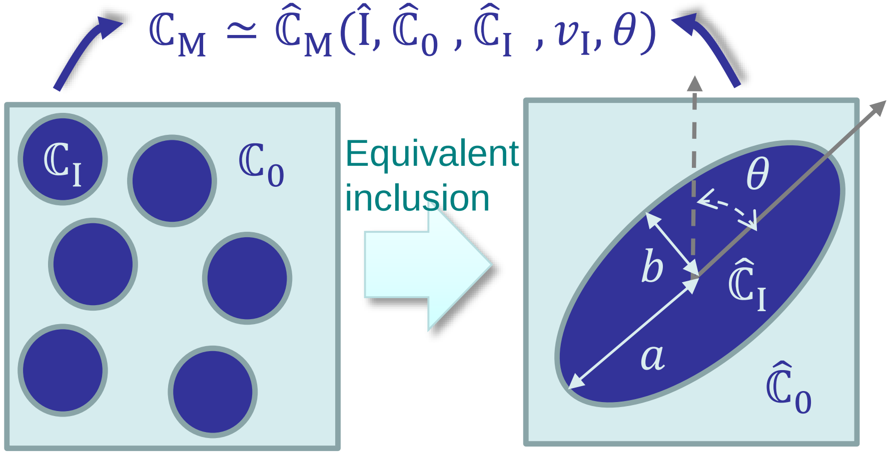


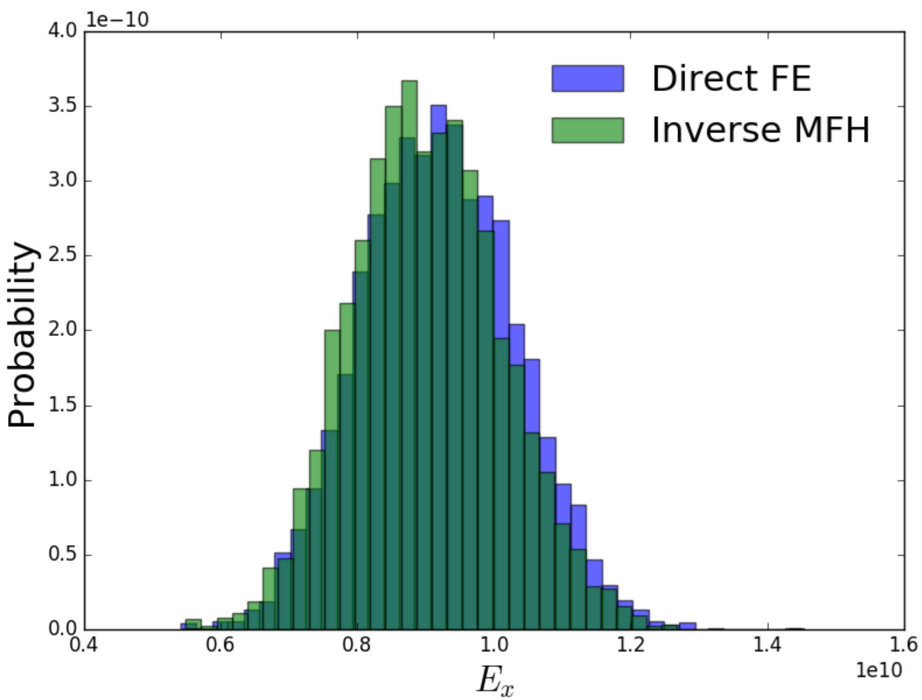
  - Remaining optimization problem:

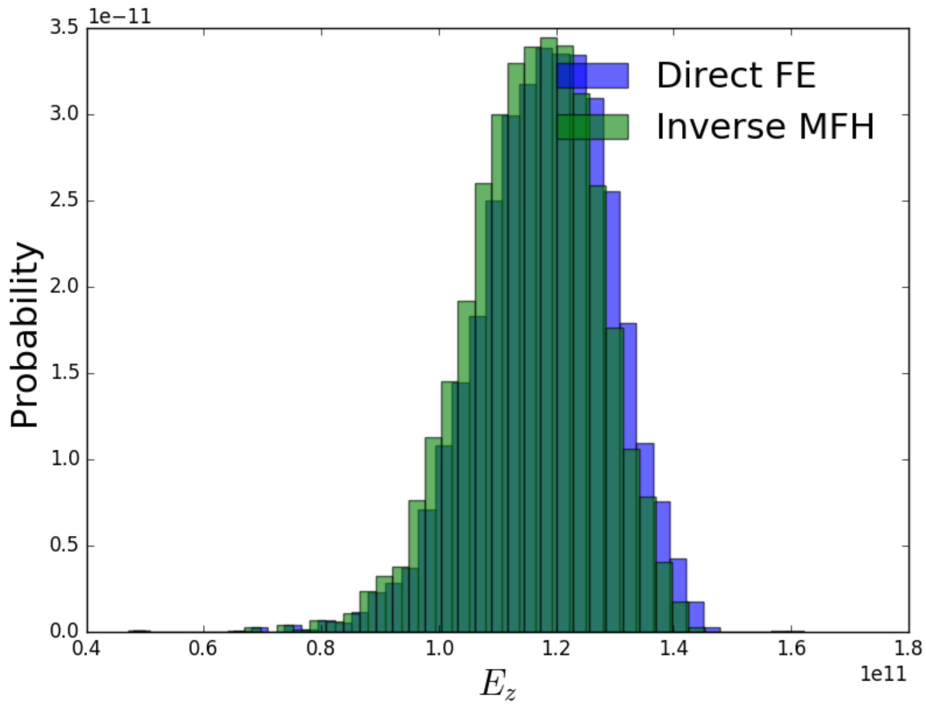
$$\min_{\frac{a}{b}, \hat{E}_0, \hat{\nu}_0} \left\| \mathbb{C}_{\mathrm{M}} - \hat{\mathbb{C}}_{\mathrm{M}}\left(\frac{a}{b}, \hat{E}_0, \hat{\nu}_0 ; v_{\mathrm{I}}, \theta, \hat{\mathbb{C}}_{\mathrm{I}}\right) \right\|$$

Stochastic Mean-Field Homogenization

- Inverse stochastic identification
  - Comparison of homogenized properties from SVE realizations and stochastic MFH

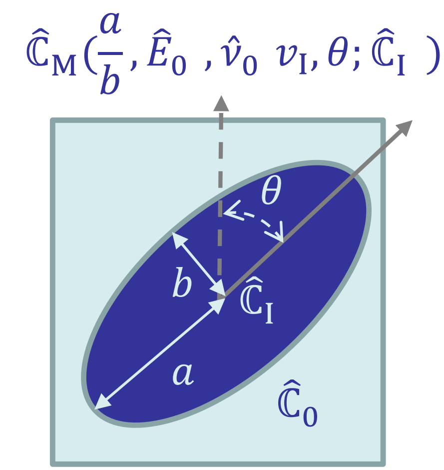






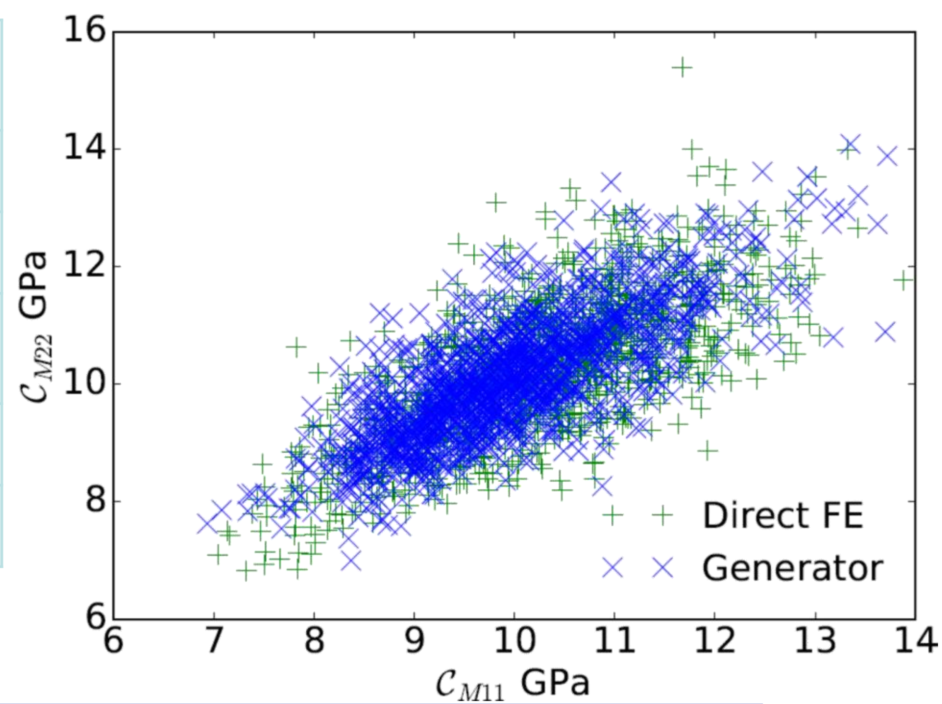
Stochastic Mean-Field Homogenization

- Stochastic MFH model:
  - Homogenized properties $\hat{\mathbb{C}}_{\mathrm{M}}(\frac{a}{b}, \hat{E}_0\ , \hat{\nu}_0\ v_{\mathrm{I}}, \theta; \hat{\mathbb{C}}_{\mathrm{I}}\ )$
  - Random vectors $\boldsymbol{\mathcal{V}}_{\mathrm{MT}}$
    - Realizations $\mathrm{v}_{\mathrm{MT}} = \left\{\frac{a}{b}, \hat{E}_0\ , \hat{\nu}_0\ v_{\mathrm{I}}, \theta\right\}$
    - Characterized by the distance correlation matrix
    - Generator using the copula method

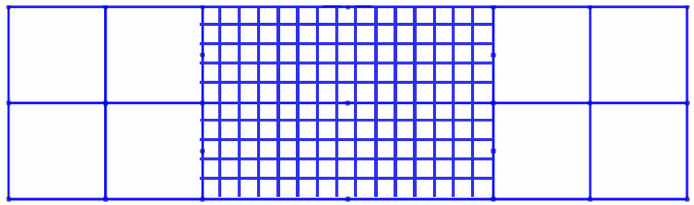


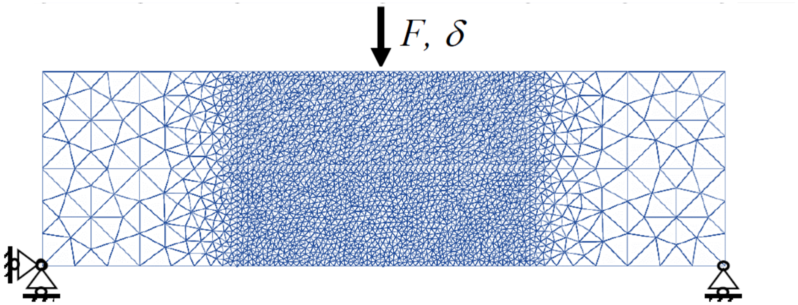
	v_{I}	θ	$\frac{a}{b}$	$\hat{E}_0$	$\hat{\nu}_0$
v_{I}	1.0	0.015	0.114	0.523	0.499
θ		1.0	0.092	0.016	0.014
$\frac{a}{b}$			1.0	0.080	0.076
$\hat{E}_0$				1.0	0.661
$\hat{\nu}_0$					1.0

Distances correlation matrix

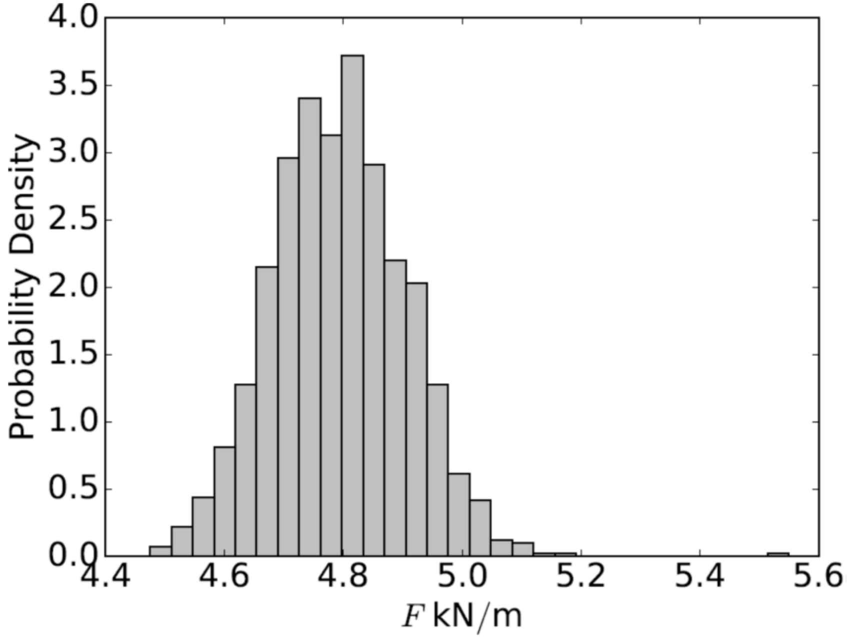


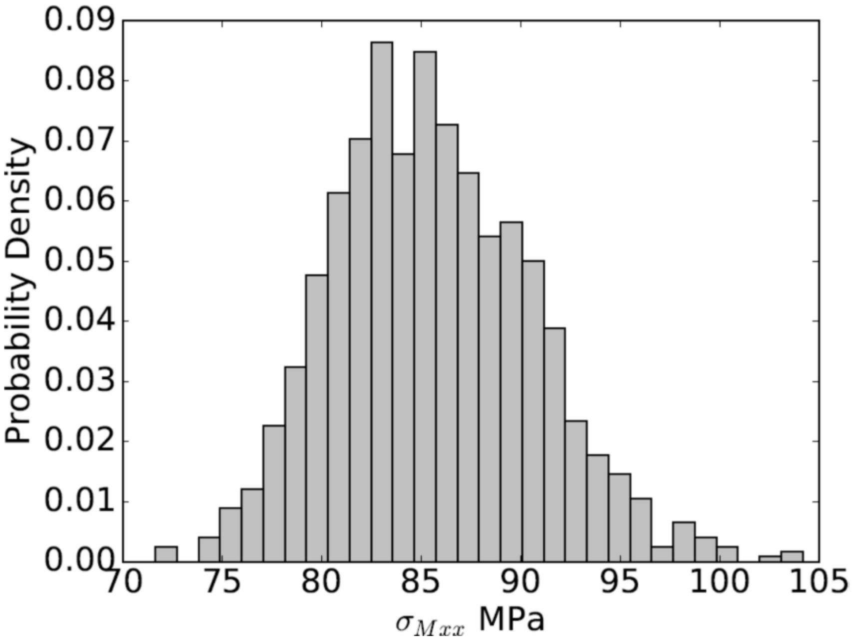
- Stochastic simulations
  - 2 discretization: Random field $\boldsymbol{\nu}_{\mathrm{MT}}$ & Stochastic finite-elements





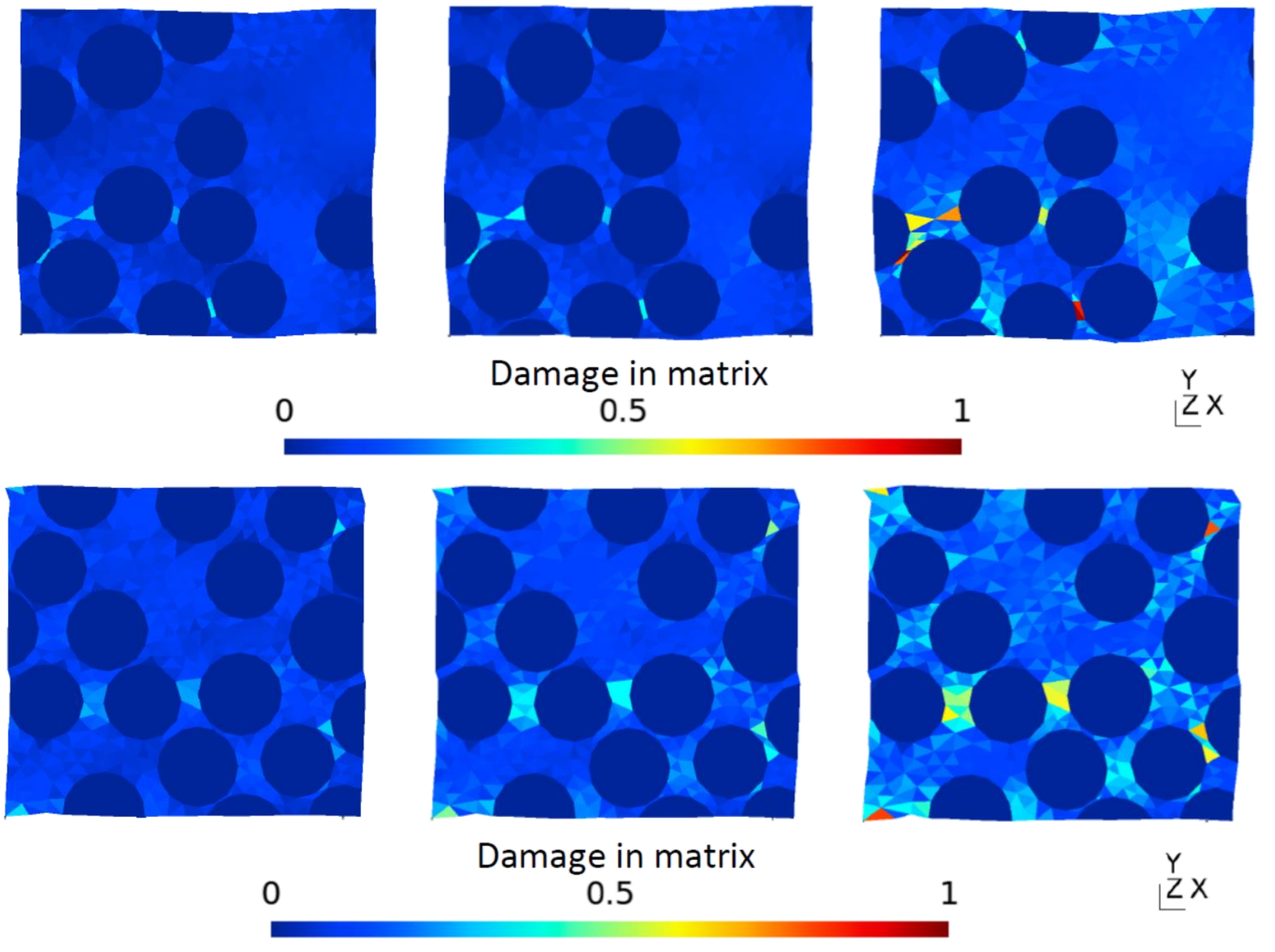
  - Realizations to reach a given deflection $\boldsymbol{\delta}$





Non-linearity

- Non-linear SVE simulations



Non-linear stochastic Mean-Field Homogenization

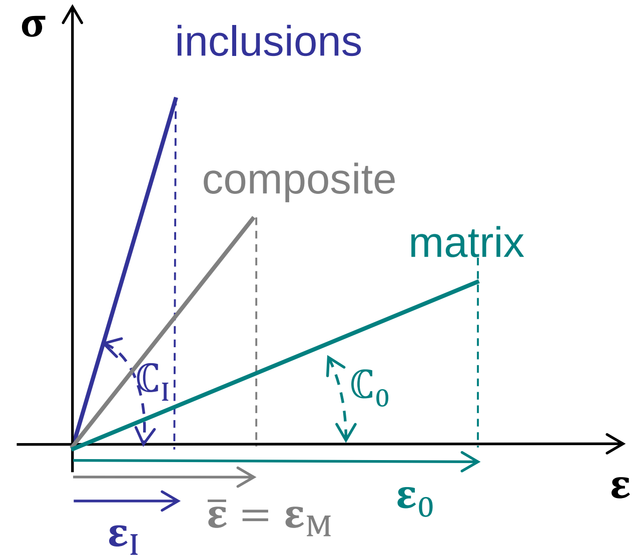
- Non-linear Mean-Field-homogenization
  - Linear composites

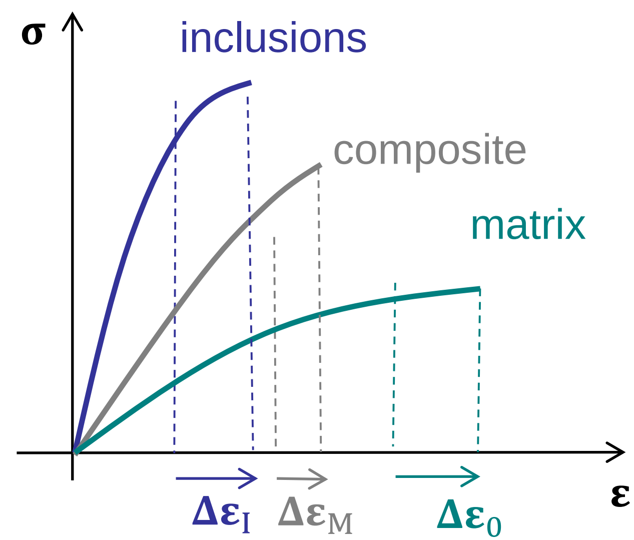
$$\begin{cases} \boldsymbol{\sigma}_M = \bar{\boldsymbol{\sigma}} = v_0\boldsymbol{\sigma}_0 + v_I\boldsymbol{\sigma}_I \\ \boldsymbol{\varepsilon}_M = \bar{\boldsymbol{\varepsilon}} = v_0\boldsymbol{\varepsilon}_0 + v_I\boldsymbol{\varepsilon}_I \\ \boldsymbol{\varepsilon}_I = \mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_I): \boldsymbol{\varepsilon}_0 \end{cases}$$

  - Non-linear composites

$$\begin{cases} \boldsymbol{\sigma}_M = \bar{\boldsymbol{\sigma}} = v_0\boldsymbol{\sigma}_0 + v_I\boldsymbol{\sigma}_I \\ \Delta\boldsymbol{\varepsilon}_M = \overline{\Delta\boldsymbol{\varepsilon}} = v_0\Delta\boldsymbol{\varepsilon}_0 + v_I\Delta\boldsymbol{\varepsilon}_I \\ \Delta\boldsymbol{\varepsilon}_I = \mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0^{LCC}, \mathbb{C}_I^{LCC}): \Delta\boldsymbol{\varepsilon}_0 \end{cases}$$

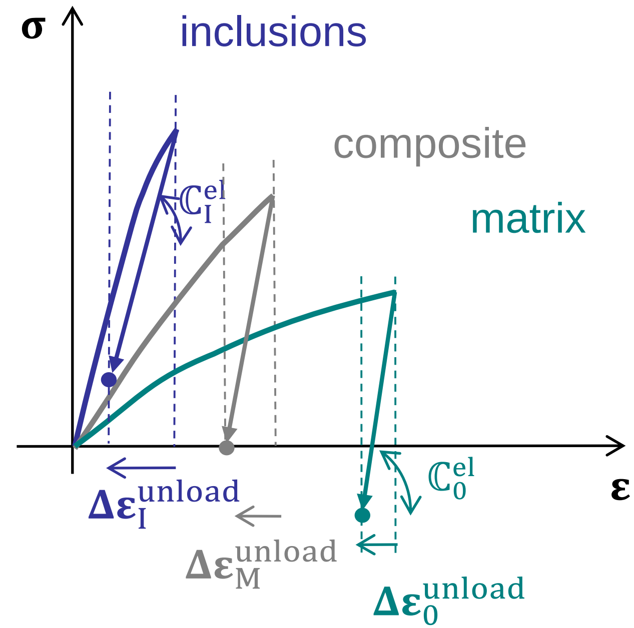
Define a linear comparison composite material





Non-linear stochastic Mean-Field Homogenization

- Incremental-secant Mean-Field-homogenization
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components



Non-linear stochastic Mean-Field Homogenization

- Incremental-secant Mean-Field-homogenization
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components
  - Define Linear Comparison Composite
    - From unloaded state

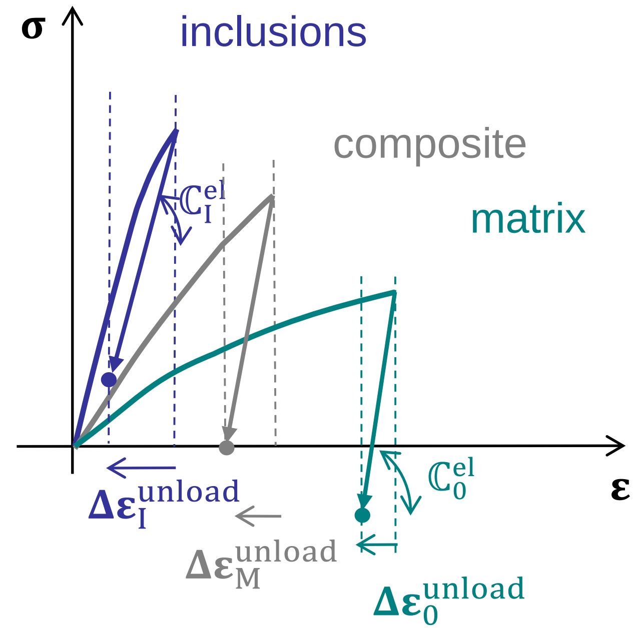
$$\boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_{\mathrm{I}/0} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}/0} + \boldsymbol{\Delta\varepsilon}^{\mathrm{unload}}_{\mathrm{I}/0}$$

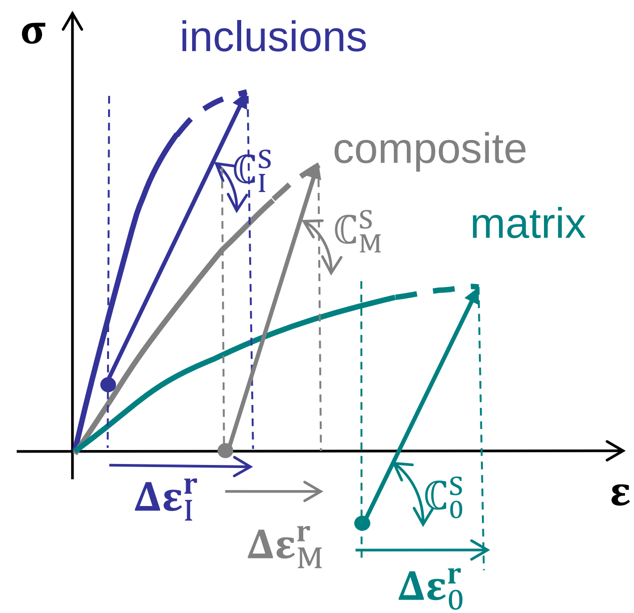
    - Incremental-secant loading

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0\boldsymbol{\sigma}_0 + v_{\mathrm{I}}\boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_{\mathrm{M}} = \overline{\boldsymbol{\Delta\varepsilon}} = v_0\boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_0 + v_{\mathrm{I}}\boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_{\mathrm{I}} \\ \boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_{\mathrm{I}} = \mathbf{B}^{\varepsilon}\left(\mathrm{I}, \mathbb{C}^{\mathrm{S}}_0, \mathbb{C}^{\mathrm{S}}_{\mathrm{I}}\right) : \boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_0 \end{cases}$$

    - Incremental secant operator

$$\Rightarrow \quad \boldsymbol{\Delta\sigma}_{\mathrm{M}} = \mathbb{C}^{\mathrm{S}}_{\mathrm{M}}\left(\mathrm{I}, \mathbb{C}^{\mathrm{S}}_0, \mathbb{C}^{\mathrm{S}}_{\mathrm{I}}, v_{\mathrm{I}}\right) : \boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_{\mathrm{M}}$$

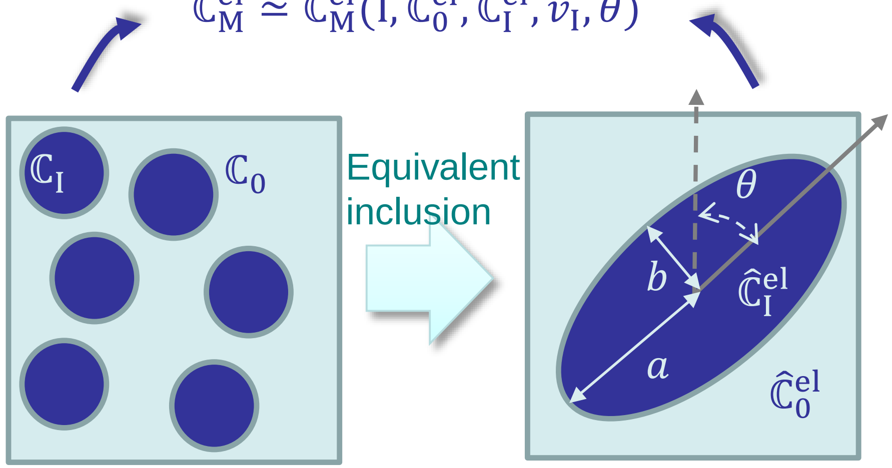


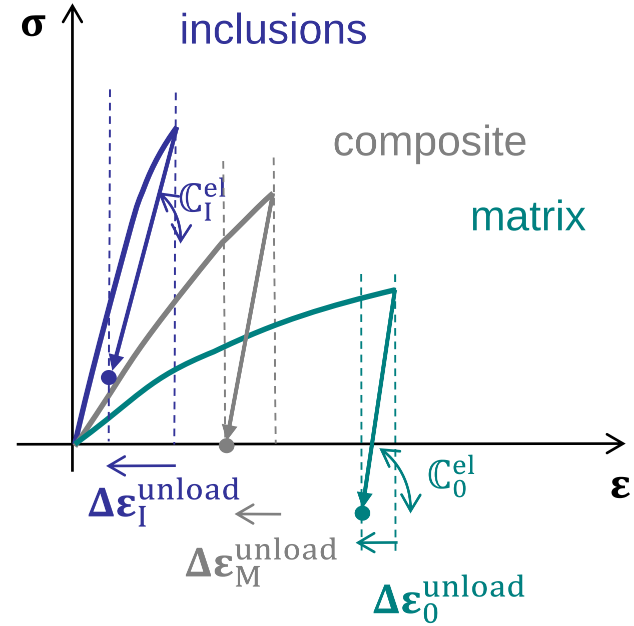


Non-linear stochastic Mean-Field Homogenization

- Non-linear inverse identification
  - First step from elastic response

$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_{0}^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta)$$

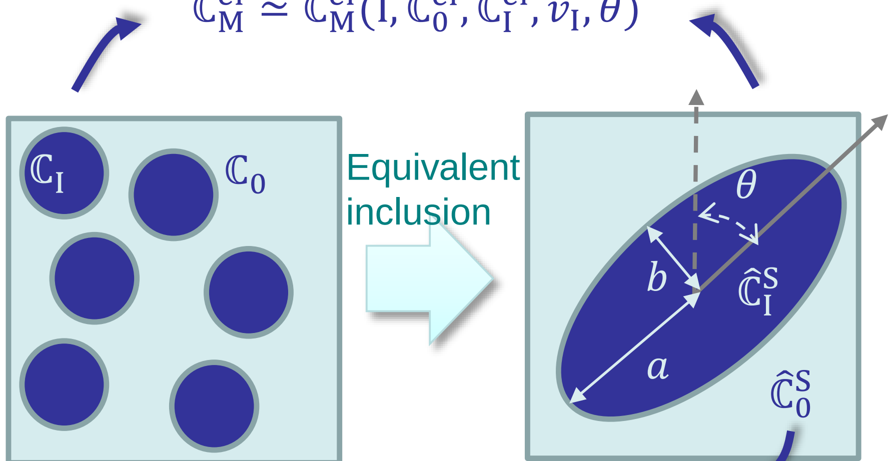




Non-linear stochastic Mean-Field Homogenization

- Non-linear inverse identification
  - First step from elastic response

$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_{0}^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta)$$



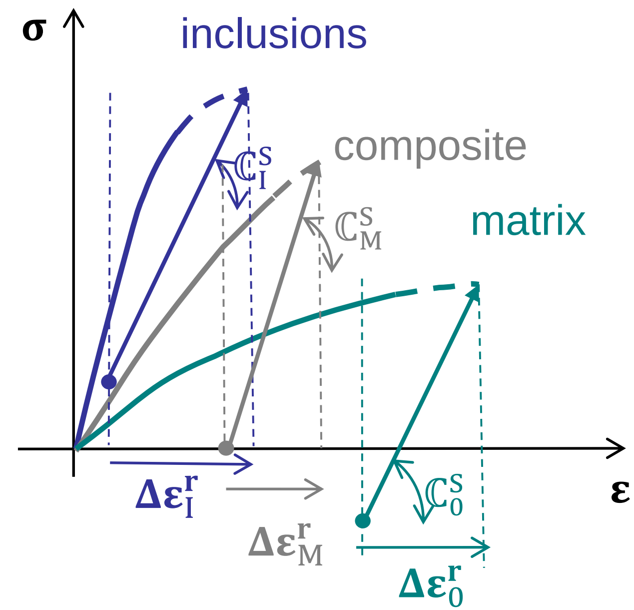
$$\mathbb{C}_{0}^{\mathrm{S}} \simeq \hat{\mathbb{C}}_{0}^{\mathrm{S}}(\hat{R}(\hat{p}_{0}); \hat{\mathbb{C}}_{0}^{\mathrm{el}})$$

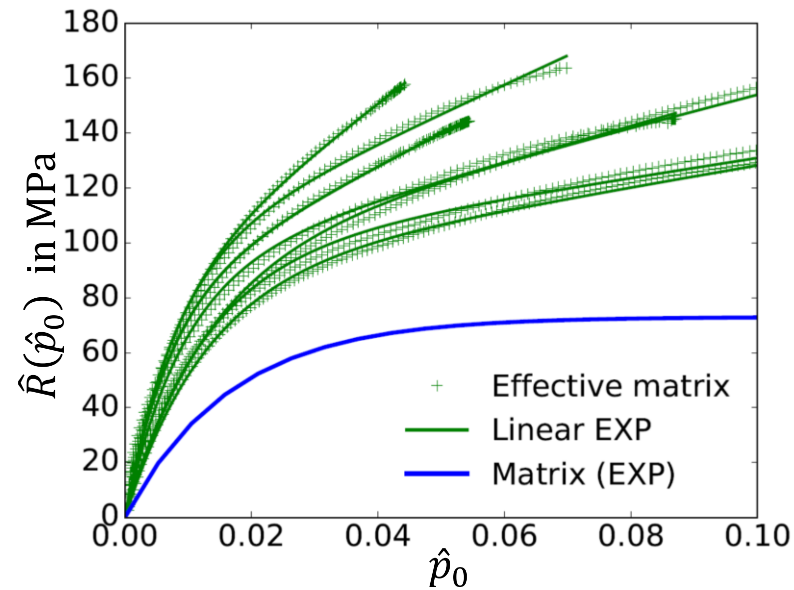
  - Second step from the LCC
    - New optimization problem

$$\boldsymbol{\Delta\sigma}_{\mathrm{M}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{S}}\left(\hat{\mathrm{I}}, \hat{\mathbb{C}}_{0}^{\mathrm{S}}, \mathbb{C}_{\mathrm{I}}^{\mathrm{S}}, v_{\mathrm{I}}, \theta\right) : \boldsymbol{\Delta\varepsilon}_{\mathrm{M}}^{\mathbf{r}}$$

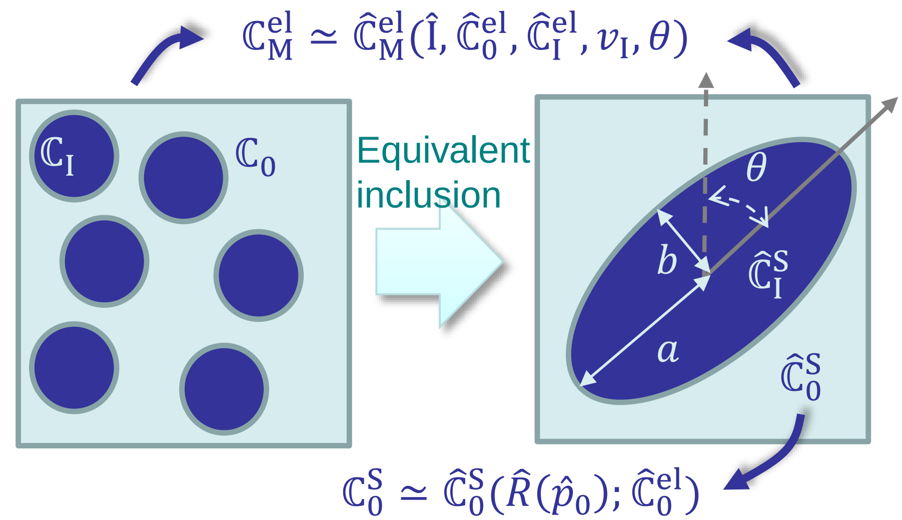
    - Extract the equivalent hardening $\hat{R}(\hat{p}_0)$ from the incremental secant tensor

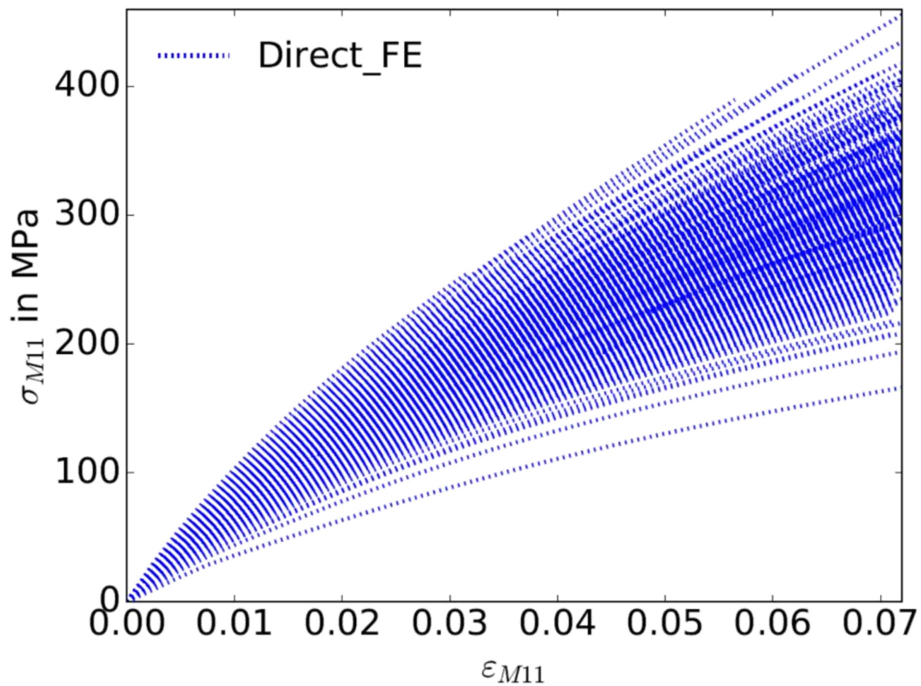
$$\mathbb{C}_{0}^{\mathrm{S}} \simeq \hat{\mathbb{C}}_{0}^{\mathrm{S}}(\hat{R}(\hat{p}_{0}); \hat{\mathbb{C}}_{0}^{\mathrm{el}})$$

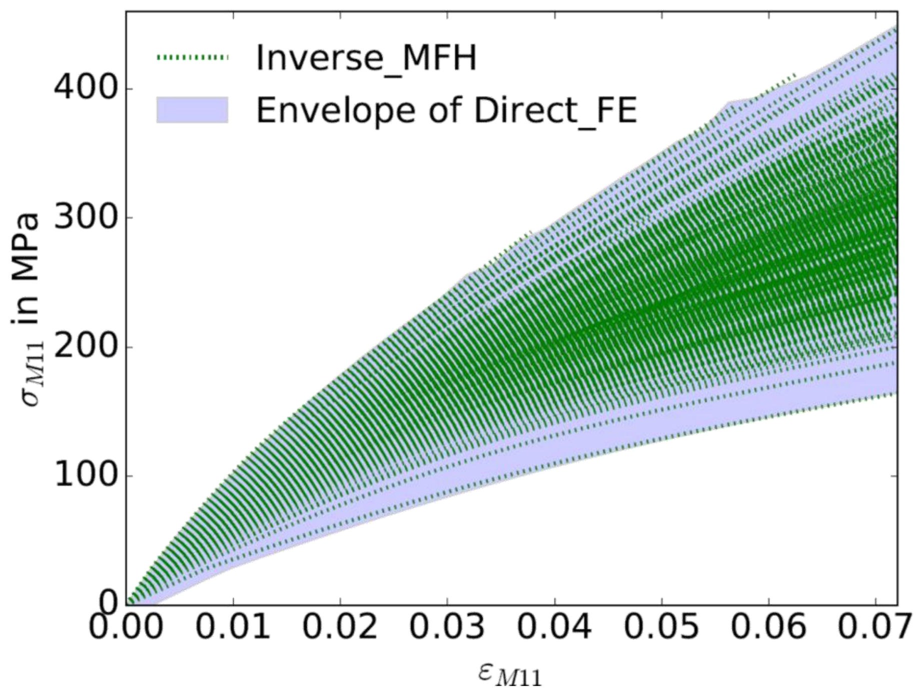




- Non-linear inverse identification
  - Comparison SVE vs. MFH







Conclusions

- Stochastic generator based on SEM measurements of unidirectional fibers-reinforced composites
- Computational homogenization on SVEs
- Definition of a Stochastic MFH method
- In progress: nonlinear and failure analyzes

Thank you for your attention !