Experimental and numerical investigation of a roots expander integrated into an ORC power system

Antoine PARTHOENS1*, Ludovic GUILLAUME2, Olivier DUMONT3, Vincent LEMORT4

1,2,3,4 Thermodynamics Laboratory, Aerospace and Mechanical Engineering Department, University of Liège, Belgium

1a.parthoens@uliege.be
2ludovic.guillaume@uliege.be
3olivier.dumont@uliege.be
4vincent.lemort@uliege.be

* Corresponding Author

ABSTRACT

The performance of internal combustion engines can be improved by valorising the waste heat by means of organic Rankine Cycle power systems (ORC). This paper focuses on an expander of a truck-embedded ORC system. The considered expander is a roots machine. The roots machine is a volumetric machine characterized by a theoretical internal volume ratio of 1. It is typically used as compressor under low pressure ratios (for instance, engine supercharging or air “blowers”).

First, a test rig has been built to perform several tests on the volumetric machine. It is an ORC power system with a typical architecture using R245fa as working fluid (and 5% in mass oil fraction), heated oil as heat source and tap water as heat sink. Maps presenting produced powers, filling factors and isentropic efficiencies versus on one side the pressure ratio (from 1.2 to 4.5) and on the other side the shaft rotational speed (from 1000 to 11000 RPM) are investigated. The maximal delivered power is slightly above 3 kW. Concerning the filling factor the range is between 0.85 and 2.75 and the isentropic efficiency reaches a maximum about 50%. Wet expansions are envisaged leading to a deterioration of the performance.

From the experimental data, a semi-empirical model is calibrated. This model is able to extrapolate the performance outside the experimental operating conditions and identify the different loss sources. Moreover, effects of overheat level and lubricating oil are also envisaged.

The actual tested machine does not have an internal volumetric ratio strictly equal to 1 but is slightly larger. Such volumetric ratio implies that best efficiencies are achieved under small pressure ratios. However, these limited pressure ratios do not lead to large produced powers. To tackle this issue, simulations based on the calibrated model are driven for two expanders in series. This allows to increase the global internal volumetric ratio and shift the best performance towards higher pressure ratios. To enhance either the efficiency or the output power, the intermediate pressure (i.e. the pressure between the two expanders in series) is numerically optimized.

1. INTRODUCTION

1.1 Background and work organization

Till date, the roots machine has not been largely investigated in expander mode and technical and scientific literature on that topic is lacking. The aim of this paper is to enrich this topic with a new study on this technology. After a short state of art, the results of an experimental investigation composed of 75 different steady-state points are presented. Based on these results, a semi-empirical model (i.e. model describing the main physical phenomena and requiring only a restricted number of parameters) is calibrated. To enhance the performance of the machine, some simulations are driven considering two expanders in series.

1.2 State of art
The roots machine is generally used as a compressor but can, however, be used as an expander. It is a positive displacement (or volumetric) machine meaning that the working fluid pressure is either increased (pump/compressor) or decreased (expander) by increasing or decreasing its volume. The roots expander is very similar to the twin-screw expander. Indeed, it also consists in two rotors having two or more lobes. The two rotors are rotating at the same speed but in opposite direction and must be maintained in a suitable relative position to always keep a fine clearance between the housing and the lobes (Ritchie & Patterson, 1968). As the working fluid is entering the suction chamber with a high pressure, the rotating lobes drive it to the outlet which is at a lower pressure. Theoretically, there is no expansion within the machine. The fluid is only driven from a high to a low pressure. It leads to a theoretical internal volumetric ratio ($r_i$) equal to 1, which is the main difference with the screw expander (Lemort & Legros, 2017).

The roots compressor is a relatively old technology which was developed in the 1860’s by Philander and Francis Roots brothers (Superchargers, 2017). Since it has evolved to enhance its performances. Concerning this machine used as an expander, there is currently no known practical application and, to the author knowledge, analytical or experimental performance has never been published. However, this volumetric machine used as a compressor is more usual and some studies were already driven. Due to the absence of volumetric compression, there is an important drawback on the machine which is a limited operating pressure ratio. This could be understandable remembering that the pressure increase is almost achieved at a constant machine volume. Practically the imposed pressure ratios are generally bounded between 1.4 and 1.6 (Vizgalov et al., 2015). Vizgalov et al. (2015) developed a mathematical model of such a machine and deduced the volumetric and isentropic efficiencies, varying the pressure ratio between 1.25 and 1.87. They observed that both efficiencies are inversely proportional to the pressure ratio. The volumetric efficiency (which is an image of the filling factor) went from about 88 to 83% and the isentropic efficiency from about 84 to 66%. In light of this model, the blower’s pressure ratio is obviously a major operating variable and to keep admissible performance, it should be kept low. Patterson and Ritchie (1969) varied the rotational speed in a range of about 1500 to 5000 RPM and the pressure ratio between 1.1 and 1.6. The maximal theoretical and experimental efficiencies reached were respectively about 45% and 43%, requiring a maximum power of 5 kW.

2. EXPERIMENTAL INVESTIGATION

2.1 Test rig description

The studied expander is called roots expander but it does not have an internal volumetric ratio strictly equal to 1. Indeed, as in a screw machine, the chamber size changes with the rotor position during a cycle, allowing a small expansion within the machine. This means that it has even more similarity with the screw expander than a classical roots. Its volumetric ratio is at first roughly estimated between 1 and 1.15. From here, the roots appellation will be kept. The studied prototype was externally synchronized and made of aluminum. The expander’s shaft was coupled to a motor/generator through a torquemeter. Knowing the shaft speed and its torque, the mechanical power delivered (or possibly consumed) was deduced. The expander’s rotation speed was piloted via a 4-quadrant frequency converter.

The roots expander was tested in an ORC power system test bench. The schematic of the test rig and the position of the sensors are represented in Figure 1 and the sensors’ basic characteristics are summed up in Table 1. The heat exchangers, the pump and the liquid receiver were described by Guillaume et al. (2016). The experimental conditions simulated the heat recovery of exhaust truck gases. These gases were represented by the means of a 150kW$_{th}$ electrical heater where thermal oil was pumped.

Oil was used to ensure the expander’s lubrication. The exact amount injected inside the circuit was estimated to 7.5% (~40 kg R245fa, ~3kg oil). However, the oil was not only present around the expansion machine but was spread out in the whole cycle. Some oil traps could had taken place, especially in the heat exchangers. No technique was implemented to evaluate the oil fraction at a specific test-rig position. An assumption was thus made for the rest of the analysis. The mass fraction of oil in the mixture going throughout the expander was considered as constant and equal to 5%. The mass flow rates are expressed as:

$$M_{\text{oil}} = X_{\text{oil}} \cdot M_{\text{tot}}$$

(1)

$$M_{\text{wf}} = (1 - X_{\text{oil}}) \cdot M_{\text{tot}}$$

(2)

where $X_{\text{oil}}$ is the oil mass fraction.
2.2 Tests description

The first aim of the experimental campaign was to characterize the expander. Three quantities were investigated, namely the isentropic efficiency, the shaft power and the filling factor, respectively defined by Equations (3), (4) and (5).

\[
\varepsilon_{is} = \frac{W_{sh}}{M_{wf} \cdot (h_{su} - h_{ex,is}) + V_{oil} \cdot (P_{su} - P_{ex})}
\]

\[
W_{sh} = \frac{2\pi}{60} \cdot N \cdot \tau
\]

\[
\Phi = \frac{V}{V_g} = \frac{M_{tot} \cdot \rho}{N \cdot \text{disp}}
\]

The experimental investigation allowed to gather directly some data about the machine and secondly to calibrate a semi-empirical model. To achieve this, a large range of conditions were envisaged. A total of 75 points were recorded in steady-state conditions. The ranges of the measured data are summed up in Table 2. All the measurements were reconciled with the method described by Dumont et al. (2016). The physical constraints used to apply the method were:

\[
M_{tot} \cdot (h_{exp,su} - h_{exp,ex}) = W_{sh} + Q_{amb}
\]

\[
Q_{amb} \geq 0
\]

The uncertainty propagation on quantities computed from the measured data was derived from the Equation (8). This equation relies on the assumption that the measurements are not correlated and the errors are random (Taylor & Kuyatt, 1994).

\[
U_y = \sqrt{\sum \left( \frac{\partial y}{\partial x_i} \right)^2 \cdot U_{x_i}^2}
\]

where \(U_y\) is the desired uncertainty of the calculated quantity \(y\) and \(U_{x_i}\) the uncertainty of the measured quantity \(x_i\).
Rotational speed | RPM | 1047 (10%) | 10992 (1%)
--- | --- | --- | ---
Torque | Nm | -0.22 (114%) | 10.4 (2%)
Shaft power | W | -217 (120%) | 3057 (4%)
Mass flow rate | g/s | 75 (0.15%) | 393 (0.15%)

2.3 Results
To have a consistent analysis, the impact of the pressure ratio and the speed must be simultaneous. The isentropic efficiency defined as in Equation (3) is represented as a function of those parameters in Figure 2.

![Figure 2: Impact of the pressure ratio and rotational speed on the isentropic efficiency](image)

Most of the tested points revealed an efficiency between 30 and 40% and reached a maximum above 50% for a pressure ratio around 1.75. Globally, the best efficiencies were reached for the small pressure ratios. However, in these conditions, for the highest tested speeds, it dropped dramatically. The combination of low pressure ratio and high speeds was not efficient.

The general trend is that the efficiency decreases linearly while the pressure ratio increases. Concerning the rotational speeds, there was a maximum between 4500 and 5000 RPM. For higher speeds (above 8000 RPM), the machine performance were poor, leading to negative efficiencies.

Another important characteristic for most of the applications is the mechanical power delivered by the machine. As for the efficiency it is represented as a function of the pressure ratio and the rotational speed.

![Figure 3: Impact of the pressure ratio and rotational speed on the shaft power](image)

Figure 3 revealed a maximum power for rotational speeds around 5000 RPM and pressure ratios around 3.2. In these conditions, the power measured was just above 3 kW. For high speeds and low pressure ratios, the power dropped significantly and could even be negative. This means that the generator worked as a motor and supplied power to the expander. For mechanical resistance reasons, no test was done for high speeds and high pressure ratios.

The last characteristic to be studied was the filling factor. It characterizes the effect of internal leakage, pressure drop and heat transfer within the expander.
The rotational speed seemed to have a big impact while the pressure ratio did not. Indeed, for a fixed speed, if the pressure ratio varied, the filling factor change was moderate. The trend was clear, slower the expander, the higher the filling factor. If the machine was going slowly the expander was more subject to leakages, which is physically consistent. Then, at higher speeds, the drop losses were more important. Another aspect is that the pressure ratio had a more important impact while considering small rotational speeds.

For similar conditions and configuration, Lemort et al. (2009) found, with a scroll expander, measured efficiencies between 42.4 and 68%, developing between 382 and 1820 W of mechanical power. For pressure ratios above 2.5, the scroll seemed to be more efficient. They found filling factors between 1.067 and 1.336 which was of the same order of magnitude. Nevertheless, here the extremes were more acute. It is explained because of the speed ranges were narrower with Lemort et al. (2009), going between 1771 and 2660 RPM. Ziviani et al., (2015) did the same exercise with a single-screw expander. It was for a bigger rated power (up to 7.3 kW) but the results were of the same order of magnitude. Indeed, they found a filling factor ranging from 1.038 to 1.331 and isentropic efficiencies 20.58 to 51.91% knowing that their lowest pressure ratio was also bigger than the one observed in this present experimental campaign.

Another aspect that was experimentally investigated was the wet expansions. The goal here was to compare performance of the machine, with an overheat and with different vapor qualities. To keep a fixed pressure ratio, the mass flow rate was tuned. As the entire ORC loop was well lubricated and as the working fluid is by nature lubricant too, the machine may not encounter any problem during these tests. The inlet pressure was set constant at 4.1 bar for all tests. Then a speed was fixed and data is recorded for an overheat of 30 K, vapor qualities of about 90%, 70% and 50%. The same exercise was realized for four different rotational speeds: 1500, 3000, 5000 and 7500 RPM. The inlet quality cannot be measured as a temperature or as a flow rate, via a sensor. Furthermore, as the fluid is inside the saturation curve, the pressure and the pair pressure temperature is not sufficient to determine the quality. The strategy adopted was to make an energy balance on the evaporator to know the working fluid enthalpy at the evaporator’s outlet and assuming that is was the same as the expander inlet. This lead to relatively high uncertainties on measurements, however, some trends could be deduced.

The trend in Figure 5 is clear, the smaller the quality, the poorer the efficiency and the produced power. As reminded before, for a fixed speed and pressure ratio, the mass flow rate was tuned. To obtain a smaller vapor quality, the mass flow rate had to be increased, inducing higher pressure drops. The machine was not adapted to these conditions and dry expansion were preferred.
3. MODEL PRESENTATION (SINGLE + SERIES)

3.1 Single expander model calibration

Based on the experimental data previously presented, a semi-empirical model was calibrated. The equations were described by Lemort et al. (2009) The global steps were: 1. isenthalpic inlet pressure drop, 2. supply heat losses, 3. adiabatic reversible and adiabatic isochoric expansion, 4. adiabatic mixing between leakages and expanded fluid, 5. exhaust heat losses, 6. isenthalpic outlet pressure drop. It should be noted that the model proposed by Lemort et al. (2009). did not take into account the outlet pressure drop which is added.

The model presents parameters of two different types. The first were fixed constant because they depend on the machine, on its geometry or on the ORC setup. The others were free to be set-up in order to fit the model predictions to the experimental data. Those are listed in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td>Oil fraction</td>
<td>$X_{oil} = 0.05$</td>
<td>-</td>
</tr>
<tr>
<td>Built-in volumetric ratio</td>
<td>$R_{v,in} = 1.12$</td>
<td>-</td>
</tr>
<tr>
<td>Machine displacement volume</td>
<td>$V_{s} = 0.0001$</td>
<td>m³</td>
</tr>
<tr>
<td>Nominal mass flow rate</td>
<td>$M_{tot,n} = 0.5$</td>
<td>kg/s</td>
</tr>
<tr>
<td>Nozzle diameter pressure drop (supply)</td>
<td>$d_{si} = 0.014$</td>
<td>m</td>
</tr>
<tr>
<td>Nozzle diameter pressure drop (exhaust)</td>
<td>$d_{se} = 0.02$</td>
<td>m</td>
</tr>
<tr>
<td>Nozzle cross section leak area</td>
<td>$A_{leak,0} = 3.25E-5$</td>
<td>m²</td>
</tr>
<tr>
<td>Mechanical efficiency</td>
<td>$\eta_{m} = 0.88$</td>
<td>-</td>
</tr>
<tr>
<td>Global heat transfer coefficient (supply)</td>
<td>$AU_{su,n} = 17$</td>
<td>W/K</td>
</tr>
<tr>
<td>Global heat transfer coefficient (exhaust)</td>
<td>$AU_{se,n} = 14$</td>
<td>W/K</td>
</tr>
<tr>
<td>Global heat transfer coefficient (ambiance)</td>
<td>$AU_{amb,n} = 7$</td>
<td>W/K</td>
</tr>
</tbody>
</table>

Those parameters lead to a model calibration that is represented in Figure 6. The measured data was set against the model output regarding the rotational speed, the filling factor, the shaft power and the working fluid exhaust temperature.

Figure 6: Model calibration results

Rotational speed which was mainly tuned with the supply pressure drop and the leakages area parameters showed a good trend with the major part of the points under 10% deviation and a maximum difference under 20%. The filling factor was mostly impacted by the leakage area as well as the supply pressure drop tuning. Its deviation order magnitude was the same as the rotational speed. Filling factors seemed to be slightly under-estimated when small
while they were over-estimated when big. The shaft power tuning relied on the one hand on the losses parameters and on the other hand on the rotational speed. For some specific points, the difference between the experimental data and the model was bigger. One explanation is the impact of the torque meter poor accuracy for small torques, which could imply a quite high uncertainty on the measurements. However, most of the points showed a difference lower than 15%. The working fluid exhaust temperature was mainly impacted by the thermal coefficients. It was well predicted with a maximum error under 3 K. Regarding these outputs, the model was considered as calibrated and reliable.

### 3.2 Single expander model results

Based on the calibrated model, two maps were generated, representing either the isentropic efficiency or the shaft power as a function of the expander’s speed and the pressure ratio.

![Figure 7: Influence of pressure ratio and rotational speed on the isentropic efficiency (left) and on the produced power (right)](image)

The goal of the extrapolation was mainly to predict performances in conditions that were not reachable with the test rig that were used. However these conditions had to be realistic for the tested machine. The first comment is that no simulation was made for combinations of high speeds and high pressure ratios. In practice, these conditions cannot take place for mechanical resistance reasons. As the experimental results suggested, best efficiencies were for small pressure ratios and speeds around 4000 RPM. However these conditions did not lead to high produced power. To increase the latter, higher pressure ratios had to be considered.

Best efficiencies were for small pressure ratios because of the internal volumetric ratio which is very small. It was set here to 1.12 whereas other volumetric expansion machines have internal volumetric ratio between, for instance, 2.85 (compliant scroll) and 10.43 (piston), (Lemort et al., 2018). If considering a perfect gas expanded in an isentropic way, the adapted pressure ratio is:

\[
r_{P,\text{adapted}} = \frac{r}{\gamma} = 1.136
\]

where \(\gamma\) is the isentropic exponent set to 1.124. For high pressure ratio, there was over-expansion leading to the fast decrease of the efficiency.

### 3.2 Series expander model

Since the single expander model revealed that the best efficiency are achieved for small pressure ratios leading to small delivered powers, two expanders were put in series, increasing the global internal volumetric ratio. As the adapted pressure ratio increased, it was closer to pressure ratios leading to higher delivered powers. Figure 8 shows the specific nomenclature used in this section.

To get the best performances, the inlet and intermediate pressures, namely \(P_{su}\) and \(P_{int}\) were optimized to maximize the isentropic efficiency. Quoilin (2011) showed analytically that for a given inlet and outlet pressure, the intermediate pressure can be optimized to maximize the efficiency for two scroll expanders in series. If the efficiency were only function of the pressure ratio, the optimum intermediate pressure ratio would be defined as the square root of the global pressure ratio. Nevertheless, the efficiency is not only dependent on the pressure ratio. Indeed, the efficiency also relies on the losses that vary, among others, with the flow rate. Quoilin (2011) described the efficiency via polynomial laws which made the analytical optimization cheap. In the frame of this paper, the efficiency was not described as polynomial laws so a numerical optimization taking the form of a minimization problem was implemented. The Matlab \textit{fmincon} solver was used with the following constraints:

\[
P_{su} > P_{int} > P_{ex}
\]
with \( T_{\text{max}} = 115^\circ \text{C} \), which is the maximal temperature admissible for mechanical resistance. As the supply and intermediate pressures varied, the outlet exhaust pressure was set constant. The choice of 2 bar was made because it was a mean value encountered for condensing pressure during the experimental campaign and in many practical applications. Additionally, the mass flow rate has been imposed and the two different rotational speeds were deduced from the model. The second machine showed a higher rotational speed as the volumetric flow rate was bigger.

From Figure 9, different conclusions could be drawn. The maxima for the machines in series were achieved for higher pressure ratios due to the higher internal volumetric ratio. The maximum for the series machines efficiency was lower than for a single machine. This is explained because of the inlet pressure drop losses. The two considered machines were of the same size, meaning that the second one was not optimized. As it had to rotate faster and as the fluid at the inlet had a larger density it was more subject to losses. This leads to the other curves of Figure 9, representing the same optimization removing the inlet pressure drop losses. In this configuration, the maxima were obviously higher and the series machines took the advantage.

Even if the maximum efficiency is smaller for machines in series, the developed power is still bigger. Figure 10 shows the shaft power as a function of the evaporator’s power. The heating power variation was simulated with a flow rate increase and imposing a constant overheat of 15 K.

The increase with the evaporator’s power was nearly linear. Depending on the regime, the increment varied but based on this simulation, the average was an increase of about 45%, which is significant. Having two machines in series was thus profitable to extract power, however, the global efficiency was degraded compared to a single machine.

5. CONCLUSIONS

An experimental study was carried out on a complete ORC power system using a mixture of R245fa and lubricating oil as working fluid. A complete acquisition was installed, especially around the expander, major interest of this research. Many parameters could be tuned to reach the desired operation conditions. The data was recorded in stable steady state conditions. A total of 75 points were considered, all presenting different conditions. The evaporation and
condensation pressures that were swept went from 3 to 10.8 bar and from 1.35 to 2.25 bar respectively, leading to pressure ratios going from about 1.1 to 4.5. Concerning the rotational speeds, the range covered was between 1500 and 11 000 RPM.

The performance criteria adopted were the produced power, the isentropic efficiency and the filling factor. Two maps could then be drawn revealing operational zone allowing for the machine to deliver the best, according to each criterion. Best efficiencies were met for speeds around 4500 RPM with small pressure ratios (around 1.5). In these conditions, the isentropic efficiency went up to 50%. However, in this zone, the delivered power was small. To increase it up to 3 kW, the pressure ratio should increase near 3.5. Wet expansions showed that the expander was not well designed for these condition leading to a decrease of the performances.

Experimental campaign being consistent, a semi-empirical model could be satisfactorily calibrated. The model considered was based on a model described by Lemort et al. (2009) that was slightly adapted, including exhaust pressure drops. Due to the model approximations and the measurement precision, the predictions were not perfect but a major part of the results stood under the 15% deviation. From this model, continuous performance maps were drawn for conditions that were similar to and beyond the empirical data.

The very next step was to evaluate the performance of the system if two machines were put in series to increase the global internal volumetric ratio. This had the effect to shift the best efficiencies towards higher pressure ratios. The supply and intermediate pressures were optimized in order to maximize the isentropic efficiency. Naturally, the available power increased with two machines in series. But it was shown that the efficiency decreased due to an inlet pressure drop impacts more the performance in the compound configuration in comparison with the single expansion one. The geometry of the low-pressure stage expander of the compound configuration should be adapted.

Even if the machine’s design is devoted to the compressor use, the performance shown by the roots machine presents some interest. Indeed, its first application is devoted to superchargers working with air. Reducing leakage losses with a geometry optimization could significantly enhance the expander performance. Moreover, few volumetric machines present good efficiencies for pressure ratios close to 1. Applications operating in such conditions may take advantage of the roots expander.

### NOMENCLATURE

<table>
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<tr>
<th>Quantities</th>
<th>Subscripts</th>
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<tr>
<td>AU</td>
<td>global heat transfer coefficient (W/K)</td>
</tr>
<tr>
<td>( M )</td>
<td>mass flow rate (kg/s)</td>
</tr>
<tr>
<td>( \dot{Q} )</td>
<td>heat transfer rate (W)</td>
</tr>
<tr>
<td>( V )</td>
<td>volumetric flow rate (m³/s)</td>
</tr>
<tr>
<td>( W )</td>
<td>mechanical or electrical power (W)</td>
</tr>
<tr>
<td>( h )</td>
<td>specific enthalpy (J/kg)</td>
</tr>
<tr>
<td>( r_p )</td>
<td>pressure ratio (-)</td>
</tr>
<tr>
<td>( r_v )</td>
<td>internal volumetric ratio (-)</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature (K or °C)</td>
</tr>
<tr>
<td>( X )</td>
<td>fraction (-)</td>
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<table>
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<tbody>
<tr>
<td>( \varepsilon )</td>
<td>efficiency (-)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>specific heat coefficient ratio (-)</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>filling factor (-)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density (kg/m³)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>torque (N.m)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>efficiency (-)</td>
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REFERENCES


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