

Study of Multiquarks Systems in a Chiral Quark Model

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We discuss the stability of multiquark systems within the recent model of Glozman *et al.* where the chromomagnetic hyperfine interaction is replaced by pseudoscalar-meson exchange contributions. In this model the (u, d) diquark $S = 0$ $I = 0$ is strongly bound (more than for the chromomagnetic interaction) and this leads to a bound heavy tetraquark $QQ\bar{q}\bar{q}$ where the light quarks are in the $S = 0$ $I = 0$ configuration. We study the stability of other multiquark systems as well.

1 Introduction

The interest for the possible existence of multiquark hadrons has been raised twenty years ago by Jaffe, who suggested that states of two quarks - two antiquarks¹ and of six quarks² could be bound.

In the following years this problem has been studied within a large variety of models. Some earlier studies in MIT bag indicated the presence of a dense spectrum of tetraquark states in the light sector¹ (and more generally of multiquarks³). Later on, tetraquark systems have been examined in potential models^{4,5,6,7} and flux tube models⁸. Weinstein and Isgur showed⁴ that there are only a few weakly bound states of resonant meson-meson structure in the light (u, d, s) sector. Usually the ground state of these systems lies closely above the lowest $(q\bar{q}) + (q\bar{q})$ threshold. The other extreme result, as compared to MIT bag model, was obtained by Carlson and Pandharipande⁸ in their flux-tube model with quarks of equal masses, where no bound state was found.

Altogether the theoretical predictions about the existence of these bound states are still unclear.

As far as the experimental situation is concerned, there are some candidates for non- $q\bar{q}$ states, but data are not yet conclusive. Furthermore, even if a resonance would be clearly identified as an exotic (not $q\bar{q}$ or qqq) state, nevertheless a careful phenomenological study would be necessary in order to

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understand its internal structure among many different possibilities (multi-quarks, hybrids, glueballs,...).

The description of these candidates and of the phenomenological properties which permit a distinction between different exotics is beyond the purposes of this proceeding and we refer to Ref.s ^{10,11} and the last issue of Review of Particle Properties ¹² for further details.

Anyway, a well-established theoretical result is that systems with a larger mass difference among their components are more easily bound ^{9,5}. For example, for a system of two heavy quarks and two light antiquarks $QQ\bar{q}\bar{q}$ ($Q = c$ or b , $q = u, d$ or s) stability can be achieved without spin-spin interaction, provided the mass ratio $m(Q)/m(q)$ is larger ⁵ than about 15, which means that Q must be a b -quark.

Recently, the interest for multiquark systems containing charm quarks has grown, considering that new experiments are being planned at Fermilab and CERN, to search for new hadrons and in particular for doubly charmed tetra-quarks ^{13,14,15}.

In this context, we have carried out ¹⁶ a study of the $QQ\bar{q}\bar{q}$ system in the framework of the chiral quark model of Glozman *et al.* ^{18,19}. This model is somehow quite “extreme” because it includes meson-exchange forces between quarks and entirely neglects the chromomagnetic interaction. However it permits a very good description of the baryon spectrum, it is thus worth testing it in further predictions.

Considering that in this model light-quark mesons are “quasiparticles” with a spectrum which must be assumed and cannot be evaluated directly (this is of course rather an unpleasant feature of the model), multiquark systems represent an obvious testing ground for it. Other possible tests could come from a careful analysis of the spectrum of charmed baryons ¹⁷.

2 The Glozman model

Before presenting our results about multiquarks, let us briefly consider the Glozman *et al.* model and compare it with other potential models used in hadron spectroscopy.

In a general Hamiltonian, which would approximate the low energy limit of QCD, one can introduce both a chromomagnetic interaction and a meson-exchange contribution, obtaining an explicit form as

$$H = \sum_i \frac{\vec{p}_i^2}{2m_i} - \frac{3}{16} \sum_{i<j} \tilde{\lambda}_i^c \cdot \tilde{\lambda}_j^c V_{\text{conf}}(r_{ij})$$

$$-\sum_{i<j} \tilde{\lambda}_i^c \cdot \tilde{\lambda}_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j V_g(r_{ij}) - \sum_{i<j} \tilde{\lambda}_i^F \cdot \tilde{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j V_F(r_{ij}), \quad (1)$$

where m_i is the constituent mass of the quark located at \vec{r}_i ; $r_{ij} = |\vec{r}_j - \vec{r}_i|$ denotes the interquark distance; $\vec{\sigma}_i$, $\tilde{\lambda}_i^c$, $\tilde{\lambda}_i^F$ are the spin, colour and flavour operators, respectively. Spin-orbit and tensor components may supplement the above spin-spin forces for studying orbital excitations (they give no contribution for $L = 0$ systems). The potential in (1) has three parts containing the confining, the chromomagnetic and the meson-exchange contribution.

Usually, the confining term V_{conf} is assumed to include a Coulomb plus a linear term,

$$V_{\text{conf}} = -\frac{a}{r} + br + c. \quad (2)$$

In the following, we shall either use the very weak linear potential of Glozman *et al.*¹⁹ corresponding to

$$(C_1) \quad a = c = 0, \quad \text{and} \quad b = 0.01839 \text{ GeV}^2, \quad (3)$$

or the more conventional choice

$$(C_2) \quad a = 0.5203, \quad b = 0.1857 \text{ GeV}^2, \quad c = -0.9135 \text{ GeV}, \quad (4)$$

which has already been applied to the study of tetraquarks by Silvestre-Brac and Semay⁶.

The term $\sum_{i<j} \tilde{\lambda}_i^c \cdot \tilde{\lambda}_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j V_g(r_{ij})$ is the chromomagnetic analogue of the Breit–Fermi term of QED. In the interaction between a quark and an antiquark (as e.g. in a meson) one finds $\tilde{\lambda}_1^c \cdot \tilde{\lambda}_2^c = -16/3$. Then a positive V_g shifts each vector meson above its pseudoscalar partner, for instance $D^* > D$ in the charm sector. For baryons, where $\tilde{\lambda}_1^c \cdot \tilde{\lambda}_2^c = -8/3$ for each quark pair, such a positive V_g pushes the spin 3/2 ground states up, and the spin 1/2 down, for instance $\Delta > N$. For the radial shape, as an example, we mention

$$V_g = \frac{a}{m_i m_j d^2} \frac{\exp -r/d}{r}, \quad (5)$$

which was used in Ref.⁶, with the same value of a as in Eq. (4) and $d = 0.454 \text{ GeV}^{-1}$.

Finally, the last term of H corresponds to meson exchange, and an explicit sum over F is understood. If the system contains light quarks only (as in Ref.s^{18,19}), the sum over F runs from 0 to 8, *i.e.* over the members of the $J^{PC} = 0^{-+}$ nonet, which represent the Goldstone bosons of the spontaneously broken $SU(3)_A$ symmetry ($1-3 \rightarrow \pi$, $4-7 \rightarrow K$, $8 \rightarrow \eta$ and $0 \rightarrow \eta'$). If a heavy

flavour is incorporated, a phenomenological extension from $SU(3)_F$ to $SU(4)_F$ would further extend the sum to $F = 9 - 12$ corresponding to a D -exchange, $F = 13 - 14$ to a D_s -exchange and $F = 15$ to an η_c -exchange (of course in this case the interpretation as Goldstone bosons is not really possible). Similar terms should then be introduced if one includes the beauty sector as well. The radial form of $V_F \neq 0$ is derived from the usual pion-exchange potential which contains a long-range part and a short-range one

$$\sum_{i < j} \vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \frac{g^2}{4\pi} \frac{1}{4m^2} \left[\mu^2 \frac{\exp(-\mu r_{ij})}{r_{ij}} - 4\pi \delta^{(3)}(r_{ij}) \right], \quad (6)$$

where μ is the pion mass.

When constructing NN forces from meson exchanges, one usually disregards the short-range term in Eq. (6), for it is hidden by the hard core, and anyhow the potential in that region is parameterized empirically. For example, when Törnqvist²¹, Manohar and Wise²² or Ericson and Karl²³ considered pion exchange in multi-quark states, they used the Yukawa term $\exp(-\mu r)/r$ acting between two well-separated quark clusters. Weber *et al.*²⁴, in their model with hyperfine plus pion-exchange interaction, studied both the cases with or without delta-term, showing that good results could be obtained without it in baryon spectroscopy. Thus the relevance of an *ad-hoc* regularized delta-term^{18,19} in the study of baryon spectroscopy is somehow surprising. Nevertheless, this ansatz permits to give a good description of the baryon spectrum and, in particular, allows one to solve the problem of the ordering of the lowest parity-odd and parity-even states of N , Λ and Σ resonances, which did not find a solution in conventional chromomagnetic models²⁰.

Incidentally, this result is not completely unexpected, in fact Buchman *et al.* had shown, already some years ago, that this term is essential for obtaining a good description of magnetic moments²⁶.

For the sake of completeness we report here the explicit form of the regularized delta-term¹⁹

$$V_\mu = \Theta(r - r_0) \mu^2 \frac{\exp(-\mu r)}{r} - \frac{4\epsilon^3}{\sqrt{\pi}} \exp(-\epsilon^2(r - r_0)^2), \quad (7)$$

where $r_0 = 2.18 \text{ GeV}^{-1}$, $\epsilon = 0.573 \text{ GeV}$, and $\mu = 0.139 \text{ GeV}$ for π , 0.547 GeV for η and 0.958 GeV for η' . Furthermore, the Yukawa-type part is cut off for $r \leq r_0$. This smearing should account for the fact that both the pseudoscalar mesons and the constituent quarks have a finite size and that boson fields cannot be described by a linear equation near their source.

The explicit form of the Hamiltonian which extends the results of Ref. ¹⁸ from $SU(3)$ to $SU(4)$ is given in Ref. ¹⁶. It includes also the exchange of D , D_s

and η_c mesons. However, considering that little $u\bar{u}$ or $d\bar{d}$ mixing is expected in η_c , this contribution can be neglected when considering systems with none or one charm quark. Moreover when the meson mass μ reaches values of a few GeV as for D or η_c the two terms in Eq. (6) basically cancel each other and one recovers practically the SU(3) form¹⁸. This is in agreement with Ref.²⁵ where it has been explicitly shown that the dominant contribution to the Σ_c and Σ_c^* masses is due to meson exchange between light quarks and the contribution of the matrix elements with the D (D_s) and D^* (D_s^*) quantum numbers (which are evaluated phenomenologically, fitting the mass difference $\Sigma_c - \Lambda_c$) play a minor rôle. In the following numerical calculations we will neglect the exchange of heavy mesons.

Finally one has to fix quark masses. The light quarks ones are fixed to 0.34 GeV according to Ref.s^{19,6}. The heavy quark masses $m_Q = m_c$ and m_b are adjusted to reproduce the experimental average mass $\bar{M} = (M + 3M^*)/4$ between the 0^- and 1^- $M = D$ or B mesons by a variational calculation, where a trial wave function of type $\phi \propto \exp(-\alpha r^2/2)$ is used (with α as a variational parameter). It has been checked that the error never exceeds a few MeV with respect to the exact value. The variational approximation is retained for consistency with the treatment of 3-, 4- and 6-body systems discussed below. This leads to $m_c = 1.35$ GeV and $m_b = 4.66$ GeV for the potential C_1 and $m_c = 1.87$ GeV and $m_b = 5.259$ GeV for C_2 .

Before proceeding further, let us briefly discuss the calculation of baryons masses in the model of Glozman *et al.* The explicit form of the Hamiltonian integrated in the spin-flavour space is :

$$H = H_0 + \frac{g^2}{48\pi m^2} \begin{cases} 15V_\pi - V_\eta - 2(g_0/g)^2 V_{\eta'} & \text{for } N \\ 3V_\pi + V_\eta + 2(g_0/g)^2 V_{\eta'} & \text{for } \Delta \end{cases} \quad (8)$$

with

$$H_0 = 3m + \sum_i \frac{\vec{p}_i^2}{2m} + \frac{b}{2} \sum_{i<j} r_{ij}, \quad (9)$$

where $g^2/4\pi = 0.67$ (which leads to the usual strength $g_{\pi NN}/4\pi \simeq 14$ for the Yukawa tail of the nucleon-nucleon potential) and $(g_0/g)^2 = 1.8$. We have performed variational estimates with a wave function $\phi \propto \exp(-\alpha(\rho^2 + \lambda^2)/2)$, where $\vec{\rho} = \vec{r}_2 - \vec{r}_3$, $\vec{\lambda} = (2\vec{r}_1 - \vec{r}_2 - \vec{r}_3)/\sqrt{3}$: our results agree with the more elaborated Faddeev calculations of Ref.¹⁹.

When the meson-exchange terms are switched off, the N and Δ ground states are degenerate at 1.63 GeV. Introducing the coupling the nucleon mass drops stronger leading to a reasonable splitting (≈ 0.3 GeV). It is interesting to notice that in this model one has an attraction both for the $S = 0$, $I = 0$

and also, albeit smaller, for the $S = 1, I = 1$ diquarks. Then both the nucleon and Δ masses decrease when the interaction is turned on. The diquarks $S = 0, I = 1$ and $S = 1, I = 0$ on the contrary are repulsive configurations. The situation is thus different from the chromomagnetic case where one has a smaller attraction for the $S = 0, I = 0$ diquark and repulsion for the $S = 1, I = 1$ one.

We have also calculated the ground state of cqq baryons using a trial wave function $\phi \propto \exp(-(\alpha\rho^2 + \beta\lambda^2)/2)$ and found $\Lambda_c = 2.32$ GeV and $\Sigma_c = \Sigma_c^* = 2.48$ GeV, close to the experimental values and consistent with the findings of Ref. ²⁵, although the Hamiltonian, its treatment, and the input parameters are somewhat different there.

3 Evaluation of multiquarks masses

Due to arguments at the beginning of this contribution, here we discuss tetraquarks containing heavy flavours, i.e. $QQ\bar{q}\bar{q}$, studying the most favourable configuration: $\bar{3}3, S = 1, I = 0$. This means that QQ is in a $\bar{3}$ colour state and $\bar{q}\bar{q}$ in a 3 colour state. The mixing with $6\bar{6}$ is neglected because one expects this to play a negligible rôle in deeply-bound heavy systems⁵. Then the Pauli principle requires $S_{12} = 1$ for QQ , and $S_{34} = 0, I_{34} = 0$ for $\bar{q}\bar{q}$ (which we have seen to be the diquark with the largest binding), if the relative angular momenta are zero for both subsystems. This gives a state of total spin $S = 1$ and isospin $I = 0$.

The tetraquark Hamiltonian integrated in the colour–spin–flavour space, and incorporating the approximations discussed in the former section, reduces to

$$H = 2(m + m_Q) + \frac{\vec{p}_x^2}{m_Q} + \frac{\vec{p}_y^2}{m} + \frac{m + m_Q}{2mm_Q} \vec{p}_z^2 + \sum_{i < j} V_{ij}, \quad (10)$$

where

$$\begin{aligned} V_{12} &= \frac{1}{2} \left(-\frac{a}{r_{12}} + b r_{12} + c \right), \\ V_{ij} &= \frac{1}{4} \left(-\frac{a}{r_{ij}} + b r_{ij} + c \right), \quad i = 1 \text{ or } 2, j = 3 \text{ or } 4, \\ V_{34} &= \frac{1}{2} \left(-\frac{a}{r_{34}} + b r_{34} + c \right) + 9V_\pi - V_\eta - 2V_{\eta'}. \end{aligned} \quad (11)$$

The momenta \vec{p}_x , etc., are conjugate to the relative distances $\vec{x} = \vec{r}_1 - \vec{r}_2$, $\vec{y} = \vec{r}_3 - \vec{r}_4$, and $\vec{z} = (\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4)/\sqrt{2}$. The wave function is parameterized as

$$\psi \propto \exp[-(\alpha x^2 + \beta y^2 + \gamma z^2)/2], \quad (12)$$

and the minimization with respect to α , β and γ shows that both the $cc\bar{q}\bar{q}$ and $bb\bar{q}\bar{q}$ systems are bound whatever is the potential, (C_1) or (C_2), provided meson exchange is incorporated. This is in contradistinction to previous studies based on conventional models where the flavour-independent confining potential is supplemented by one gluon exchange. For example the authors of Ref.⁶ found that the $cc\bar{q}\bar{q}$ state is about 20MeV above threshold (while the $bb\bar{q}\bar{q}$ is bound of 135 MeV). A similar situation is also obtained in Ref. ²⁷, where studying the four quarks states in a diquark model only the $bb\bar{q}\bar{q}$ is found to be bound (by 50MeV, while $cc\bar{q}\bar{q}$ is 30 MeV above the threshold).

Quantitatively in our model we find that using the confining potential C_1 plus meson exchange the double charmed tetraquark is bound by 185 MeV and the double bottom one by 226 MeV. Using the potential C_2 the results are nearly two times larger (332 and 497 MeV respectively) and also much more different from each other. The reason is that (C_2) contains a Coulomb part which binds more, heavier is the system, leading thus to a larger separation among levels as well. This is related to the fact that the potential (C_2) has been fitted to reproduce the J/Ψ and the Υ meson masses (anyway it also gives overall good results both for other mesons and baryons) while, by construction ^{18,19}, the potential (C_1) was designed and fitted to light baryons only. Altogether, our results shows that the prediction of binding for the $cc\bar{q}\bar{q}$ is substantially independent on the choice of the confining potential.

Considering this result one could rise the question if in the model of Glozman *et al.* a proliferation of multi-quark systems appears. We have therefore tried to investigate $QQqqqq$ and q^6 systems as well.

As a general procedure, for a given multi-quark system, one searches for the spin-isospin wave functions corresponding to a colour singlet, then selects the most favourable configuration. The contribution of global spin-flavour-averaged interaction is reduced to the calculation of the matrix elements of the two body operator $(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j)$; this is accomplished by using Clebsch–Gordan coefficients of the permutation group according to Ref.s ^{28,29}.

Let us begin with $QQqqqq$. In this case the most favourable configuration is the one where the light quark subsystem has $S = 1$, $I = 0$, which leads to the spin-flavour-averaged interaction

$$\langle V \rangle = 10V_\pi - 2/3V_\eta - 4/3(g_0/g)^2V_{\eta'}. \quad (13)$$

In the numerical calculation we have used the variational Gaussian wave function

$$\Psi \propto \exp[-(\alpha x^2 + \beta y^2 + \gamma(u^2 + v^2 + w^2))] \quad (14)$$

where appear the Jacobi variables $\vec{x} = \vec{r}_1 - \vec{r}_2$, $\vec{y} = \vec{R}_q - \vec{R}_Q$, in terms of $\vec{R}_q = (\vec{r}_1 + \vec{r}_2)/2$ and $\vec{R}_Q = (\vec{r}_3 + \vec{r}_4 + \vec{r}_5 + \vec{r}_6)/4$, $\vec{u} = (\vec{r}_3 + \vec{r}_4 - \vec{r}_5 - \vec{r}_6)/\sqrt{2}$,

$\vec{v} = (\vec{r}_3 - \vec{r}_4 + \vec{r}_5 - \vec{r}_6)/\sqrt{2}$, $\vec{w} = (\vec{r}_3 - \vec{r}_4 - \vec{r}_5 + \vec{r}_6)/\sqrt{2}$, where indices 1, 2 refer to the heavy quarks and 3, 4, 5, 6 to the light ones.

The result of the numerical calculation is that this potential is largely insufficient to bind both the $ccqqqq$ (e.g. of 500 MeV with the potential C_1) and the $bbqqqq$ systems. Incidentally, at this workshop results about heavy hexaquarks were presented also by Lichtenberg *et al.*³⁰, in their diquark model. Albeit they do not treat directly the case $ccqqqq$, they find that the system $csqqqq$ is unbound, while the one containing a beauty quark instead of the charm is bound.

For the sake of completeness we have also made a study of the q^6 system. The most favourable configuration is $S = 1$, $I = 0$, leading to

$$\langle V \rangle = 11V_\pi - 5/3V_\eta - 10/3(g_0/g)^2V_{\eta'} \quad (15)$$

Our result shows that, also in this case, the system is largely insufficiently bound (by almost 1.5GeV with the potential C_1) for being under the two baryons threshold.

4 Conclusions

In conclusion, considering the success of the Glozman *et al.* model in describing the baryon spectrum, we have searched for new possible tests of this model. Unluckily the model does not permit an analysis of mesons (light-quark mesons at least), which are interpreted as “quasiparticles” related to the breaking of flavour $SU(3)$. Further tests of it would then involve multiquarks systems. We have thus studied four and six-quark systems involving two heavy quarks, showing that the Glozman *et al.* model leads to predictions for the $cc\bar{q}\bar{q}$ which differ from the ones of more conventional models. In fact this system is found to be strongly bound. The search of such a resonance in forthcoming experiments will therefore be a good test for understanding the dynamics which produces the hadron spectrum.

We have also found that the $bb\bar{q}\bar{q}$ state is bound, but this result is obtained also in other models and the possibility to observe this resonance is relegated to a more remote future.

Finally we have found that the six quark system with two heavy quarks is unbound, also this result does not differ from the one of other more conventional models. However, together with the outcome that also the q^6 system is unbound, it is a useful indication that in the framework of the model under investigation there is no proliferation of multiquark bound states.

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1. R.L. Jaffe, *Phys. Rev. D* 15, 267 (1977); *Phys. Rev. D* 17, 1444 (1978).
2. R.L. Jaffe, *Phys. Rev. Lett.* 38, 195 (1977).
3. A. T. M. Aerts *et al.*, *Phys. Rev. D* 17, 260 (1977).
4. J. Weinstein and N. Isgur, *Phys. Rev. D* 27, 588 (1983), *Phys. Rev. D* 41, 2236 (1990).
5. S.Zouzou, B. Silvestre-Brac, C. Gignoux and J.-M. Richard, *Z. Phys. C* 30, 457 (1986).
6. B. Silvestre Brac and C. Semay, *Z. Phys. C* 59, 457 (1993); *Z. Phys. C* 61, 271 (1994).
7. D.M. Brink and Fl. Stancu, *Phys. Rev. D* 49, 4665 (1994).
8. J. Carlson and V.R. Pandharipande, *Phys. Rev. D* 43, 1652 (1991).
9. J.P. Ader, J.-M. Richard and P. Taxil, *Phys. Rev. D* 25, 2370 (1982).
10. G. Karl, *Int. J. of Mod. Phys. E*1, 491 (1992); *Nucl. Phys. A* 558, 113c (1993); N.A. Törnqvist, Proc. of “Int. Europh. Conf. on High Energy Phys.”, Brussels (Belgium), edit. J. Lemonne *et al.*, 84 (1995); G. Landsberg, *Phys. of Atom. Nucl.* 57, 42 (1994).
11. M. Genovese, *Nuovo Cimento A* 107, 1249 (1994) and references therein.
12. Particle Data Group, *Phys. Rev. D* 54, 1 (1996).
13. M.A. Moinester, *Z. Phys. A* 355, 349 (1996).
14. D.M. Kaplan, Proc. Int. Workshop “Production and Decay of Hyperons, Charm and Beauty Hadrons”, Strasbourg (France) September 5–8, 1995.
15. COMPASS Collaboration (G. Baum *et al.*), CERN-SPSLC-96-14, March 1996.
16. S. Pepin, Fl. Stancu, M. Genovese and J.-M.Richard, hep-ph 9609348, ISN-96.99, to be published in *Phys. Lett. B*.
17. S. Pepin, Fl. Stancu, M. Genovese, J.-M.Richard and S. Zouzou, work in progress.
18. L.Ya. Glozman and D.O. Riska, *Phys. Rep.* 268, 263 (1996).
19. L.Ya. Glozman, Z. Papp and W. Plessas, *Phys. Lett. B* 381, 311 (1996).
20. L. A. Copley, N. Isgur and G. Karl, *Phys. Rev. D* 20, 768 (1979); S. Capstick and N. Isgur, *Phys. Rev. D* 34, 2809 (1986); C. S. Kalman and D. Pfeffer, *Phys. Rev. D* 27, 1648 (1983).
21. N. Törnqvist, *Phys. Rev. Lett.* 67, 556 (1991); *Z. Phys. C* 61, 525 (1994).

22. A.V. Manohar and M.B. Wise, *Nucl. Phys. B* 399, 17 (1993).
23. T.E.O. Ericson and G. Karl, *Phys. Lett. B* 309, 426 (1993).
24. M. Weyrauch and H.J. Weber, *Phys. Lett. B* 171, 13 (1986); H.J. Weber and H.T. Williams, *Phys. Lett. B* 205, 118 (1988).
25. L.Ya. Glozman and D.O. Riska, *Nucl. Phys. A* 603, 326 (1996).
26. A. Buchman *et al.*, *Nucl. Phys. A* 569, 661 (1994).
27. D. B. Lichtenberg, R. Roncaglia and E. Predazzi, IUHET-344, published in these proceedings.
28. Fl. Stancu, *Group Theory in Subnuclear Physics*, Oxford University Press, 1996, chapter 4;
29. S. Pepin and Fl. Stancu Preprint ULG-PNT-96-1-J.
30. D. B. Lichtenberg, R. Roncaglia and E. Predazzi, DFTT 65/96, published in these proceedings.