Energy Transition from a Linear Oscillator to an Attached Mass through an Essential Nonlinearity

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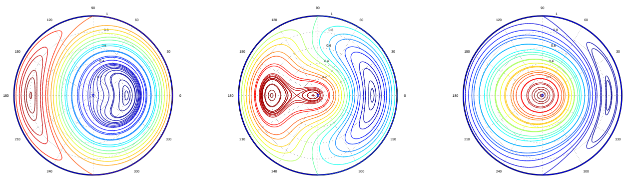
We consider the response of a two degree-of-freedom nonlinear system. The equations of motion can be viewed to describe a weakly damped linear oscillator coupled to a second mass through a weakly nonlinear spring-damper. This work focuses on the use of the secondary nonlinear attachment as an energy absorber for impulse loads to the primary linear oscillator.

The nondimensional equations of motion for this system can be written as

$$\begin{aligned}\ddot{y}_1 + 2\,\varepsilon\,\zeta\,\gamma\,\dot{y}_1 + 2\,\varepsilon\,\zeta(\dot{y}_1 - \dot{y}_2) + y_1 & \\ + \frac{4}{3}\,\varepsilon\,\alpha(y_1 - y_2)^3 &= 0,\\ \varepsilon\,\ddot{y}_2 + 2\,\varepsilon\,\zeta(\dot{y}_2 - \dot{y}_1) + \frac{4}{3}\,\varepsilon\,\alpha(y_2 - y_1)^3 &= 0,\end{aligned}$$

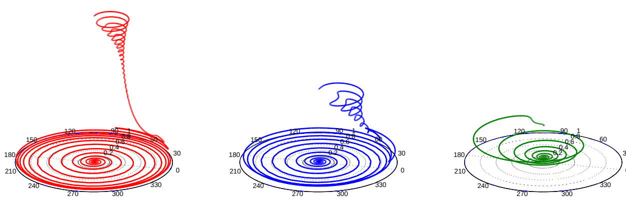
where y_1 describes the linear oscillator with y_2, while the response of the nonlinear attachment is measured with y_2. The nondimensional parameter ε represents the mass ratio of the two components while ζ and γ are damping parameters. Finally the quantity α represents the strength of the nonlinear coupling between the linear oscillator and the attachment, here taken to be cubic in the relative displacement between the two.

In the above system, the resonant frequency of the linear oscillator is unity. In contrast, the linearization of the coupling vanishes, so that the response of the attachment exhibits strong period-amplitude dependence despite the weak nature of the coupling. Thus there exists an amplitude for the attachment so that the system exhibits a $1:1$ resonance. Application of the method of averaging in the neighborhood of this resonant state yields a reduced-order system of equations written in terms of (r, ψ, ϕ). In these new coordinates r and ψ describe the amplitude of each mass and ϕ describes the phase difference between y_1 and y_2. The resulting averaged equations can be formally represented as



a. $r = 1.00$ b. $r = 0.375$ c. $r = 0.25$

Figure 1: Phase space projection ($\alpha = 1.00$, $\varepsilon = 0.10$, $\zeta = 0.00$, $\gamma = 1$).



a. $r(0) = 2.00$ b. $r(0) = 1.00$ c. $r(0) = 0.50$

Figure 2: Phase space projection ($\alpha = 1.00$, $\varepsilon = 0.10$, $\zeta = 0.05$, $\gamma = 1$).

$$\begin{aligned}\dot{\psi} &= f_\psi(\psi,\phi,r) + \zeta\,g_\psi(\psi,\phi,r),\\ \dot{\phi} &= f_\phi(\psi,\phi,r) + \zeta\,g_\phi(\psi,\phi,r),\\ \dot{r} &= \zeta\,g_r(\psi,\phi,r),\end{aligned}$$

where the functions $f_{(\bullet)}$ and $g_{(\bullet)}$ arise from the method of averaging (not shown for brevity). For $\zeta = 0$ the total energy of the system, defined as h, is conserved. In addition, the value of r is constant and can be viewed as a parameter of the system. Therefore for $\zeta = 0$, h and r represent two conserved quantities. In Figure 1 the phase space of the system is shown for three different values of r. In this projection of phase space ϕ describes the angular coordinate while ψ corresponds to the radial distance.

For nonzero damping ($\zeta \neq 0$) the system is no longer conservative and r varies slowly in time. As seen in Figure 2 the response initially localizes near the equilibria of the undamped system, parameterized by r while the value of r slowly decreases. In this figure r is shown as the height of the response while ψ and ϕ are identical to Figure 1.

The efficiency of the nonlinear attachment represents

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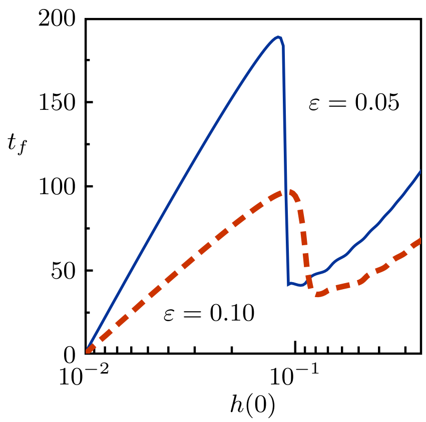

Figure 3: Time required for the impulsive orbit to reach $h_f = 0.01$ ($\alpha = 1.00$, $\zeta = 0.05$, $\gamma = 1$).

its ability to reduce the energy of the system when initially localized in the linear oscillator, defined as the impulsive orbit. In Figure 3 the efficiency is measured as the time t_f required for the energy of the impulsive orbit to decay to a value $h(t_f) = h_f = 0.01$. While one might expect that this time would increase monotonically with increasing values of the initial energy $h(0)$, the numerical results show a dramatic increase in the efficiency of the nonlinear attachment for intermediate energy values, identified with the decrease of t_f.

By carefully considering the structure of phase space for zero damping, together with the stability of the trajectories of the damped system that perturb from the equilibria of the undamped system, one can describe and understand these observed results. Moreover the role of the damping and mass ratio can be clearly identified in the response of this system.