HEAVY HEXAQUARKS IN A CHIRAL CONSTITUENT QUARK MODEL

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Abstract

We discuss the stability of hexaquark systems of type uuddsQ (Q=c or b) within a chiral constituent quark model which successfully describes the baryon spectra including the charmed ones. We find these systems highly unstable against strong decays and give a comparison with some of the previous literature.

I. INTRODUCTION

According to QCD rules, systems such as tetraquarks, pentaquarks or hexaquarks can, in principle, exist. Their study is important in disentangling between various QCD inspired models. Here we are mainly concerned with non-relativistic models, which simulate the low-energy limit of QCD. In these models, the central part of the interquark potential usually contains a linear term which describes the QCD confinement and a Coulomb term generated by the long-range one-gluon exchange (OGE) interaction. The spin part is usually described by the chromo-magnetic part of the one-gluon exchange interaction, analogous to the Fermi-Breit interaction of QED [1,2].

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An interest in the constituent quark model has recently been revitalized [3] after recognition of the role of the spontaneous chiral symmetry of the QCD vacuum. This implies that the valence quarks acquire a constituent dynamical mass related to the quark condensate $< q\bar{q} >$ and that the Goldstone bosons π, K, η couple directly to constituents quarks [4]. It has been shown that the hyperfine splitting and especially the correct ordering of positive and negative parity states of baryons with u,d and s quarks are produced by the short-range part of the Goldstone boson exchange (GBE) interaction [3,5,6], instead of the OGE interaction.

In Ref. [7], we studied the stability of the H-particle, a uuddss system, with $J^P = 0^+$ and I=0, in the frame of the chiral constituent quark model of Ref. [8]. We found that the GBE interaction induces a strong repulsion in the flavour singlet uuddss system with $J^P = 0^+$ and I=0, i.e. this system lies 847 MeV above the $\Lambda\Lambda$ threshold. This implies that the model of Ref. [8] predicts that the H-particle should not exist, in contrast to Jaffe's [9] or many other studies based on conventional one-gluon exchange models [10]. In the model used by Jaffe, the chromomagnetic interaction, gave more attraction for the flavour singlet state than for two well separated lambda baryons. In Jaffe's picture the H-particle should be a compact object, in contrast to the molecular-type deuteron.

In a recent study by Lichtenberg, Roncaglia and Predazzi [11], the uuddss system is discussed in the context of a diquark model and it is found unstable, in contrast to Jaffe's result. The above authors also discuss hexaquarks in which one of the s quarks is replaced by a c quark, the H_c particle, or by a b quark, the H_b particle. The charmed hexaquark is found unstable but the bottom hexaquark is found stable by about 10 MeV with respect to the $\Lambda + \Lambda_b$ threshold. Based on the concept of dynamical hadron supersymmetry, Ref. [11] predicts the exotic masses from the ones of ordinary mesons and baryons as input, with no free parameters. On the other hand, more sophisticated calculations within a constituent quark model with chromomagnetic interaction give both H_c(I=0, J=3) and H_b(I=0, J=2 or 3) stable by 7.7 MeV up to 13.8 MeV [12]. In fact, from general grounds [13,14] one expects that the stability of multiquark systems should increase with the mass asymetry of the constituent quarks and this was tested for tetraquarks systems, in a conventional constituent quark model with chromomagnetic interaction [15–17].

Here we focus our attention on hexaquarks and in particular on the uuddsQ (Q=c or b) system, which results as a promising candidate from the above models. We study the stability of the uuddsQ system within the chiral constituent quark model [8] used previously in the study of the H-particle [7] and of heavy tetraquarks as well [18]. An essential difference with respect to Refs [12,15–17] is that the spin-spin interaction of [8] is flavour-dependent. In Ref. [18] we found that the Goldstone boson exchange interaction between quarks binds strongly both the $cc\bar{q}\bar{q}$ system and the $bb\bar{q}\bar{q}$ system. Within conventional models based on one-gluon exchange, the $cc\bar{q}\bar{q}$ was found unstable and $bb\bar{q}\bar{q}$ stable [16].

In section 2, we establish the basis states required by the internal symmetries of the system under discussion and, based on simple group theory arguments, we indicate the most important basis states for a given isospin I and total angular momentum J. In section 3, we briefly describe the Hamiltonian of the chiral constituent quark model used in the calculations. In section 4, we present our results and the last section is devoted to a summary.

II. BASIS STATES

In principle the GBE interaction contains all pairs ij of particles. But the exchange of a heavy pseudoscalar meson, between a light and a heavy quark Q, can in practice be neglected [18]. For example, it was explicitly shown in Ref. [19] that the dominant contribution to the masses of C=+1 charmed baryons is due to meson exchange between light quarks and that the exchange of D and D_s mesons is negligible, of the order of few MeV. Inasmuch as the quark-quark interaction [3,8] is inverse proportional to the masses of the interacting quarks, the exchange of B or B_s meson can further be neglected. Neglecting the contribution of D, D_s, B and B_s mesons, the GBE interaction in the uuddsQ system, with Q=c or b, reduces to the interaction between light quarks q=u, d or s.

We adopt this point of view, i.e. we neglect the contribution of heavy meson-exchange.

Then similarly to the study of the H-particle or of the NN system [20] it is useful to have a qualitative insight about the uuddsQ system by first considering a schematic quark-quark interaction which simplifies the GBE interaction of Ref. [8], by removing its radial dependence. The schematic interaction reads:

$$V_\chi = -C_\chi \sum_{i<j} \lambda_i^F.\lambda_j^F \vec{\sigma}_i.\vec{\sigma}_j, \tag{1}$$

where λ_i^F (F=1,2,...,8) are the quark-flavour Gell-Mann matrices (with an implied summation over F) and $\vec{\sigma}$ are the spin matrices. The minus sign of the interaction (1) is related to the sign of the short-range part of the GBE interaction, crucial for the hyperfine splitting in baryon spectroscopy. This feature of the short-range part of the GBE interaction is clearly discussed at length by Glozman and Riska [3]. A typical order of magnitude for the constant C_χ is about 30 MeV.

In order to calculate the expectation value of (1) for the remaining light pentaquark system q^5 we have to give a classification of its states. In the colour space this system is described by the state $[221]_C$ compatible with the colourless state $[222]_C$ of q^5Q. Here and below [f] stands for the corresponding Young diagram in the colour (C), spin (S) or flavour (F) space.

We first assume that u,d and s are identical. If the quarks are all in the ground state the orbital part of the wave function is symmetric. Then the only q^5 flavour-spin state allowed by the Pauli principle is $[32]_{FS}$. By using inner product rules [21] one can find the flavour $[f]_F$ and spin $[f]_S$ symmetries compatible with $[32]_{FS}$. These are listed in Table 1, together with the corresponding isospin I and spin S associated with these states. We also give the total angular momentum $\vec{J} = \vec{S} + \vec{S}_Q$ of the q^5Q system. The last column reproduces the expectation value of (1) in units of C_χ. This has been calculated using the formula given in the Appendix A of Ref. [20], containing the Casimir operators of $SU(6)_{FS}$, $SU(3)_F$ and $SU(2)_S$. The multiplicity of a given IJ state is consistent with Table 3 (Y=2/3) of Ref. [12]. From Table 1, one can see that the most favourable candidate for stability (the most negative eigenvalue of (1)) should have I=0 (flavour symmetry $[221]_F$) and J=0 or 1. In the numerical calculations given below, different flavour symmetries will be mixed by the GBE interaction (1).

In the diagonalization procedure of the GBE Hamiltonian given below, we truncate the basis for various sectors IJ, by retaining only the lowest states. The mixture with the others is expected to be small. Note also that the flavour spin interaction of type (1) does not mix $[f]_S \neq [f']_S$. In numerical calculations, we therefore restrict the basis states to the following ones :

<u>I=0, J=0 or 1</u>

$$|1>=|[221]_F[32]_S>; |2>=|[32]_F[32]_S> \tag{2}$$

<u>I=1, J=0 or 1</u>

$$|1>=|[311]_F[32]_S>; |2>=|[32]_F[32]_S> \tag{3}$$

<u>I=2, J=0 or 1</u>

$$|1>=|[41]_F[32]_S>; |2>=|[5]_F[32]_S> \tag{4}$$

III. HAMILTONIAN

The Hamiltonian to be diagonalized is [19] :

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^2}{2m_i} - \frac{(\sum_i \vec{p}_i)^2}{2\sum_i m_i} + \sum_{i<j} V_{conf}(r_{ij}) + \sum_{i<j} V_\chi(r_{ij}) \tag{5}$$

with the linear confining interaction :

$$V_{conf}(r_{ij}) = -\frac{3}{8}\lambda_i^c \cdot \lambda_j^c \, C \, r_{ij} \tag{6}$$

and the spin-spin component of the GBE interaction in its $SU_F(3)$ form :

$$\begin{aligned} V_\chi(\vec{r}_{ij}) = \Bigg\{ & \sum_{F=1}^{3} V_\pi(\vec{r}_{ij})\lambda_i^F \lambda_j^F \\ & + \sum_{F=4}^{7} V_{\rm K}(\vec{r}_{ij})\lambda_i^F \lambda_j^F + V_\eta(\vec{r}_{ij})\lambda_i^8 \lambda_j^8 + V_{\eta'}(\vec{r}_{ij})\lambda_i^0 \lambda_j^0 \Bigg\} \vec{\sigma}_i \cdot \vec{\sigma}_j, \end{aligned} \tag{7}$$

with $\lambda^0 = \sqrt{2/3}\,\mathbf{1}$, where $\mathbf{1}$ is the 3×3 unit matrix. The interaction (7) contains $\gamma = \pi, K, \eta$ and η' exchanges and the form of $V_\gamma(r_{ij})$ is given explicitly in Ref. [8] as the sum of two distinct contributions : a Yukawa type potential containing the mass of the exchanged meson and a short-range contribution, of opposite sign, the role of which is crucial in baryon spectroscopy. For a given meson γ, the meson exchange potential is :

$$V_\gamma(\vec{r}_{ij}) = \frac{g_\gamma^2}{4\pi}\frac{1}{3}\frac{1}{4m_i m_j}\{\mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \frac{4}{\sqrt{\pi}}\alpha^3 \exp(-\alpha^2(r-r_0)^2)\}, \quad (\gamma = \pi, K, \eta, \eta') \tag{8}$$

For the Hamiltonian (5)-(8), we use the parameters of Ref. [8]. These are :

$$\frac{g_{\pi q}^2}{4\pi} = \frac{g_{\eta q}^2}{4\pi} = \frac{g_{Kq}^2}{4\pi} = 0.67; \ \frac{g_{\eta' q}^2}{4\pi} = 1.206$$

$$r_0 = 0.43\,\mathrm{fm},\ \alpha = 2.91\,\mathrm{fm}^{-1},\quad C = 0.474\,\mathrm{fm}^{-2}, m_{u,d} = 340\,\mathrm{MeV}.$$

$$\mu_\pi = 139\,\mathrm{MeV},\ \mu_\eta = 547\,\mathrm{MeV},\ \mu_{\eta'} = 958\,\mathrm{MeV},\ \mu_K = 495\,\mathrm{MeV}. \tag{9}$$

They provide a very satisfactory description of low-lying nonstrange baryons, extended to strange baryons in [6] in a fully dynamical three-body calculations as well. The latter reference gives $m_s = 0.440$ GeV. For the masses of the heavy quarks we take $m_c = 1.35$ GeV, $m_b = 4.66$ GeV in agreement with Ref. [18] where these masses are adjusted to reproduce the average mass $\bar{M} = (M + 3M^*)/4$ of M = D and B mesons respectively. Note that within the spirit of the model of Glozman and Riska there is no meson exchange between a quark and a antiquark.

IV. RESULTS

First we discuss the spin S=1/2 baryons needed to calculate the threshold energy. Using the Hamiltonian (5)-(8), we have performed variational estimates with a general wave function of the form $\psi \sim \exp[-(ax^2 + by^2)]$ where $\vec{x} = \vec{r}_1 - \vec{r}_2$, $\vec{y} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{3}$. To the nucleon mass, only π, η and η' exchange contribute. To the mass of Λ or Σ, there is a contribution from K-exchange as well. In the case of heavy baryons $\Lambda_c, \Lambda_b, \Sigma_c, \Sigma_b, \Xi_c$

and Ξ_b we neglect any meson exchange between a light and a heavy quark, in the spirit of the above discussion. The results are presented in Table 2, where for N, Λ and Σ we made the simplification a=b. The expectation values lie typically about 50 MeV above the experimental value. The results for Σ could be improved by taking a$\neq$b and those for Λ_b made more realistic by tuning the mass of m_b. But, for the present purpose, as will be seen below, these estimates are quite satisfactory.

For the hexaquarks discussed here, it is useful to introduce the following system of Jacobi coordinates (where i=1,2,...,5 are associated with light quarks and i=6 with the heavy one):

$$\begin{aligned}
\vec{x} &= \vec{r}_1 - \vec{r}_2 \\
\vec{y} &= \vec{r}_3 - \vec{r}_4 \\
\vec{z} &= \tfrac{1}{\sqrt{2}}(\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4) \\
\vec{t} &= \tfrac{1}{\sqrt{10}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5) \\
\vec{w} &= \tfrac{1}{\sqrt{15}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \vec{r}_5 - 5\vec{r}_6) \\
\vec{R}_{CM} &= (m\vec{r}_1 + m\vec{r}_2 + m\vec{r}_3 + m\vec{r}_4 + m_s\vec{r}_5 + m_Q\vec{r}_6)/(4m + m_s + m_Q)
\end{aligned} \tag{10}$$

Moreover, in the kinetic term only, we use the average mass :

$$\bar{m} = (4m + m_s)/5 \tag{11}$$

for all light quarks.

By assuming a ground state variational wave function of the form:

$$\psi = (\frac{a}{\pi})^3(\frac{b}{\pi})^{3/4} \exp[-\frac{a}{2}(x^2 + y^2 + z^2 + t^2) - \frac{b}{2}w^2] \tag{12}$$

the expectation value E_0 of the spin-independent part of the Hamiltonian becomes:

$$\begin{aligned}
E_0 = \; & \frac{6}{\bar{m}}\frac{\hbar^2 c^2}{2a^2} + (\frac{1}{\bar{m}} + \frac{5}{m_b})\frac{\hbar^2 c^2}{8b^2} \\
& + \frac{4}{5}C[10\sqrt{\frac{1}{\pi a}} + 5\sqrt{\frac{1}{5\pi}(\frac{2}{a} + \frac{3}{b})}]
\end{aligned} \tag{13}$$

For the matrix elements of the spin-dependent part, the fractional parentage technique [21] has been used. Details are given in Appendix A.

In Table 3, we present results from the diagonalization of the Hamiltonian (5)-(8) for the bases (2)-(4). The column-heading M represents the lowest expectation value obtained in that sector. The next column gives the lowest theoretical threshold compatible with a given IJ. We also indicate the next to the lowest threshold whenever it is close to the lowest one. Note that using the experimental masses, Table 2, last column, the two thresholds may interchange their position. The last column is the lowest eigenvalue from which the threshold mass M_T obtained from Table 2 has been subtracted. The lowest eigenvalue is the equilibrium value obtained by minimizing with respect to the variational parameters a and b of (12). In each case it turns out that b at equilibrium is approximately equal to the value of b given in Table 2, associated to the heavier of the threshold baryons. At equilibrium, we also find that the off-diagonal matrix elements of the GBE interaction are typically one order of magnitude smaller than the diagonal ones so that the lowest state does not change much through the coupling to the next state. A typical change is of a few MeV. This also proves that the truncation of the bases as in (2)-(3) is safe. The smallest $M - M_T$ corresponds to IJ=00 or 01, as expected from the discussion following Table 1. In all cases $M - M_T$ is positive and very large which means that none of the considered system is stable against strong decays. Actually, there is a substantial amount of repulsion, similar to the case of the H-particle [7]. Thus within the GBE model used here, the heavy compact hexaquark uuddsb is highly unstable, contrary to the findings of Ref. [11] or [12]. In the latter reference the system with I=0, J=2 is bound by 13.8 MeV for the most favourable choice of the model parameters.

The amount of repulsion found depends, of course, on the approximations used, and in particular on the treatment of the kinetic energy. A better treatment, where instead of the mass average (11) the kinetic energy is expressed in terms of the average of the inverse of the reduced masses , may slightly decrease the kinetic contribution but certainly will not change the above conclusion. The expectation is that the incorporation of the D or B meson exchange will not change the conclusion either. But in cases where uuddsQ is found to have a mass close to the lowest threshold, as for example Ref [12], a proper treatment of the kinetic energy is much more important in drawing a conclusion about stability under strong interactions. In a shell model description where one assumes that all the quarks are in an s state, as in [12], one can make an estimate of the kinetic energy of a system of six identical quarks from which one can subtract the kinetic energy of two separate clusters of three quarks each. This gives $3/4\hbar\omega$ [7], i.e. a positive contribution, which may counterbalance the binding found in Ref. [12].

One can also raise the question whether or not by increasing the number of heavy quarks the stability would increase. In [18], we also investigated the system qqqqQQ in the Glozman et al. model [8], by using a similar procedure. There the most favourable configuration has I=0, J=1. For Q=c we obtained $M - M_T = 0.523$ GeV and for Q=b, $M - M_T = 0.515$ GeV. In both cases M_T corresponds to the lowest threshold qQQ + qqq where m(ccu) = 3.514 GeV and m(bbu) = 10.066 GeV. Therefore, these systems are also unbound in a compact configuration.

V. SUMMARY

In a chiral constituent quark model which successfully describes the light, strange and the presently known charmed and b-baryons, we have calculated the mass M of the hexaquarks H_c (uuddsc) and H_b (uuddsb) for various IJ sectors and compared it to the mass M_T of the corresponding lowest threshold. We found that the smallest $M - M_T$ value is associated to the IJ=00 or 01 sector. The quantity $M - M_T$ is always positive and of the order of few hundreds MeV. This indicates that H_c and H_b cannot exist as compact systems.

However, the existence of a weakly bound, molecular-type heavy hexaquark system, like the deuteron, cannot be excluded. The GBE interaction [3] generates a long-range attraction due to its Yukawa-potential tail and in principle, it can also produce a medium-range attraction from correlated two-pseudoscalar meson exchange. It is certainly interesting to pursue investigations in this direction, in a dynamical approach as the resonating group or the generator coordinate method, by incorporating six-quark states with orbital excitations, as in the NN case [20].

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APPENDIX A:

The calculation of the matrix elements of the interaction potential (7) between five light quarks is based on the fractional parentage technique described in Ref. [21]. Through this technique each five-body matrix element reduces to a linear combination of two-body matrix elements of the pair, 4 and 5, of quarks. This is possible due to the fact that each part (orbital, spin, flavour, colour) of the wave function is written as a sum of products of the first three and of the pair (45) wave functions. These functions have a definite permutation symmetry [f] and [f'] respectively, where [f']=[2] or [11]. The coefficients of these linear combinations have been obtained from the isoscalar factors of the Clebsch-Gordan coefficients of S_5 as calculated in Ref. [22].

The calculation of the spin-spin matrix elements is trivial. Below we give the flavour wave functions which we derived in the Rutherford-Young-Yamanouchi representation, where the pair 45 is either in a symmetric or an antisymmetric state, as mentioned above. We denote by p and q the row of the 5th and 4th particle in a Young tableau and by $\bar{p}q$ and $\tilde{p}q$ a symmetric and an antisymmetric state respectively. Then the uudds states required in these calculations are:

$$\begin{aligned} |[5]\bar{1}1> \;=\;& \sqrt{\tfrac{1}{5}}\psi_{[3]}(uud)\phi_{[2]}(ds) + \sqrt{\tfrac{1}{5}}\psi_{[3]}(udd)\phi_{[2]}(us) \\ &+\sqrt{\tfrac{2}{5}}\psi_{[3]}(uds)\phi_{[2]}(ud) + \sqrt{\tfrac{1}{10}}\psi_{[3]}(dds)\phi_{[2]}(uu) \\ &+\sqrt{\tfrac{1}{10}}\psi_{[3]}(uus)\phi_{[2]}(dd) \end{aligned} \tag{A1}$$

$$\begin{aligned} |[41]\bar{1}1> \;=\;& -\sqrt{\tfrac{4}{6}}\psi^{\rho,\Sigma_0}_{[21]}(uds)\phi_{[2]}(ud) + \sqrt{\tfrac{1}{6}}\psi^{\rho}_{[21]}(dsd)\phi_{[2]}(uu) \\ &+\sqrt{\tfrac{1}{6}}\psi^{\rho}_{[21]}(usu)\phi_{[2]}(uu) \end{aligned} \tag{A2}$$

$$|[41]\bar{1}\bar{2}> \; = \; \sqrt{\tfrac{3}{10}}\psi_{[3]}(uud)\phi_{[2]}(ds) + \sqrt{\tfrac{3}{10}}\psi_{[3]}(udd)\phi_{[2]}(us) - \sqrt{\tfrac{4}{15}}\psi_{[3]}(uds)\phi_{[2]}(ud) - \sqrt{\tfrac{1}{15}}\psi_{[3]}(dds)\phi_{[2]}(uu) - \sqrt{\tfrac{1}{15}}\psi_{[3]}(uus)\phi_{[2]}(dd) \tag{A3}$$

$$|[41]\tilde{1}\tilde{2}> = -\sqrt{\frac{1}{2}}\psi_{[3]}(uud)\phi_{[11]}(ds) - \sqrt{\frac{1}{2}}\psi_{[3]}(udd)\phi_{[11]}(us) \tag{A4}$$

$$|[32]\bar{2}\bar{2}> \; = \; \tfrac{1}{3}\psi_{[3]}(uud)\phi_{[2]}(ds) - \tfrac{1}{3}\psi_{[3]}(udd)\phi_{[2]}(us) - \tfrac{\sqrt{2}}{3}\psi_{[3]}(uds)\phi_{[2]}(ud) + \tfrac{1}{\sqrt{2}}\psi_{[3]}(dds)\phi_{[2]}(uu) + \tfrac{1}{3\sqrt{2}}\psi_{[3]}(uus)\phi_{[2]}(dd) \tag{A5}$$

$$|[32]\bar{1}\bar{2}> \; = \; \tfrac{2}{3}\psi^{\rho}_{[21]}(udu)\phi_{[2]}(ds) + \tfrac{1}{3}\psi^{\rho}_{[21]}(dud)\phi_{[2]}(us) - \tfrac{\sqrt{2}}{3}\psi^{\rho}_{[21]}(uds)\phi_{[2]}(ud) + \tfrac{\sqrt{2}}{3}\psi^{\rho}_{[21]}(usu)\phi_{[2]}(dd) \tag{A6}$$

$$|[32]\tilde{1}\tilde{2}> = \sqrt{\frac{1}{3}}\psi^{\rho}_{[21]}(dud)\phi_{[11]}(us) - \sqrt{\frac{2}{3}}\psi^{\rho}_{[21]}(uds)\phi_{[11]}(ud) \tag{A7}$$

$$|[311]\bar{1}\bar{1}> \; = \; \psi_{[111]}(uds)\phi_{[2]}(ud) \tag{A8}$$

$$|[311]\bar{1}\bar{3}> \; = \; \tfrac{\sqrt{15}}{10}\psi^{\rho}_{[21]}(udu)\phi_{[2]}(ds) - \tfrac{\sqrt{15}}{10}\psi^{\rho}_{[21]}(dud)\phi_{[2]}(us) - \sqrt{\tfrac{1}{10}}\psi^{\rho,\Lambda^0}_{[21]}(uds)\phi_{[2]}(ud) + \sqrt{\tfrac{3}{10}}\psi^{\rho}_{[21]}(dsd)\phi_{[2]}(uu) - \sqrt{\tfrac{3}{10}}\psi^{\rho}_{[21]}(usu)\phi_{[2]}(dd) \tag{A9}$$

$$|[311]\tilde{1}\tilde{3}> \; = \; -\tfrac{1}{2}\psi^{\rho}_{[21]}(udu)\phi_{[11]}(ds) + \tfrac{1}{2}\psi^{\rho}_{[21]}(dud)\phi_{[11]}(us) - \sqrt{\tfrac{1}{2}}\psi^{\rho,\Sigma_0}_{[21]}(uds)\phi_{[11]}(ud) \tag{A10}$$

$$|[311]\tilde{2}\tilde{3}> \; = \; \sqrt{\tfrac{2}{5}}\psi_{[3]}(uud)\phi_{[11]}(ds) - \sqrt{\tfrac{2}{5}}\psi_{[3]}(udd)\phi_{[11]}(us) + \sqrt{\tfrac{1}{5}}\psi_{[3]}(uds)\phi_{[11]}(ud) \tag{A11}$$

$$|[221]2\bar{3}> \; = \; \tfrac{\sqrt{2}}{4}\psi^{\rho}_{[21]}(udu)\phi_{[2]}(ds) + \tfrac{\sqrt{2}}{4}\psi^{\rho}_{[21]}(dud)\phi_{[2]}(us)$$
$$-\tfrac{1}{2}\psi^{\rho,\Sigma^0}_{[21]}(uds)\phi_{[2]}(ud) - \tfrac{1}{2}\psi^{\rho}_{[21]}(dsd)\phi_{[2]}(uu) \tag{A12}$$
$$-\tfrac{1}{2}\psi^{\rho}_{[21]}(usu)\phi_{[2]}(dd)$$

$$|[221]2\tilde{3}> \; = \; -\tfrac{\sqrt{6}}{4}\psi^{\rho}_{[21]}(udu)\phi_{[11]}(ds) - \tfrac{\sqrt{6}}{4}\psi^{\rho}_{[21]}(dud)\phi_{[11]}(us)$$
$$-\tfrac{1}{2}\psi^{\rho,\Lambda^0}_{[21]}(uds)\phi_{[11]}(ud) \tag{A13}$$

$$|[221]1\tilde{2}> \; = \; \psi_{[111]}(uds)\phi_{[11]}(ud) \tag{A14}$$

The states $\psi_{[2]}(ab)$ and $\psi_{[11]}(ab)$ are the symmetric and antisymmetric two-particle states. The $\psi_{[3]}(abc)$ is the symmetric three particle states. For mixed symmetry states $\psi^{\rho}_{[21]}$ some care should be taken. As usually [21] one has :

$$\psi^{\rho}_{[21]}(udu) = \frac{1}{2}(udu - duu) \tag{A15}$$

$$\psi^{\rho,\Lambda_0}_{[21]}(uds) = \frac{1}{\sqrt{12}}(2uds - 2dus + sdu - sud + usd - dsu) \tag{A16}$$

$$\psi^{\rho,\Sigma_0}_{[21]}(uds) = -\frac{1}{2}(usd + dsu - sdu - sud) \tag{A17}$$

However, it turns out that the states $|[32]1\bar{2}>$ and $|[32]1\tilde{2}>$ contain the function $\psi^{\rho}_{[21]}$, the definition of which is :

$$\psi^{\rho}_{[21]}(uds) = \frac{\sqrt{3}}{2}\psi^{\rho,\Lambda_0}_{[21]} - \frac{1}{2}\psi^{\rho,\Sigma_0}_{[21]} \tag{A18}$$

i.e. a linear combination of (A16) and (A17). While the states (A15)- (A17) have a definite isospin the state (A18) is a mixture of I=0 and I=1. Thus the states $|[32]1\bar{2}>$ and $|[32]1\tilde{2}>$ do not have a definite isospin. Therefore one has to project into a specific value of I in the calculation of matrix element of these two states. Calculation with or without projection indicate a difference of few MeV which is insignificant in the context of the present study.

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TABLES

TABLE I. Expectation value of the operator (1) in units of C_χ for all flavour $[f]_F$ and spin $[f]_S$ symmetries compatibles with $[32]_{FS}$. The corresponding spin S and isospin I, together with the total angular momentum J are also given.

$[f]_F$	$[f]_S$	S	J	I	$< V_\chi >$
[221]	[41]	3/2	1,2	0	-12
[221]	[32]	1/2	0,1	0	-16
[311]	[32]	1/2	0,1	1	-12
[32]	[32]	1/2	0,1	0,1	-8
[311]	[41]	3/2	1,2	1	-8
[32]	[41]	3/2	1,2	0,1	-4
[41]	[32]	1/2	0,1	1,2	-2
[41]	[41]	3/2	1,2	1,2	2
[32]	[5]	5/2	2,3	0,1	8/3
[5]	[32]	1/2	0,1	2	8

TABLE II. Variational solution of the Hamiltonian (5)-(7) for spin S=1/2 low lying baryons compared to experimental masses

Baryon	Variational parameters (fm)		Expectation value	Experimental mass (GeV)
	a	b	(GeV)	[23]
N	0.4376		0.9696	0.940
Λ	0.4486		1.1654	1.1156
Σ	0.4625		1.2354	1.193
Λ_c	0.4683	0.7099	2.3268	2.2849
Σ_c	0.6320	0.7201	2.4889	2.452
Ξ_c	0.5705	0.6967	2.5494	2.470
Λ_b	0.4678	0.6509	5.6147	5.641
Σ_b	0.6292	0.6623	5.7775	?
Ξ_b	0.5683	0.6339	5.83629	?

TABLE III. Masses M and mass differences $M - M_T$ (M_T - the threshold mass) of heavy hexaquark systems uuddsQ (Q=c or b) of given isospin I and total angular momentum J

System	I	J	M (GeV)	Threshold	$M - M_T$ (GeV)
uuddsc	0	0,1	4.144	$N + \Xi_c$	0.625
				$\Lambda + \Lambda_c$	0.652
	1	0,1	4.304	$\Sigma + \Lambda_c$	0.742
				$N + \Xi_c$	0.785
	2	0,1	4.496	$\Sigma + \Sigma_c$	0.772
uuddsb	0	0,1	7.425	$N + \Xi_b$	0.619
				$\Lambda + \Lambda_b$	0.645
	1	0,1	7.586	$\Sigma + \Lambda_b$	0.736
				$N + \Xi_b$	0.780
	2	0,1	7.780	$\Sigma + \Sigma_b$	0.767