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Tetraquarks with colour-blind forces in chiral quark models

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Abstract

We discuss the stability of multiquark systems within the recent model of Glozman *et al.* where the chromomagnetic hyperfine interaction is replaced by pseudoscalar-meson exchange contributions. We find that such an interaction binds a heavy tetraquark systems $QQ\bar{q}\bar{q}$ ($Q = c$, b and $q = u$, d) by $0.2 - 0.4$ GeV. This is at variance with results of previous models where $cc\bar{q}\bar{q}$ is unstable.

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The existence of tetraquark hadrons —two quarks and two antiquarks— has been raised about twenty years ago by Jaffe [1] and has been studied within a variety of models. The MIT bag study indicated the presence of a dense spectrum of tetraquark states in the light sector [1]. Later on, tetraquark systems have been examined in potential models [2-4] and flux tube models [5]. In particular the question of stability has been raised, that is whether the tetraquark ground state lies below or above the lowest $(q\bar{q}) + (q\bar{q})$ threshold. Weinstein and Isgur showed [2] that there are only a few weakly–bound states of resonant meson–meson structure in the light (u, d, s) sector. On the other hand, no bound state was found by Carlson and Pandharipande [5] in their flux-tube model with quarks of equal masses.

One important result in the MIT bag or potential models is that the chromomagnetic interaction plays a crucial role [2,6] in lowering the ground state energy of a light system. Otherwise, in a system of two heavy quarks and two light antiquarks $QQ\bar{q}\bar{q}$ ($Q = c$ or b, $q = u$, d or s) stability can be achieved without spin–spin interaction, provided the mass ratio $m(Q)/m(q)$ is larger [3] than about 15, which means that Q must be at least a b-quark.

In the light sector, there are several candidates for non-$q\bar{q}$ states, but the experimental situation is not yet conclusive. For a review, see for example Ref. [7] and the last issue of Review of Particle Properties [8]. In the heavy sector, experiments are being planned at Fermilab and CERN, to search for new hadrons and in particular for doubly charmed tetraquarks [9-11].

Recently, the baryon spectrum has been analysed by Glozman *et al.* [12,13] within a chiral potential model which includes meson-exchange forces between quarks and entirely neglects the chromomagnetic interaction. In view of its intriguing success in the description of the baryon spectrum, it seems to us natural to apply this model to multiquark hadrons with more than three quarks, and in particular to tetraquarks.

A general Hamiltonian containing both chromomagnetic interaction and meson-exchange contributions has the form

$$H = \sum_i \frac{\vec{\mathrm{p}}_i^{\,2}}{2m_i} - \frac{3}{16}\sum_{i<j} \tilde{\lambda}^{\mathrm{c}}_i\cdot\tilde{\lambda}^{\mathrm{c}}_j\, V_{\mathrm{conf}}(r_{ij}) \\ - \sum_{i<j} \tilde{\lambda}^{\mathrm{c}}_i\cdot\tilde{\lambda}^{\mathrm{c}}_j\, \vec{\sigma}_i\cdot\vec{\sigma}_j\, V_{\mathrm{g}}(r_{ij}) - \sum_{i<j} \tilde{\lambda}^{\mathrm{F}}_i\cdot\tilde{\lambda}^{\mathrm{F}}_j\, \vec{\sigma}_i\cdot\vec{\sigma}_j\, V_{\mathrm{F}}(r_{ij}), \tag{1}$$

where m_i is the constituent mass of the quark located at $\vec{\mathrm{r}}_i$; $r_{ij} = |\vec{\mathrm{r}}_j - \vec{\mathrm{r}}_i|$ denotes the interquark distance; $\vec{\sigma}_i$, $\tilde{\lambda}^{\mathrm{c}}_i$, $\tilde{\lambda}^{\mathrm{F}}_i$ are the spin, colour and flavour operators, respectively. Spin-orbit and tensor components may supplement the above spin-spin forces for studying orbital excitations. The potential in H has three parts containing the confining, the chromomagnetic and the meson-exchange contributions, respectively.

The confining term V_{conf} usually consists of a Coulomb plus a linear term,

$$V_{\mathrm{conf}} = -\frac{a}{r} + br + c. \tag{2}$$

In the following, we shall use either the very weak linear potential of Glozman *et al.* [13] corresponding to

$$(C_1) \qquad a = c = 0, \qquad \text{and} \qquad b = 0.01839\ \mathrm{GeV}^2, \tag{3}$$

or a more conventional choice

$$(C_2) \qquad a = 0.5203, \qquad b = 0.1857\ \mathrm{GeV}^2, \qquad c = -0.9135\ \mathrm{GeV}, \tag{4}$$

which has already been used in the study of tetraquarks by Silvestre-Brac and Semay [4].

The third term in H is often understood as the chromomagnetic analogue of the Breit–Fermi term of atomic physics. For mesons, $\tilde{\lambda}^{\mathrm{c}}_1\cdot\tilde{\lambda}^{\mathrm{c}}_2 = -16/3$, and a positive V_{g}, as in the one-gluon-exchange model, shifts each vector meson above its pseudoscalar partner, for instance $D^* > D$ in the charm sector. For baryons, where $\tilde{\lambda}^{\mathrm{c}}_1\cdot\tilde{\lambda}^{\mathrm{c}}_2 = -8/3$ for each quark pair, such a positive V_{g} pushes the spin 3/2 ground states up, and the spin 1/2 down, for instance $\Delta > N$. In Ref. [4], the following radial shape has been used

$$V_{\mathrm{g}} = \frac{a}{m_i m_j d^2}\,\frac{\exp -r/d}{r}, \tag{5}$$

with the same value of a as in Eq. (4) and $d = 0.454$ GeV^{-1}.

The last term of H corresponds to meson exchange, and an explicit sum over F is understood. If the system contains light quarks only (as in Refs. [12,13]), the sum over F runs from 0 to 8 ($1-3 \to \pi$, $4-7 \to K$, $8 \to \eta$ and $0 \to \eta'$). If a heavy flavour is incorporated, a phenomenological extension from $\mathrm{SU}(3)_\mathrm{F}$ to $\mathrm{SU}(4)_\mathrm{F}$ would further extend the sum to $\mathrm{F} = 9-12$ corresponding to a D-exchange, $\mathrm{F} = 13-14$ to a D_s-exchange and $\mathrm{F} = 15$ to an η_c-exchange. If $V_\mathrm{F} = 0$, one recovers a standard constituent quark model. The radial form of $V_\mathrm{F} \neq 0$ is derived from the usual pion-exchange potential which contains a long-range part and a short-range one

$$\sum_{i<j} \vec{\tau}_i \cdot \vec{\tau}_j \, \vec{\sigma}_i \cdot \vec{\sigma}_j \frac{g^2}{4\pi} \frac{1}{4m^2} \left[\mu^2 \frac{\exp(-\mu r_{ij})}{r_{ij}} - 4\pi \delta^{(3)}(r_{ij}) \right], \tag{6}$$

where μ is the pion mass. A coupling constant $g^2/4\pi = 0.67$ at the quark level corresponds to the usual strength $g_{\pi NN}/4\pi \simeq 14$ for the Yukawa tail of the nucleon–nucleon (NN) potential.

When constructing NN forces from meson exchanges, one disregards the short-range term in Eq. (6), for it is hidden by the hard core, and anyhow the potential in that region is parameterized empirically. Similarly, when Törnqvist [14], Manohar and Wise [15] or Ericson and Karl [16] considered pion exchange in multiquark states, they had in mind the Yukawa term $\exp(-\mu r)/r$ acting between two well-separated quark clusters. For similar reasons, Weber *et al.*[17], in their model with hyperfine plus pion-exchange interaction ignored the delta-term too. Therefore it is somewhat of a surprise to see the delta-term of Eq. (6) taken seriously, and with an *ad-hoc* regularisation playing a crucial role in the quark dynamics [12,13]. This regularised form is [13]

$$V_\mu = \Theta(r - r_0)\mu^2 \frac{\exp(-\mu r)}{r} - \frac{4\epsilon^3}{\sqrt{\pi}} \exp(-\epsilon^2 (r - r_0)^2), \tag{7}$$

with the Yukawa-type part cut off for $r \leq r_0$, where $r_0 = 2.18$ GeV^{-1}, $\epsilon = 0.573$ GeV, and $\mu = 0.139$ GeV for π, 0.547 GeV for η and 0.958 GeV for η'.

Incorporating in H both light quarks and charm (one can add bottom similarly) and working out the flavour matrix elements of the mesons-exchange interaction between two quarks (or antiquarks), one obtains:

$$\langle V_{i,j}\rangle = \vec{\sigma}_i\cdot\vec{\sigma}_j \begin{cases} V_\pi + \frac{1}{3}V_\eta^{uu} + \frac{1}{6}V_{\eta_c}^{uu} & ; \quad [2]_{\rm F} \quad I=1 \\ 2V_K - \frac{2}{3}V_\eta^{us} + 2V_D^{uc} + V_{D_s}^{sc} & ; \quad [2]_{\rm F} \quad I=1/2 \\ \frac{4}{3}V_\eta^{ss} + \frac{3}{2}V_{\eta_c}^{cc} & ; \quad [2]_{\rm F} \quad I=0 \\ -2V_K - \frac{2}{3}V_\eta^{us} - 2V_D^{uc} - V_{D_s}^{sc} & ; \quad [11]_{\rm F} \quad I=1/2 \\ -3V_\pi + \frac{1}{3}V_\eta^{uu} + \frac{1}{6}V_{\eta_c}^{uu} & ; \quad [11]_{\rm F} \quad I=0 \end{cases} \tag{8}$$

for F $= 1, \ldots 15$, and

$$\langle V_{ij}\rangle = \frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j V_{\eta'} \tag{9}$$

for F $= 0$. This is an extension of Eq. (3.3) of Ref. [12] from SU(3) to SU(4). Here I is the isospin and in each case it is specified whether the pair is in a symmetric $[2]_{\rm F}$ state or in an antisymmetric $[11]_{\rm F}$ state. Actually little $u\bar{u}$ or $d\bar{d}$ mixing is expected in η_c so that the contribution of $V_{\eta_c}^{uu}$ can be safely neglected. Moreover when the meson mass μ reaches values of a few GeV as for D or η_c the two terms in Eq. (6) basically cancel each other so the expressions (7) reduce practically to their SU(3) form [12]. This is in agreement with Ref. [18] where it has been explicitly shown that the dominant contribution to the Σ_c and Σ_c^* masses is due to meson exchange between light quarks and the contribution of the matrix elements with the D (D_s) and D^* (D_s^*) quantum numbers (which are evaluated phenomenologically, fitting the mass difference $\Sigma_c - \Lambda_c$) play a minor role. In the following numerical calculations we will neglect the exchange of heavy mesons.

The remaining parameters are the quark masses. They are indicated in Table 1, in conjunction with the two choices. The light quark masses $m = m_u = m_d$ are from the corresponding literature [13,4]. The heavy quark mass $m_Q = m_c$ or m_b is adjusted to reproduce the experimental average mass $\overline{M} = (M + 3M^*)/4$ of $M = D$ or B mesons. The meson mass is obtained from a trial wave function of type $\phi \propto \exp(-\alpha r^2/2)$, with α as a variational parameter. It has been checked that the error never exceeds a few MeV with respect to the exact value. The variational approximation is retained for consistency with the treatment of 3- and 4-body systems discussed below.

Table 1. Quark masses and average heavy meson masses $\overline{M} = (M + 3M^*)/4$ ($M = D$, B) used for the potentials (C_1) and (C_2). The units are GeV.

Model	m	m_c	m_b	$\overline{D}$	$\overline{B}$
(C_1)	0.340[a)]	1.350	4.660	2.001	5.302
(C_2)	0.337[b)]	1.870[b)]	5.259[b)]	2.006	5.350

[a)] Ref. [13] [b)] Ref. [4]

We now briefly discuss the baryons. In the model of Glozman *et al.* the explicit form of the Hamiltonian integrated in the spin–flavour space is :

$$H = H_0 + \frac{g^2}{48\pi m^2} \begin{cases} 15V_\pi - V_\eta - 2\,(g_0/g)^2\, V_{\eta'} & \text{for} \quad N \\ 3V_\pi + V_\eta + 2\,(g_0/g)^2\, V_{\eta'} & \text{for} \quad \Delta \end{cases} \tag{10}$$

with

$$H_0 = 3m + \sum_i \frac{\vec{\mathrm{p}}_i^{\,2}}{2m} + \frac{b}{2}\sum_{i<j} r_{ij}, \tag{11}$$

where $g^2/4\pi = 0.67$, $(g_0/g)^2 = 1.8$ and $V_\eta = V_\eta^{uu}$ of Eq. (8).

We have performed variational estimates with a wave function $\phi \propto \exp(-\alpha(\rho^2+\lambda^2)/2)$, where $\vec{\rho} = \vec{\mathrm{r}}_2 - \vec{\mathrm{r}}_3$, $\vec{\lambda} = (2\vec{\mathrm{r}}_1 - \vec{\mathrm{r}}_2 - \vec{\mathrm{r}}_3)/\sqrt{3}$, and reproduced the results of the more elaborate Faddeev calculations of Ref. [13]. When the meson–exchange terms are switched off, the N and Δ ground states are degenerate at 1.63 GeV. When the coupling is introduced, the wave function is modified. For the nucleon, the spin-independent part H_0 of the Hamiltonian gives a contribution of 2.11 GeV, and receives a large -1.14 GeV correction from meson exchange. For the Δ ground state, the contribution of H_0 and meson exchange parts are 1.91 GeV and -0.63 GeV, respectively. Thus one ends up with a reasonable value for the $\Delta - N$ splitting, close to 0.3 GeV.

We have also calculated the ground state baryons of content cqq with a trial wave function $\phi \propto \exp(-(\alpha\rho^2 + \beta\lambda^2)/2)$ and found $\Lambda_c = 2.32$ GeV and $\Sigma_c = \Sigma_c^* = 2.48$ GeV, close to the experimental values and consistent with the findings of Ref. [18], although the Hamiltonian, its treatment, and the input parameters are somewhat different there.

Due to arguments at the beginning of this letter, here we discuss tetraquarks containing heavy flavours, i.e. $QQ\bar{q}\bar{q}$, studying the most favourable configuration $\bar{3}3$, $S=1$, $I=0$. This means QQ is in a $\bar{3}$ colour state and $\bar{q}\bar{q}$ in a 3 colour state. The mixing with $6\bar{6}$ is neglected because one expects this plays a negligible role in deeply bound heavy systems [3]. Then the Pauli principle requires $S_{12}=1$ for QQ, and $S_{34}=0$, $I_{34}=0$ for $\bar{q}\bar{q}$, if the relative angular momenta are zero for both subsystems. This gives a state of total spin $S=1$ and isospin $I=0$.

The tetraquark Hamiltonian integrated in the colour–spin–flavour space, and incorporating the approximations discussed in relation to Eq. (8), reduces to

$$H = 2(m+m_Q) + \frac{\vec{\mathrm{p}}_x^2}{m_Q} + \frac{\vec{\mathrm{p}}_y^2}{m} + \frac{m+m_Q}{2mm_Q}\vec{\mathrm{p}}_z^2 + \sum_{i<j} V_{ij}, \tag{12}$$

where

$$\begin{aligned} V_{12} &= \frac{1}{2}\left(-\frac{a}{r_{12}} + b\, r_{12} + c\right), \\ V_{ij} &= \frac{1}{4}\left(-\frac{a}{r_{ij}} + b\, r_{ij} + c\right), \qquad i = 1 \text{ or } 2,\ j = 3 \text{ or } 4, \\ V_{34} &= \frac{1}{2}\left(-\frac{a}{r_{34}} + b\, r_{34} + c\right) + 9V_\pi - V_\eta - 2V_{\eta'}. \end{aligned} \tag{13}$$

The momenta $\vec{\mathrm{p}}_x$, etc., are conjugate to the relative distances $\vec{\mathrm{x}} = \vec{\mathrm{r}}_1 - \vec{\mathrm{r}}_2$, $\vec{\mathrm{y}} = \vec{\mathrm{r}}_3 - \vec{\mathrm{r}}_4$, and $\vec{\mathrm{z}} = (\vec{\mathrm{r}}_1 + \vec{\mathrm{r}}_2 - \vec{\mathrm{r}}_3 - \vec{\mathrm{r}}_4)/\sqrt{2}$. The wave function is parameterized as

$$\psi \propto \exp[-(\alpha x^2 + \beta y^2 + \gamma z^2)/2], \tag{14}$$

and the minimization with respect to α, β and γ leads to the results displayed in Table 2 for $Q=c$ and b. Columns 2 and 3 corresponds to results derived from Eqs. (12) and (13). This shows that both the $cc\bar{q}\bar{q}$ and $bb\bar{q}\bar{q}$ systems are bound whatever is the potential, (C_1) or (C_2), provided meson exchange is incorporated. This is in contradistinction to previous studies based on conventional models where the flavour-independent confining potential is supplemented by one gluon exchange. This remark is illustrated by column 4 which shows that $cc\bar{q}\bar{q}$ is unbound in such a case.

Table 2. Energy (GeV) $\Delta E = QQ\bar{q}\bar{q} - 2(Q\bar{q})$ for heavy tetraquarks with $Q = c$ or b, $q = u$, d in three different models with parameters defined in the text (OME = one meson exchange).

System	$(C_1) +$ OME	$(C_2) +$ OME	Ref. [4]
$cc\bar{q}\bar{q}$	−0.185	−0.332	0.019
$bb\bar{q}\bar{q}$	−0.226	−0.497	−0.135

Considering this result one could rise the question if in the model of Glozman *et al.* a proliferation of multiquarks systems appears. We have therefore tried to investigate $QQqqqq$ and q^6 systems as well. We have proceeded analogously to the previous case using a Gaussian wave function. In the case of $QQqqqq$, group–theory analysis [19] shows that the most favourable configuration is the one where the light quark subsystem has $S = 1$, $I = 0$, corresponding to a global spin-flavour-averaged interaction $\langle V \rangle = 10V_\pi - 2/3V_\eta - 4/3(g_0/g)^2 V_{\eta'}$. Our numerical calculation shows that this potential is largely insufficient to bind the system. Also the q^6 system, the most favourable configuration of which is $S = 1$, $I = 0$, leading to $\langle V \rangle = 11V_\pi - 5/3V_\eta - 10/3(g_0/g)^2 V_{\eta'}$, is not sufficiently bound for being under the two baryons threshold.

It is also interesting to notice that the binding energy of $cc\bar{q}\bar{q}$ and $bb\bar{q}\bar{q}$ are nearly twice larger for the potential (C_2) as compared to those of (C_1) and also much more different from each other. The reason is that (C_2) contains a Coulomb part which binds more, heavier is the system, leading thus to a larger separation among levels as well. The potential (C_2) has been fitted to reproduce the J/Ψ and the Υ meson masses. It also gives overall good results both for other mesons and baryons. By construction [12,13], the potential (C_1) was designed and fitted to light baryons only. It is desirable to construct a chiral potential model with a wider range of validity, covering the light and heavy sector as well, and to apply it to the study of mesons and multiquarks systems. Our results indicate that the chiral model of Glozman *et al.* leads to qualitatively different results for tetraquarks, with respect to commonly used quark models. We hope that future experimental investigations might distinguish among various approaches to quark dynamics.

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