

SURVEY OF ALTERNATIVE BARYON MODELS

Fl. Stancu¹

*Université de Liège, Institut de Physique B5, Sart Tilman, B-4000
Liège 1, Belgium*

Abstract:

I critically review aspects of the Goldstone Boson Exchange model and of the algebraic model used in baryon spectroscopy. I discuss their performances with regard to the Roper resonance.

1 Introduction

There is a continuous debate about the most adequate degrees of freedom in baryon spectroscopy. Accordingly, there is a variety of models which are being used to derive the baryon spectra. Some of them are quite thoroughly described in Bhaduri's text book [1]. Besides those models there are new ones. Two of them will be reviewed in the following. One can roughly classify the baryon models in several categories:

- Constituent quark models (3 valence quarks)
 - One gluon exchange models (OGE) (presented by S. Capstick)
 - Meson exchange, also called Goldstone boson exchange (GBE) models
 - Hybrid models with OGE + GBE interactions combined together
 - Algebraic models
- Relativistic constituent quark modes (3 valence quarks)
 - The MIT bag model and the cloudy bag model
 - The chromodielectric model, which can be viewed as a more realistic version of the MIT bag model (see e.g. [2])
 - The instanton model (presented by B. Ch. Metsch)
- Phenomenological lagrangians
 - Topological soliton models, and in particular the Skyrme model (presented by G. Holzwarth, see also P. Morsch's introductory talk. An elementary, pedagogical description can be found in Bhaduri's book)
 - Chiral field models (see e.g. [3]). These models imply an infinite number of particles. The role of the Dirac sea is important. They are well suited to describe ground

¹e-mail: fstancu@ulg.ac.be

state baryons which appear as solitons (quarks bound by chiral fields).. A generic example is the Nambu-Jona-Lasinio model.

2 THE LEVEL ORDERING PROBLEM

In this talk I shall review two constituent quark models, namely the GBE and the algebraic model. To my knowledge, apart from the Skyrme model, these are the only ones which reproduce correctly the order of positive and negative parity levels in baryons and in particular the position of the Roper resonance with respect to the lowest negative parity states. The lowest part of the experimental spectrum of N, Δ and Λ baryons shows that for N and Δ the parity order is $+, +, -$ and for Λ $+, -, +$ respectively. Apart from a kinetic (nonrelativistic or relativistic) part, the spin independent part of the Hamiltonian describing three interacting quarks usually contains a confinement part of a harmonic oscillator type or a linear type. They both provide levels with the parity order $+, -, +$ while the desired order in nonstrange baryons is $+, +, -$ as mentioned above. The reason is that the Laplacian of such potentials is positive. In Ref. [4] the question has been raised whether another kind of spin independent potential can provide the correct order. Let us consider the following semirelativistic Hamiltonian

$$H_0 = \sum_{i=1}^3 (m_i^2 + p_i^2)^{1/2} + \frac{1}{2} \sqrt{\sigma} \sum_{i < j} r_{ij} - \frac{g_\sigma^2}{4\pi} \sum_{i < j} \left[\frac{\exp(-\mu_\sigma r_{ij})}{r_{ij}} - \frac{\exp(-\Lambda r_{ij})}{r_{ij}} \right], \quad (1)$$

where $m_i = 340$ MeV and $r_{ij} = |\vec{r}_{ij}| = |\vec{r}_i - \vec{r}_j|$. The second term is a linear confinement potential with a string tension

$$\sqrt{\sigma} = 1 \text{ GeV fm}^{-1}. \quad (2)$$

The third term is due to a σ -exchange interaction, the form of which corresponds to the Pauli-Villars regularisation. In this term the σ -meson mass is fixed to be $\mu_\sigma = 600$ MeV. The coupling constant $g_\sigma^2/(4\pi)$ and the regularisation parameter Λ are taken as variable parameters. In Ref. [4] it was found that in the limit $\Lambda \rightarrow \infty$ the level order becomes $1S, 2S, 1P$ after the critical value of the coupling constant $g_\sigma^2/(4\pi) \approx 0.75$ has been reached, as shown in Fig. 1.

In this case the second term in the σ -exchange potential vanishes and in the resulting potential every S state is lowered with respect to P states. The reason is that the wave function of the latter being small around the origin, it is less sensitive to the short-range attraction due to the σ -meson exchange potential. In mathematical terms in Eq. (1) one deals with a potential having components with $\Delta V < 0$ when $\Lambda \rightarrow \infty$. The linear potential has a positive Laplacian and the Yukawa-type potential has a negative Laplacian. Once the coupling constant raises above a critical value the total potential acquires a negative Laplacian which allows the level crossing in Fig. 1. In this way the desired parity order is obtained. However when a low cut-off parameter $\Lambda \sim 1$ GeV is used, although the same trend remains, the effect becomes quite marginal, leading essentially to an overall shift of the spectrum. In that case a proper order of levels can be obtained from a flavour-dependent spin-spin interaction.

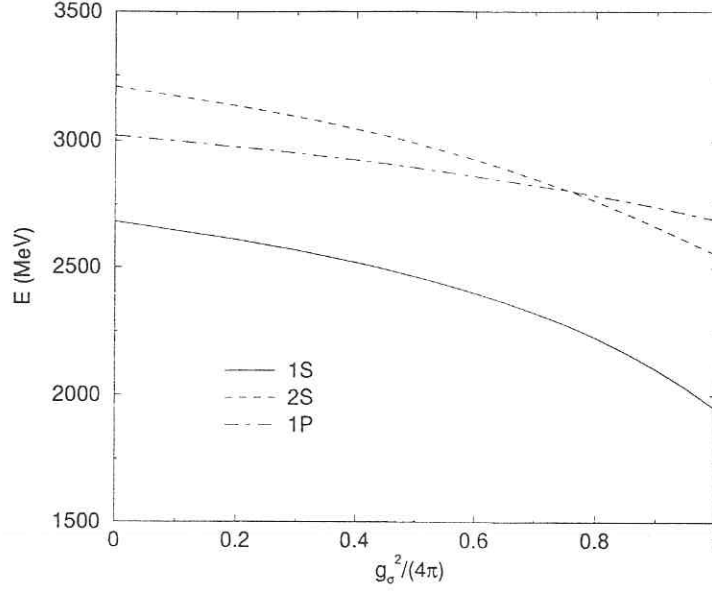


Figure 1: Energies of the first three eigenvalues of the Hamiltonian (1) for increasing values of the coupling constant $g_\sigma^2/(4\pi)$ in the limit $\Lambda \rightarrow \infty$ (from Ref. [4]).

A schematic SU(2) version of a flavour dependent spin-spin interaction is:

$$O^{IS} = - \sum_{i < j} \vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (3)$$

with $\vec{\tau}_i$ and $\vec{\sigma}_i$ related to the isospin and spin operators. The effect of this interaction is similar to that shown in Fig. 1 as implied by Table I where three expectation values of O^{IS} are indicated. One can see that the ground state and the first radially excited state, both of orbital symmetry $[3]_O$ are lowered by 15 units while the first negative parity state of either spin 1/2 (i.e., $[21]_S$) or 3/2 (i.e., $[3]_S$) are shifted up or down by a quantity five times smaller. Then the ground state and the Roper resonance are both lowered while the negative parity states are much less affected by the interaction (3), leading in practice to a good level ordering.

Table I *The expectation value of the interaction (3) for a colour singlet $3q$ system.*

state	$\langle O^{IS} \rangle$
$[3]_O[3]_{FS}[21]_F[21]_S$	-15
$[21]_O[21]_{FS}[21]_F[21]_S$	-3
$[21]_O[21]_{FS}[21]_F[3]_S$	3

The schematic operator (3) can be extended to the SU(3) flavour symmetry. This is precisely what the GBE model does, as described in the next section.

3 The Goldstone Boson Exchange Model

The GBE model exploits the spontaneous breaking of the chiral symmetry in the QCD vacuum for deriving a flavour dependent hyperfine interaction. The history of the model is rather short. Manohar and Georgi [5] introduced an effective Lagrangian with gluons plus Goldstone boson fields and quarks with an internal structure resulting from spontaneous breaking of the chiral symmetry. The Goldstone boson fields were supposed to play a role in low energy QCD in the range between 100 MeV (the confinement scale) and about 1 GeV (the spontaneous breaking symmetry scale). More drastically, Glozman and Riska [6] assumed that the only degrees of freedom relevant for baryon spectroscopy are the Goldstone bosons (pseudoscalar mesons) and the constituent quarks (no gluons). The quark masses are assumed to be dynamically generated by the spontaneous breaking of the chiral symmetry. Accordingly they proposed an effective hyperfine interaction between quarks as due to the exchange of pseudoscalar mesons between quarks and showed that such an interaction, due to its spin-flavour symmetry, reproduces the correct level ordering in light nonstrange and strange baryons. Its schematic form is

$$V_{\chi} = - \sum_{i < j} \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j \quad (4)$$

with $\lambda_i^F \cdot \lambda_j^F$ instead of $\vec{\tau}_i \cdot \vec{\tau}_j$ of Eq. (3). It was first noticed by Robson [7] that such an interaction gives the position of the Roper resonance below the first negative parity states, as the experiment requires. However the conclusion of Ref. [7] that any contact interaction, irrespective of its unitary symmetry structure, would give the desired order, was incorrect. It is in fact the $SU_F(3) \times SU_S(2)$ structure of the operator (4) which is decisive for the level order. Several successive papers of the Graz group [8, 9, 10] brought the GBE model to a realistic form. The model is supported by lattice calculations [11] and is consistent with the model independent analysis of the mass spectrum of nonstrange $L = 1$ baryons in large N_c QCD [12]. The present status of the model is summarized in Ref. [13]. In its extended form [14], besides pseudoscalar, it also contains scalar and vector meson exchanges. The Hamiltonian reads

$$H = \sum_{i=1}^3 (m_i^2 + p_i^2)^{1/2} + \sum_{i < j} [V_{conf}(r_{ij}) + V_{\chi}(r_{ij})], \quad (5)$$

where the chiral interaction is

$$\begin{aligned} V_{\chi}(r_{ij}) = & \sum_{a=1}^3 [V_{\pi}(r_{ij}) + V_{\rho}(r_{ij})] \lambda_i^a \lambda_j^a + \sum_{a=4}^7 [V_K(r_{ij}) + V_{K^*}(r_{ij})] \lambda_i^a \lambda_j^a \\ & + [V_{\eta}(r_{ij}) + V_{\omega_8}(r_{ij})] \lambda_i^8 \lambda_j^8 + \frac{2}{3} [V_{\eta'}(r_{ij}) + V_{\omega_0}(r_{ij})] \end{aligned} \quad (6)$$

The confinement $V_{conf}(r_{ij})$ and the chiral potential V_{γ} ($\gamma = \pi, K, \eta, \eta', \rho, K^*, \omega_8$ and ω_0) have analytic forms containing several parameters. The meson masses and the coupling constants are predetermined parameters. The cut-off parameters obey some scaling laws which contain three parameters to be fitted. In practical calculations only the spin-spin and tensor parts of the pseudoscalar and vector meson exchanges are considered. The

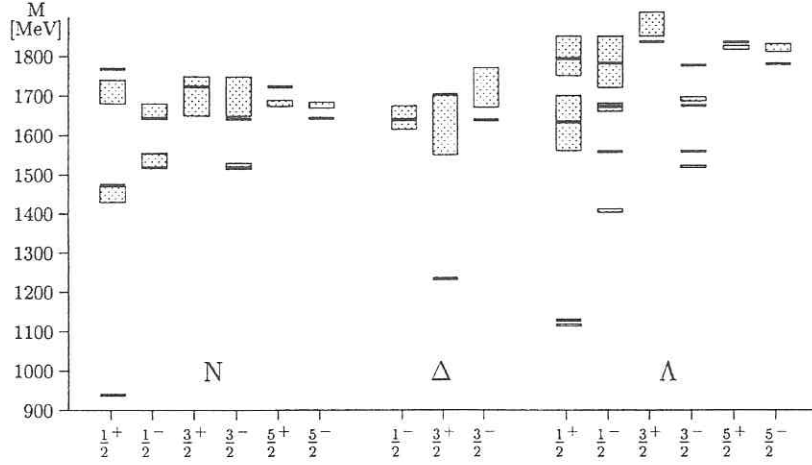


Figure 2: The lowest N , Δ and Λ baryons in the GBE model from Ref. [14]. The shadowed boxes represent the experimental values with their uncertainties from Ref. [20].

spin-orbit interaction is neglected. It is expected to be cancelled out by the Thomas term [15]. The tensor term is small, inasmuch as the pseudoscalar and vector meson exchange give contributions of opposite signs, which largely cancel each other. The calculated spectrum of N , Δ and Λ is shown in Fig. 2 together with the experimental values within their uncertainties. One can see that the level order is correct and the Roper resonance lies at the edge of the experimental uncertainty box.

Table II *Predictions for π decay widths of N and Δ resonances from Ref. [19] based on a modified 3P_0 decay model employing a Gaussian meson wave function with a radius $r_\pi = 0.565$ fm. The strength parameter γ is fixed for each model to reproduce the $\Delta \rightarrow N\pi$ width. Experimental data are from the PDG[20].*

N^*	J^π	$\Gamma(N^* \rightarrow N\pi)$ [MeV]				Exp.
		OGE NR	OGE SR	GBE NR	GBE SR	
N_{1440}	$\frac{1}{2}^+$	161	1064	262	530	$(227 \pm 18)^{+70}_{-59}$
N_{1710}	$\frac{1}{2}^+$	8	202	16	55	$(15 \pm 5)^{+30}_{-5}$
Δ_{1232}	$\frac{3}{2}^+$	120	120	120	120	$(119 \pm 1)^{+5}_{-5}$
Δ_{1600}	$\frac{3}{2}^+$	14	174	40	45	$(61 \pm 26)^{+26}_{-10}$
N_{1520}	$\frac{3}{2}^-$	168	108	228	45	$(66 \pm 6)^{+9}_{-5}$
N_{1535}	$\frac{1}{2}^-$	109	462	167	140	$(67 \pm 15)^{+55}_{-17}$
N_{1650}	$\frac{1}{2}^-$	8	87	13	9	$(109 \pm 26)^{+36}_{-3}$
N_{1675}	$\frac{5}{2}^-$	52	40	23	4	$(68 \pm 8)^{+14}_{-4}$
N_{1700}	$\frac{3}{2}^-$	9	7	4	1	$(10 \pm 5)^{+3}_{-3}$
Δ_{1620}	$\frac{1}{2}^-$	5	41	6	4	$(38 \pm 8)^{+8}_{-6}$
Δ_{1700}	$\frac{3}{2}^-$	38	20	16	3	$(45 \pm 15)^{+20}_{-10}$
N_{1680}	$\frac{5}{2}^+$	313	149	88	80	$(85 \pm 7)^{+6}_{-6}$
N_{1720}	$\frac{3}{2}^+$	238	689	109	367	$(23 \pm 8)^{+9}_{-5}$
γ		11.868	18.015	14.643	8.871	

The strong decay widths have been calculated with the model version of Ref. [10], which does not contain the tensor term. The decay mechanism was chosen to be a modified version [16] of the 3P_0 model [17]. The 3P_0 mechanism is rooted in a flux tube picture of confinement. Its microscopic origin has been discussed in Ref. [18]. The calculated widths [19] for the decay $N^* \rightarrow N\pi$ obtained from the nonrelativistic GBE model (GBE NR) [9] and the semirelativistic GBE model (GBE SR) [10] are shown in Table II. They are compared with those obtained from a nonrelativistic and a relativistic version of the OGE model of Ref. [21]. One can see that the GBE NR performs better than the other models, especially for the Roper resonance. Some widths are well reproduced. In all models there are however striking discrepancies with respect to data. A particularly critical case is the $N(1720)3/2^+$ resonance.

Fig. 3 reproduces the more popular OGE results of Fig. 6 of Capstick and Roberts [22]. This figure shows the amplitudes of the decay $N^* \rightarrow N\pi$ of positive parity resonances as compared to the data. I added to it the GBE NR [9] amplitudes (circles) extracted from Table II. One can see that this version of the OGE model and the GBE model lead to results of similar quality.

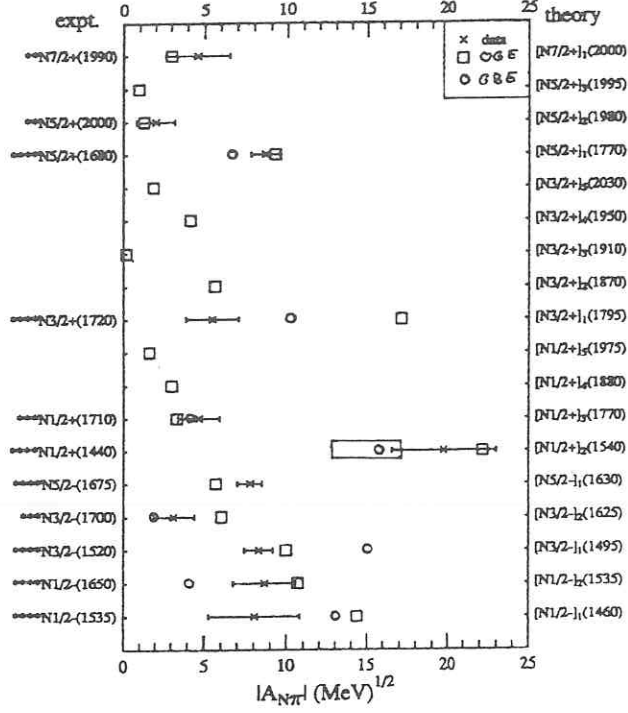


Figure 3: The decay amplitudes $|A_{N\pi}| = \sqrt{\Gamma_{N\pi}}$ of positive parity resonances. The OGE results (squares) are from Ref. [22] and the GBE results (filled circles) are from Ref. [19]. The rectangle represents the experimental uncertainty associated to the Roper resonance updated to the latest Particle Data Group [20]

4 The algebraic model

The algebraic model discussed here is due to Bijker, Iachello and Leviatan. It has been applied to nonstrange baryons in Ref. [23] and extended to strange ones in Ref. [24]. The basic general method is the spectrum generating algebra [25, 26]. Here the method is associated with the dynamical symmetries of a $3q$ system. The novelty of the model is that the algebraic method is extended to space degrees of freedom while Refs. [25, 26] are concerned with spin and flavour only. In this way one introduces collective radial excitations which are interpreted either as vibrations of the string-like configurations as shown in Fig. 4 or as rotational excitations. The vibrational quantum numbers are

n_u, n_v and n_w . The angular momentum L is related to rotations. The resulting mass

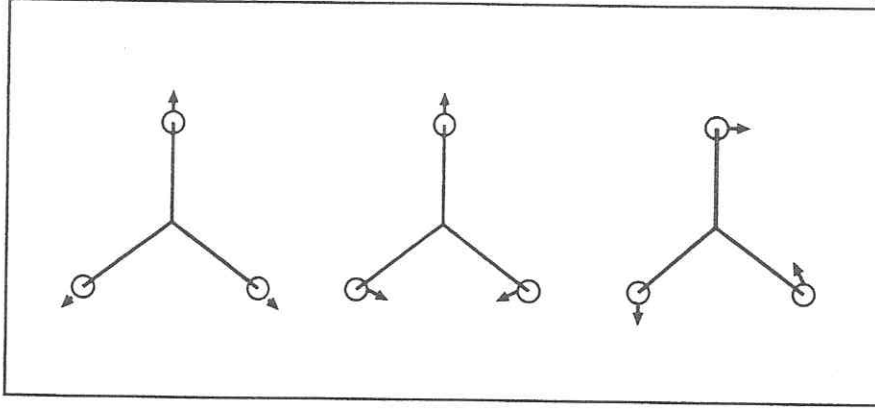


Figure 4: Vibrations of string-like configurations from Ref. [23].

formula has the form

$$\begin{aligned}
 M^2 = & M_0^2 + \kappa_1 v_1 + \kappa_2 v_2 + \alpha L \\
 & + a [2f_1(f_1 + 5) + 2f_2(f_2 + 3) + 2f_3(f_3 + 1) - \frac{1}{3}(f_1 + f_2 + f_3)^2 - 45] \\
 & + b \left[\frac{3}{2}(g_1(g_1 + 2) + g_2^2 - \frac{1}{3}(g_1 + g_2)^2 - 9) \right] + c \left[S(S + 1) - \frac{3}{4} \right] \\
 & + d(Y - 1) + e(Y^2 - 1) + f \left[I(I + 1) - \frac{3}{4} \right]
 \end{aligned} \tag{7}$$

The coefficient M_0^2 is determined by the nucleon mass $M_0^2 = 0.882 \text{ GeV}^2$. The next two terms represent the vibrational part with $v_1 = n_u$ and $v_2 = n_v + n_w$. The linear term in L , stemming from the rotational part, reproduces the Regge trajectories. The bracket multiplied by the coefficient a comes from $SU(6)$ or equivalently the S_3 symmetry. The bracket multiplying the coefficient b corresponds to $SU_F(3)$ and that multiplying c is related to $SU_S(2)$. The last line containing the coefficients d, e and f stem from $U_Y(1)$ and $SU_I(2)$ symmetries. As an example, introducing in (7) the correct quantum numbers for the ground state nucleon N and Δ , one obtains

$$M_\Delta^2 - M_N^2 = 9(b + \frac{1}{3}) + 3c \tag{8}$$

This is precisely the linear combination of coefficients of the flavour and spin dependent terms which determine the mass splitting between N and Δ . The 9 coefficients $\kappa_1, \kappa_2, a, b, \dots, f$ appearing in (7) are determined in a simultaneous fit to the three and four star nonstrange and strange resonances. Their total number is 48. The resulting r.m.s. deviation is $\delta = 33 \text{ MeV}$ [24]. As an example, the $N = 2$ band N^* resonances are shown in Table III. One can see that the Roper resonance belongs to the vibrational band $(v_1, v_2) = (1, 0)$ and the $N(1710)$ resonance to the vibrational band $(v_1, v_2) = (0, 1)$. Due to new these new collective degrees of freedom, in the algebraic model the number of calculated states is larger than in potential models.

Tentative assignments have been made. For example the third P_{11} state found at 1713 MeV (not shown in the table) is associated to the recently discovered resonance of mass 1740 ± 11 MeV [27].

There are however some problems. 1) In the presents approach the spin-orbit coupling has not been taken into account. For this reason the states grouped into multiplets labelled by the same L, S are degenerate, as for example $N(1520)$ and $N(1535)$. 2) The resonance $\Lambda(1405)$ which is not included in the fitting procedure is overpredicted by a large amount of 236 MeV. 3) The lowest "missing" resonances of the octet are associated with the $^2 8_f[20, 1^+]$ configuration, in contrast to all potential model results. The search for missing resonances is therefore very important to distinguish between different models and to check previous assignments.

Within this model strong decays of baryons have also been calculated. The mechanism is rather simple and based on a point-like emission model. There are three adjustable parameters. As a first approximation the results are encouraging. Finite size emitted mesons, as in the 3P_0 decay model, should be the next step.

Table III *Mass spectrum of the $N = 2$ band four and three star N^* resonances from Ref. [24]. The experimental masses are from Ref. [20].*

Baryon	M_{exp}	State	(v_1, v_2)	M_{calc}
$N(939)P_{11}$	939	$^2 8_{1/2}[56, 0^+]$	(0,0)	939
$N(1440)P_{11}$	1430-1470	$^2 8_{1/2}[56, 0^+]$	(1,0)	1444
$N(1520)D_{13}$	1515-1530	$^2 8_{3/2}[70, 1^-]$	(0,0)	1563
$N(1535)S_{11}$	1520-1555	$^2 8_{1/2}[70, 1^-]$	(0,0)	1563
$N(1650)S_{11}$	1640-1680	$^4 8_{1/2}[70, 1^-]$	(0,0)	1683
$N(1675)D_{15}$	1670-1685	$^4 8_{5/2}[70, 1^-]$	(0,0)	1683
$N(1680)F_{15}$	1675-1690	$^2 8_{5/2}[56, 2^+]$	(0,0)	1737
$N(1700)D_{13}$	1650-1750	$^4 8_{3/2}[70, 1^-]$	(0,0)	1683
$N(1710)P_{11}$	1680-1740	$^2 8_{1/2}[70, 0^+]$	(0,1)	1683
$N(1720)P_{13}$	1650-1750	$^2 8_{3/2}[56, 2^+]$	(0,0)	1737

5 Conclusions

The calculation of the spectrum is only a partial test of any model. Wave functions can be tested via strong and electromagnetic decays. Apart from decays into $N +$ pseudoscalar mesons, for both models discussed here it would be interesting to study the decays $N +$ vector mesons for which experiments are being made at TJLAB (see for example [28]).

Also the 3P_0 decay mechanism for strong decays should be more thoroughly investigated. It could help in checking the present assignments and for making further predictions.

Finally, it is desirable that a good model of the nucleon to be also able to describe the NN system, i. e. the nucleon-nucleon phase shifts and the deuteron properties. The

first studies based on the GBE model and employing the resonanting group method give promising results [29] for the 3S_1 and 1S_0 phase shifts..

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