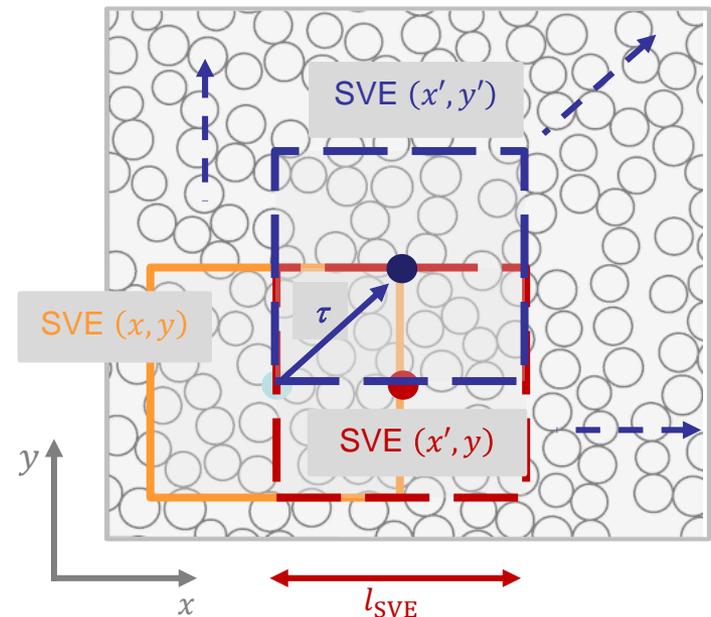# A stochastic Mean Field Homogenization model of

Unidirectional composite materials

Wu Ling, Noels Ludovic
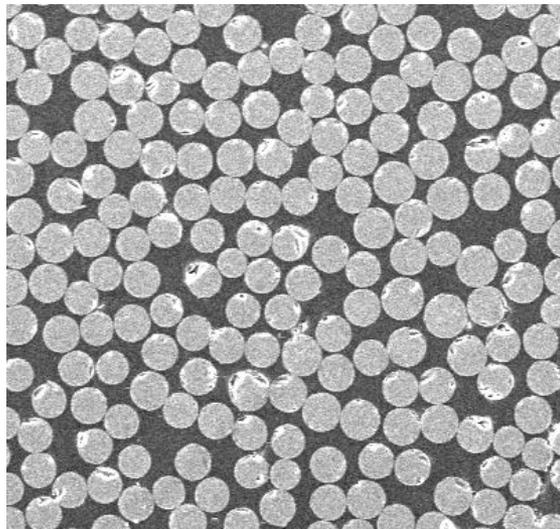
SVE (x', y')

SVE (x, y)

τ

SVE (x', y)

y

x

l_{SVE}

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Multi-scale modelling

- ## Two-scale modelling
  - One method: homogenization
  - 2 problems are solved (concurrently)
    - The macro-scale problem
   - The meso-scale problem (on a meso-scale Volume Element)

Material response

Macro-scale

Extraction of a meso-scale Volume Element

BVP

$P, \sigma, q, \dots$

$F, \mathcal{E}, T, \nabla T, \dots$

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université
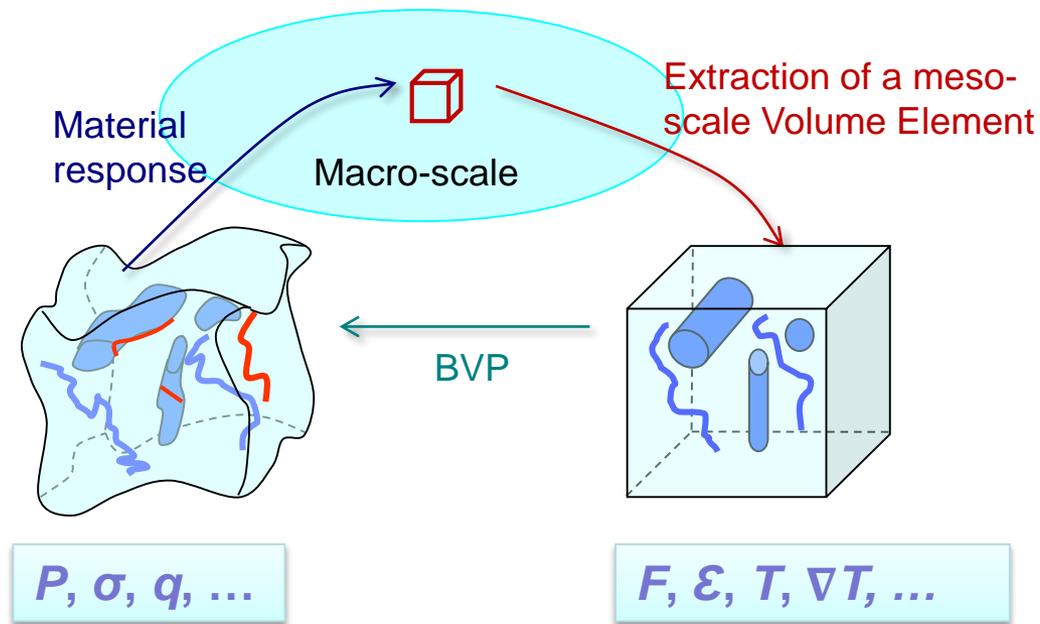
The problem

- Material uncertainties affect structural behaviors

Probability — E_{Fibers}

Probability — E_{matrix}

Probability — Fiber orientation (α)

UQ

Probabilistic homogenization

ω_I α ω_0

Probability — *Composite stiffness*

Homogenized material properties distribution

Loading

Probability — *Failure load*

Stochastic structural analysis
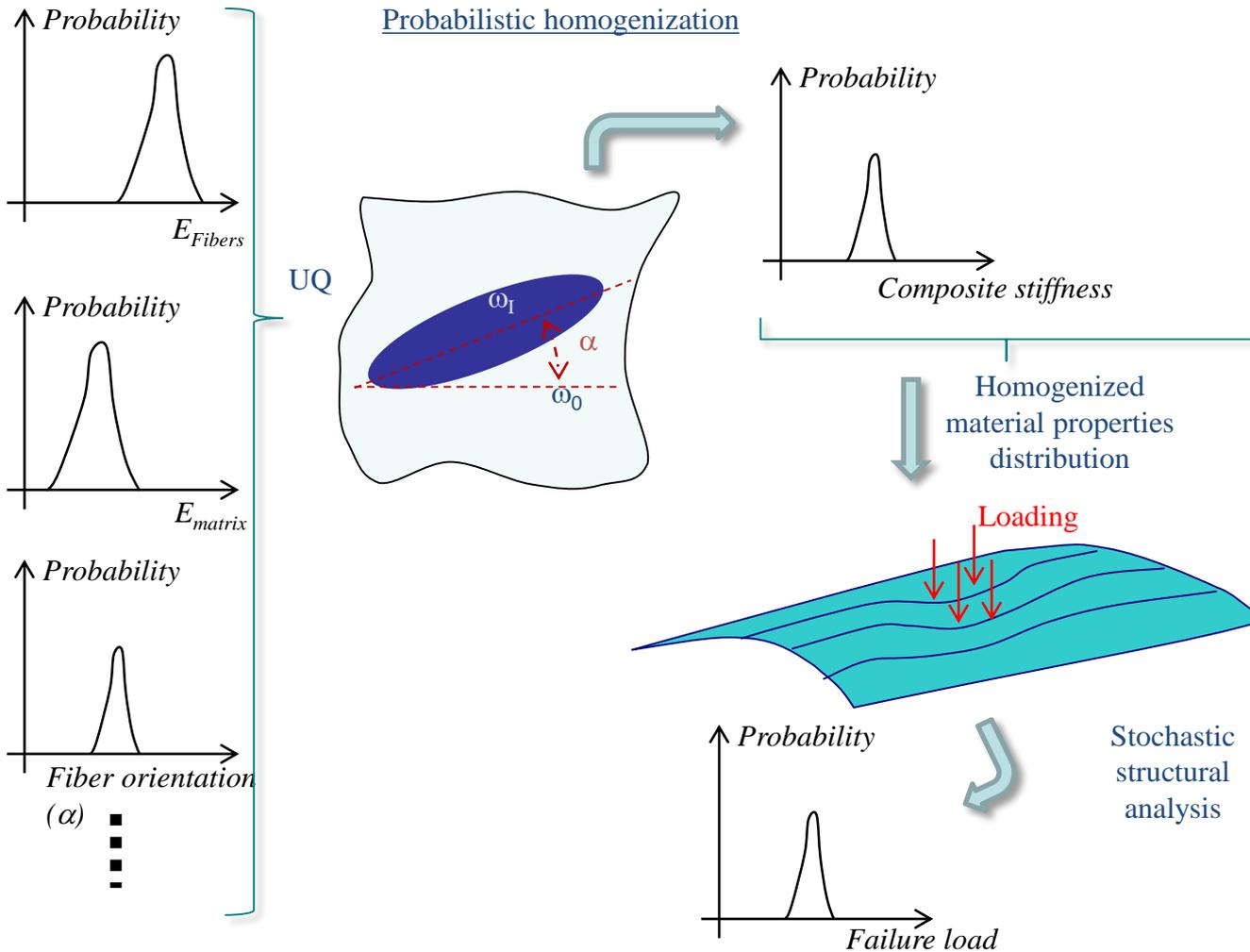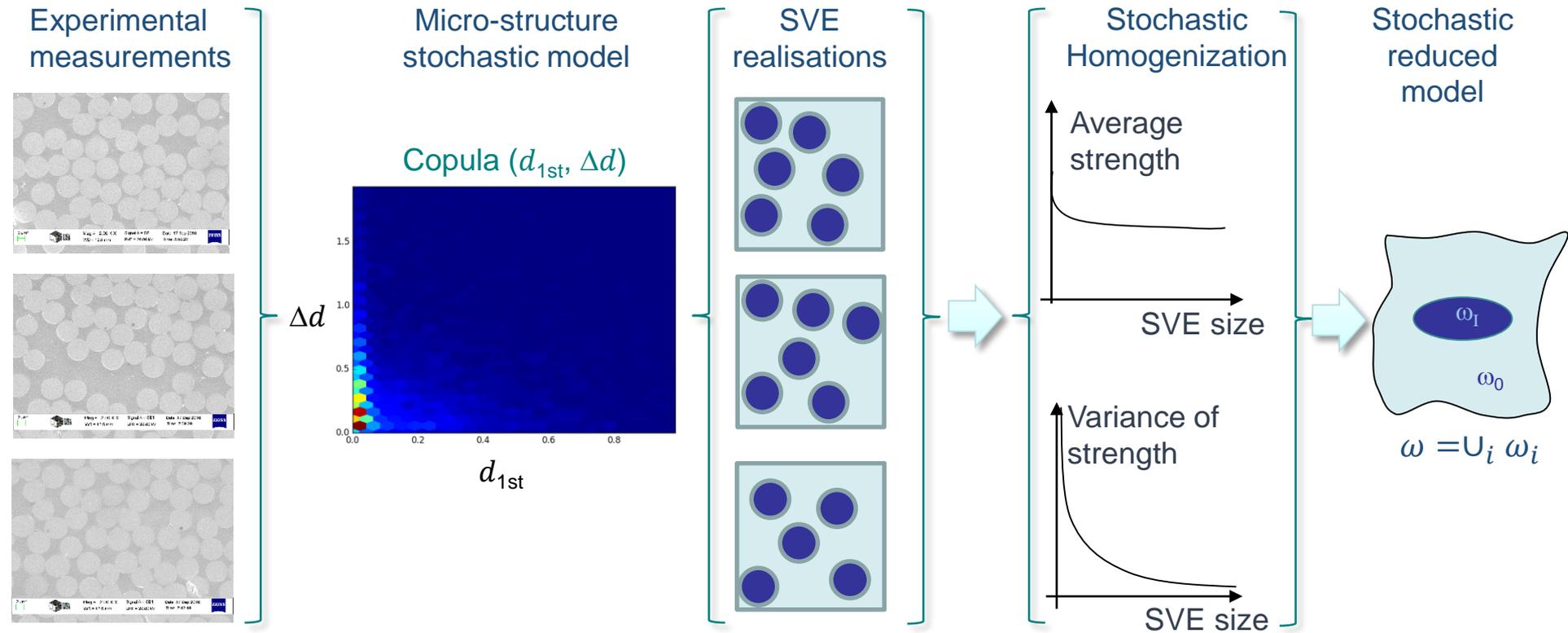
The problem

- Illustration assuming a regular stacking
  - 60%-UD fibers
  - Damage-enhanced matrix behavior
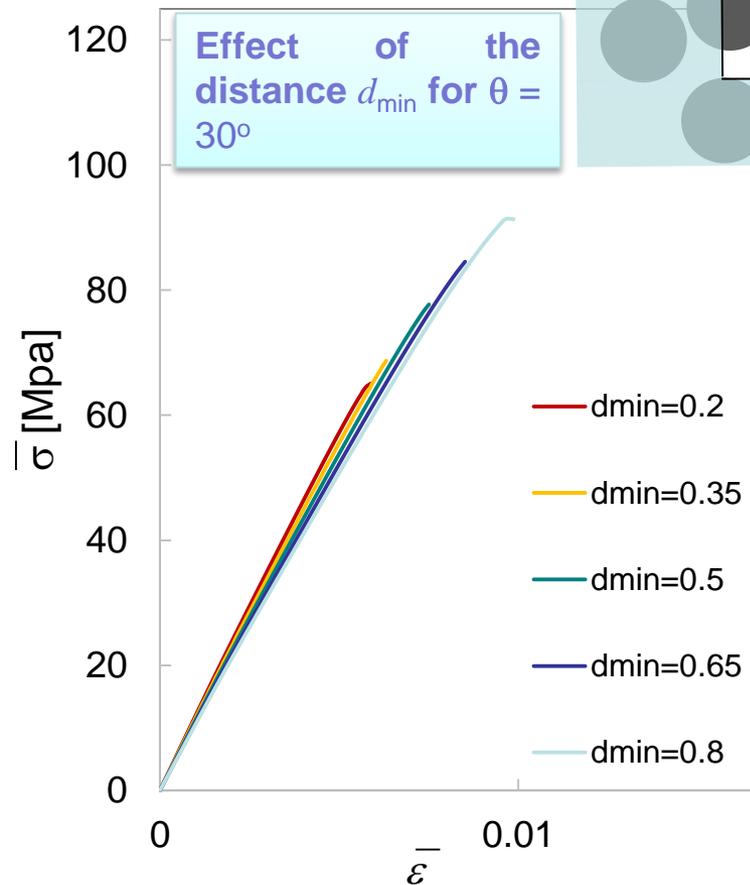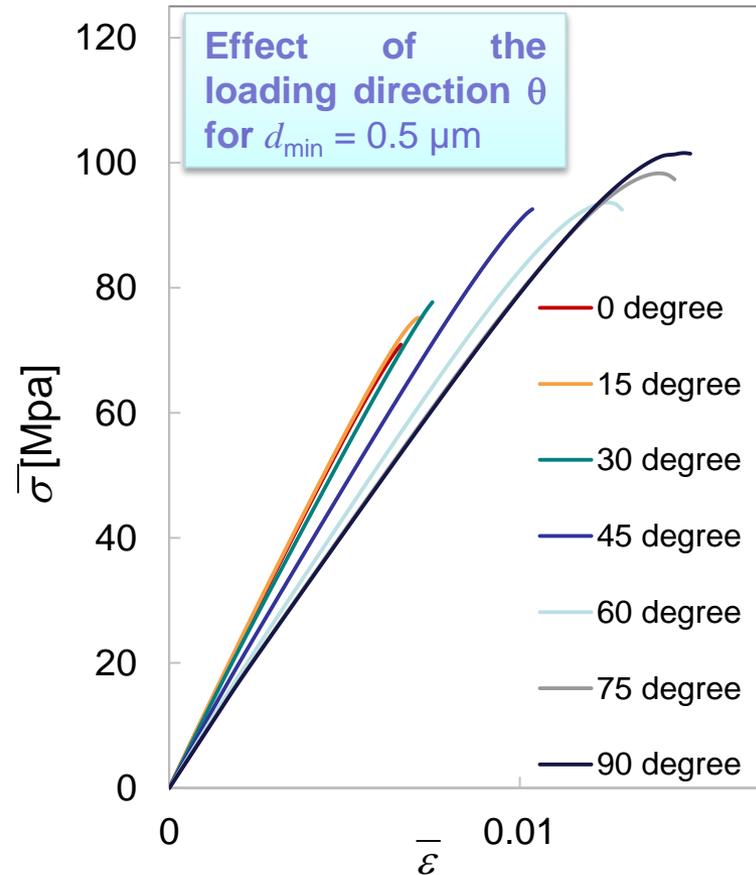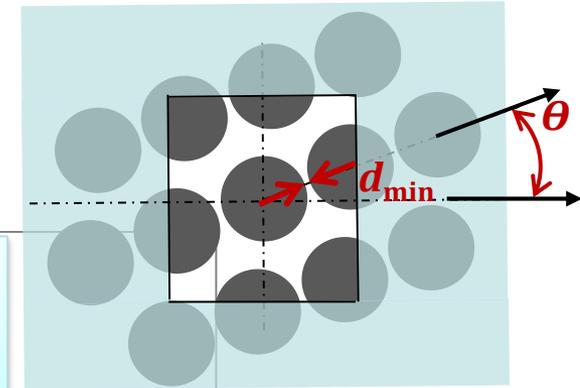
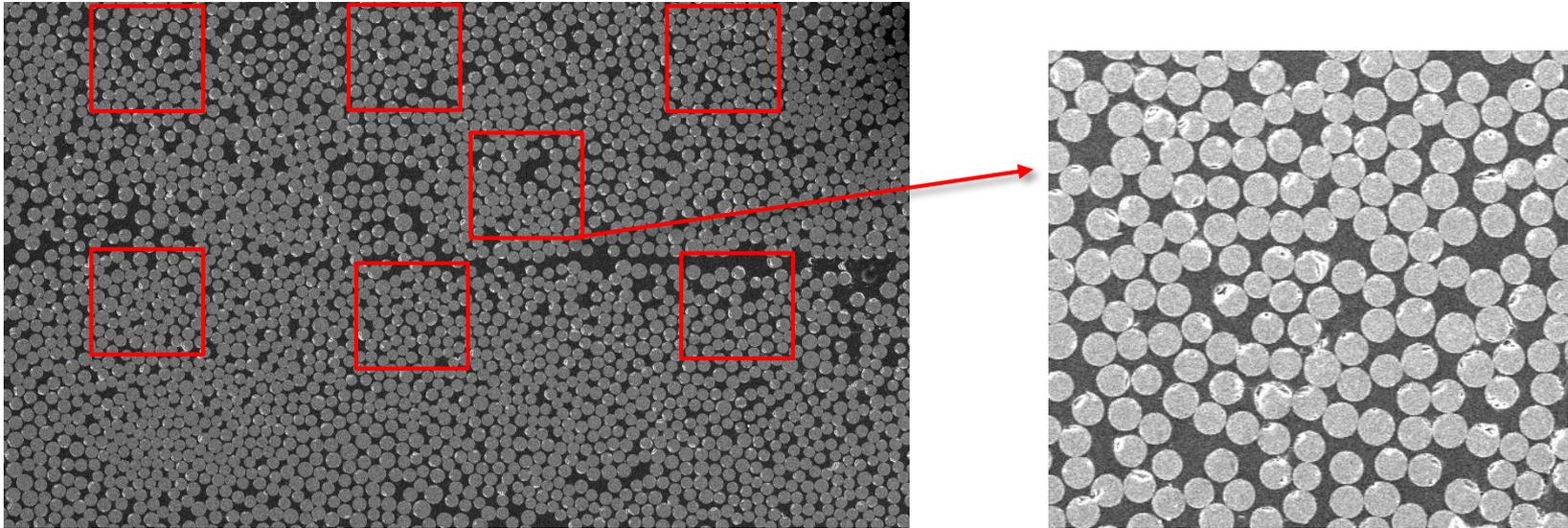- Question: what does happen for a realistic fiber stacking?

The problem

- Proposed methodology:

Experimental measurements

Micro-structure stochastic model

Copula $(d_{1st}, \Delta d)$

Δd

d_{1st}

SVE realisations

Stochastic Homogenization

Average strength

SVE size

Variance of strength

SVE size

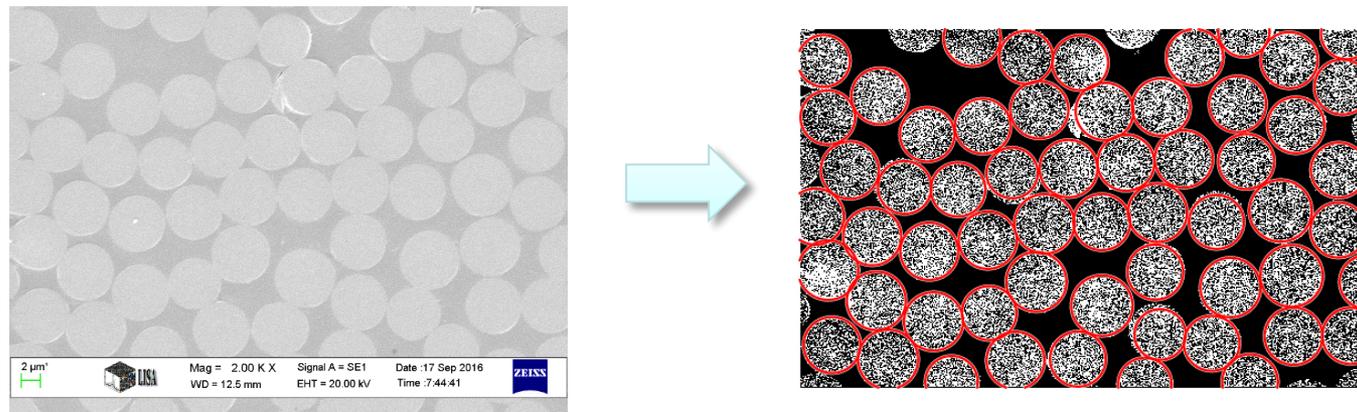Stochastic reduced model

ω_I

ω_0

$\omega = \cup_i \omega_i$

Experimental measurements

- 2000x and 3000x SEM images

- Fibers detection

Micro-structure stochastic model

- Basic geometric information of fibers' cross sections
  - Fiber radius distribution $p_R(r)$

- Basic spatial information of fibers
  - The distribution of the nearest-neighbor net distance function $p_{d_{1st}}(d)$
  - The distribution of the orientation of the undirected line connecting the center points of a fiber to its nearest neighbor $p_{\vartheta_{1st}}(\theta)$
  - The distribution of the difference between the net distance to the second and the first nearest-neighbor $p_{\Delta d}(d)$ with $\Delta d = d_{2nd} - d_{1st}$
  - The distribution of the second nearest-neighbor's location referring to the first nearest-neighbor $p_{\Delta\vartheta}(\theta)$ with $\Delta\vartheta = \vartheta_{2nd} - \vartheta_{1st}$
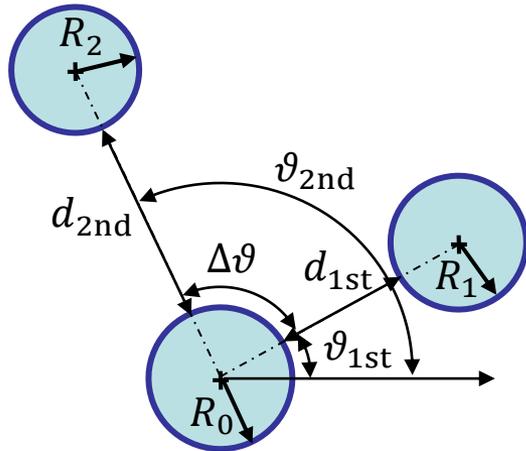
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Micro-structure stochastic model
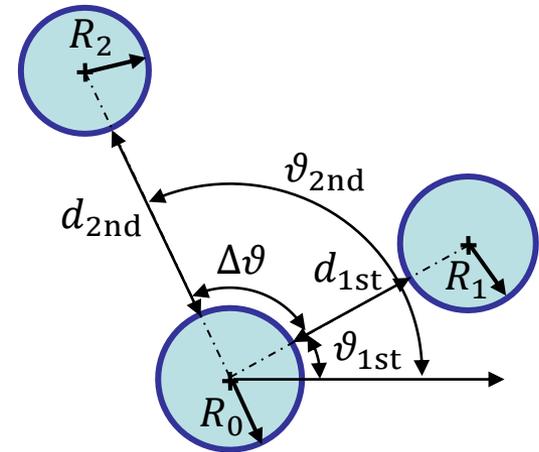
- Histograms of random micro-structures' descriptors
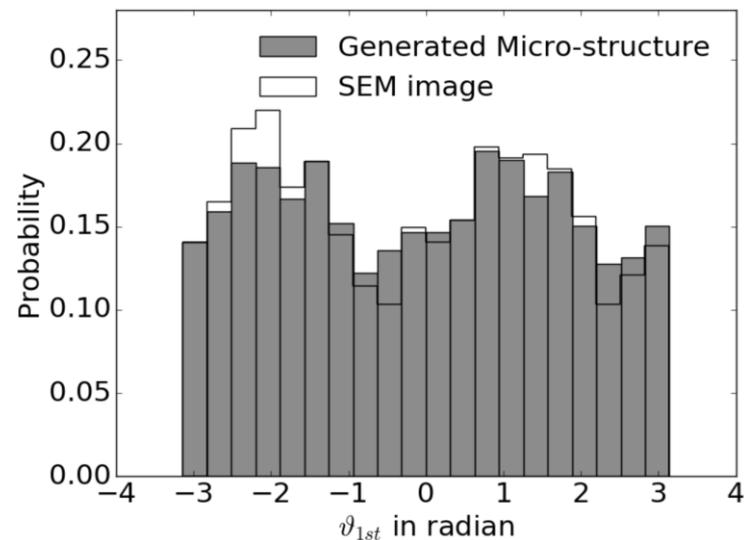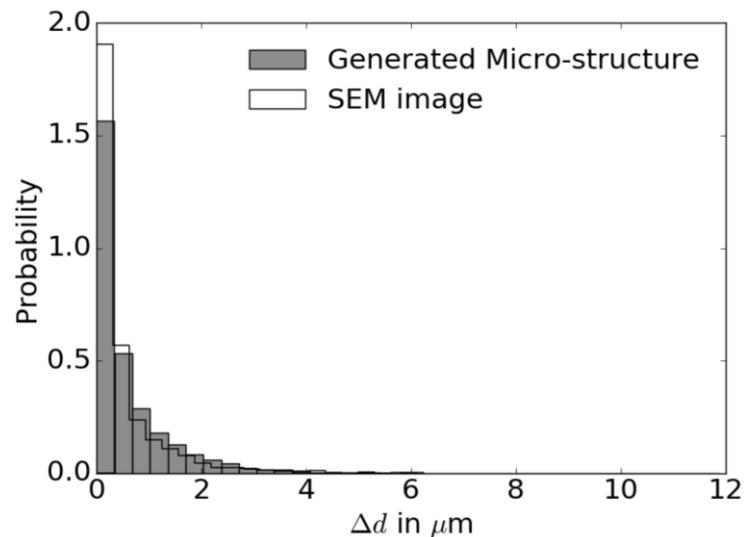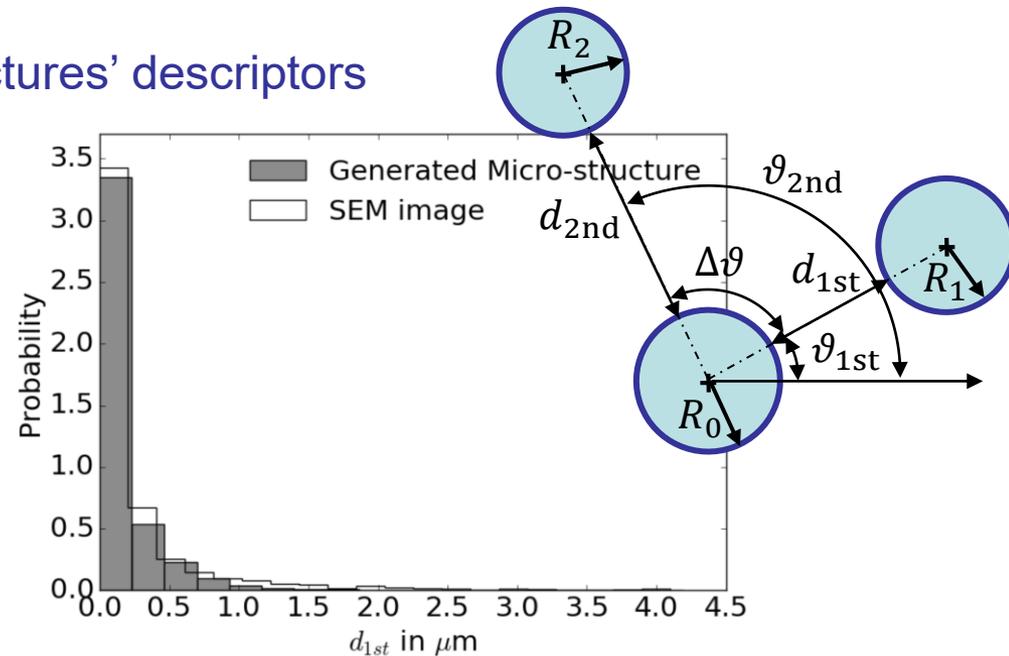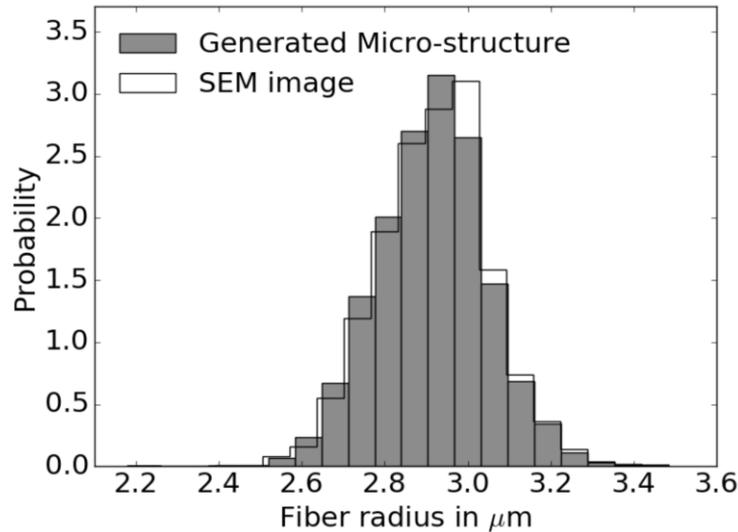
Micro-structure stochastic model

- Dependency of the four random variables d_{1st}, Δd, ϑ_{1st}, $\Delta\vartheta$

- Correlation matrix

	d_{1st}	Δd	ϑ_{1st}	$\Delta\vartheta$
d_{1st}	1.0	0.21	0.01	0.02
Δd		1.0	0.002	-0.005
ϑ_{1st}			1.0	0.02
$\Delta\vartheta$				1.0

- Distances correlation matrix

d_{1st} and Δd are dependent
⟹ they will have to be generated from their empirical copula

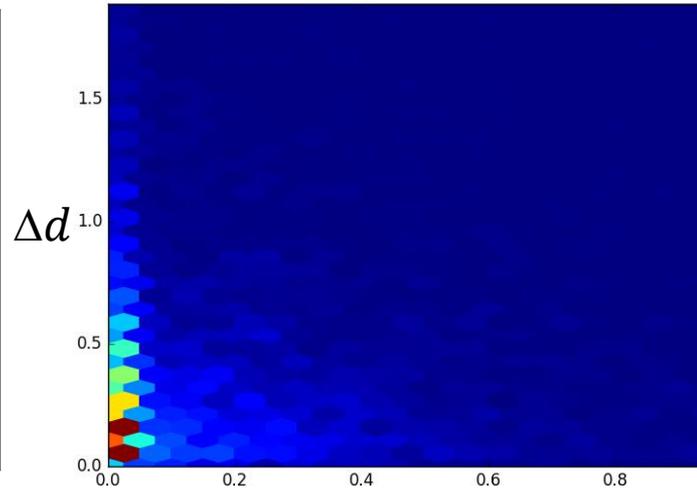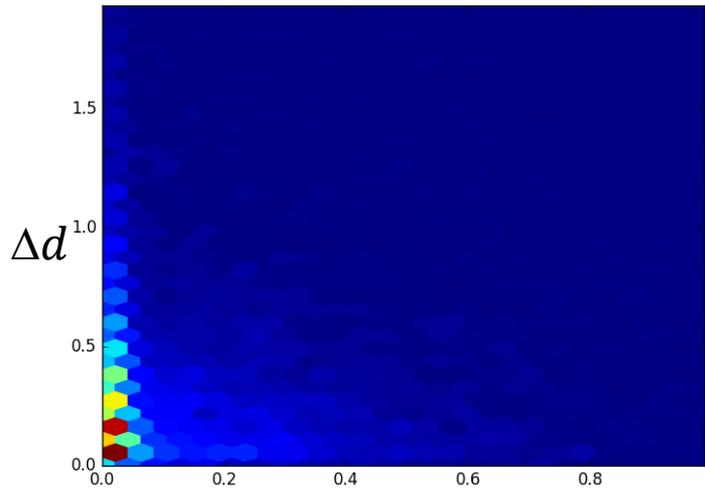|	d_{1st}	Δd	ϑ_{1st}	$\Delta\vartheta$
d_{1st}	1.0	0.27	0.04	0.08
Δd		1.0	0.05	0.06
ϑ_{1st}			1.0	0.05
$\Delta\vartheta$				1.0

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Micro-structure stochastic model
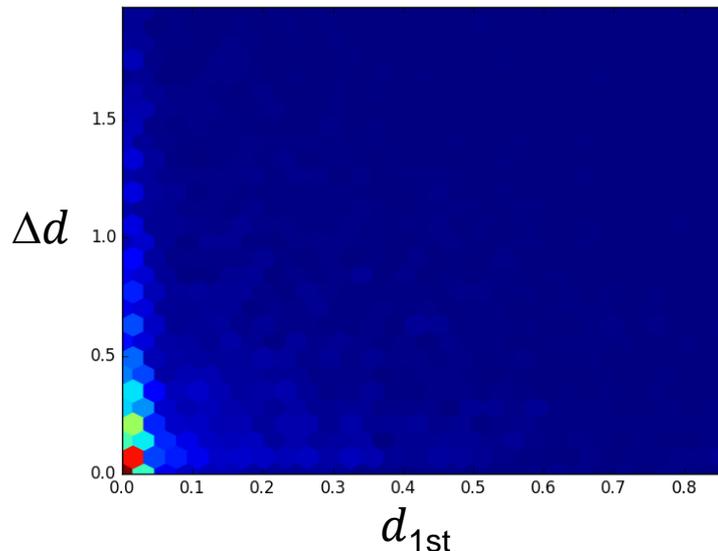
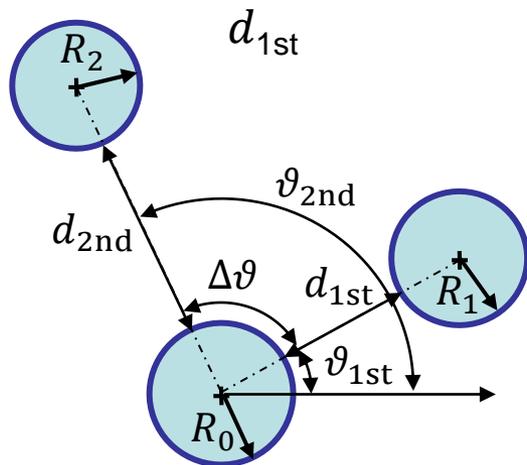- d_{1st} and Δd should be generated using their empirical copula

<div align="center">
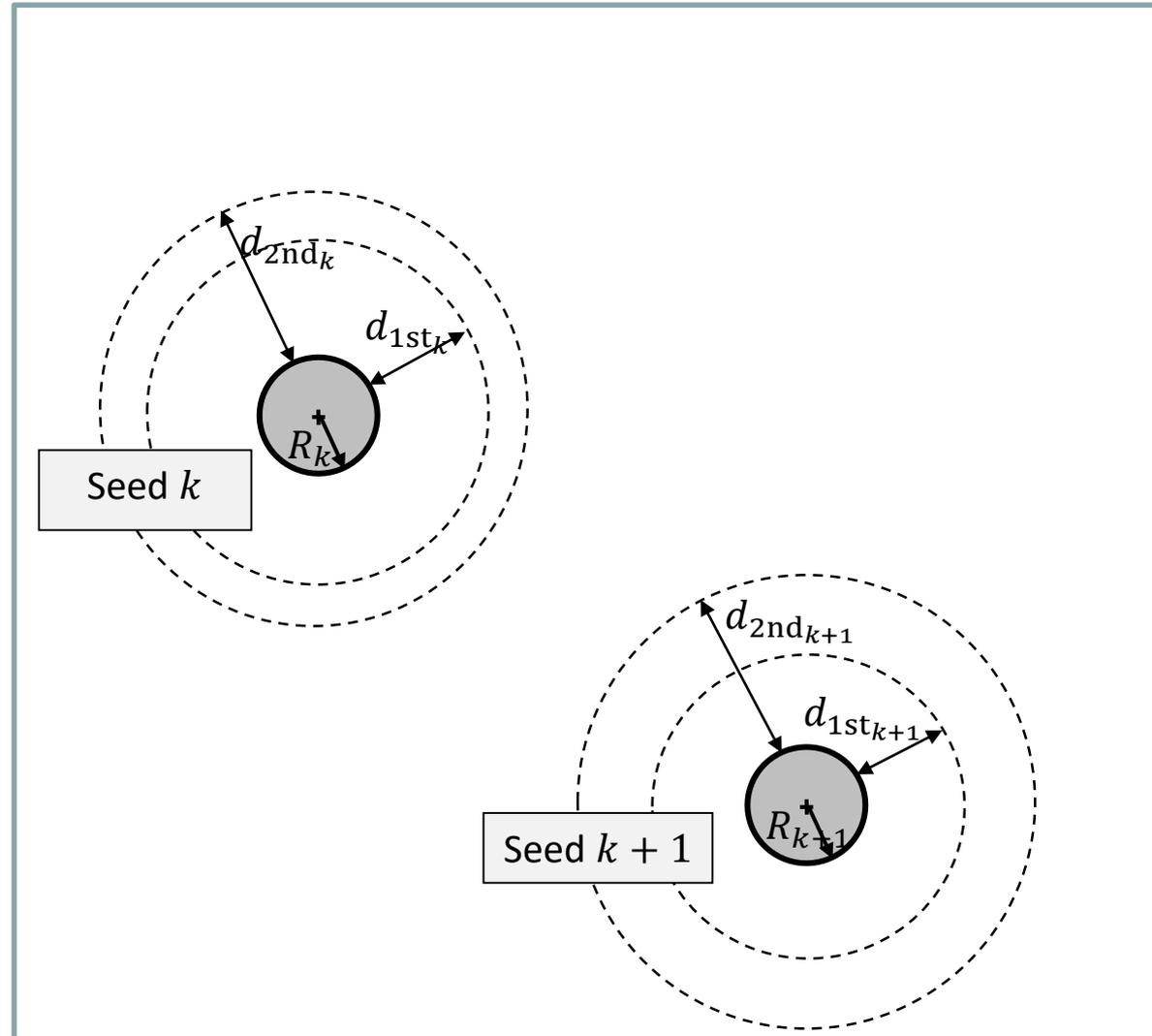
SEM sample   Generated sample

</div>

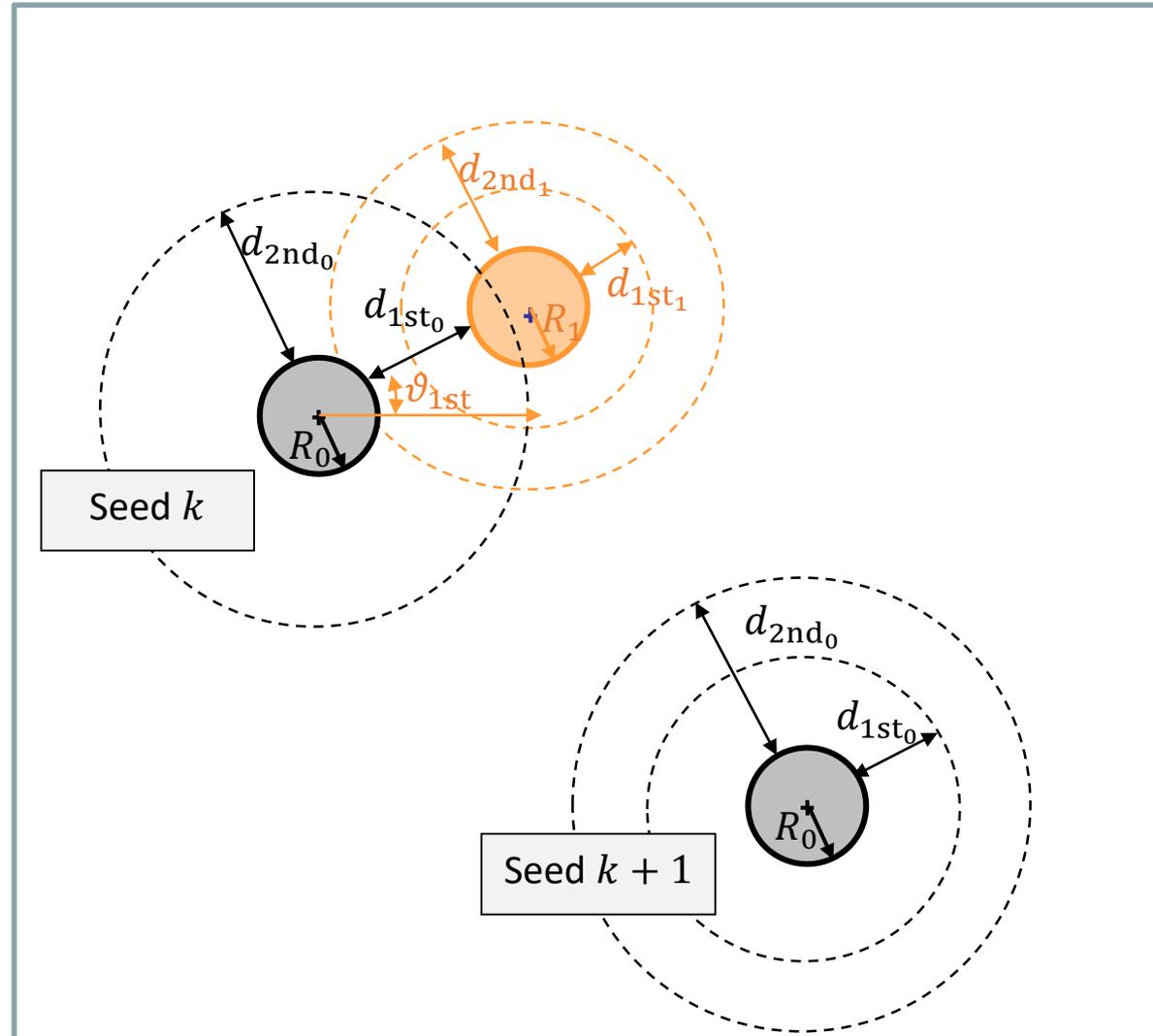Directly from copula generator

Statistic result from generated SVE

- The numerical micro-structure is generated by a fiber additive process

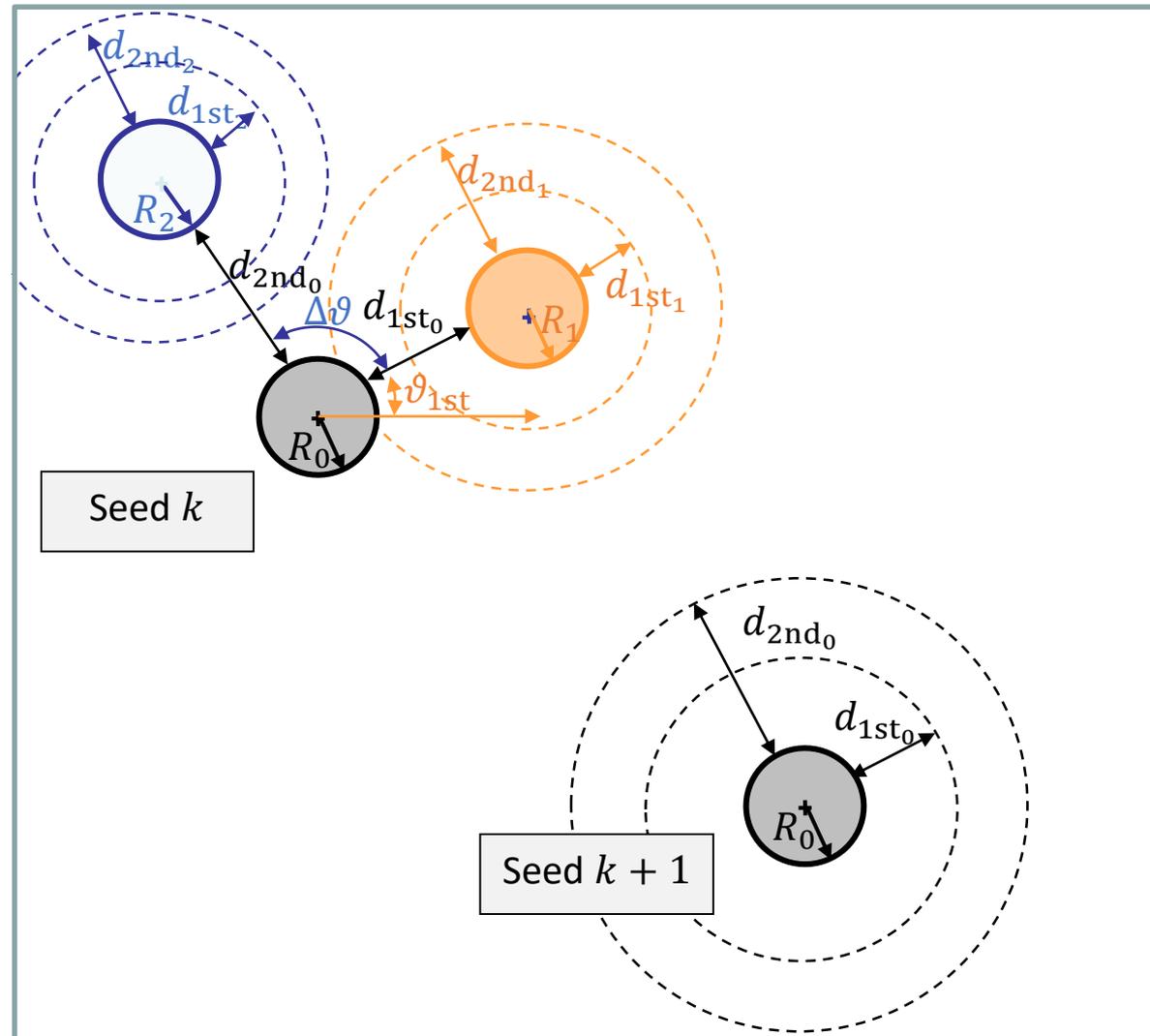 1) Define N seeds with first and second neighbors distances

Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process

  1) Define N seeds with first and second neighbors distances

  2) Generate first neighbor with its own first and second neighbors distances
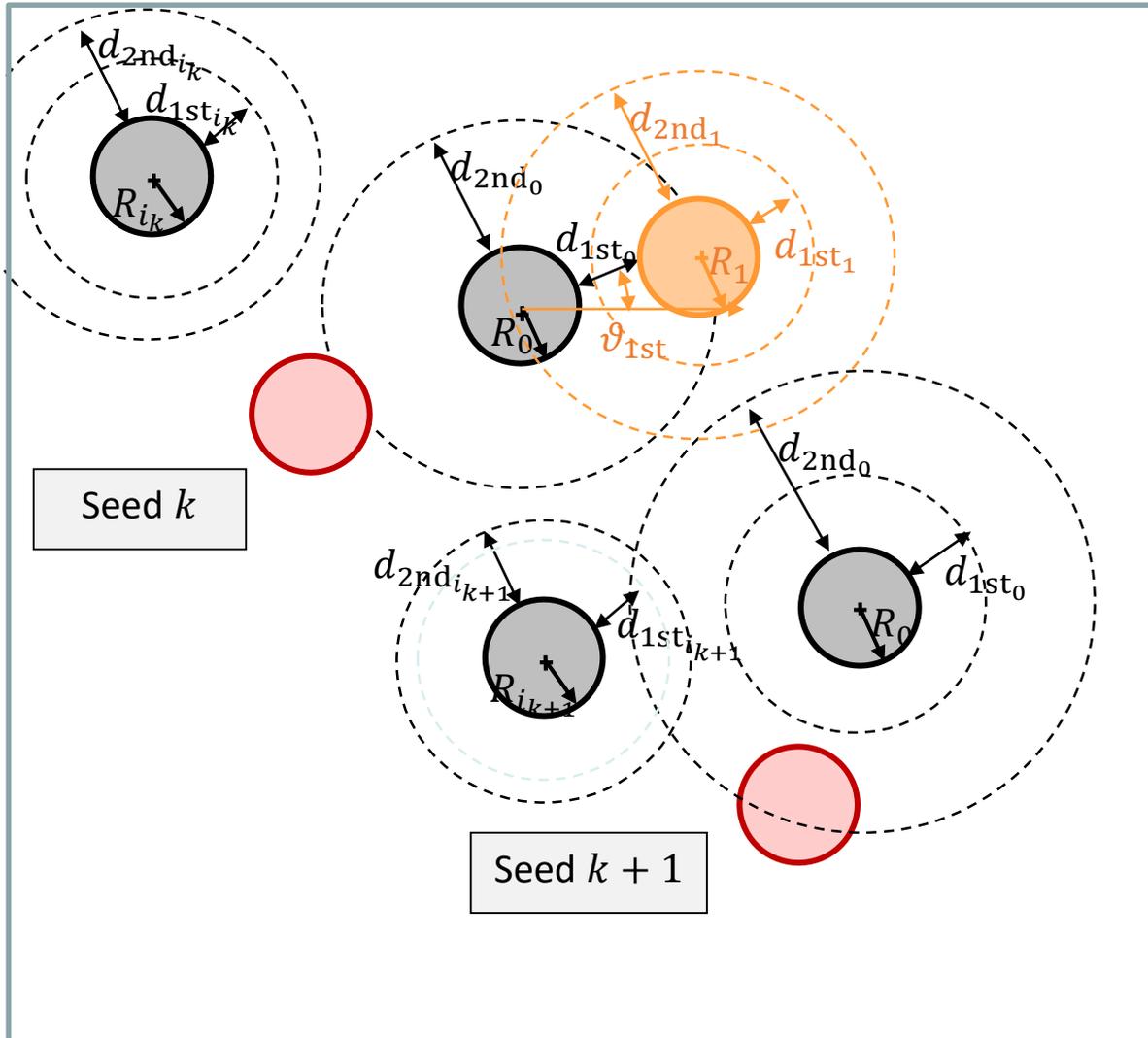
- The numerical micro-structure is generated by a fiber additive process

    1) Define N seeds with first and second neighbors distances

    2) Generate first neighbor with its own first and second neighbors distances

    3) Generate second neighbor with its own first and second neighbors distances

Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process

  1) Define N seeds with first and second neighbors distances

  2) Generate first neighbor with its own first and second neighbors distances

  3) Generate second neighbor with its own first and second neighbors distances

 4) Change seeds & then change central fiber of the seeds

Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process
  - The effect of the initial number of seeds N and
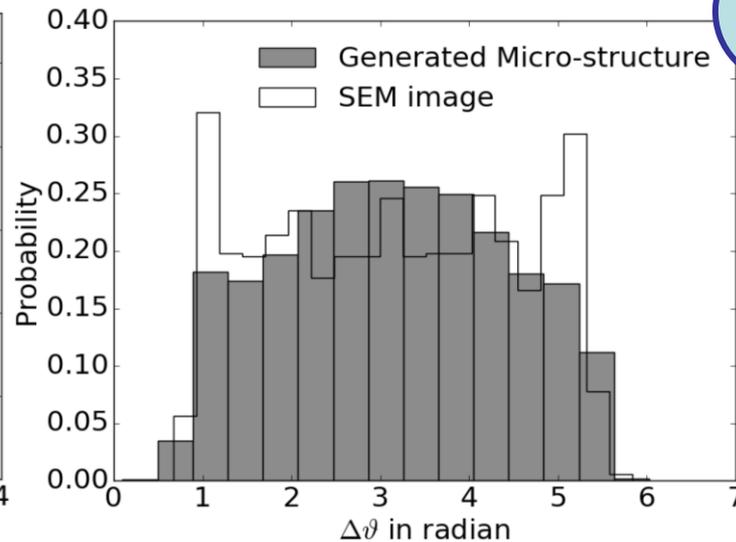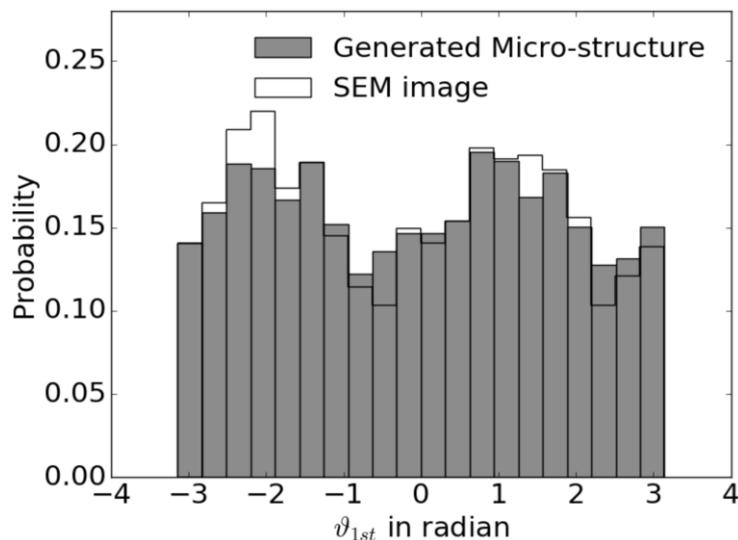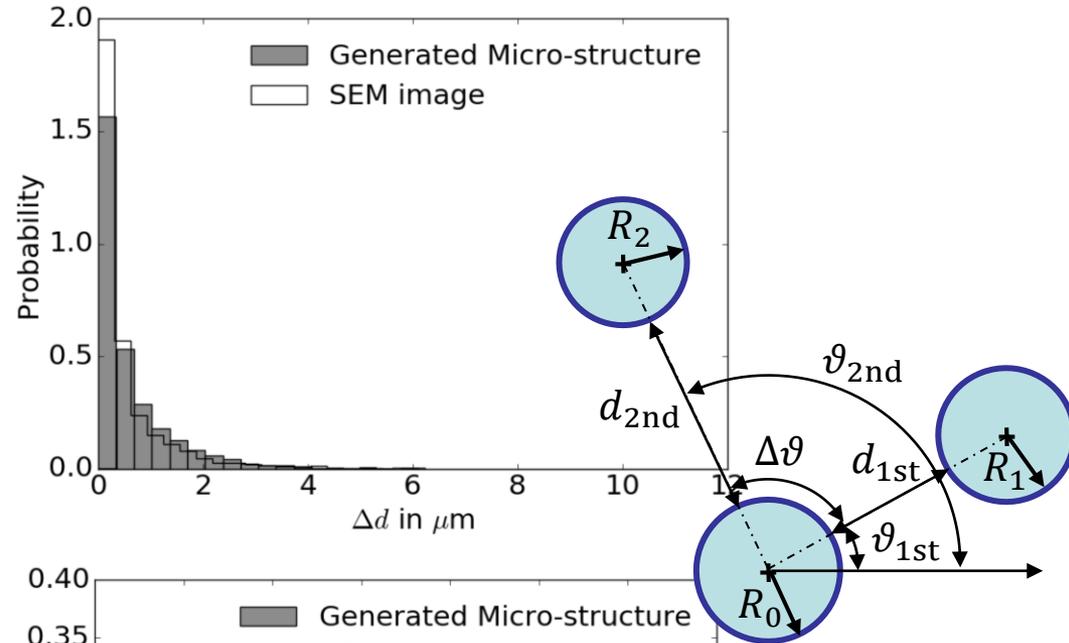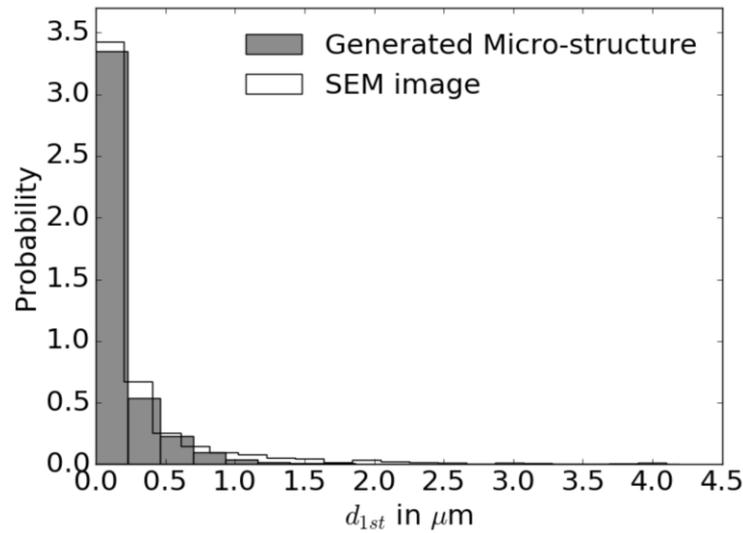 - The effect of the maximum regenerating times n_{max} after rejecting a fiber due to overlap

SEM: Average V_f of 103 windows;

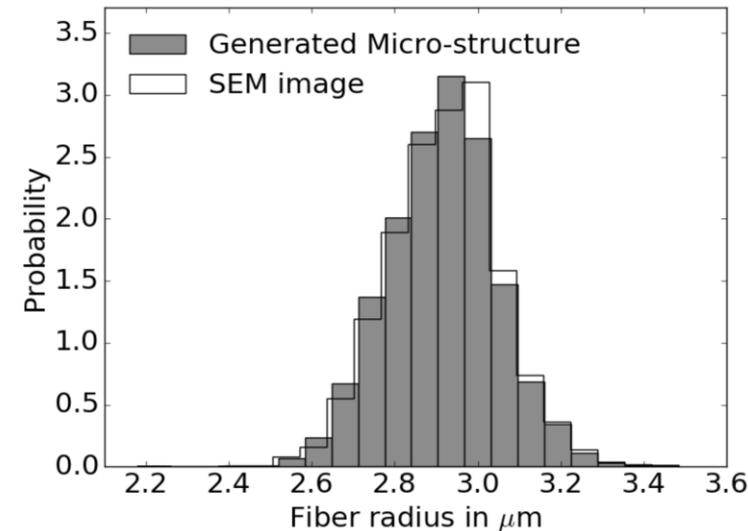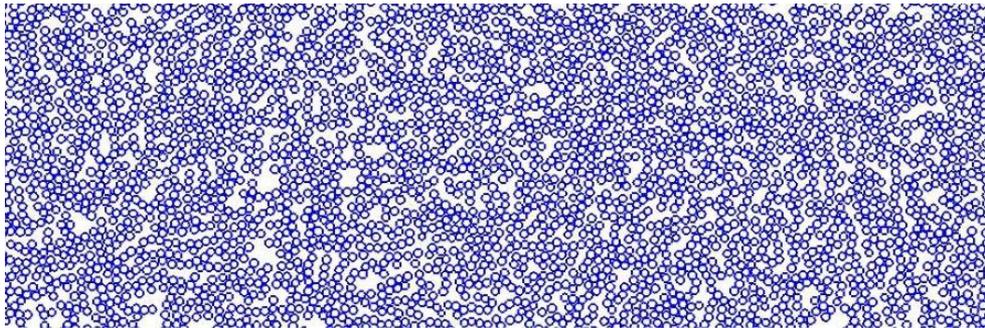Numerical micro-structures: Average V_f of 104 windows.
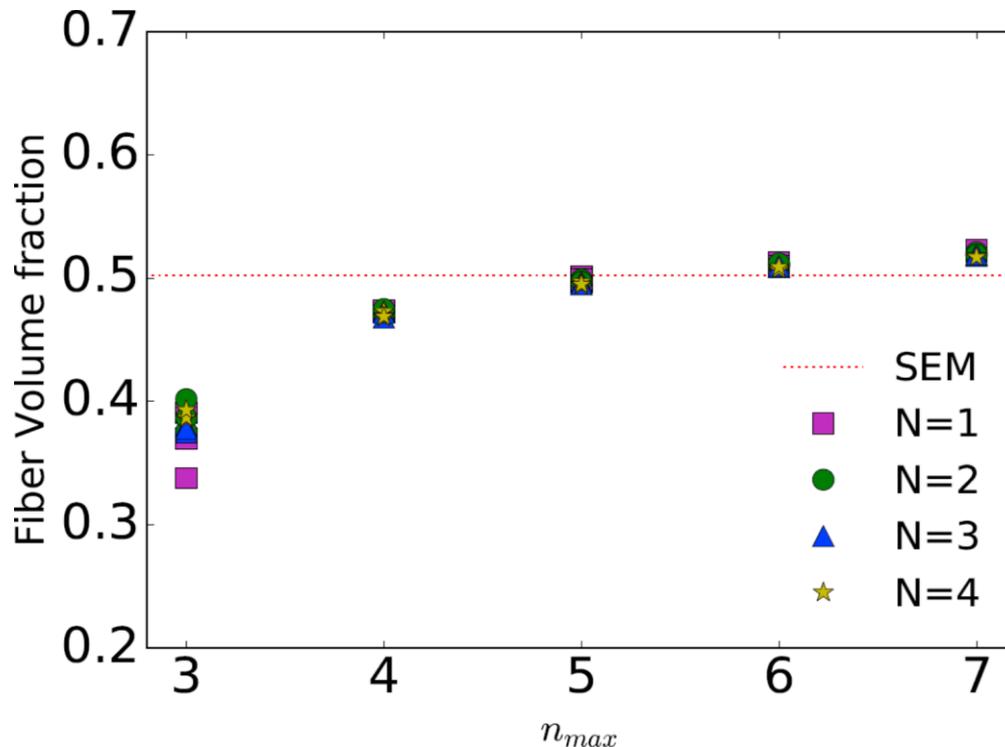
Micro-structure stochastic model

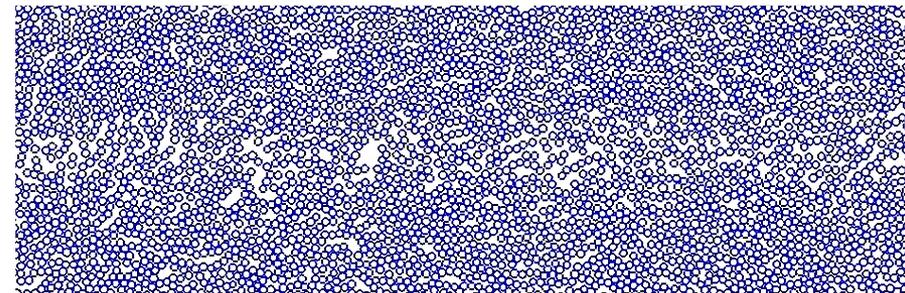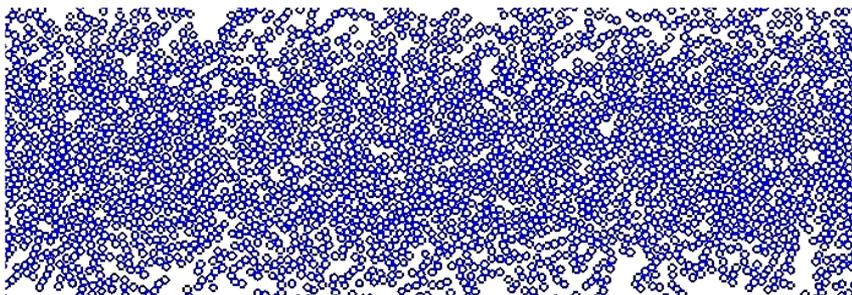- Comparisons of fibers spatial information

Micro-structure stochastic model

- **Numerical micro-structures are generated by a fiber additive process**
  - Arbitrary size
  - Arbitrary number

  - Possibility to generate non-homogenous distributions

Stochastic homogenization on the SVEs

- **Stochastic homogenization**

  - Extraction of Stochastic Volume Elements

    - 2 sizes considered: $l_{\text{SVE}} = 10 \ \mu m$ & $l_{\text{SVE}} = 25 \ \mu m$
    - Window technique to capture correlation

$$R_{\mathbf{rs}}(\boldsymbol{\tau}) = \frac{\mathbb{E}\big[(r(\boldsymbol{x}) - \mathbb{E}(r))(s(\boldsymbol{x}+\boldsymbol{\tau}) - \mathbb{E}(s))\big]}{\sqrt{\mathbb{E}\big[(r - \mathbb{E}(r))^2\big]}\sqrt{\mathbb{E}\big[(s - \mathbb{E}(s))^2\big]}}$$

  - For each SVE

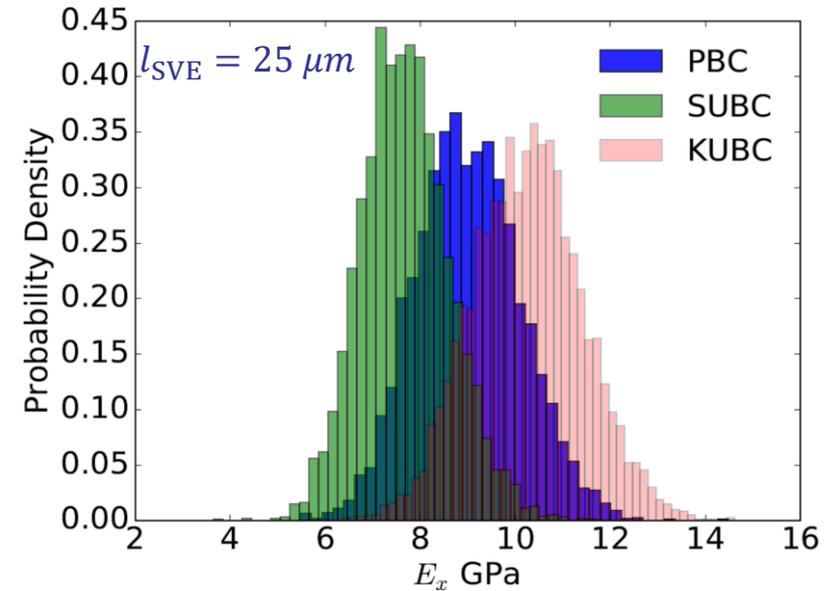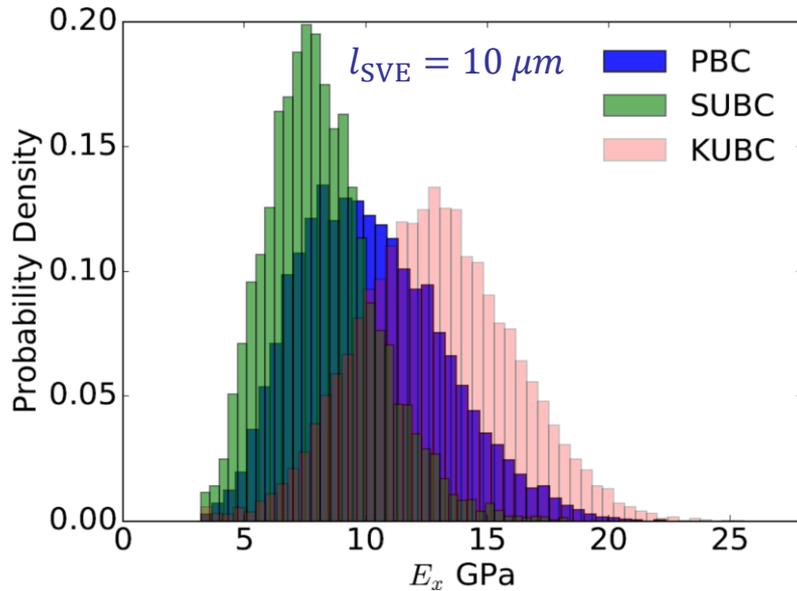   - Extract apparent homogenized material tensor $\mathbb{C}_{\text{M}}$

$$\begin{cases} \boldsymbol{\varepsilon}_{\text{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \boldsymbol{\varepsilon}_{\text{m}} d\omega \\[3mm] \boldsymbol{\sigma}_{\text{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \boldsymbol{\sigma}_{\text{m}} d\omega \\[3mm] \mathbb{C}_{\text{M}} = \dfrac{\partial \boldsymbol{\sigma}_{\text{M}}}{\partial \boldsymbol{u}_{\text{M}} \otimes \boldsymbol{\nabla}_{\text{M}}} \end{cases}$$

    - Consistent boundary conditions:
      - Periodic (PBC)
      - Minimum kinematics (SUBC)
      - Kinematic (KUBC)

Stochastic homogenization on the SVEs
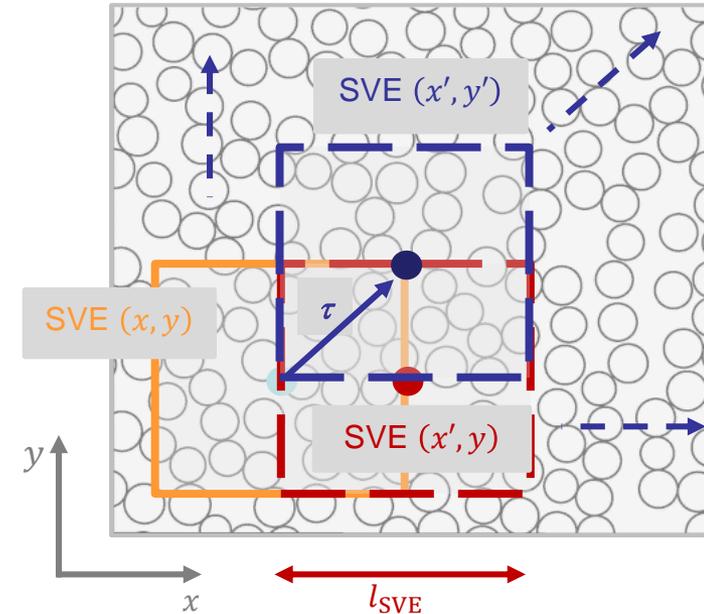
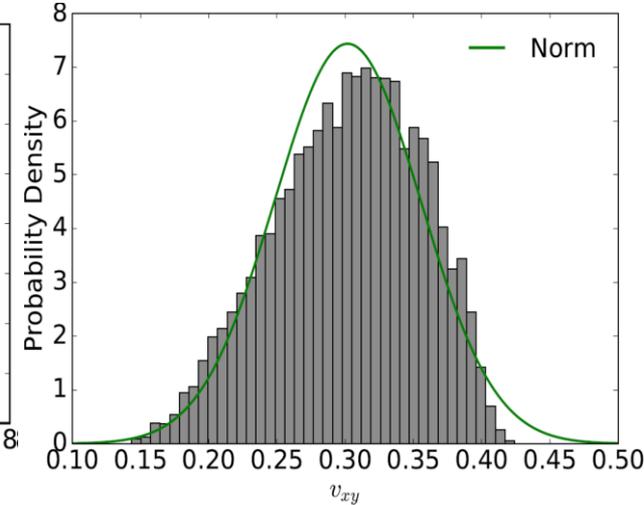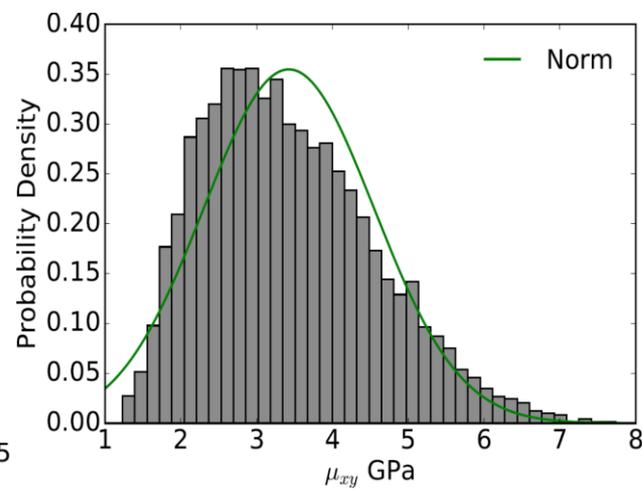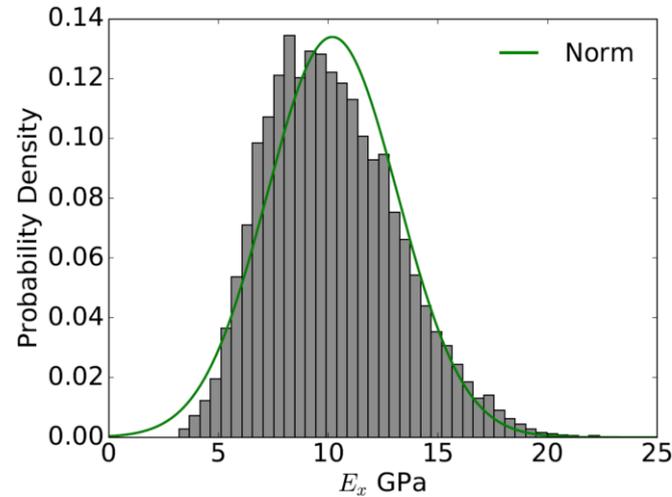- Apparent properties

Increasing l_{SVE}

When l_{SVE} increases
- Average values for different BCs get closer (to PBC one)
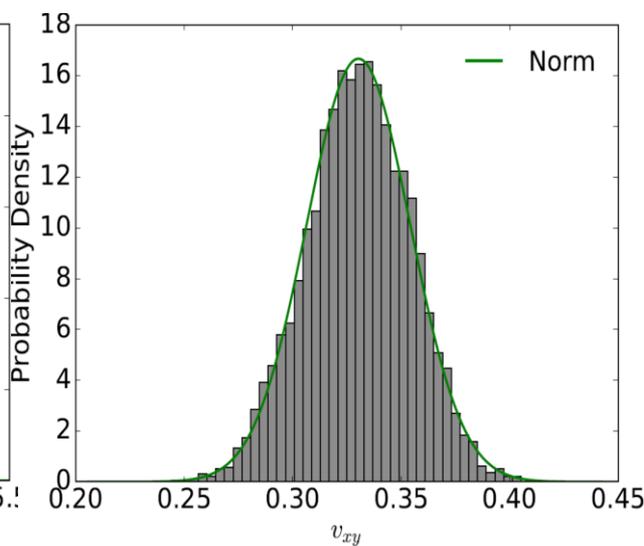- Distributions narrow
- Distributions get closer to normal

- When l_{SVE} increases: marginal distributions of random properties closer to normal
    - l_{SVE} = 10 µm

    - l_{SVE} = 25 µm

Stochastic homogenization on the SVEs

- Correlation

Increasing l_{SVE}
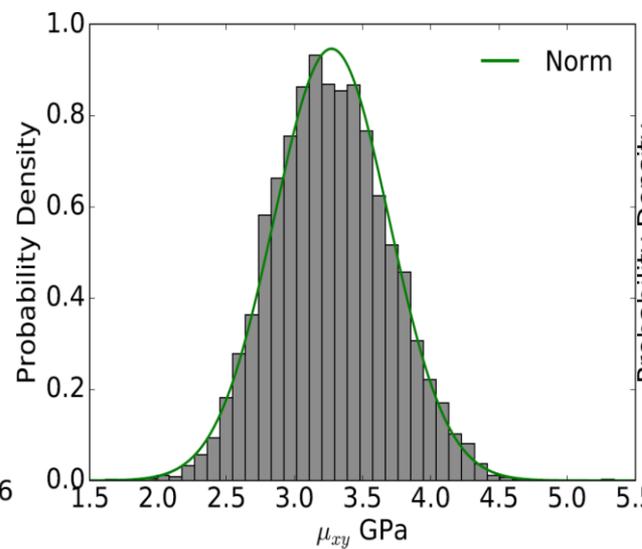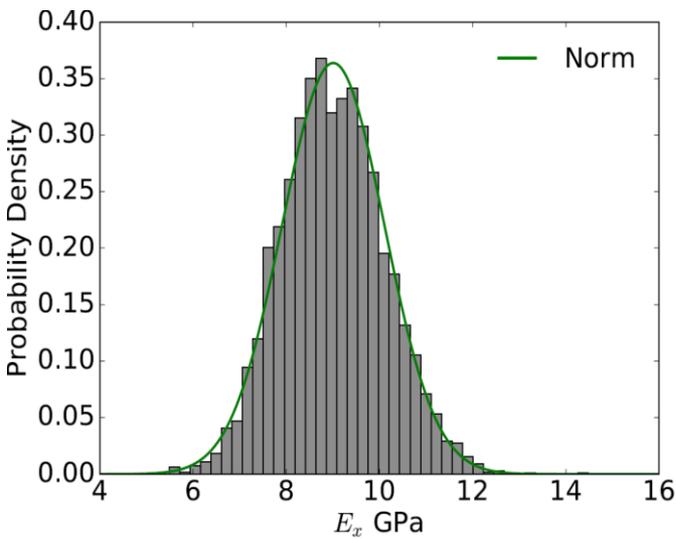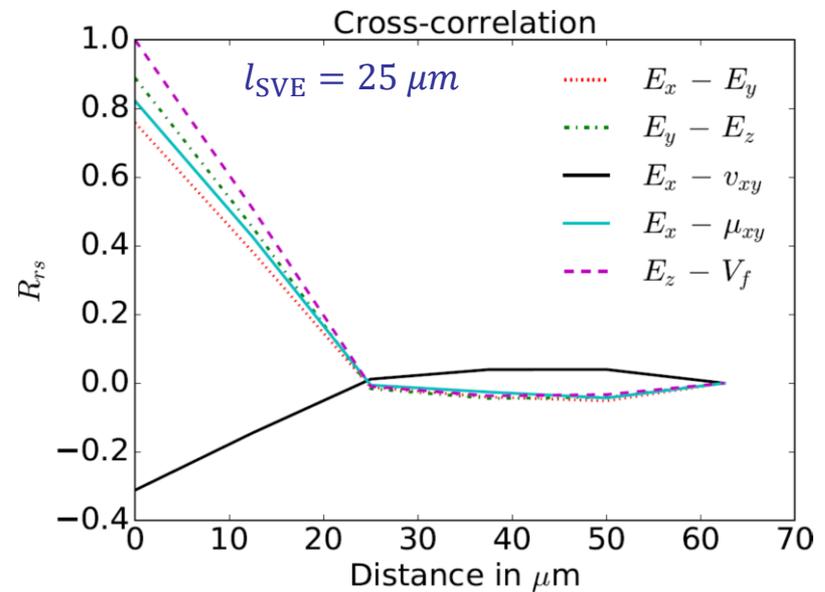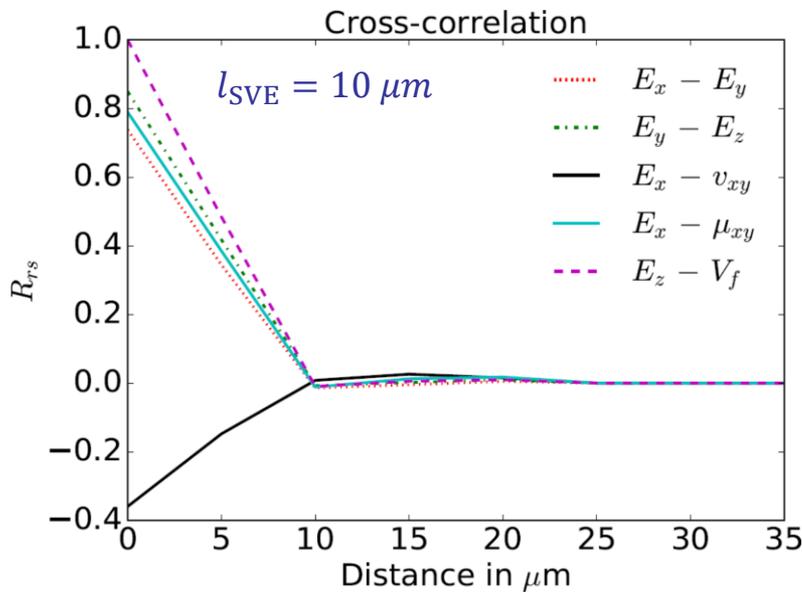
(1) Auto/cross correlation vanishes at $\tau = l_{\text{SVE}}$
(2) When l_{SVE} increases, distributions get closer to normal

(1)+(2) Apparent properties are independent random variables
However the distribution depend on
- l_{SVE}
- The boundary conditions

- Quid larger SVEs?
  - Computational cost affordable in linear elasticity
  - Computational cost non affordable in failure analyzes
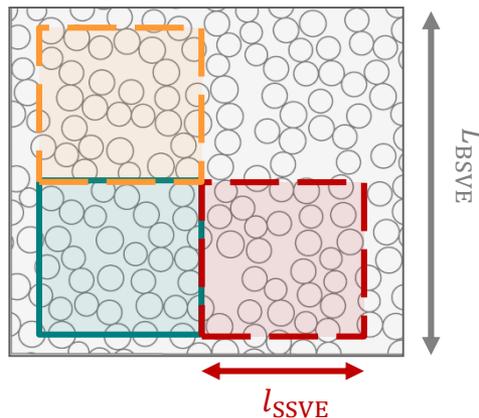
- How to deduce the stochastic content of larger SVEs?
  - Take advantages of the fact that the apparent tensors can be considered as random variables

Stochastic homogenization on the SVEs

- **Quid larger SVEs?**
  - Computational cost affordable in linear elasticity
  - Computational cost non affordable in failure analyzes

- **How to deduce the stochastic content of larger SVEs?**
  - Take advantages of the fact that the apparent tensors can be considered as random variables

L_{BSVE}

l_{SSVE}

Level I   Level II

Computational homogenization

Stochastic homogenization on the SVEs
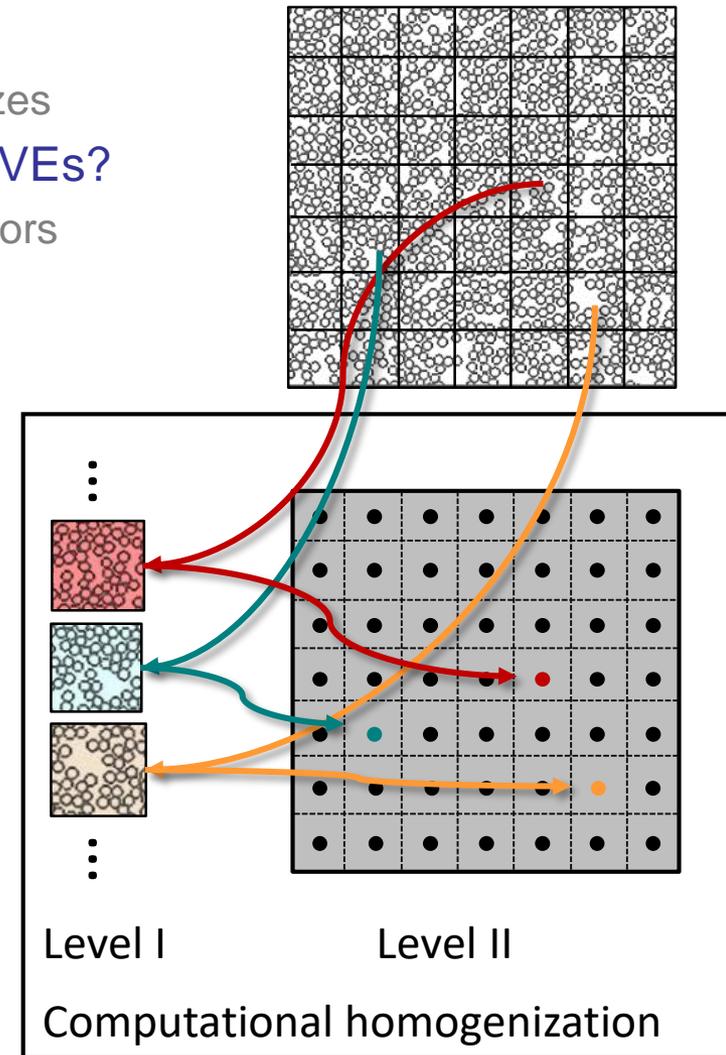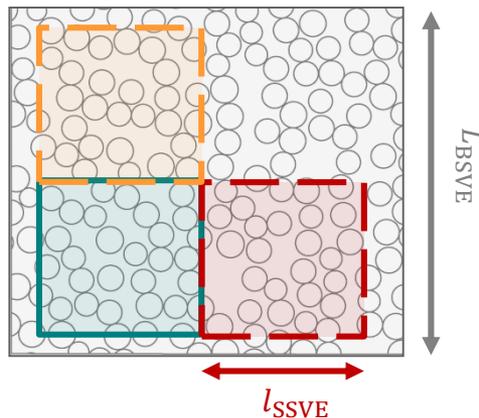
- **Quid larger SVEs?**
  - Computational cost affordable in linear elasticity
  - Computational cost non affordable in failure analyzes
- **How to deduce the stochastic content of larger SVEs?**
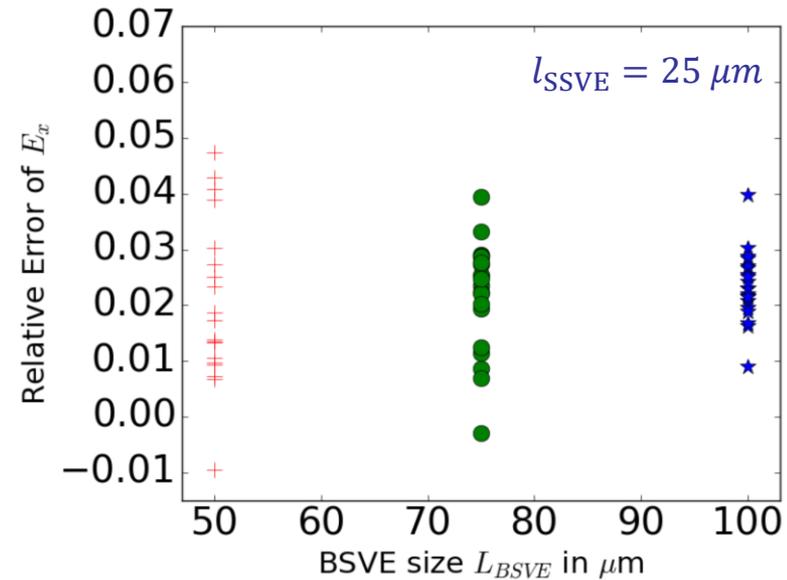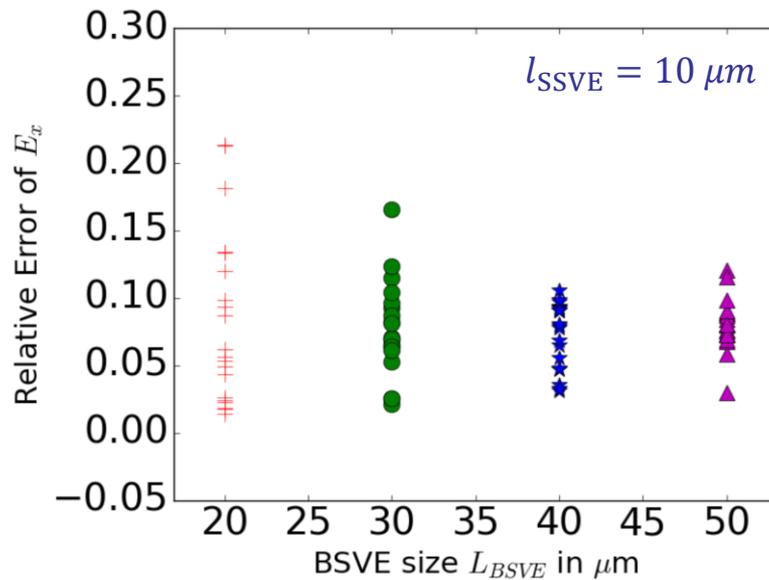 - Take advantages of the fact that the apparent tensors can be considered as random variables
  - Accuracy depends on Small/Large SVE sizes

$l_{SSVE} = 10 \ \mu m$

$l_{SSVE} = 25 \ \mu m$
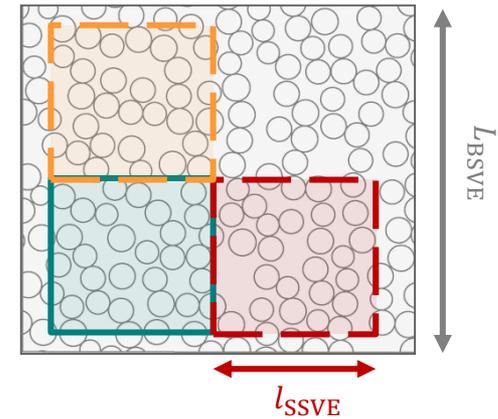
Stochastic homogenization on the SVEs

- Numerical verification of 2-step homogenization
  - Direct homogenization of larger SVE (BSVE) realizations
  - 2-step homogenization using BSVE subdivisions
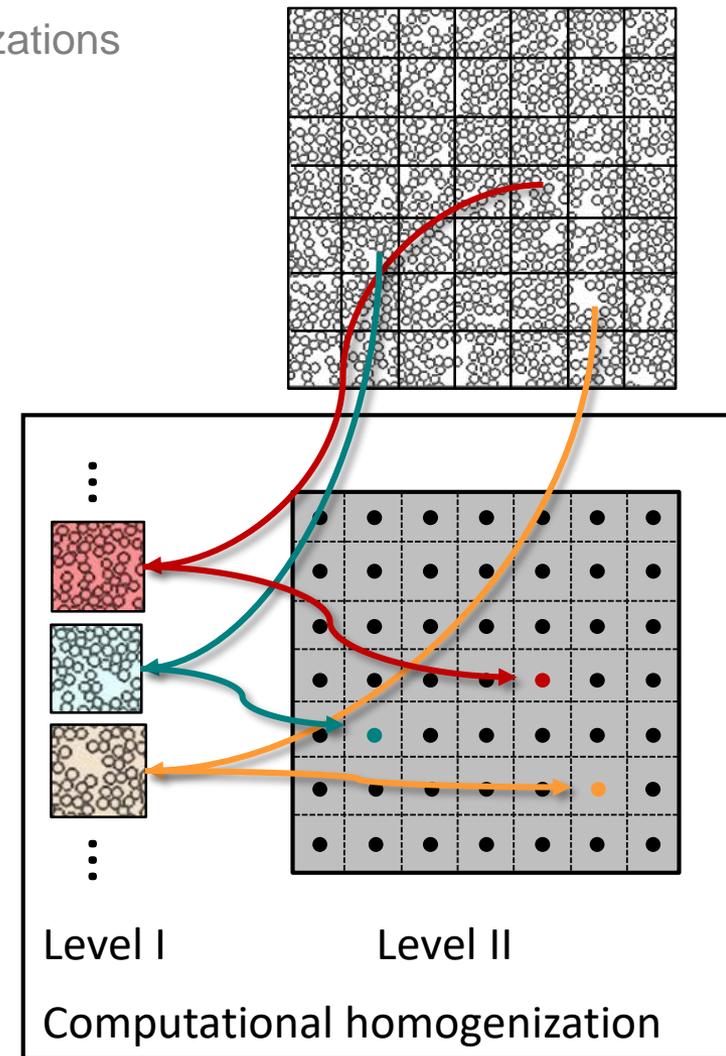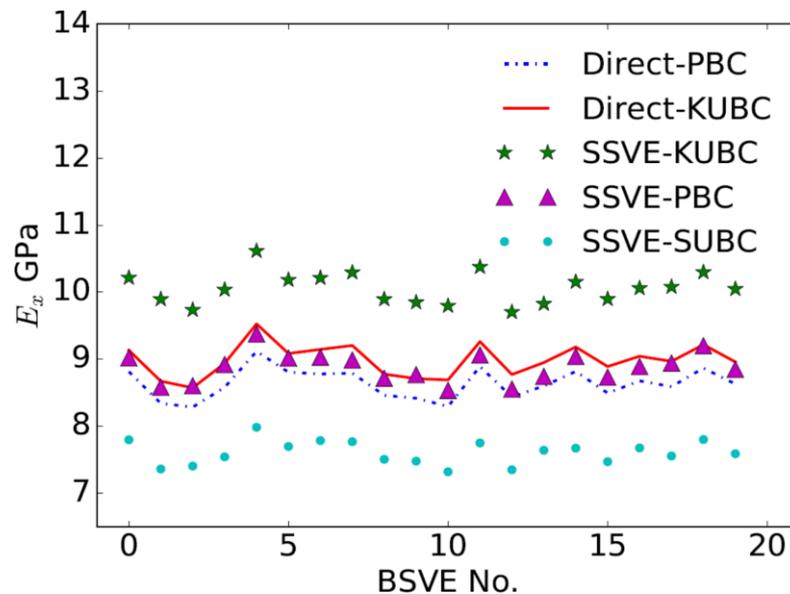
Level I   Level II

Computational homogenization

- Stochastic model of the anisotropic elasticity tensor
  - Extract (uncorrelated) tensor realizations $\mathbb{C}_M^i$
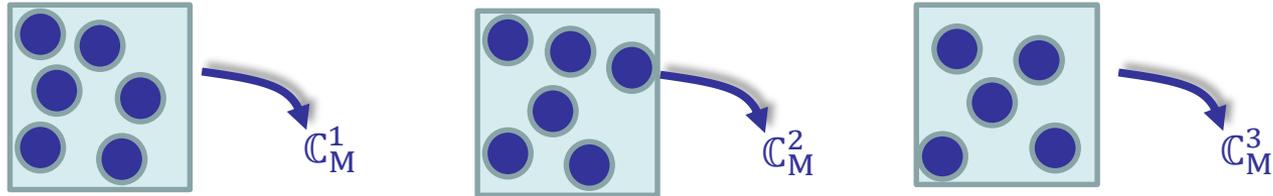
  - Represent each realization $\mathbb{C}_M^i$ by a vector $\mathcal{V}$ of 9 (dependant) $\mathcal{V}^{(r)}$ variables
  - Generate random vectors $\mathcal{V}$ using the Copula method

Stochastic reduced order model

- Stochastic model of the anisotropic elasticity tensor
  - Extract (uncorrelated) tensor realizations $\mathbb{C}_M^i$

$\mathbb{C}_M^1$ $\mathbb{C}_M^2$ $\mathbb{C}_M^3$

  - Represent each realization $\mathbb{C}_M^i$ by a vector $\mathcal{V}$ of 9 (dependant) $\mathcal{V}^{(r)}$ variables
  - Generate random vectors $\mathcal{V}$ using the Copula method
- Simulations require two discretizations
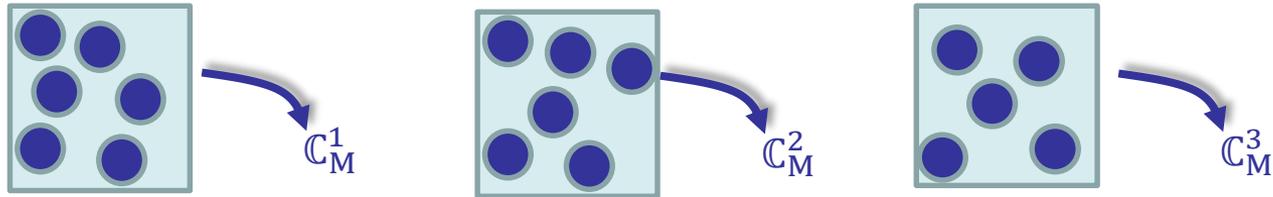 - Random vector discretization
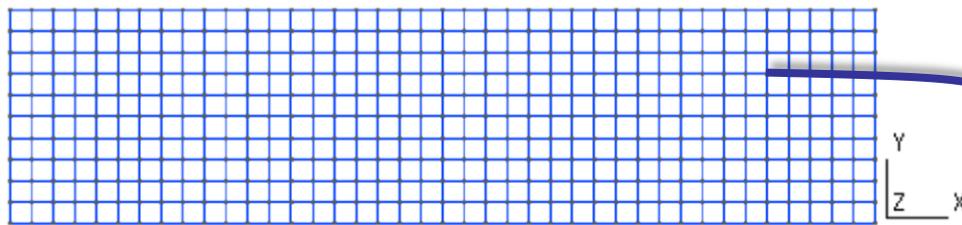
  - Finite element discretization
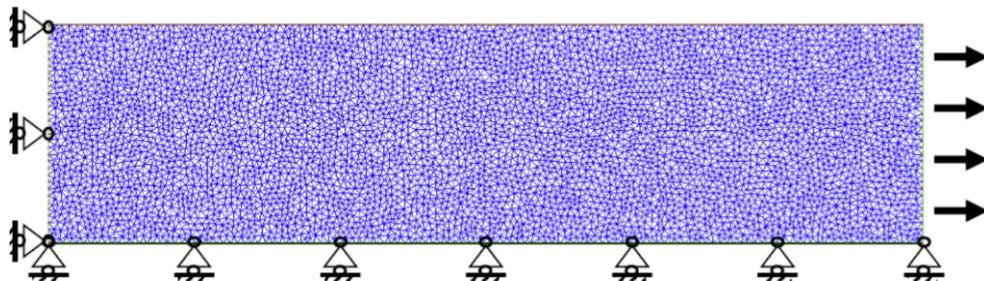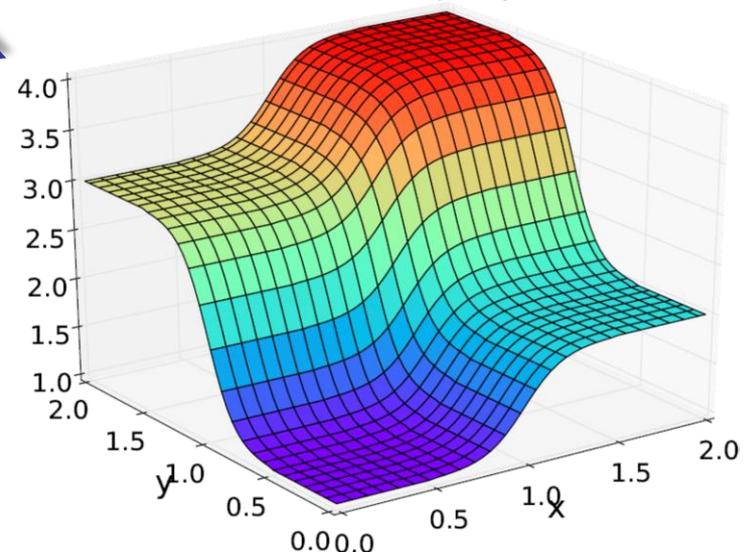
Stochastic reduced order model

- **Ply loading realizations**

  – Non-uniform homogenized stress distributions

  – Different realizations yield different solutions
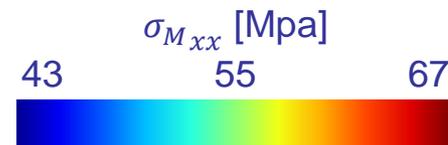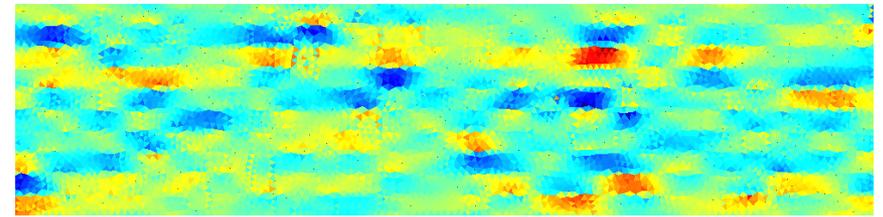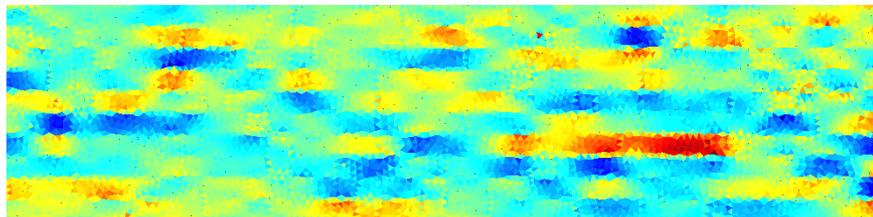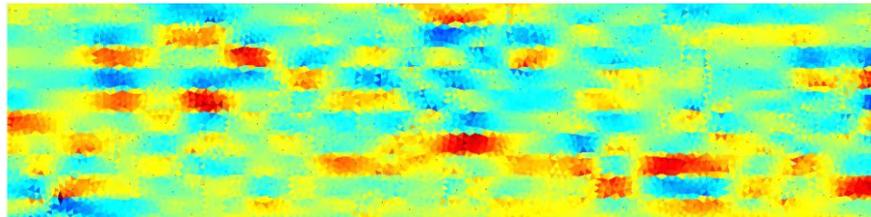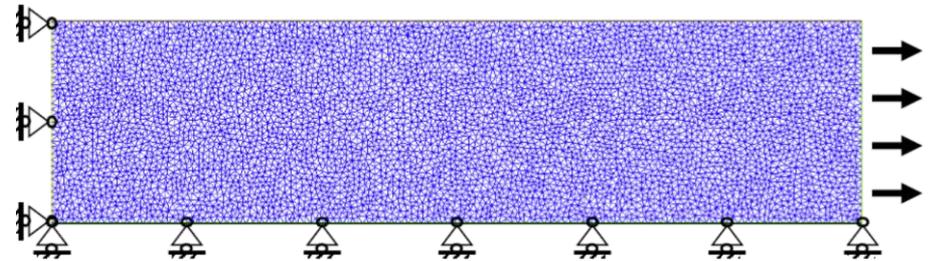
$\sigma_{M_{xx}}$ [Mpa]

43   53   63

$\sigma_{M_{xx}}$ [Mpa]

43   55   67

$\sigma_{M_{xx}}$ [Mpa]

44   55   66

$\sigma_{M_{xx}}$ [Mpa]

43   55   67

Stochastic Mean-Field Homogenization

- Mean-Field-homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \overline{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \overline{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$\Longrightarrow \quad \mathbb{C}_{\mathrm{M}} = \mathbb{C}_{\mathrm{M}}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}})$

  - We use Mori-Tanaka assumption for $\mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}})$

- Stochastic MFH
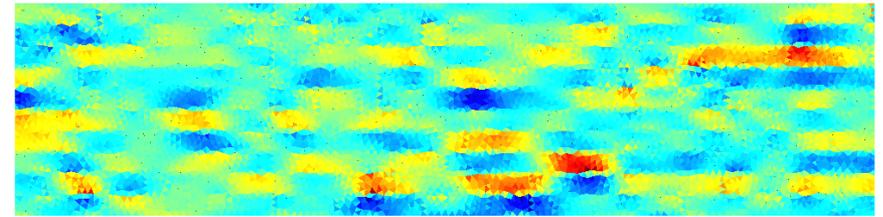 - How to define random vectors $\boldsymbol{\mathcal{V}}_{\mathrm{MT}}$ of $\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}}$?
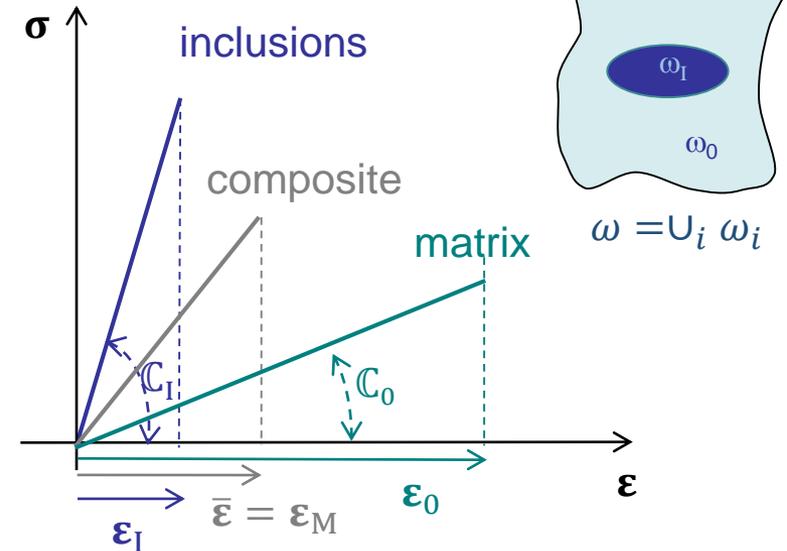
Stochastic Mean-Field Homogenization

- Mean-Field-homogenization (MFH)
  - Linear composites

$$\sigma_M = \overline{\sigma} = v_0 \sigma_0 + v_I \sigma_I$$

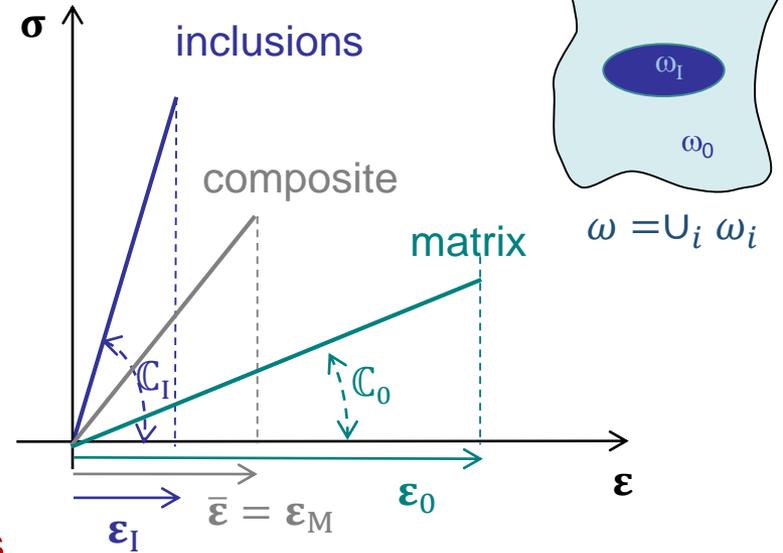$$\varepsilon_M = \overline{\varepsilon} = v_0 \varepsilon_0 + v_I \varepsilon_I$$

$$\varepsilon_I = B^\varepsilon(I, \mathbb{C}_0, \mathbb{C}_I): \varepsilon_0$$

$$\mathbb{C}_M = \mathbb{C}_M(I, \mathbb{C}_0, \mathbb{C}_I, v_I)$$

Defined as random fields

$$\omega = \cup_i \omega_i$$

- Consider an equivalent system
  - For each SVE realization i:

    $\mathbb{C}_M$ and v_I known

  - Anisotropy from $\mathbb{C}_M^i$

    θ is evaluated

  - Fiber behavior uniform

    $\hat{\mathbb{C}}_I$ for one SVE

$$\mathbb{C}_M \simeq \hat{\mathbb{C}}_M(\hat{I}, \hat{\mathbb{C}}_0, \hat{\mathbb{C}}_I, v_I, \theta)$$

Equivalent inclusion

- Remaining optimization problem:
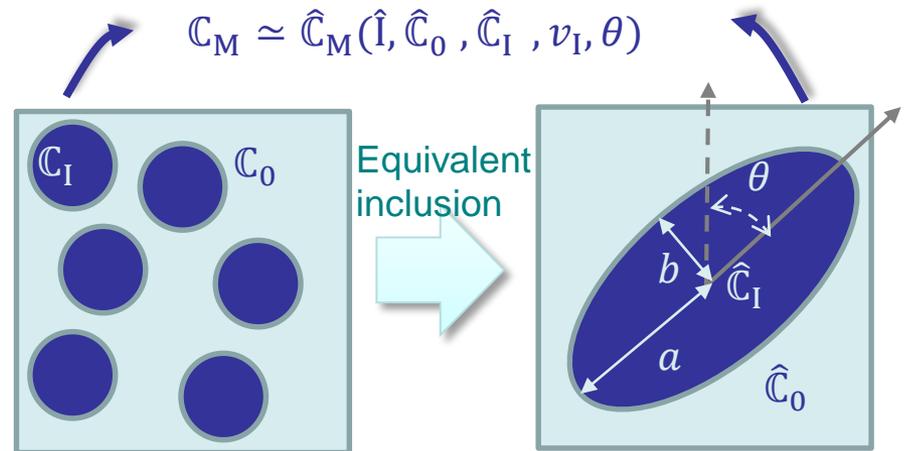
$$\min_{\frac{a}{b}, \hat{E}_0, \hat{v}_0} \left\| \mathbb{C}_M - \hat{\mathbb{C}}_M\left(\frac{a}{b}, \hat{E}_0, \hat{v}_0; v_I, \theta, \hat{\mathbb{C}}_I\right) \right\|$$

Stochastic Mean-Field Homogenization

$$\mathbb{C}_{\mathrm{M}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}(\hat{I}, \hat{\mathbb{C}}_0, \hat{\mathbb{C}}_{\mathrm{I}}, v_{\mathrm{I}}, \theta)$$

- Inverse stochastic identification
  - Comparison of homogenized properties from SVE realizations and stochastic MFH

Equivalent inclusion
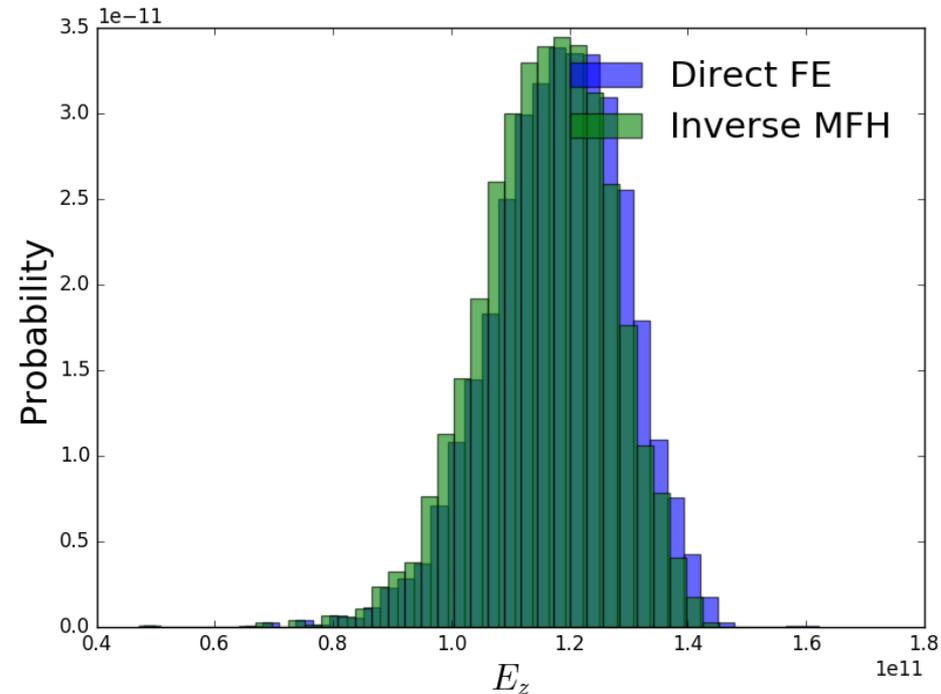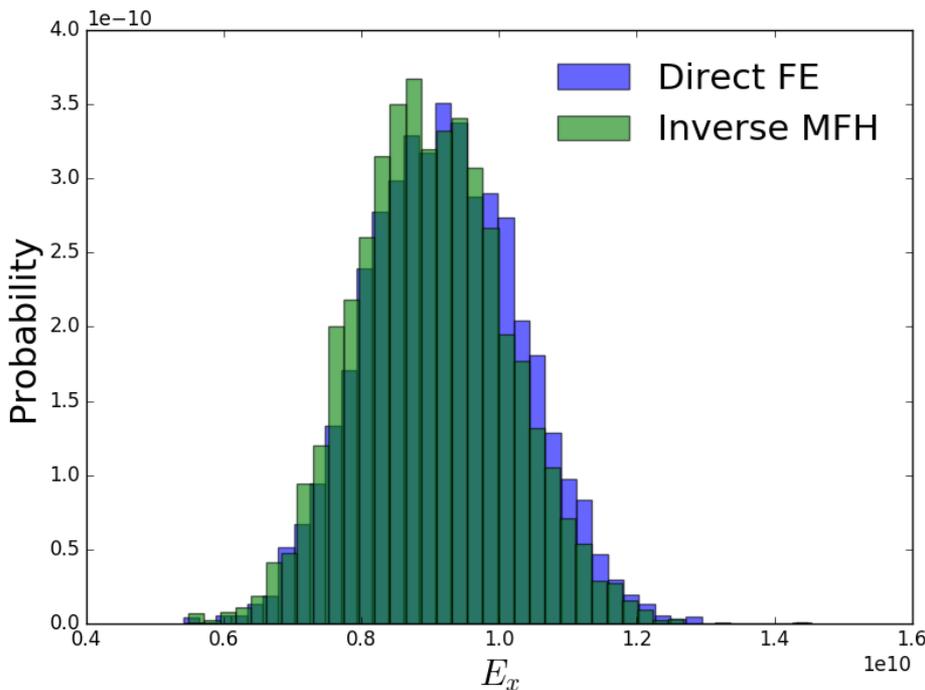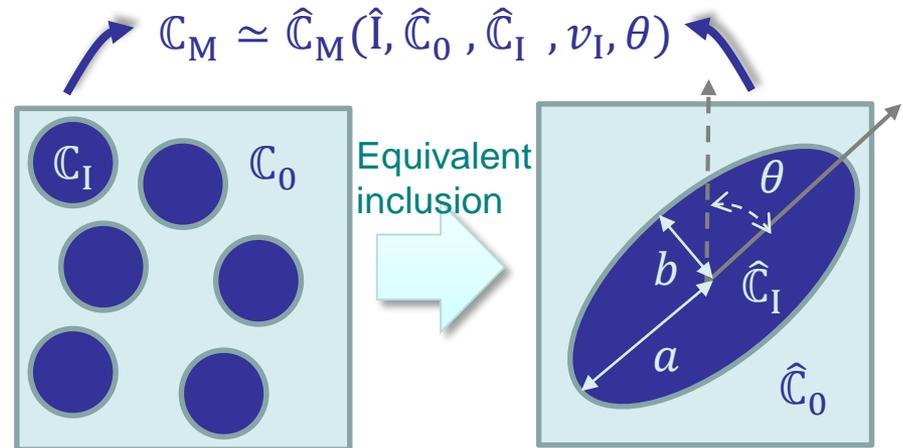
Stochastic Mean-Field Homogenization

- Comparison Random fields *vs.* Stochastic elastic MFH

Random anisotropic material tensor

σ_{xx} [Mpa]

0   33   66

σ^{eq}[Mpa]

0   30   60

Stochastic MFH

σ_{xx} [Mpa]

0   33   66

σ^{eq}[Mpa]

0   30   60

Stochastic Mean-Field Homogenization

- Stochastic MFH model:

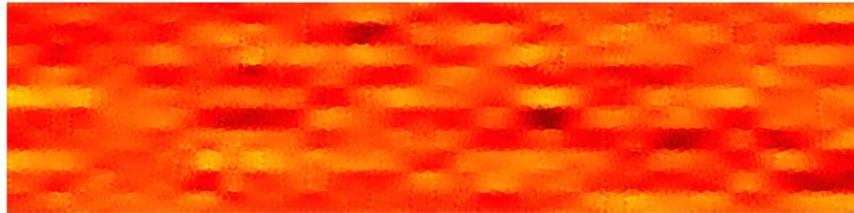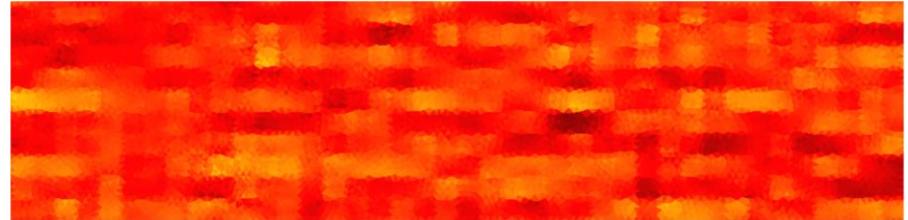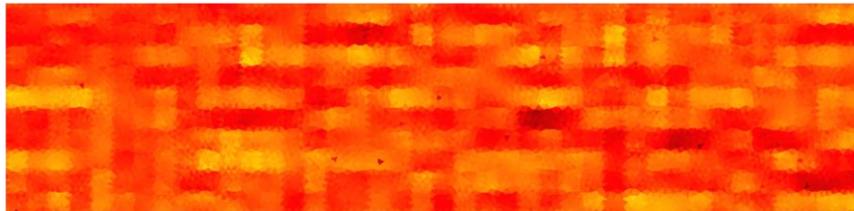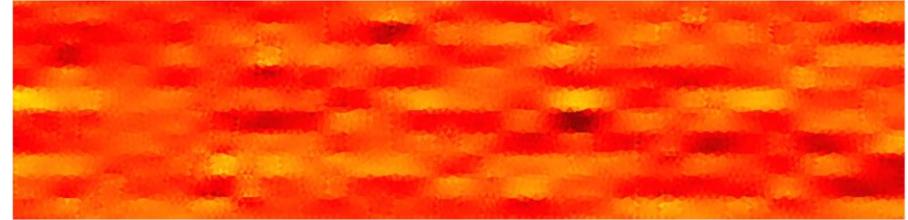 - Homogenized properties $\hat{\mathbb{C}}_\mathrm{M}(\frac{a}{b}, \hat{E}_0, \hat{v}_0 \; v_\mathrm{I}, \theta; \hat{\mathbb{C}}_\mathrm{I})$

  - Random vectors $\boldsymbol{\mathcal{V}}_\mathrm{MT}$

    - Realizations $\mathrm{v}_\mathrm{MT} = \left\{\frac{a}{b}, \hat{E}_0, \hat{v}_0 \; v_\mathrm{I}, \theta\right\}$

    - Characterized by the distance correlation matrix

    - Generator using the copula method

$$\hat{\mathbb{C}}_\mathrm{M}(\frac{a}{b}, \hat{E}_0, \hat{v}_0 \; v_\mathrm{I}, \theta; \hat{\mathbb{C}}_\mathrm{I})$$
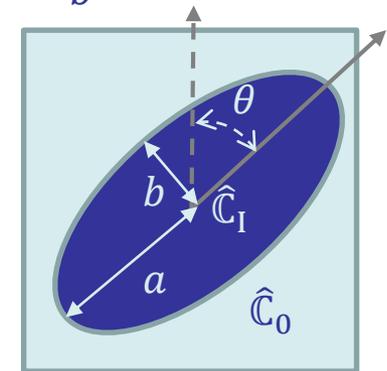
	v_I	θ	$\dfrac{a}{b}$	$\widehat{E}_0$	$\widehat{v}_0$
v_I	1.0	0.015	0.114	0.523	0.499
θ		1.0	0.092	0.016	0.014
$\dfrac{a}{b}$			1.0	0.080	0.076
$\widehat{E}_0$				1.0	0.661
$\widehat{v}_0$					

Distances correlation matrix

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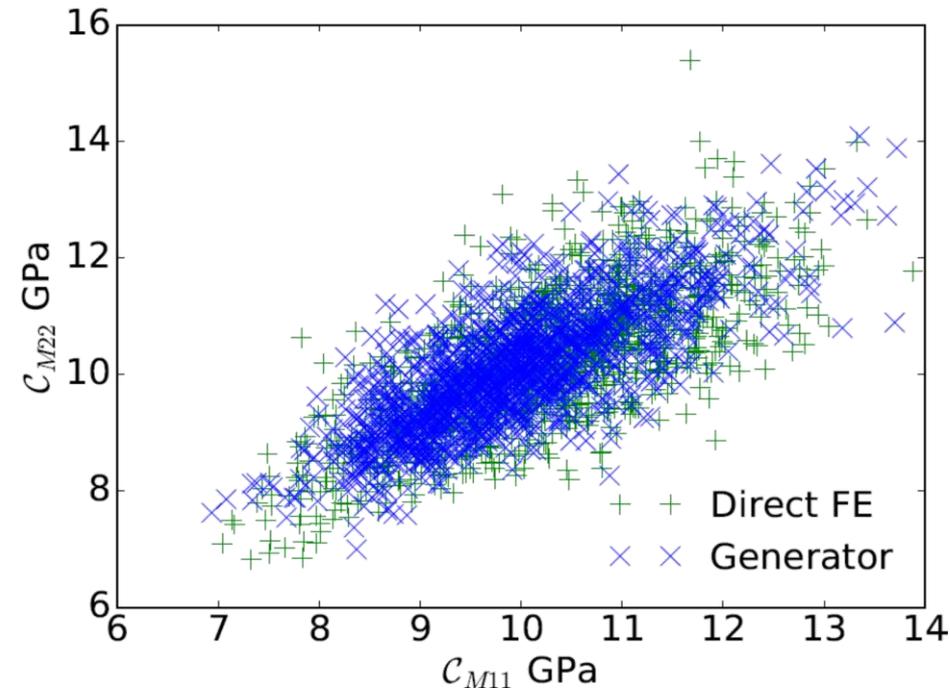
Stochastic Mean-Field Homogenization

- **Stochastic simulations**
  - 2 discretization: Random field $\mathcal{V}_{MT}$ & Stochastic finite-elements

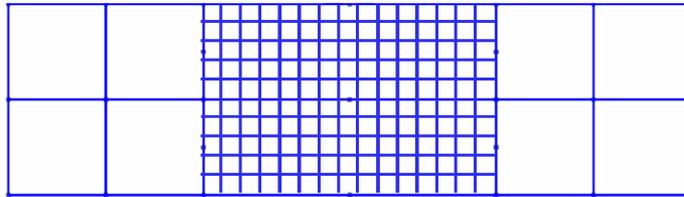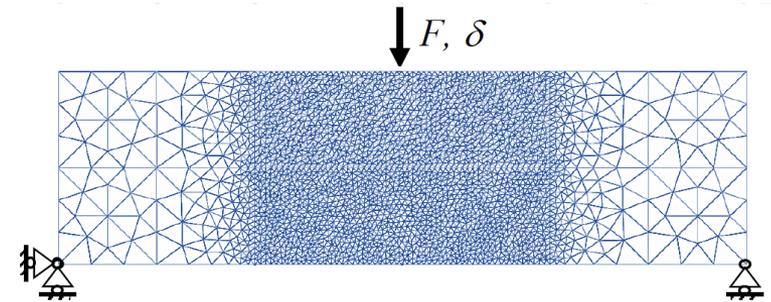

  - Realizations to reach a given deflection δ
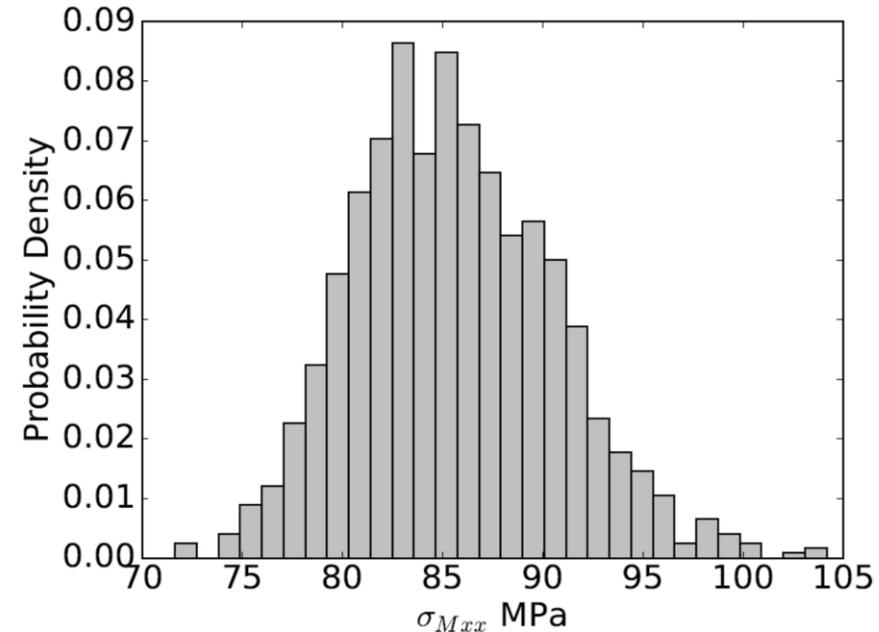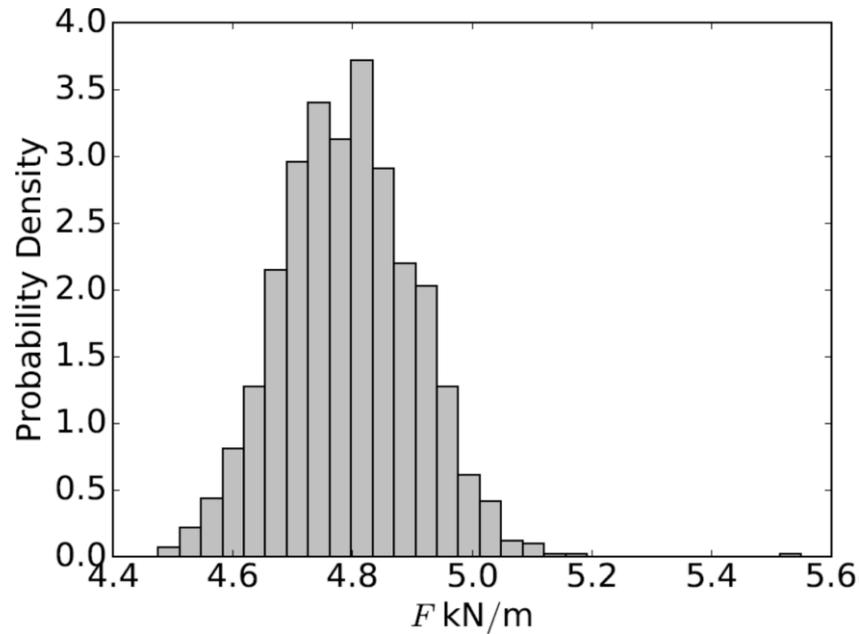
- Non-linear SVE simulations

Damage in matrix

0   0.5   1

Damage in matrix

0   0.5   1
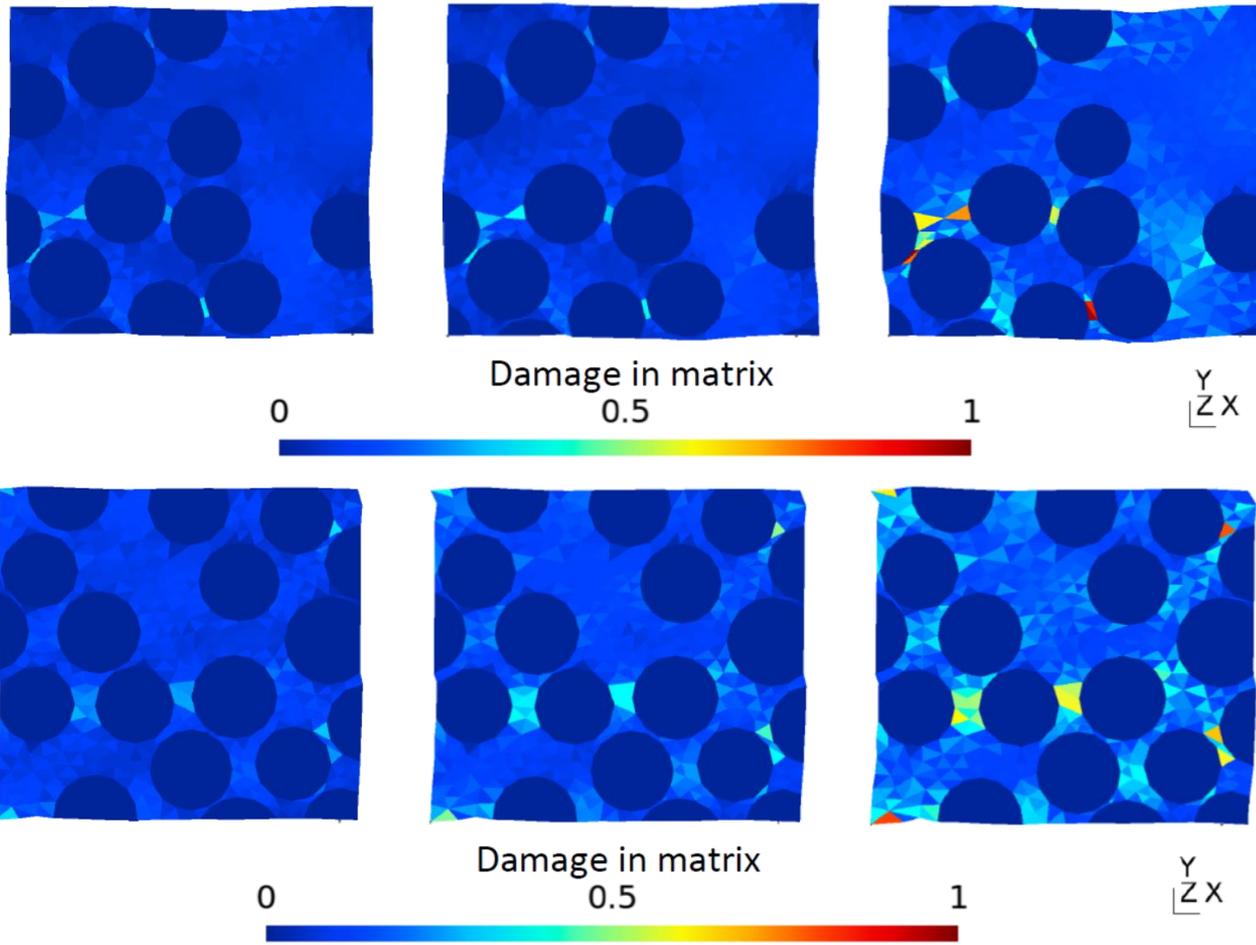
Non-linear stochastic Mean-Field Homogenization

- **Non-linear Mean-Field-homogenization**
  - Linear composites

$$\sigma_M = \overline{\sigma} = v_0\sigma_0 + v_I\sigma_I$$
$$\varepsilon_M = \overline{\varepsilon} = v_0\varepsilon_0 + v_I\varepsilon_I$$
$$\varepsilon_I = \mathbf{B}^\varepsilon(I, \mathbb{C}_0, \mathbb{C}_I) : \varepsilon_0$$

  - Non-linear composites

$$\sigma_M = \overline{\sigma} = v_0\sigma_0 + v_I\sigma_I$$
$$\Delta\varepsilon_M = \overline{\Delta\varepsilon} = v_0\Delta\varepsilon_0 + v_I\Delta\varepsilon_I$$
$$\Delta\varepsilon_I = \mathbf{B}^\varepsilon(I, \mathbb{C}_0^{LCC}, \mathbb{C}_I^{LCC}) : \Delta\varepsilon_0$$

Define a linear comparison composite material
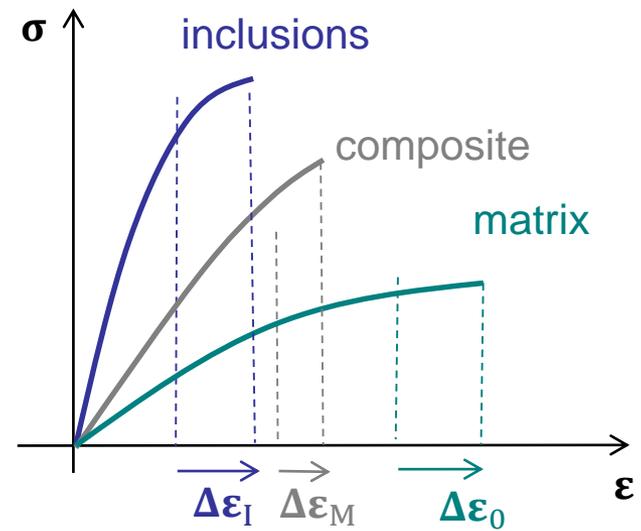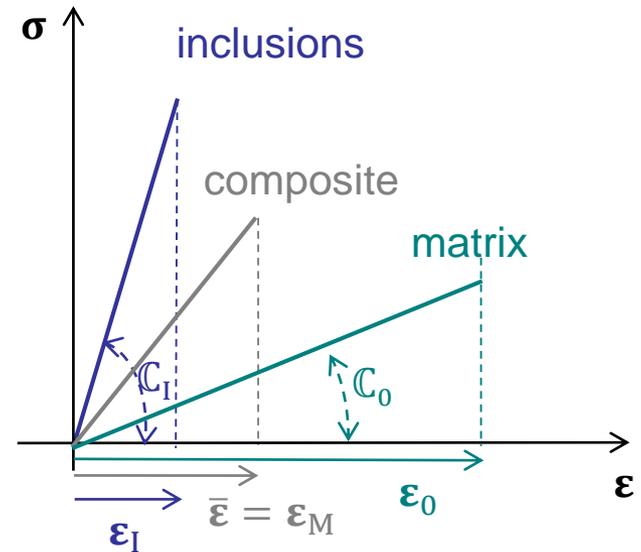
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Non-linear stochastic Mean-Field Homogenization

- **Incremental-secant Mean-Field-homogenization**
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components
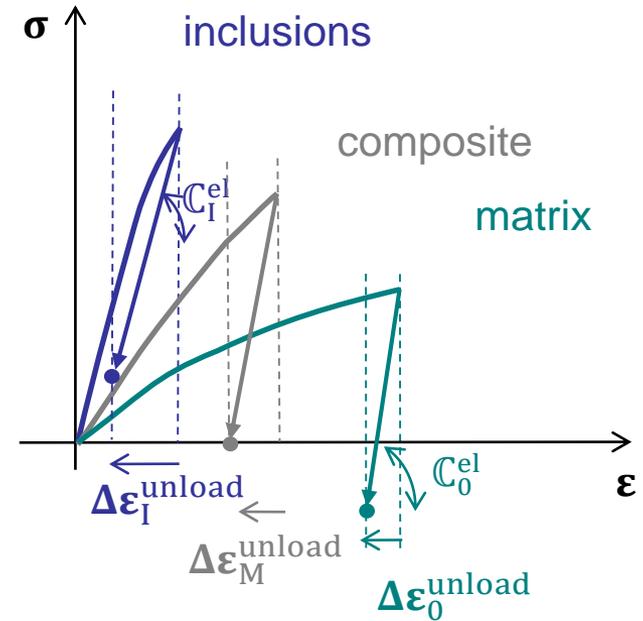
Non-linear stochastic Mean-Field Homogenization

- **Incremental-secant Mean-Field-homogenization**
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

  - Define Linear Comparison Composite
    - From unloaded state

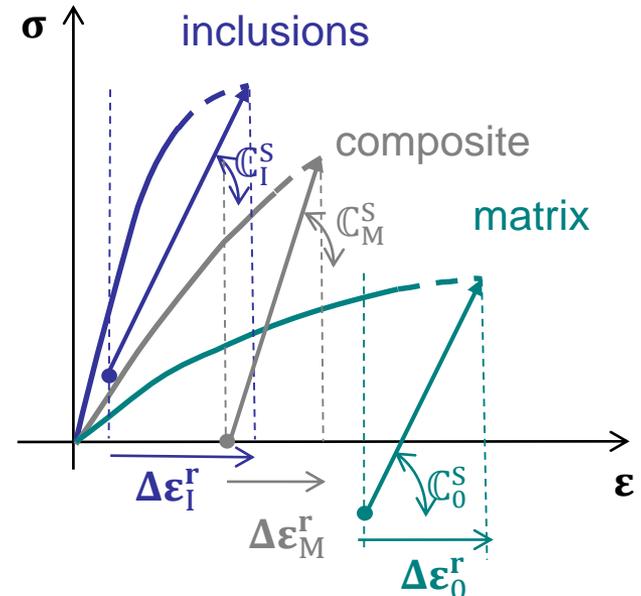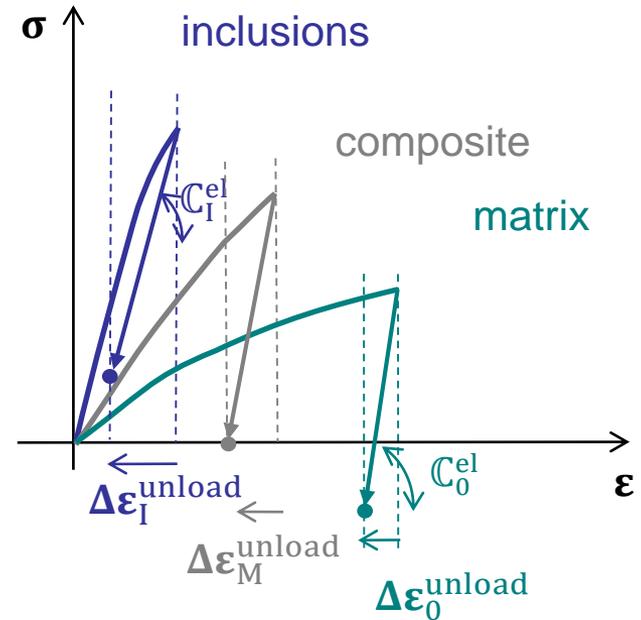 $$\Delta \boldsymbol{\varepsilon}_{I/0}^{r} = \Delta \boldsymbol{\varepsilon}_{I/0} + \Delta \boldsymbol{\varepsilon}_{I/0}^{\text{unload}}$$

    - Incremental-secant loading

    $$\begin{cases} \boldsymbol{\sigma}_M = \overline{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_I \boldsymbol{\sigma}_I \\ \Delta \boldsymbol{\varepsilon}_M^{r} = \overline{\Delta \boldsymbol{\varepsilon}} = v_0 \Delta \boldsymbol{\varepsilon}_0^{r} + v_I \Delta \boldsymbol{\varepsilon}_I^{r} \\ \Delta \boldsymbol{\varepsilon}_I^{r} = \mathbf{B}^{\varepsilon}\big(I, \mathbb{C}_0^{S}, \mathbb{C}_I^{S}\big) : \Delta \boldsymbol{\varepsilon}_0^{r} \end{cases}$$

    - Incremental secant operator

    $$\Rightarrow \quad \Delta \boldsymbol{\sigma}_M = \mathbb{C}_M^{S}\big(I, \mathbb{C}_0^{S}, \mathbb{C}_I^{S}, v_I\big) : \Delta \boldsymbol{\varepsilon}_M^{r}$$

Non-linear stochastic Mean-Field Homogenization

- ## Non-linear inverse identification
  - First step from elastic response

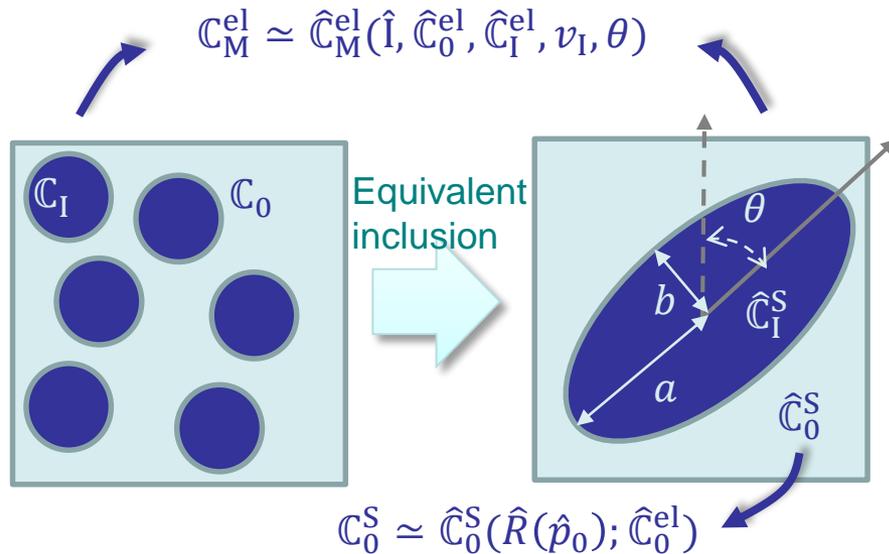$$\mathbb{C}_M^{el} \simeq \hat{\mathbb{C}}_M^{el}(\hat{I}, \hat{\mathbb{C}}_0^{el}, \hat{\mathbb{C}}_I^{el}, v_I, \theta)$$

Non-linear stochastic Mean-Field Homogenization

- **Non-linear inverse identification**
  - First step from elastic response

$$\mathbb{C}_M^{el} \simeq \hat{\mathbb{C}}_M^{el}(\hat{I}, \hat{\mathbb{C}}_0^{el}, \hat{\mathbb{C}}_I^{el}, v_I, \theta)$$

Equivalent inclusion

$$\mathbb{C}_0^S \simeq \hat{\mathbb{C}}_0^S(\hat{R}(\hat{p}_0); \hat{\mathbb{C}}_0^{el})$$
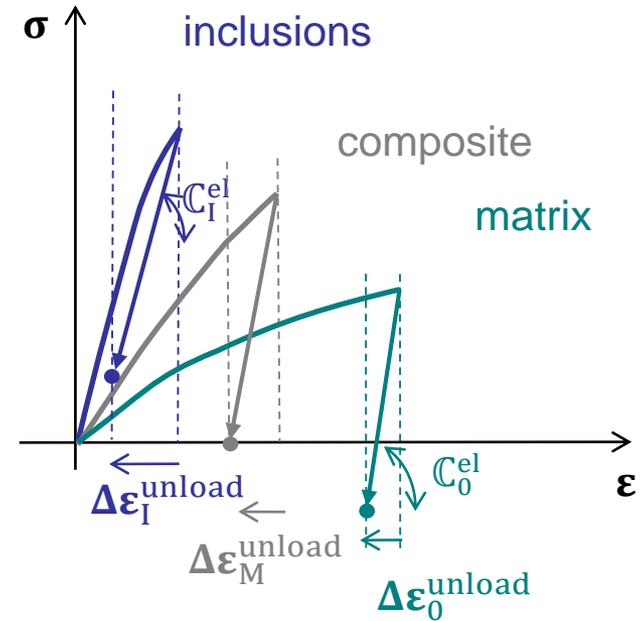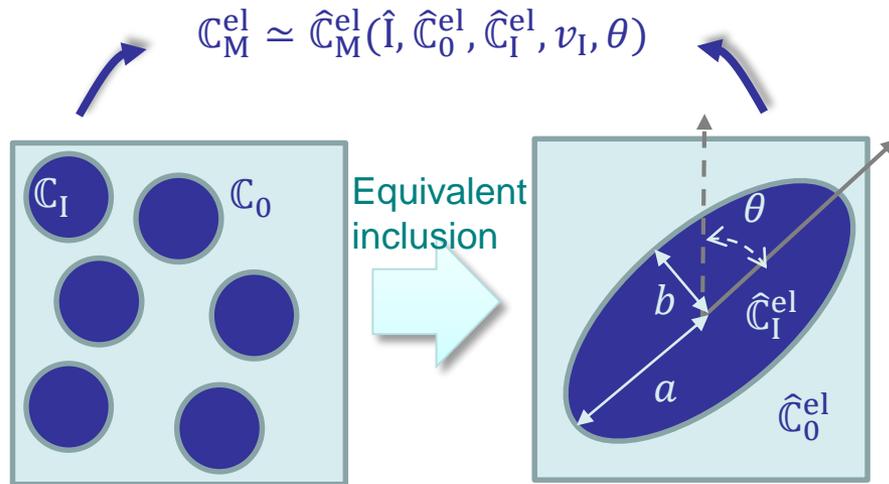
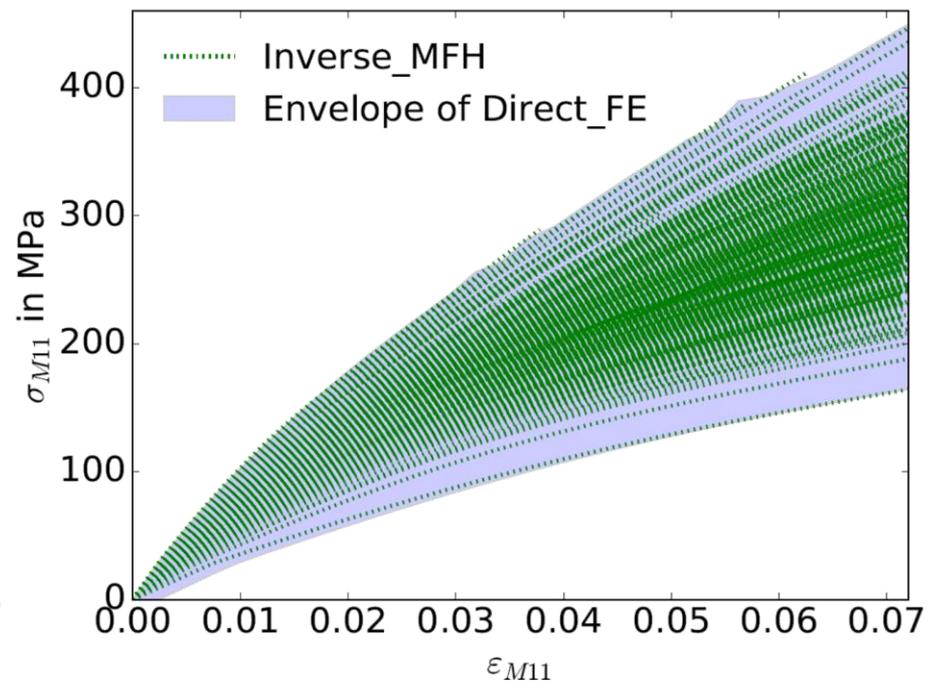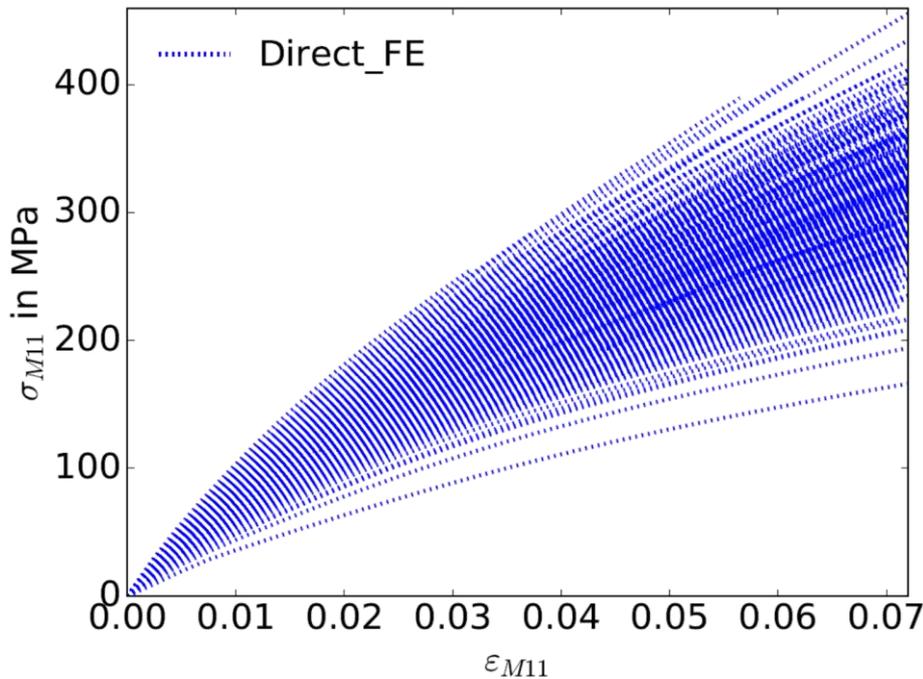- Second step from the LCC
  - New optimization problem

$$\Delta\boldsymbol{\sigma}_M \simeq \hat{\mathbb{C}}_M^S(\hat{I}, \hat{\mathbb{C}}_0^S, \mathbb{C}_I^S, v_I, \theta):\Delta\boldsymbol{\varepsilon}_M^r$$

  - Extract the equivalent hardening $\hat{R}(\hat{p}_0)$ from the incremental secant tensor

$$\mathbb{C}_0^S \simeq \hat{\mathbb{C}}_0^S(\hat{R}(\hat{p}_0); \hat{\mathbb{C}}_0^{el})$$

Non-linear stochastic Mean-Field Homogenization

- Non-linear inverse identification
  - Comparison SVE vs. MFH

$$\mathbb{C}_M^{el} \simeq \hat{\mathbb{C}}_M^{el}(\hat{I}, \hat{\mathbb{C}}_0^{el}, \hat{\mathbb{C}}_I^{el}, v_I, \theta)$$

Equivalent inclusion

$$\mathbb{C}_0^{S} \simeq \hat{\mathbb{C}}_0^{S}(\hat{R}(\hat{p}_0); \hat{\mathbb{C}}_0^{el})$$

- **Damage-enhanced Mean-Field-homogenization**
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components
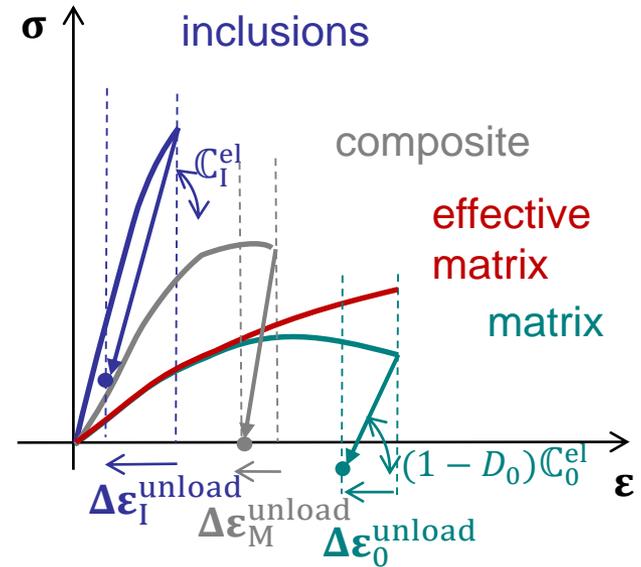
- **Damage-enhanced Mean-Field-homogenization**
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

  - Define Linear Comparison Composite
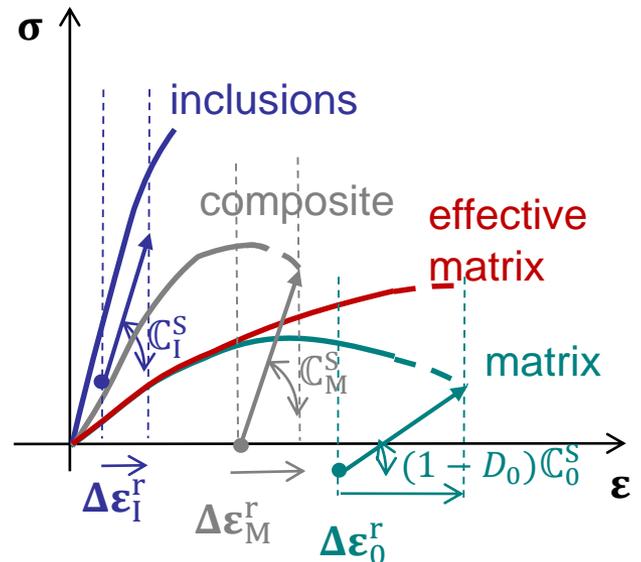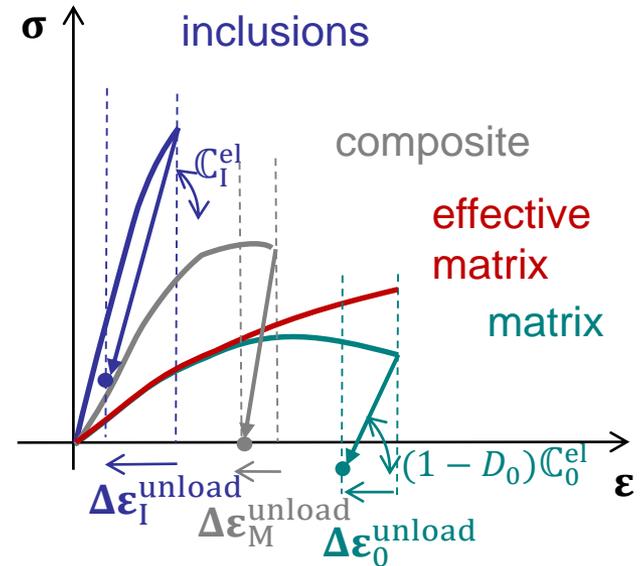   - From elastic state

$$\Delta\boldsymbol{\varepsilon}_{I/0}^{r} = \Delta\boldsymbol{\varepsilon}_{I/0} + \Delta\boldsymbol{\varepsilon}_{I/0}^{unload}$$

    - Incremental-secant loading

$$
\begin{cases}
\boldsymbol{\sigma}_M = \overline{\boldsymbol{\sigma}} = v_0\boldsymbol{\sigma}_0 + v_I\boldsymbol{\sigma}_I \\
\Delta\boldsymbol{\varepsilon}_M^{r} = \overline{\Delta\boldsymbol{\varepsilon}} = v_0\Delta\boldsymbol{\varepsilon}_0^{r} + v_I\Delta\boldsymbol{\varepsilon}_I^{r} \\
\Delta\boldsymbol{\varepsilon}_I^{r} = \mathbf{B}^{\varepsilon}\big(I, (1 - D_0)\mathbb{C}_0^{S}, \mathbb{C}_I^{S}\big) : \Delta\boldsymbol{\varepsilon}_0^{r}
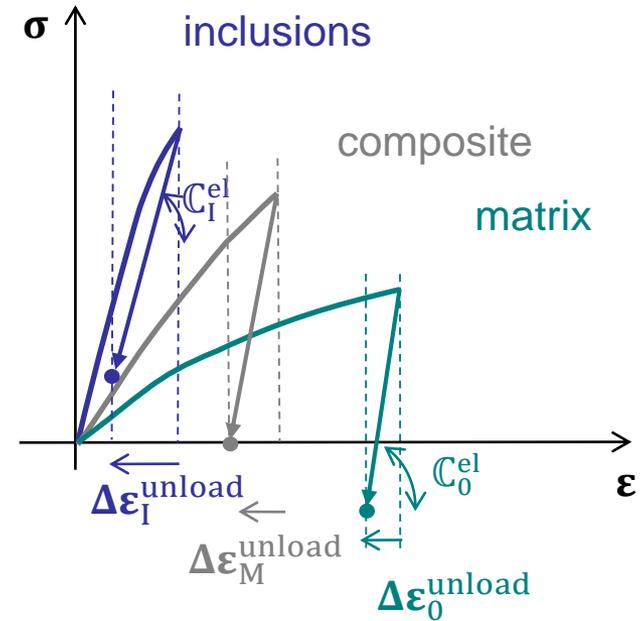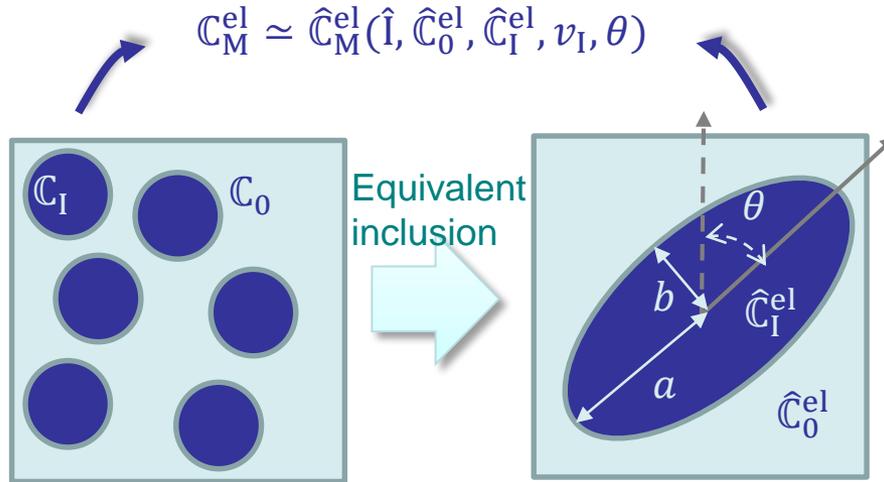\end{cases}
$$

    - Incremental secant operator

$$\Rightarrow \quad \Delta\boldsymbol{\sigma}_M = \mathbb{C}_M^{S}\big(I, (1 - D_0)\mathbb{C}_0^{S}, \mathbb{C}_I^{S}, v_I\big) : \Delta\boldsymbol{\varepsilon}_M^{r}$$
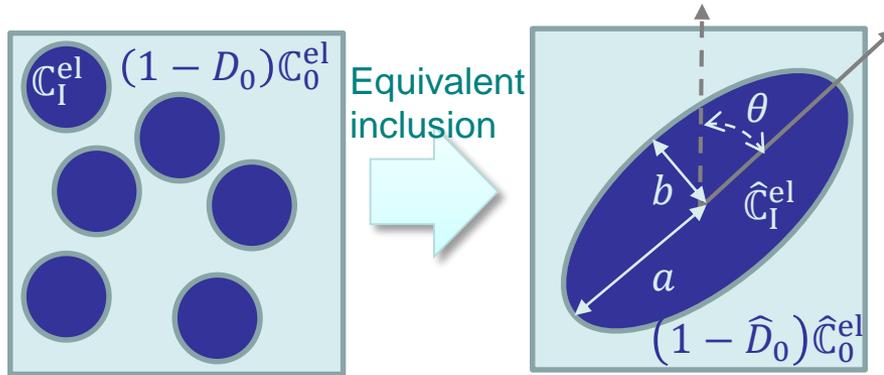
Non-linear stochastic Mean-Field Homogenization

- **Damage-enhanced inverse identification**
  - First step from elastic response
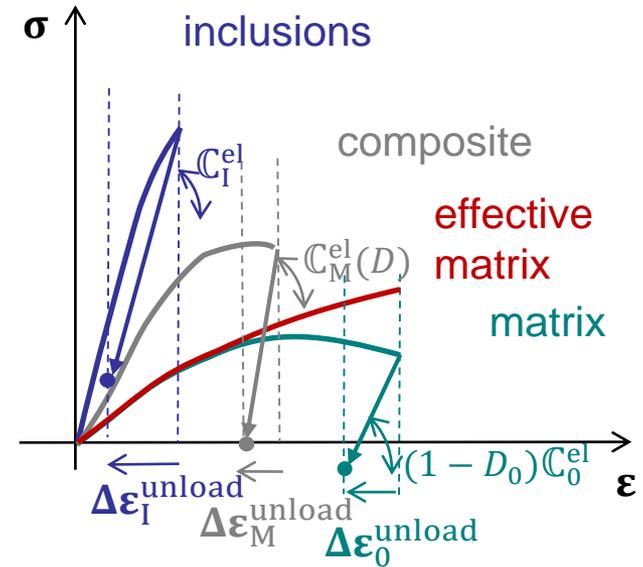   - Before damage occurs

$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}(\hat{I}, \hat{\mathbb{C}}_0^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta)$$

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Non-linear stochastic Mean-Field Homogenization

- **Damage-enhanced inverse identification**
  - – Second step: elastic unloading

- Identify damage evolution $\widehat{D}_0$

$$\mathbb{C}_M^{el}(D) \simeq \widehat{\mathbb{C}}_M^{el}\left(\hat{I}, \left(1 - \widehat{D}_0\right)\widehat{\mathbb{C}}_0^{el}, \widehat{\mathbb{C}}_I^{el}, v_I, \theta\right)$$

Non-linear stochastic Mean-Field Homogenization

- **Damage-enhanced inverse identification**

  – Second step: elastic unloading

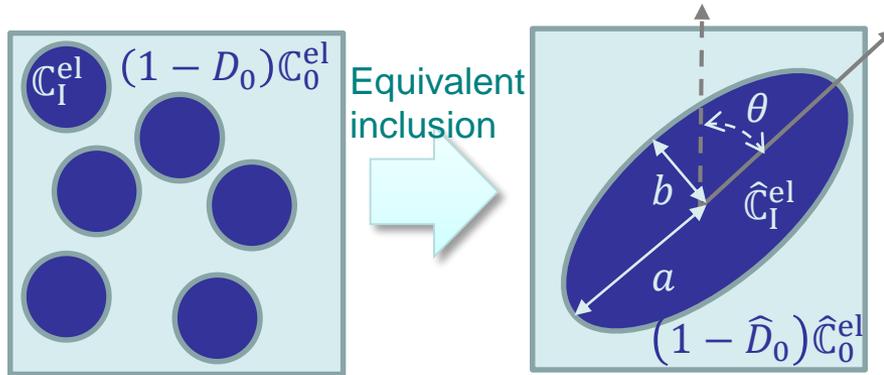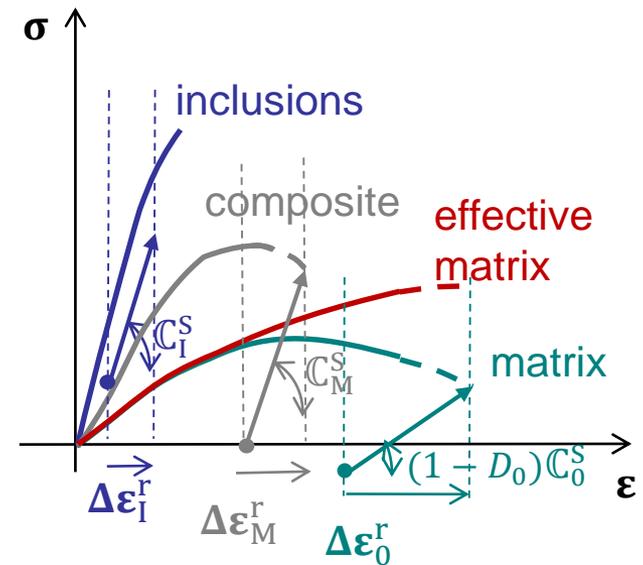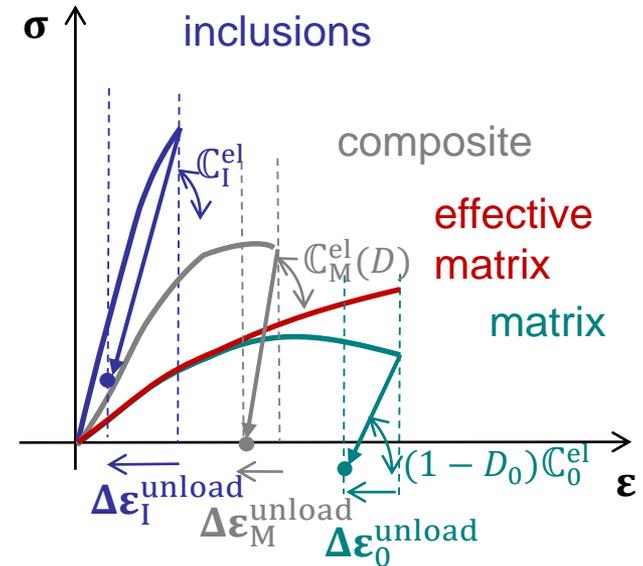 - Identify damage evolution $\widehat{D}_0$

  $$\mathbb{C}_M^{el}(D) \simeq \widehat{\mathbb{C}}_M^{el}\big(\hat{I}, \big(1 - \widehat{D}_0\big)\widehat{\mathbb{C}}_0^{el}, \widehat{\mathbb{C}}_I^{el}, v_I, \theta\big)$$

  – Third step from the LCC

  - $\boldsymbol{\Delta\sigma}_M = \mathbb{C}_M^S\big(I, (1 - D_0)\mathbb{C}_0^S, \mathbb{C}_I^S, v_I\big) : \boldsymbol{\Delta\varepsilon}_M^r$

  - Extract the equivalent hardening $\hat{R}(\hat{p}_0)$ & damage evolution $\widehat{D}_0(\hat{p}_0)$ from incremental secant tensor:

  $$(1 - D_0)\mathbb{C}_0^S \simeq \big(1 - \widehat{D}_0(\hat{p}_0)\big)\widehat{\mathbb{C}}_0^S\big(\hat{R}(\hat{p}_0); \widehat{\mathbb{C}}_0^{el}\big)$$

Non-linear stochastic Mean-Field Homogenization

- **Damage-enhanced inverse identification**
  - Comparison SVE vs. MFH

$$(1 - D_0)\mathbb{C}_0^S \simeq \left(1 - \widehat{D}_0(\hat{p}_0)\right) \widehat{\mathbb{C}}_0^S(\widehat{R}(\hat{p}_0); \widehat{\mathbb{C}}_0^{el})$$
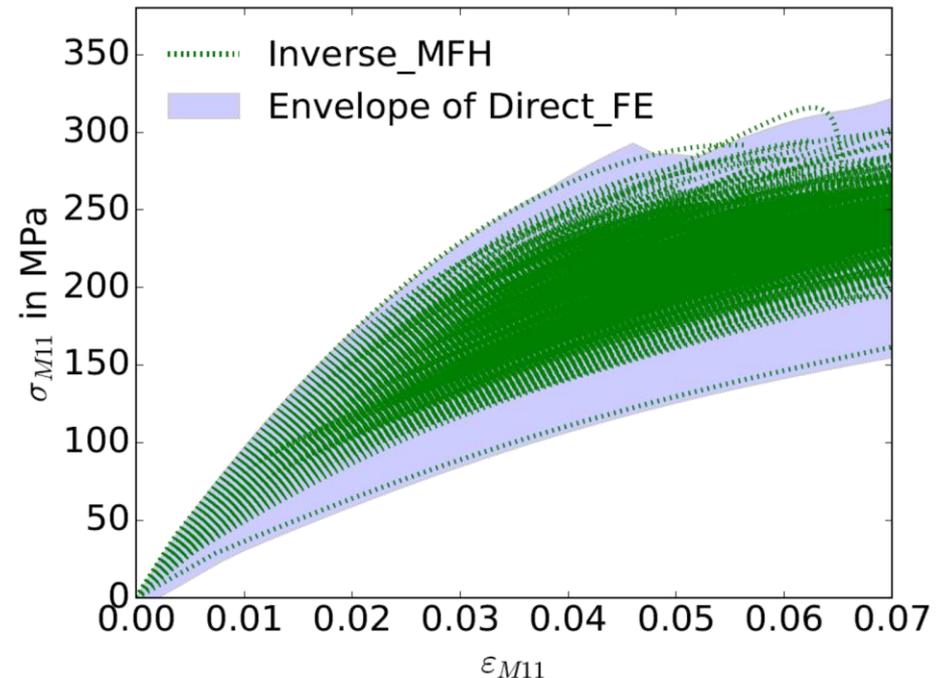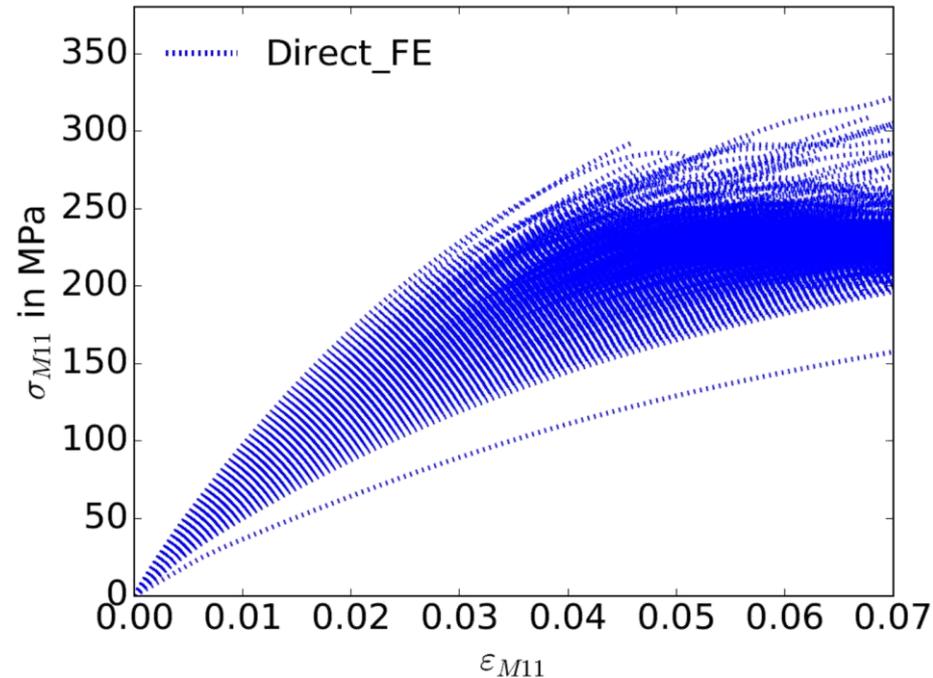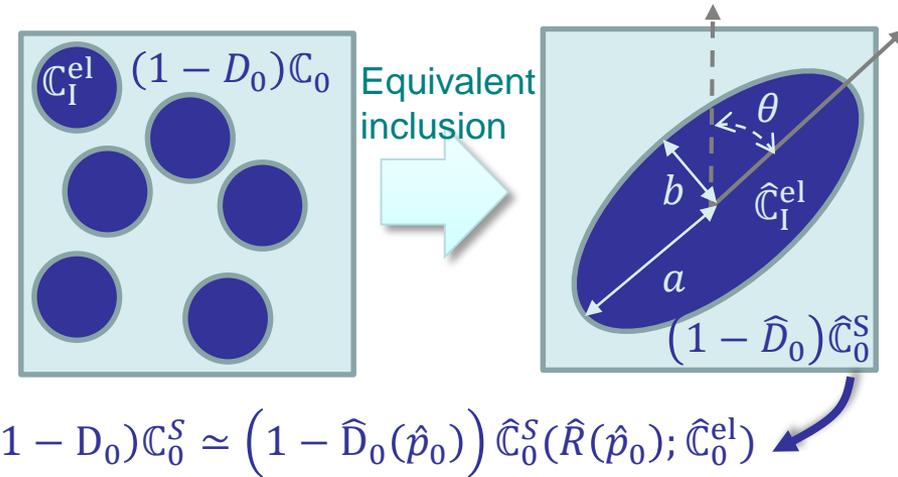
Conclusions

- Stochastic generator based on SEM measurements of unidirectional fibers-reinforced composites

- Computational homogenization on SVEs

- Two-step computational homogenization for Big SVEs

- Definition of a Stochastic MFH method

- In progress: nonlinear and failure analyzes

Thank you for your attention !