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INTERACTION POTENTIAL BETWEEN TWO ^{16}O NUCLEI DERIVED FROM THE SKYRME INTERACTION

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Abstract: The energy functional for the Skyrme density-dependent force is used to calculate the interaction potential between two ^{16}O nuclei. A two-centre harmonic oscillator potential is employed to construct the density and kinetic energy density of the ground state of the combined system and of the separated nuclei. The antisymmetrization effects are investigated. The relative motion of the nuclei is taken approximately into account and the energy dependence of the potential derived from the Skyrme force is presented.

1. Introduction

Several microscopic studies of the interaction energy between two heavy ions have been carried out recently. We refer to those which belong to one of the following two groups. In the first group the density functional of Brueckner *et al.*[1]) is employed in the sudden [2]) or adiabatic [3]) approximation. The interaction potential is determined by minimization with respect to the density parameters. In the second group the nuclear interaction between two overlapping nuclei is determined from molecular [4]) or shell model [5]) orbitals and an effective two-body interaction. The choice of this interaction depends on how well it reproduces the binding energies of both the interacting system and the separated nuclei. Up to now the Brink-Boeker [6]) and Volkov [7]) forces have been most extensively used. The problems principally investigated in these papers are the antisymmetrization and the distortion effects. Another important problem is the treatment of the relative motion between nuclei. Some investigations have been made by using simplified versions of the generator coordinate method [8,9]).

In the present paper we present calculations of the real part of the interaction potential between two nuclei based on an approach which has similarities with both groups of works mentioned above. The potential is defined as the binding energy

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of the total system less the binding energies of the isolated nuclei. For the binding energies we choose the energy functional derived by Vautherin and Brink [10]) for the Skyrme interaction. Extensive Hartree-Fock calculations carried out with this interaction [11]) provide sets of parameters which give good binding energies and reproduce well the single-particle properties of nuclei. For the Skyrme interaction the energy density is an algebraic function of the nucleon density ρ, the kinetic energy density τ and a spin-orbit density J.

There are two ways of evaluating the interaction energy of two nuclei. One is to perform a self-consistent calculation. Such an approach introduces difficult numerical problems [12]). The second way, which will be adopted in this paper, is to describe the densities ρ and τ by a model. For the purpose of our work we consider that a two-centre shell model is satisfactory. The ground state of the two interacting nuclei is taken as a Slater determinant built from a non-orthogonal basis of harmonic oscillator wave functions centred at two different points separated by a distance R. The same wave function has been used in ref. [5]).

In self-consistent calculations the interaction energy of two nuclei is determined through a variational calculation where the averaged hamiltonian describing the composite system is minimized under a constraint on a physical quantity relevant to the problem. Such a quantity could be for instance the quadrupole moment [12]) and then the ground state wave function and the binding energy are determined as a function of it. In model calculations, for any value of the quadrupole moment the binding energy should be obviously higher than that obtained in a self-consistent way. But if the model wave function of the ground state is not too different from the self-consistent wave function one should expect that the difference between the corresponding binding energy and the self-consistent result should be small of second order.

We apply the formalism to the interaction between two ^{16}O nuclei and present results for the potential as a function of R. We discuss in detail the exchange effects due to antisymmetrization and show that a large part of the exchange contribution comes from the modifications produced by antisymmetrization on the kinetic energy density. An approximate way of including exchange effects is suggested by this result.

Treating the c.m. motion in an approximate way we find an energy-dependent potential which can be compared with that obtained by Fliessbach [5]). The implications of using the Skyrme force on the energy dependence of the potential are pointed out.

In sect. 2 we briefly describe the formalism. Sect. 3 is devoted to the construction of the density and kinetic energy density and some of their properties. In sect. 4 we present the results and make a comparison with other works. Sect. 5 contains concluding remarks.

2. The formalism

We define the interaction potential between two nuclei as the energy expectation value of the whole system minus the energy expectation value of the two nuclei

separated at infinity

$$V = E(R) - E(\infty). \tag{1}$$

Here R is a parameter which measures the separation of the two nuclei. We shall define R explicitly in sect. 3. To calculate $E(R)$ and $E(\infty)$ we use the energy functional derived by Vautherin and Brink [10]) for the Skyrme interaction. There are two main reasons for using this force. First, the Hartree-Fock calculations performed with the Skyrme interaction provide sets of parameters which give good binding energies and radii for the whole mass table [11]). It is obvious from definition (1) that an interaction which gives good binding energies will yield a reasonable depth for the potential. An advantage of the Skyrme interaction is that it can be used for any pair of nuclei. Also, it gives good nuclear density distributions. This is important because the interaction energy of two nuclei is sensitive to their radii [13]). The second reason for using the Skyrme interaction is the mathematical simplicity of its energy functional. This simplicity makes calculations easier and gives a better physical insight because it connects directly the behaviour of the potential to the properties of the density.

For the Skyrme interaction the expectation value of the total energy E of a system whose ground state is described by a Slater determinant can be written as

$$E = \int H(\boldsymbol{r}) \mathrm{d}\boldsymbol{r}, \tag{2}$$

where the energy density $H(\boldsymbol{r})$ is an algebraic function [10]) of the nucleon densities $\rho_n, \rho_p, \rho = \rho_n + \rho_p$, the kinetic energy densities $\tau_n, \tau_p, \tau = \tau_n + \tau_p$, and the spin-orbit densities $\boldsymbol{J}_p, \boldsymbol{J}_p, \boldsymbol{J} = \boldsymbol{J}_p + \boldsymbol{J}_n$. If we neglect the spin-orbit coupling and Coulomb potentials, the densities of the $N = Z$ nuclei will satisfy $\rho_n = \rho_p = \frac{1}{2}\rho$, $\tau_n = \tau_p = \frac{1}{2}\tau$ and $H(\boldsymbol{r}) = H(\rho, \tau)$ may be written as

$$H(\rho, \tau) = \frac{\hbar^2}{2m}\tau + \tfrac{3}{8}t_0\rho^2 + \tfrac{1}{16}t_3\rho^3 + \tfrac{1}{16}(3t_1 + 5t_2)\rho\tau + \tfrac{1}{64}(9t_1 - 5t_2)(\nabla\rho)^2. \tag{3}$$

The parameters of the Skyrme interaction are t_0, t_1 and t_2, t_3. In this paper the two sets of parameters SI and SII provided by ref. [10]) are used and a comparison of the results obtained with the two sets is made.

In expression (3) different terms contribute to different effects. The second and third term are responsible for the volume part of the binding energy and the term proportional to $9t_1 - 5t_2$ gives the surface effects. While the first term is a pure kinetic one the fourth comes from the exchange part of the nucleon-nucleon interaction and determines the density dependence of the nucleon effective mass [10]). This term is modified when the nuclei are in relative motion with respect to each other. The modification requires that $\rho\tau$ should be replaced by $\rho\tau$-$\boldsymbol{j}^2$ where $\boldsymbol{j}$ is the momentum density and is given by eq. (20). The momentum density $\boldsymbol{j}$ vanishes in a static Hartree-Fock calculation because of time reversal invariance but is non-zero in our case where the nuclei have a relative motion [14]).

To calculate the potential between two ^{16}O nuclei 1 and 2 according to definition (1) it is necessary to evaluate the expression

$$V(R, k) = \int [H(\rho, \tau) - H(\rho_1, \tau_1) - H(\rho_2, \tau_2)] \mathrm{d}\boldsymbol{r}, \tag{4}$$

where H is given by eq. (3) and k by eq. (6). The densities ρ, τ refer to the whole system and ρ_1, τ_1 and ρ_2, τ_2 to the isolated nuclei.

3. Densities

We consider a shell model description for the ground state of the system ^{16}O+^{16}O. For simplicity we ignore spin and isospin and let the 32 nucleons occupy the s- and p-states of two harmonic oscillator wells centred at $-\frac{1}{2}R$ and $\frac{1}{2}R$ along the z-axis. If in the c.m. system the two nuclei are moving towards each other with momenta $\boldsymbol{K}$ and $-\boldsymbol{K}$, respectively, this motion can approximately be described by a plane wave $\mathrm{e}^{i\boldsymbol{K}\cdot\boldsymbol{R}}$ where R is the distance between the centres of the two potentials. In fact here we ignore the difference between R as the distance between the centres of mass and as the distance between the centres of potential wells. This could be corrected by the treatment of the c.m. motion within the generator coordinate method, but we assume that such corrections would not essentially modify our results. Fliessbach has shown [5]) that the dependence of the angle between K and R is very small so that it is reasonable to take $\boldsymbol{K}$ along the z-axis. The system has cylindrical symmetry and ρ and τ will be functions of the coordinates z and r and parameters R and k.

With this approximation of the relative motion the ground state of ^{16}O+^{16}O is described by a Slater determinant wave function built from the following single-particle states:

$$\psi_{\alpha(1)} = \phi_{\alpha(1)} \mathrm{e}^{ik(z+\frac{1}{2}R)}, \qquad \psi_{\beta(2)} = \phi_{\beta(2)} \mathrm{e}^{-ik(z-\frac{1}{2}R)}, \tag{5}$$

where $\phi_{\alpha(1)}$ and $\phi_{\beta(2)}$ are s- or p-eigenstates of harmonic oscillator wells centred around $-\frac{1}{2}R$ and $\frac{1}{2}R$, respectively, and k is related to K by

$$k = K/A. \tag{6}$$

To find the expression of the density ρ and the kinetic energy density τ from this non-orthogonal basis we use the appendix of ref. [15]). Accordingly, these two quantities are

$$\rho = \sum_{\beta(j),\alpha(i)} B^{-1}_{\beta(j),\alpha(i)}(R, k) \psi^*_{\alpha(i)} \psi_{\beta(j)}, \tag{7}$$

$$\tau = \sum_{\beta(j),\alpha(i)} B^{-1}_{\beta(j),\alpha(i)}(R, k) \nabla\psi^*_{\alpha(i)} \nabla\psi_{\beta(j)}, \tag{8}$$

where $B_{\beta(j),\alpha(i)} = \langle\psi_{\beta(j)}|\psi_{\alpha(i)}\rangle$ are the matrix elements of an 8×8 complex matrix. When $R \to \infty$ or $k \to \infty$ the matrix elements $B_{\alpha(1),\beta(2)}$ and their transpose tend to zero and B becomes the unit matrix. Consequently in either of these limits

ρ and τ are the superposition of individual densities i.e. $\rho = \rho_1 + \rho_2$ and $\tau = \tau_1 + \tau_2$. In other words the antisymmetrization effects on ρ and τ tend to zero when the two nuclei are far apart or move very fast with respect to each other. In the high energy limit as well as for large separation distances ρ becomes a constant with respect to the relative energy but τ is linear with k^2 through the energy dependence of τ_1 and τ_2,

$$\tau_{1,2} = \tau_{1,2}(0) + k^2 \rho_{1,2}, \tag{9}$$

where

$$\rho_1 = \sum_{\alpha(1)} |\varphi_{\alpha(1)}|^2, \qquad \tau_1(0) = \sum_{\alpha(1)} |\nabla \varphi_{\alpha(1)}|^2, \tag{10}$$

$$\rho_2 = \sum_{\beta(2)} |\varphi_{\beta(2)}|^2, \qquad \tau_2(0) = \sum_{\beta(2)} |\nabla \varphi_{\beta(2)}|^2.$$

Fig. 1 represents ρb^3 as a function of z/b where b is the oscillator parameter. The density is normalized to $\int \rho d\boldsymbol{r} = 32$. The three curves correspond to three values 0,1 and 2 of the cylindrical coordinate r/b. The nuclei are at rest. i.e. $k = 0$ and are separated by a distance of $R/b = 3$. In fig. 2 the kinetic energy density τ in b^{-5} units is plotted as a function of z/b for $R/b = 3$, $k = 0$ and $r = 0$. Curve (1) represents τ given by the formula (8). Curve (1′) corresponds to an alternative definition of τ,

$$\tau' = - \sum_{\beta(j), \alpha(i)} B^{-1}_{\beta(j), \alpha(i)}(R, k) \psi^*_{\alpha(i)} \nabla^2 \psi_{\beta(j)}. \tag{11}$$

The total kinetic energy is the volume integral of either τ or τ'. The two densities give the same volume integral because they differ only by a surface term, which

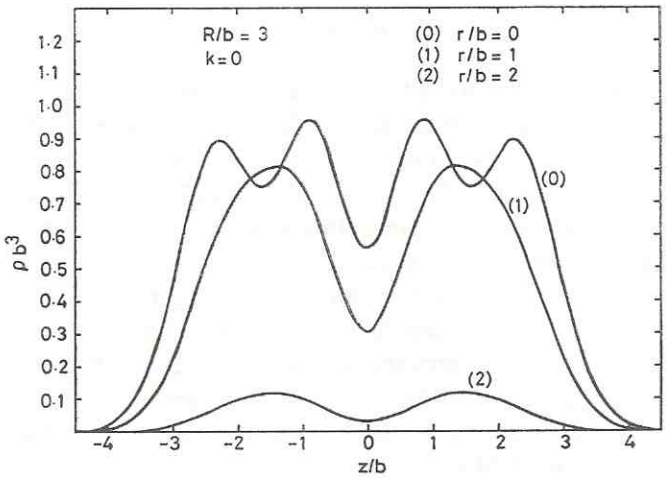

Fig. 1. Density of $^{16}O + ^{16}O$ in b^{-3} units as a function of z/b calculated from formula (7) for $R/b = 3$, $k = 0$. The three curves correspond to values 0, 1 and 2 of the coordinate r/b. Density is normalized to $\int \rho d\boldsymbol{r} = 32$.

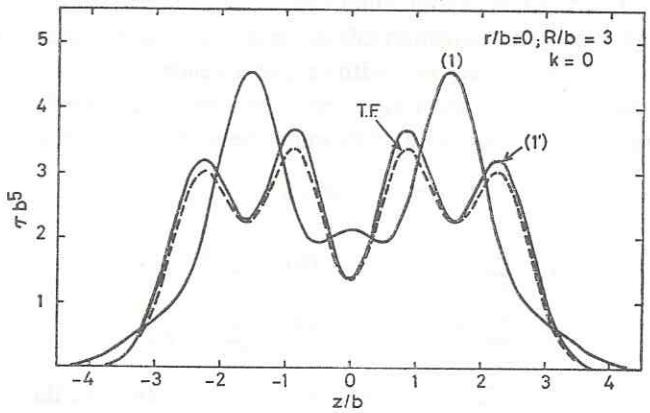

Fig. 2. Kinetic energy density of $^{16}O+^{16}O$ in b^{-5} units as a function of z/b calculated at $R/b = 3$, $r = 0$, $k = 0$ from formula (7) — curve (1), formula (11) — curve (1'), formula (12) — curve TF.

vanishes at infinity. The dashed curve TF is the Thomas-Fermi approximation,

$$\tau_{TF} = \tfrac{3}{5}(\tfrac{3}{2}\pi^2)^{\frac{2}{3}}\rho^{\frac{5}{3}}, \tag{12}$$

with ρ given by formula (7). As can be seen from fig. 2 the Thomas-Fermi formula is a good approximation to τ' for $R/b = 3$. In fact calculations show that it is an even better approximation for greater distances. This remark will be used in the next section in estimating the exchange effects due to antisymmetrization.

4. Results

One of the purposes of our work is to find out how antisymmetrization can influence the interaction potential between two nuclei. To this end we have calculated $V(R, 0)$ according to formula (4) in two different versions. In the first one we used ρ and τ given by eqs. (7) and (8). The result is plotted as V_{EX} in figs. 3 and 4 for the sets of parameters SI and SII respectively. In the second version we took values for ρ and τ ignoring antisymmetrization i.e. $\rho = \rho_1 + \rho_2$ and $\tau = \tau_1(0) + \tau_2(0)$. The corresponding results are represented by the curve V_{NOEX} in the same figures. By comparing V_{EX} with V_{NOEX} one notices that for both sets of parameters the potential is largely influenced by the antisymmetrization for all values of R. Some numerical values of V_{EX} and V_{NOEX} obtained for SII are given in table 1. The ratio V_{NOEX}/V_{EX} is ≈ 1.8 even at $R = 8$ fm whereas the barrier (sum of Coulomb and nuclear potential) is positioned at about 7.74 fm.

Let us call $\delta\rho$ and $\delta\tau$ the modifications produced in ρ and τ by the antisymmetrization, i.e. $\delta\rho = \rho - \rho_1 - \rho_2$ and $\delta\tau = \tau - \tau_1 - \tau_2$. By taking $\delta\rho = 0$ and $\delta\tau \neq 0$ we found that a large part of the exchange effects in V_{EX} comes from $\delta\tau$. If the correction $\delta\rho$

TABLE 1

Values of the interaction potential (MeV) between two ^{16}O nuclei for different separation distances R calculated with the interaction SII

R (fm)	4.22	4.58	5.28	6.34	7.39	7.74	8.10
V_{NOEX}	−118.84	−94.66	−53.41	−16.17	−3.18	−1.67	−0.85
V_{EX}	−7.91	−9.77	−9.82	−5.35	−1.53	−0.88	−0.47
$V_{\mathrm{EX}}-V_{\mathrm{NOEX}}$	110.93	84.89	43.59	10.82	1.65	0.79	0.38
ΔV	84.72	67.67	39.32	13.24	2.99	1.65	0.87
$V_{\mathrm{EX}}^{\mathrm{TF}}$	−22.97	−25.44	−20.59	−7.84	−1.59	−0.70	−0.38
$V_{\mathrm{EX}}^{\mathrm{DTF}}$	−1.49	−11.32	−15.16	−6.69	−1.43	−0.62	−0.34

V_{NOEX} and V_{EX} are results obtained by ignoring and including antisymmetrization respectively. ΔV is defined by eq. (13) and $V_{\mathrm{EX}}^{\mathrm{TF}}$ and $V_{\mathrm{EX}}^{\mathrm{DTF}}$ are approximations explained in the text.

is neglected the difference $V_{\mathrm{EX}}-V_{\mathrm{NOEX}}$ would become

$$V_{\mathrm{EX}}-V_{\mathrm{NOEX}} \approx \Delta V = \int \left[\frac{\hbar^2}{2m} + \tfrac{1}{16}(3t_1+5t_2)(\rho_1+\rho_2)\right] \delta\tau \, \mathrm{d}\boldsymbol{r}. \tag{13}$$

The fifth row in table 1 gives values for the integral ΔV. For distances 4.22 fm $\leqq R \leqq$ 5.28 fm ΔV accounts for 75–90% of the difference $V_{\mathrm{EX}}-V_{\mathrm{NOEX}}$. Around the barrier ΔV overestimates the exact $V_{\mathrm{EX}}-V_{\mathrm{NOEX}}$. Hence it seems that the correction $\delta\rho$ plays a more important role in the barrier region than for other distances but still it modifies only slightly the position and the height of the barrier. The curves V_{EX} for both SI and SII are similar to one another but the curve V_{NOEX} for SI shows much less attraction than that for SII. This difference can be understood from eq. (13). Neglecting $\delta\rho$ the integrand defining ΔV is $\delta\tau\hbar^2/2m^*$ where m^* is the effective mass [10]) and this quantity is almost two times larger for SI than for SII.

In the previous section we have shown that for distances larger than two times the rms radius of ^{16}O the Thomas-Fermi expression is a very good approximation to the exact kinetic energy density τ'. This result plus the one for ΔV suggest that one could estimate the exchange effects due to antisymmetrization by calculating $V(R, 0)$ putting $\rho = \rho_1 + \rho_2$ and $\tau' = \frac{3}{5}(\frac{3}{2}\pi^2)^{\frac{2}{3}}(\rho_1 + \rho_2)^{\frac{5}{3}}$ in eq. (3) where τ is related to τ' by $\tau = \tau' + \frac{1}{2}\nabla^2\rho$. This result is shown in figs. 3 and 4 as the curves $V_{\mathrm{EX}}^{\mathrm{TF}}$. Some numerical values of $V_{\mathrm{NOEX}}^{\mathrm{TF}}$ show how well the Thomas-Fermi approximation works for the individual kinetic energy densities. In fact also in $V_{\mathrm{EX}}^{\mathrm{TF}}$ τ_1 and τ_2 are given by the Thomas-Fermi approximation.

When the overlap between the two nuclei is small one might expect that the usual Thomas-Fermi approximation for τ' is not adequate. The Pauli principle should differentiate between different components of the internal momentum of each nucleon. When the two oxygen are pushed together [17]) along the z-axis the Pauli principle would force an increase of k_z but not of k_x and k_y as the single-particle wave functions are not changed. Hence a modified Thomas-Fermi kinetic energy density with an ellipsoidal distribution in k-space would be more adequate. One can get such a

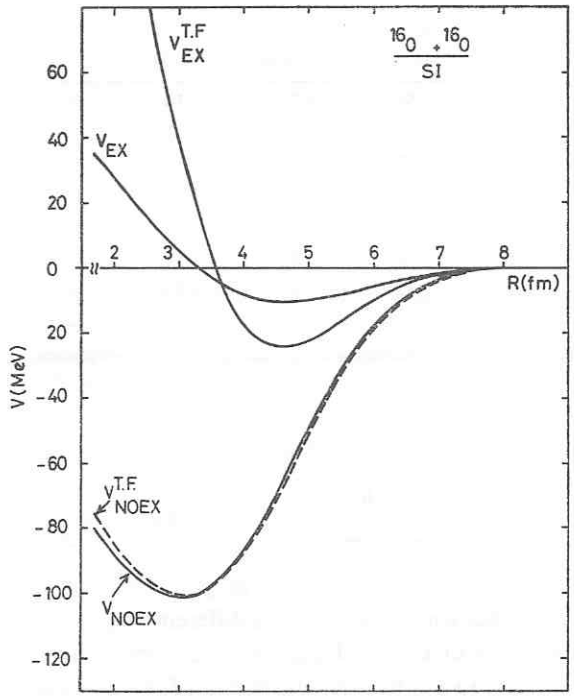

Fig. 3. Interaction potential between two ^{16}O nuclei as a function of the separation distance R, calculated with the interaction SI: $V_{\mathrm{NOEX}}(V_{\mathrm{EX}})$ are results without (with) antisymmetry included; $V^{\mathrm{TF}}_{\mathrm{NOEX}}$ is calculated as V_{NOEX} plus Thomas-Fermi approximation for the kinetic energy densities of the isolated nuclei; $V^{\mathrm{TF}}_{\mathrm{EX}}$ is an approximation of the exchange effects as explained in the text.

deformed Thomas-Fermi approximation by writing for τ',

$$\tau' = \tfrac{1}{5}\rho(k_x^2+k_y^2+k_z^2), \tag{14}$$

where k_x, k_y and k_z are the Fermi momenta along the axes x, y and z and the total density $\rho = \rho_1+\rho_2$ is given by

$$\rho = \frac{2}{3\pi^2}\, k_x k_y k_z . \tag{15}$$

The initial density (when nuclei are far apart) is

$$\rho_{\mathrm{i}} = \frac{2}{3\pi^2}\, k_0^3 , \tag{16}$$

where k_0 is the initial Fermi momentum. If we assume that k_x and k_y are unchanged i.e. $k_x = k_y = k_0$, then

$$k_z = k_0\, \rho/\rho_{\mathrm{i}} . \tag{17}$$

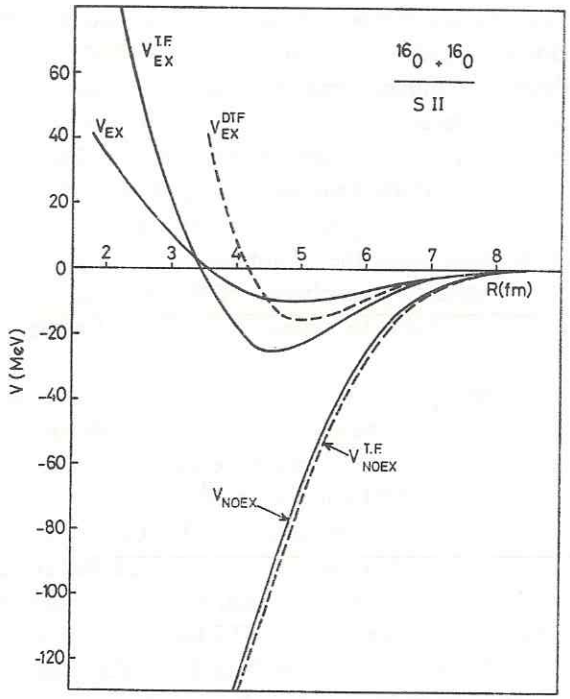

Fig. 4. Interaction potential between two ^{16}O nuclei as a function of the separation distance R, calculated with the interaction SII: $V_{NOEX}(V_{EX})$ are results without (with) antisymmetry included; V_{NOEX}^{TF} is calculated as V_{NOEX} plus Thomas-Fermi approximation for the kinetic energy densities of the isolated nuclei; V_{EX}^{TF} and V_{EX}^{DTF} are approximations of the exchange effects as explained in the text.

Hence

$$\tau' = \tfrac{1}{5}(\tfrac{3}{2}\pi^2)^{\frac{2}{3}}\rho\left[2+\left(\frac{\rho}{\rho_i}\right)^2\right]\rho_i^{\frac{2}{3}}. \tag{18}$$

It remains to choose ρ_i. We take

$$\begin{aligned}\rho_i &= \rho_1 \text{ in the region where } \rho_1 > \rho_2 (z<0)\\ &= \rho_2 \text{ in the region where } \rho_2 > \rho_1 (z>0).\end{aligned} \tag{19}$$

The curve V_{EX}^{DTF} in fig. 4 shows the interaction potential calculated with expression (18) for τ' and (12) for τ_1 and τ_2. The last row in table 1 contains some numerical values of V_{EX}^{DTF} for several separation distances.

The asymmetry of the Fermi surface is most important near $z = 0$ where $\rho_1 \approx \rho_2$. At $z = 0$ $\rho = 2\rho_i$ and formula (18) implies that τ' is four times larger than its initial value. This is equivalent to a variation of τ' as ρ^2 instead of $\rho^{\frac{5}{3}}$. The extra repulsion introduced by such a dependence of ρ makes V_{EX}^{DTF} a better approximation of the exact result V_{EX}.

The curves V_{EX} of figs. 3 and 4 can be compared with the results of Reidemeister [4]. In ref. [4]) the interaction energy of two ^{16}O nuclei has been derived from a variational calculation with a Slater determinant trial wave function built up from molecular orbitals. For the two-body interaction the force B1 [ref. [6])] was used. The calculations have been done with and without allowing for deformation. The spherical case has similarities to our curves V_{EX}. Both Reidemeister's and our calculations produce a shallow potential, with a minimum depth of about 10 MeV for V_{EX} and 20 MeV for Reidemeister's result. In both cases the minimum depth is situated at about two times the rms radius. An interesting conclusion of Reidemeister's calculation is that beyond this point the distortion due to mutual interaction of nuclei has no essential effect.

We also attempt a comparison of our results with the self-consistent calculations performed by Flocard [12]) for the same system but with the Skyrme interaction SIII [ref. [18])]. To make a more precise comparison we have repeated the calculation of V_{EX} with the Skyrme interaction SIII and found out that the result differs insignificantly from V_{EX} of figs. 3 or 4. We have also added a Coulomb interaction of the form $V_C = Z_1 Z_2 e^2/R$. This should be satisfactory provided that the charge distributions of the two nuclei do not overlap too much, that is provided $R > 6$ fm. (The rms radius of the ^{16}O charge distribution is ≈ 2.7 fm.) In fig. 5 we sketch $V_C + V_{EX}$ as a function of R where V_{EX} is calculated with SIII. On the same figure the results of ref. [12]) are also shown. The large difference between the two curves for distances R smaller than two times the rms radius is expected essentially because the self-consistent approach allows for nuclear distortion while our calculations leave the single-particle wave functions unchanged. This feature of constrained Hartree-Fock calculations is pointed out by Flocard.

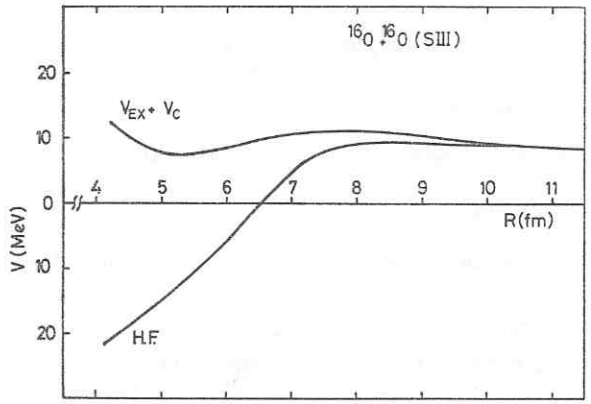

Fig. 5. Nuclear plus Coulomb interaction potential between two ^{16}O nuclei as a function of separation distance R. The curve $V_{EX}+V_C$ is the potential V_{EX} calculated with the interaction SIII to which a Coulomb potential of the form $V_C = Z_1 Z_2 e^2/R$ was added. Curve HF represents the result of ref. [12]).

For larger separations, outside our Coulomb barrier ($R > 8$ fm), there is also a difference between the two calculations which persists up to about 11 fm. The long range character of this difference suggests that it might be due to the induced polarization of the nuclei produced by the Coulomb field. Flocard's calculation includes this effect and he shows that each fragment has an induced quadrupole moment $Q_F \approx 0.5\,\mathrm{fm}^2$ even when $R \approx 14$ fm. However, an estimate of the polarization energy gives a result which is much smaller than the discrepancy mentioned above. Therefore it would be interesting to make a comparison with a self-consistent calculation of the $^{16}O+^{16}O$ potential performed without Coulomb interaction.

Besides the antisymmetry we consider now the effect of the relative motion between nuclei, that is, we calculate $V(R, k)$ for various values of the momentum k or alternatively of the relative energy $E_{c.m.} = 16\hbar^2k^2/m$ (m is the nucleon mass). At this point we need to specify the expression we have used for $\boldsymbol{j}$. According to ref. [14]) we write momentum density $\boldsymbol{j}$ in terms of the single-particle states (5)

$$\boldsymbol{j} = \frac{1}{2i}\sum_{\beta(j),\alpha(i)} B^{-1}_{\beta(j),\alpha(i)}(R, k)(\psi^*_{\alpha(i)}\nabla\psi_{\beta(j)} - \nabla\psi^*_{\alpha(i)}\psi_{\beta(j)}). \tag{20}$$

As for ρ and τ, the momentum density can be written as a sum of a direct term

$$\boldsymbol{j}_1 + \boldsymbol{j}_2 = k(\rho_1 - \rho_2), \tag{21}$$

plus a correction δj due to antisymmetrization. Here $|k|$ is given by eq. (6) and the direction k is parallel to the direction of the relative motion. If the nuclei do not overlap much or if k is large the correction δj is small. We expect that it is not important and have neglected it.

The result we have obtained for the interaction SII is shown in fig. 6. One can see that the potential becomes more and more attractive when $E_{c.m.}$ varies from 0 to 280 MeV. After this point the curve $V(R, k)$ starts to move in the opposite direction and even rises above $V(R, 0) = V_{EX}$, becoming entirely repulsive for energies larger than 2000 MeV. This repulsion comes both from the terms $\rho\tau$ and j^2. In fact the latter brings repulsion proportional to $E_{c.m.}$ at any energy. In the interval $E_{c.m.}$ = 0–280 MeV this repulsion decreases the energy dependence of the potential. In the interval 0–30 MeV the potential depth may be represented as $(10.3+\alpha E_{c.m.})$ MeV with $\alpha = 0.17$. In the same interval, the interaction SI gives a stronger energy dependence through a coefficient $\alpha = 0.21$. The difference in the energy dependence is due to the repulsion brought by the term j^2 which has a strength $3t_1 + 5t_2$ about seven times larger for SII than for SI.

The relative motion of two ^{16}O nuclei has been treated here in the same way as by Fliessbach [5]). Moreover, the total wave function of the combined system in ref. [5]) and the present paper are the same (up to an orthogonal transformation). The difference with Fliessbach's work consists in the choice of the two-body interaction. In using the B1 force [6]) Fliessbach finds an energy-dependent potential with a depth $V_0 + \alpha E_{c.m.}$ where $\alpha = 0.18$ when 15 MeV $< E_{c.m.} <$ 30 MeV. This value of α lies

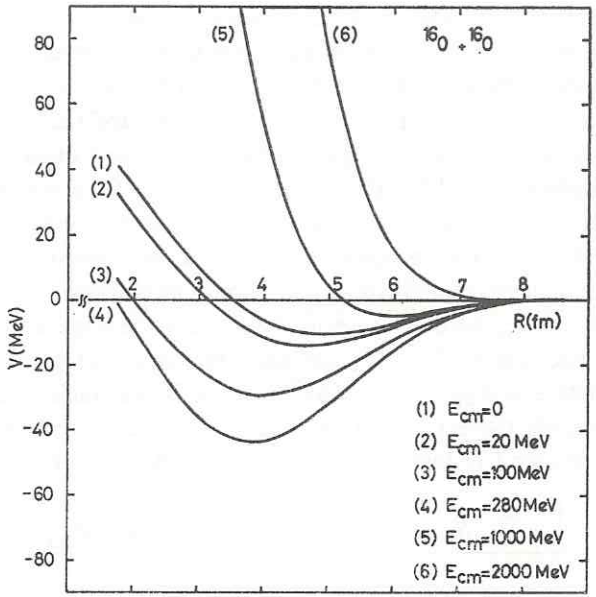

Fig. 6. The energy dependence of the interaction potential between two ^{16}O nuclei calculated with the Skyrme interaction SII. The results obtained for six different values of the relative energy ($E_{c.m.}$) are presented.

between the values of the linear coefficient which we have obtained with the Skyrme interactions SII and SI.

A large difference between Fliessbach and our results comes at high energies $E_{c.m.} > 280$ MeV. According to Fliessbach [5]) the B1 force gives a potential which becomes more and more attractive with the increase of the energy due to the fact that the Pauli principle which brings repulsion becomes less and less important. The Skyrme force brings an extra repulsive effect proportional to the relative kinetic energy which becomes dominant at higher energies and makes the potential entirely repulsive beyond about $E_{c.m.} = 2000$ MeV for SII.

5. Conclusions

We have calculated the interaction potential between two ^{16}O nuclei by using the energy functional of the Skyrme interaction and a two-centre oscillator potential to describe the ground state of the combined system. In such a model antisymmetrization can easily be taken into account and its effect has been shown explicitly. Due to antisymmetrization both the density and the kinetic energy density are modified in the Skyrme energy functional and these modifications give rise to exchange effects in the interaction potential. It has been found that a large part of the exchange effects comes from modification produced by antisymmetrization on the kinetic

energy density. Hence, a way of estimating the exchange effects due to antisymmetrization was suggested. Such an approximation could be used in order to avoid a large amount of numerical work. In a subsequent paper we shall show its practical value for the case where the nuclear densities of each of the two interacting nuclei are described by Hartree-Fock results [11]).

The relative motion of the nuclei has been described in a simple way which leads to an energy-dependent interaction potential. At all energies there is a repulsion linear in E, the energy of relative motion, which comes from the k^2 dependence of τ and j^2, eqs. (9), (21). This term has a coefficient proportional to $3t_1+5t_2$, and it makes the only contribution to the energy dependence at high energies. According to Dover and Van Giai [16]) the same combination of t_1 and t_2 determines the energy dependence of the nucleon-nucleus optical potential. In both cases the energy-dependent term is repulsive and is related to the same property of the nucleon-nucleon force.

There is an additional energy dependence at low energies which comes from the effect of antisymmetry on ρ and τ and which appears to have no analogue in the nucleon-nucleus optical potential. We found that this energy dependence comes mainly from the first and fourth terms in expression (3) for $H(\rho, \tau)$. With the interaction SII the antisymmetry effect is dominant up to $E \approx 280$ MeV ($kb = 1.14$) and makes the potential more attractive with increasing energy. For higher energies the repulsive term proportional to E becomes more important and for $E \approx 2000$ MeV there is no longer any attraction.

At zero relative energy the two Skyrme interactions SI and SII give almost identical potentials between two ^{16}O nuclei but the energy dependence of the two potentials is very different due to the fact that the parameter $3t_1+5t_2$ is seven times larger for SII than SI.

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