THE TENSOR PART OF SKYRME'S INTERACTION

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A tensor part is added to central and spin-orbit parts of Skyrme's effective interaction and its contribution to the Hartree–Fock spin-orbit coupling of spherical spin-unsaturated nuclei is analysed. Minor improvements to spin-orbit splittings can be achieved if the strengths of the tensor interaction are treated as free parameters, but the fit to experiment deteriorates if the tensor interaction contribution is estimated for a realistic interaction.

Several authors [1–6] have pointed out the important effect of the tensor part of the nucleon-nucleon effective interaction on the spin-orbit splitting of the Hartree–Fock single-particle spectrum.

In spin-unsaturated nuclei the tensor force or the exchange part of the central force may affect the spin-orbit splitting obtained from the two-body spin orbit force considerably [4]. Shell model calculations for nuclei in the $A = 50$ region support these conclusions [7]. In spite of these indications of the importance of a tensor term in the interaction there is no systematic study on its influence on properties of spin-unsaturated nuclei. The reason probably comes from the numerical difficulties involved in calculations using a tensor force. However, in the case of Skyrme's interaction, including a tensor term does not make the calculations more difficult, at least not for spherical nuclei. Such terms were included in the original paper of Skyrme [8]. In earlier studies of the properties of Skyrme's interaction they were suppressed in order to restrict the number of free parameters. In view of the good results obtained with this restricted parametrization it is interesting to see if improvements can be achieved by including tensor terms.

In this letter we use the set of parameters SIII determined in ref. [9] as a starting point and study the influence of an additional tensor term on the Hartree–Fock results for single particle energies.

Using the notations of refs. [4,8] we consider the following tensor interaction v_T in configuration space

$$
\begin{aligned}
v_T = \tfrac{1}{2} T \{ & [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}')(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}') - \tfrac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) k'^2] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \\
& + \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) - \tfrac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) k^2] \} \\
& + U \{ (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}') \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) \\
& - \tfrac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) [\boldsymbol{k}' \cdot \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \boldsymbol{k}] \} ,
\end{aligned} \tag{1}
$$

where the operator $\boldsymbol{k} = (\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)/2\mathrm{i}$ acts on the right and $\boldsymbol{k}' = -(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)/2\mathrm{i}$ on the left. The term proportional to T acts in even states of relative motion and contributes to mixing of relative states S and D, while the term proportional to U acts in P states of relative motion. The strengths T and U of the even and odd parts of the tensor interaction can be treated as free parameters. They can also be estimated from the tensor part of a nucleon-nucleon interaction.

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Table 1
Parameters α and β from eqs. (10) and (12) calculated with the renormalized interaction G–O of Sprung [5] and Sprung and Banerjee [12] ($K_F = 1.36\ \mathrm{fm}^{-1}$); the units are MeV. fm^5.

q (fm^{-1})	α_T	β_T	α_c	β_c
0.7	134.76	238.20	121	23.73
1.0	67.73	115	86.66	24.91

In a short range limit we have

$$T = -\frac{8\pi}{5}\int_0^\infty r^4 V_T^{(e)}(r)\,\mathrm{d}r\,, \tag{2}$$

$$U = \frac{8\pi}{5}\int_0^\infty r^4 V_T^{(o)}(r)\,\mathrm{d}r\,, \tag{3}$$

where $V_T^{(e)}(r)S_{12}$ and $V_T^{(o)}(r)S_{12}$ are the tensor potentials in even and odd states of relative motion and $S_{12} = (3/r^2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r}) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. Tensor potentials such as the one given by Sprung [5] are not of short enough range to justify using the short range limit. Estimates taking finite range into account can be made by a method analogous to the one given by Scheerbaum [6] for spin-orbit forces. This method de-emphasizes the role of the long range part of the interaction. The expressions we use are

$$T = -\frac{24\pi}{q^2}\int_0^\infty j_2(qr)\,V_T^{(e)}(r)\,j_0(qr)\,\mathrm{d}r\,, \tag{4}$$

$$U = \frac{72\pi}{5q^2}\int_0^\infty j_1(qr)\,V_T^{(o)}(r)\,j_1(qr)\,\mathrm{d}r\,. \tag{5}$$

In the limit $q \to 0$ eqs. (4) and (5) reduce to (2) and (3). Scheerbaum argued that q should be a typical relative momentum of a pair collision in the nucleus. He chose $q = 0.7\ \mathrm{fm}^{-1}$. Table 1 gives the combinations α_T and β_T (eq. (10)) of T and U calculated from eqs. (4), (5) with Sprung's effective interaction [5], taking $q = 0.7\ \mathrm{fm}^{-1}$ and $q = 1.0\ \mathrm{fm}^{-1}$.

To write down the contribution of the tensor interaction to the binding energy and spin-orbit splitting of spherical nuclei we introduce the quantity $J_q(r)$ called spin density [4].

$$J_q(r) = \frac{1}{4\pi r^3}\sum_\alpha (2j_\alpha+1)\,[j_\alpha(J_\alpha+1) - l_\alpha(l_\alpha+1) - \tfrac{3}{4}]R_\alpha^2(r)\,, \tag{6}$$

where $\alpha = \mathrm{n, l, j}$ runs over all occupied neutron ($\mathrm{q=n}$) or proton ($\mathrm{q=p}$) states described by the single particle wave functions $R_\alpha(r)$.

Besides the contribution from the tensor interactions we also include an additional contribution from the exchange part of the central interaction and ignored previously for various reasons [4, 9, 10]. These two contributions amount to a change ΔH in the energy density

$$\Delta H = \tfrac{1}{2}\alpha(J_n^2 + J_p^2) + \beta J_n J_p\,, \tag{7}$$

and a change $\Delta W_q(\mathrm{q = n, p})$ in the spin-orbit potential W_q of ref. [4]

$$\Delta W_n = \alpha J_n + \beta J_p\,, \tag{8}$$

$$\Delta W_p = \alpha J_p + \beta J_n\,, \tag{9}$$

where $\alpha = \alpha_T + \alpha_c$ and $\beta = \beta_T + \beta_c$. The tensor contribution is

$$\alpha_T = \tfrac{5}{12}U;\qquad \beta_T = \tfrac{5}{24}(T+U)\,, \tag{10}$$

and it is derived in appendix I of ref. [11] in the general case where no special symmetry is assumed. For Skyrme's interaction the central contribution is

$$\begin{aligned}\alpha_c &= \tfrac{1}{8}(t_1 - t_2) - \tfrac{1}{8}(t_1x_1 + t_2x_2)\,,\\ \beta_c &= -\tfrac{1}{8}(t_1x_1 + t_2x_2)\,,\end{aligned} \tag{11}$$

where the parameters t_1, t_2, x_1 and x_2 are defined in ref. [8]. These central force contributions have been neglected in previous calculations. In the set of parameters SIII, which we use in the present work, the parameters x_1 and x_2 are zero, hence $\beta_c = 0$ and therefore the central part of the interaction brings contributions through α_c only. To have a more general idea about the relative contribution of the central and tensor interactions to the spin-orbit coupling we estimate the values of α_c and β_c (table 1) corresponding to the interaction of Sprung and Banerjee [12] by a method similar to that used for T and U,

$$\begin{aligned}\alpha_c &= -\frac{\pi}{6q^2}\int_0^\infty [3(1-j_0^2(qr))^1\mathrm{even} + qj_1^2(qr)^3\mathrm{odd}]r^2\,\mathrm{d}r\,,\\ \beta_c &= \frac{\pi}{12q^2}\int_0^\infty [3(1-j_0^2(qr))(^3\mathrm{even} - {}^1\mathrm{even})\\ &\quad + qj_1^2(qr)(^1\mathrm{odd} - {}^3\mathrm{odd})]\,r^2\,\mathrm{d}r\,,\end{aligned} \tag{12}$$

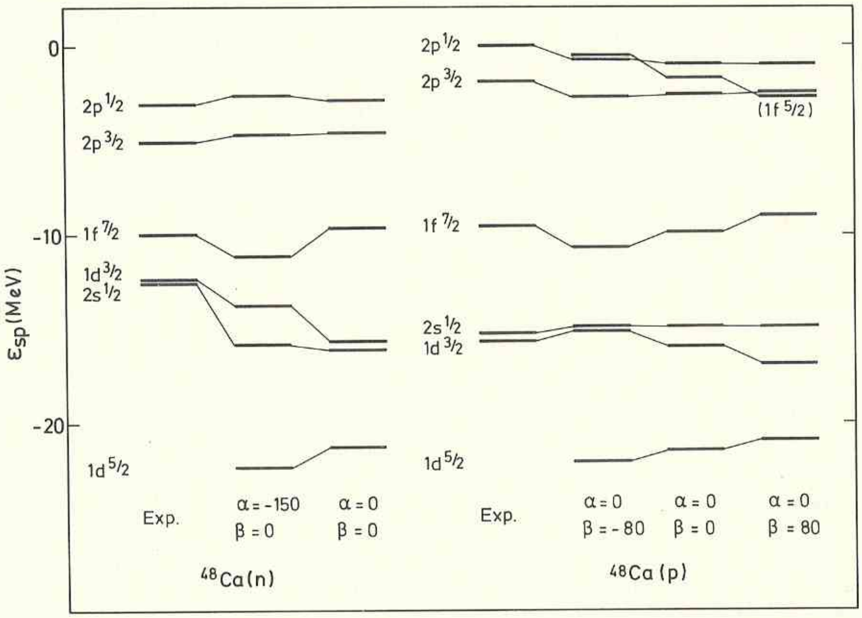

Fig. 1. Neutron and proton levels of ^{48}Ca for several values of α and β. Experimentally known single particle energies are also indicated (ref. [14]).

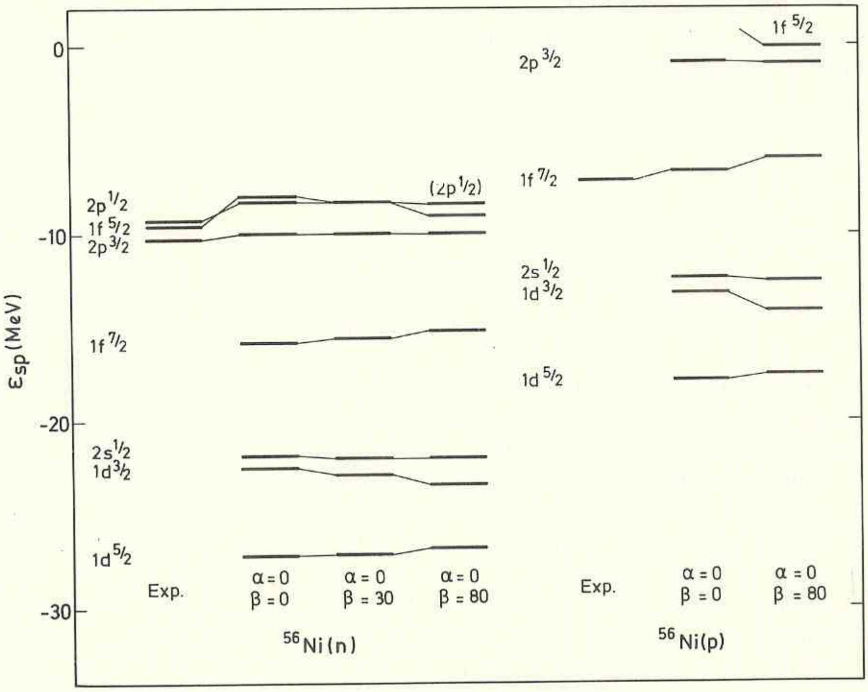

Fig. 2. Same as fig. 1 but for ^{56}Ni.

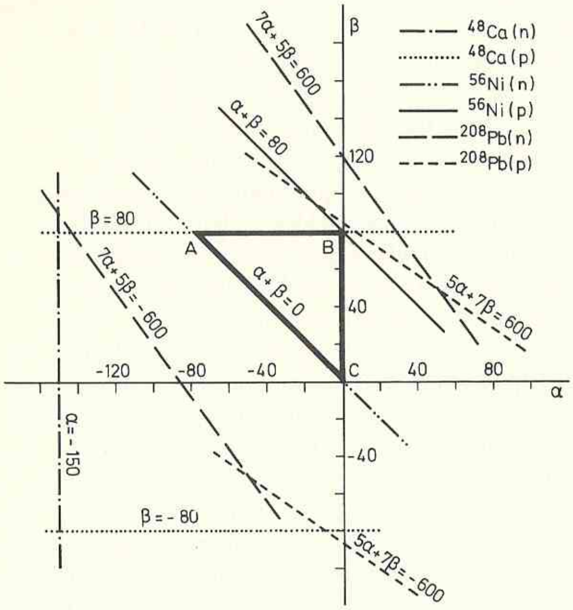

Fig. 3. Diagrams indicating limitations on α and β obtained from the single particle spectra of ^{48}Ca, ^{56}Ni and ^{208}Pb. The triangle ABC is the domain (α, β) compatible with all single particle spectra (see text).

where we follow the notations of ref. [12] (^{3}even stands for the triplet even part of the central potential and so on). We should point out that this is only one possible way to make a short range approximation to a finite range interaction. Another quite different approach was used by Treiner and Krivine [13] to obtain an equivalent Skyrme-type interaction from the interaction G–O of ref. [12]. These authors calculate only the central part of the effective interaction and use a phenomenological form for the spin orbit part.

For a spin saturated nucleus as e.g., ^{40}Ca the quantities ΔH and ΔW_q are negligible because, as one can see from eq. (2), $J_q \simeq 0$. For a spin unsaturated nucleus the lower member of a spin-orbit doublet is filled before the upper member and this implies $J_q > 0$. For example the nucleus ^{48}Ca where only the neutron $f_{7/2}$ level is filled has $J_p \simeq 0, J_n > 0$. In addition J_q is surface peaked because the main contribution in eq. (2) comes from the most external wave function (the largest l in a shell).

The Hartree–Fock solutions minimizing the total energy defined in ref. [4] plus the additional term $\int \Delta H \, d^3r$ have been studied as a function of the parameters α and β for the nuclei ^{48}Ca, ^{56}Ni, ^{90}Zr and ^{208}Pb. As we mentioned already, the parameter set III was chosen for the central and spin-orbit part of the interaction. The Coulomb potential and the centre of mass corrections were included as in ref. [9].

For each nucleus we were looking for values of α and β which might improve the single particle spectra found in ref. [9] ($\alpha = 0, \beta = 0$). We found a series of restrictions on α and β which produced spectra comparable with or better than the spectra obtained with $\alpha = \beta = 0$. These restrictions are:

i) $-150 \lesssim \alpha \lesssim 0$: The neutron spectrum of ^{48}Ca can be modified only if $\alpha \neq 0$. An improvement is achieved for $\alpha < 0$ because this increases the spin-orbit splitting for neutron levels, raising the $d_{3/2}$ level and depressing the $f_{7/2}$ level (c.f. fig. 1). A very large negative value of α would depress the $f_{7/2}$ level too much. A compromise is achieved for $\alpha = -150$ MeV fm^{-5} when both the levels $1d_{3/2}$ and $1f_{7/2}$ deviate by ~ 1.3 MeV from the experimental result. The above range of values of α also produce an improvement in both the occupied and unoccupied neutron single particle levels of ^{90}Zr.

ii) $-80 \lesssim \beta \lesssim 80$: A non zero value of β modifies the proton spectrum of ^{48}Ca. The proton spectrum is quite well reproduced with $\beta = 0$. Values in the range $-80 < \beta < 80$ gives single particle energies deviating from experimental values by at most 1.2 MeV.

iii) $0 \lesssim \alpha + \beta \lesssim 80$: As $J_n \simeq J_p$ for ^{56}Ni the change in both neutron and proton spin-orbit potentials is approximately the same and proportional to $\alpha + \beta$. When $\alpha + \beta < 0$ the spin-orbit splitting is too large so that the $1\,f_{5/2}(n)$ level is too high and lies above the $2p_{1/2}(n)$ levels. When $\alpha + \beta > 0$ the spin-orbit splitting is reduced and the $1\,f_{5/2}(n)$ level moves towards its experimental value. It lies below the $2p_{1/2}(n)$ level for $\alpha + \beta > 30$. An upper limit for $\alpha + \beta$ comes from the proton levels. The $1f_{7/2}(p)$ level is within 1 MeV of the experimental value if $\alpha + \beta < 80$ (c.f. fig. 2).

iv) $-600 \lesssim 7\alpha + 5\beta \lesssim 600$ and $-600 \lesssim 5\alpha + 7\beta \lesssim 600$. These restrictions come from the neutron and proton spectra of ^{208}Pb respectively. In this nucleus due to the similarity between the radial distributions of the $i_{13/2}(n)$ and $h_{11/2}(p)$ levels one expects the contributions ΔW_n and ΔW_p to be proportional to $7\alpha + 5\beta$ and $5\alpha + 7\beta$ respectively. These two contributions produce modifications mainly of the 1h(p), 1h(n), 1i(n) and 1j(n) levels and in general do not significantly improve any level without adversly perturbing others.

The above restrictions on $7\alpha + 5\beta$ and $5\alpha + 7\beta$ give roughly the same quality spectrum as $\alpha = \beta = 0$. Perhaps it is not necessary to insist too much on the agreement with the experiment [14] for ^{208}Pb because as was pointed out in ref. [15] levels of this nucleus might be perturbed significantly by collective effects. All the inequalities mentioned above are satisfied in a triangle A(−80, 80), B(0, 80), C(0, 0) in the plane α, β of fig. 3. From this triangle one can extract values for the tensor interaction strengths T and U compatible with the single particle spectra. We point out that the limits on α and β are somewhat arbitrary and the allowed region in fig. 3 can be extended by permitting a larger discrepancy with respect to experimental single particle energies.

The change in the total binding energy due to terms depending on α and β is given by the volume integral of eq. (7). For the values of α and β from the triangle ABC in fig. 3 this change in binding energy of spin unsaturated nuclei can almost be compensated by a small change in the parameter x_0 of Skyrme's interaction to give good agreement with experimental values.

Our values of α and β can be compared with those estimated from eqs. (4, 5, 10) for the tensor interaction of Sprung [5] and from eq. (12) for the central interaction of Sprung and Banerjee [12]. These effective interactions are obtained from Reid's soft core potential which is fitted to nucleon-nucleon scattering data.

The values of α and β given in table 1 for two choices of the parameter q correspond to the G–O interaction renormalized as in table 5 of ref. [12]. The result is sensitive to q but for both q values the parameters α and β are several times larger than those resulting from our empirical analyses and indicated in fig. 3. Moreover the values of α in the allowed region of fig. 3 are negative while those in table 1 are positive. The effect of positive α and β is to reduce the spin-orbit splitting in spin unsaturated nuclei. The values of α and β for $q = 1\ \mathrm{fm}^{-1}$ from table 1 would reduce the spin-orbit splittings in ^{208}Pb by almost a factor of 2. About 1/3 of this reduction is due to the central interaction and the remaining 2/3 to the tensor interaction. Such a reduction is in agreement with results of other Hartree–Fock calculations [1, 2].

In the present theory the contributions of tensor forces and the exchange part of central forces to spin-orbit splittings in spin unsaturated nuclei are collected in the coefficients α and β of eqs. (7)–(9). We draw two main conclusions from our calculations:

1) If the coefficients α and β are treated as free parameters then the additional terms (8) and (9) in the spin-orbit potential can give minor improvements to single-particle energies in spin-unsaturated closed shell nuclei. The results presented here relate to the SIII interaction, but similar improvements can be achieved if other parameter sets are used for the central and spin orbit parts of the interaction.

2) If the coefficients α and β are estimated from the Sprung and Sprung–Banerjee interactions then the fit to single particle energies deteriorates markedly and the spin-orbit splittings in spin-unsaturated nuclei become too small. This effect could be compensated by increasing the strength W_0 of the nucleon-nucleon spin-orbit interaction [4] but then spin-orbit splittings in light spin-saturated nuclei (^{16}O and ^{40}Ca) would be too large. Other authors [1, 2] using realistic effective interactions have also found spin-orbit splittings which are too small in spin-unsaturated nuclei. As these authors do not make any short range approximation there is an indication that our result is not due to the short range form (1) which is used here for the tensor force. The results suggest that either there is something wrong with the tensor force in some effective interactions derived from nucleon-nucleon scattering data, or that there is a structure effect which produces a quenching of the tensor interaction in spin-unsaturated nuclei.

[1] C.W. Wong, Nucl. Phys. A108 (1968) 481.
[3] R.M. Tarbutton and K.T.R. Davies, Nucl. Phys. A120 (1968) 1.
[3] J.W. Negele, Phys. Rev. C1 (1970) 1260.
[4] D. Vautherin and D.M. Brink, Phys. Rev. C5 (1972) 626.
[5] D.W.L. Sprung, Nucl. Phys. A182 (1972) 97.
[6] R.R. Scheerbaum, Nucl. Phys. A257 (1976) 77; Phys. Lett. 61B (1976) 151.
[7] H. Horie and K. Ogawa, Progs. Theor. Phys. 46 (1971) 439.
[8] T.H.R. Skyrme, Nucl. Phys. 9 (1959) 615.
[9] M. Beiner et al., Nucl. Phys. A238 (1975) 29.
[10] D. Vautherin, Phys. Rev. C7 (1973) 6.
[11] H. Flocard, thesis Orsay 1975.
[12] D.W.L. Sprung and P.K. Banerjee, Nucl. Phys. A168 (1971) 273.
[13] J. Treiner and H. Krivine, J. Phys. G2 (1976) 285.
[14] A. Bohr and B. Mottelson, Nuclear structure (Benjamin, New York, 1969) Vol. 1.
[15] I. Hamamato and P. Siemens, preprint Nordita-75/13.