STATIC POLARIZATION EFFECTS IN THE NUCLEUS-NUCLEUS POTENTIAL

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ABSTRACT. We study a simple way to relax the frozen configuration assumption in the calculation of the real part of the nucleus-nucleus potential. The proposed model allows the thickness of the surface layer of each nucleus to vary from the outer side to the interacting side giving rise to polarized shapes which simulate a neck formation. The result is compared to other potentials available in the literature.

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The calculation of the nucleus-nucleus potential starting from an energy-density formalism is very convenient when simplifying assumptions about the density of the composite system are made. A common starting point is the frozen configuration assumption or the sudden approximation according to which the density of the whole system is taken as the sum of the asymptotic individual densities at any separation distance between the interacting nuclei. Such a simplification allows in practice to calculate the interaction potential between any pair of nuclei once the densities are known.

In this letter we propose a way to relax the frozen configuration assumption by trying to maintain the simplicity of the calculations. Under mutual influence the nuclei deviate from their spherical shape and we try to describe this polarization effect by a gradual change of the surface layer thickness from an equilibrium value \( a_o \) at the outer side to a value \( a \) at the interacting side of each nucleus. This value is found by minimizing the potential with respect to \( a \) at each separation distance \( s_o \) between the surfaces

\[
s_o = R - R_1 - R_2 \quad ,
\]

where \( R \) is the distance between the centres of the nuclei and \( R_i \) \((i = 1, 2)\) the nuclear radii. In this way we are extending the formalism of Ref. 4 according to which we define the nucleus-nucleus potential \( \tilde{V}(s_o, a) \) as the difference between the binding energy of the composite system
at \( a_0 \) and the binding energy of the nuclei separated at infinity. The Skyrme interaction density functional\(^6\) is used to calculate the binding energies. For the composite system the density is taken as the sum of the individual densities and the kinetic energy densities of the composite system and of the individual nuclei are described by the Thomas-Fermi approximation. The advantage of these approximations is that the Skyrme interaction energy density \( H \) reduces to an algebraic function of the densities \( \rho_1 \) and \( \rho_2 \) only. Then the potential can be written as the volume integral

\[
\hat{\mathcal{V}}(a_o,a) = \int [H(\rho_1^a + \rho_2^a) - H(\rho_1^{a_0}) - H(\rho_2^{a_0})] dv , \tag{2}
\]

The parameter \( a_0 \) is the equilibrium value of the surface layer thickness for an individual nucleus. As Skyrme interaction parameters we have chosen the set SII \(^6\) which gives for a semi-infinite slab \( a_0 = .48 \text{ fm} \) and \( E_{\text{surf}} = 22.38 \text{ MeV/fm}^2 \) when the Thomas-Fermi approximation is used for the kinetic energy density as defined in Ref. 6. At infinity the densities \( \rho_1 \) and \( \rho_2 \) are described by Fermi-type distributions with parameters which reproduce the Hartree-Fock r.m.s. radii of spherical nuclei \(^6\) and describe well the surface region as explained in Ref. 4. The value \( a_0 = .48 \text{ fm} \) seems to be a good choice also for the surface thickness of finite nuclei as \(^40\text{Ca}\) or heavier. At a finite separation distance the thickness parameter is made to vary continuously as a function of the polar angle \( \theta \) defined with
respect to a z-axis passing through the centres of the nuclei. For simplicity this variation is supposed to be equal for both nuclei and have the form

\[ a = a_0 (1 + b_0 \cos^2 \frac{\theta}{2}) \quad \text{and} \quad a = a_0 (1 + b_0 \sin^2 \frac{\theta}{2}) \],

(3)

for the left and right nucleus respectively where \( b_0 \) is a step variation parameter. The nucleus \( {}^{160}O \) does not fit well into this description being too small. We have considered a selection of pairs of nuclei in order to cover a large interval of values for \( Z_1 Z_2 \) and \( A_1^{1/3} + A_2^{1/3} \). These are: \( {}^{40}Ca + {}^{40}Ca \), \( {}^{40}Ca + {}^{90}Zr \), \( {}^{48}Ca + {}^{208}Pb \), \( {}^{56}Ni + {}^{208}Pb \), \( {}^{90}Zr + {}^{90}Zr \) and \( {}^{90}Zr + {}^{208}Pb \). Some preliminary calculations have been presented elsewhere.\(^7\)

In the barrier region Figs. 1 and 2 compare the results given by crosses of the polarization effect with the results based on the frozen configuration assumption represented by points each connected by a line with the corresponding cross. Fig. 1 shows the position barrier \( r_B = R_B/(A_1^{1/3} + A_2^{1/3}) \) as a function of \( Z_1 Z_2/(A_1^{1/3} + A_2^{1/3}) \) and Fig. 2 gives the value of the nuclear potential at \( r_B \). One can see that for the limiting cases the polarization effect consists either in shifting the position barrier towards larger values (\( \sim .3 \) fm for \( {}^{40}Ca + {}^{40}Ca \)) or in decreasing the potential at the barrier (about 10% for \( Zr + {}^{208}Pb \)).

A much larger influence of the polarization of the interacting nuclei can be seen at negative separation distances \( s_0 \) between the nuclear surfaces. Typical cases
are those presented in Figs. 4 and 5, where the interaction potential is given as a function of \( s_0 \). For \(^{40}\text{Ca} + ^{40}\text{Ca}\) the potential \( \tilde{V} \) of eqs. (2) and (3) reaches a minimum at \( a = 0.73 \, \text{fm} \) for \( s_0 = 0. \, \text{fm} \), at \( a = 1.40 \, \text{fm} \) for \( s_0 = -1. \, \text{fm} \) and at \( a = 1.66 \, \text{fm} \) for \( s_0 = -2. \, \text{fm} \). These polarized shapes bring a large amount of attraction producing an almost linear steep decrease of the potential as a function of \( s_0 \). It contrasts with the repulsive core of the potential \( V \) based on the frozen configuration assumption and calculated from eq. (2) with \( a = a_0 \). Potentials with a shape similar to \( \tilde{V} \) have been introduced by Randrup as modified proximity potentials. The justification lies on geometrical grounds and the extra attraction included at negative \( s_0 \) expresses the disappearance of a part of the nuclear surfaces and of their interaction after the nuclei touch. Our result represents a more microscopical estimate of the contribution brought by a neck formation.

In fact our calculations seem to indicate that the presence of a neck manifests differently when one of the partners is very heavy. This is illustrated in Fig. 4 for the system \(^{48}\text{Ca} + ^{208}\text{Pb}\). In this case the nuclei polarize less, with \( a = 0.55 \, \text{fm} \) at \( s_0 = 0. \, \text{fm} \), \( a = 0.62 \, \text{fm} \) at \( s_0 = -1. \, \text{fm} \) and \( a = 0.90 \, \text{fm} \) at \( s_0 = -2. \, \text{fm} \) producing a potential more similar in shape with a Woods-Saxon potential, as e.g. that proposed by Siwek-Wilczyńska and Wilczyński. It seems that a saturation limit is attained for \(-2.5 < s_0 < -1.0\), probably because the change in the central density of
$^{208}\text{Pb}$ is still small (typical leptodermous system). For $s_0 < -2.5$ the polarization becomes more important ($a = 1.61$ fm at $s_0 = -3.5$ fm) and the attraction seems to increase again.

As far as the validity of the calculated potentials is concerned we expect a better agreement with the elastic cross section data than in previous calculations\textsuperscript{4,8} where the potential was not deep enough at the strong absorption radius. For $^{40}\text{Ca} + ^{40}\text{Ca}$ the elastic scattering data can be fitted with a potential which at the strong absorption radius of 10.63 fm is $-1.3$ MeV.\textsuperscript{11} In the present calculations at this separation distance the polarization effect doubles the potential from $V = -0.72$ MeV to $\tilde{V} = -1.14$ MeV and therefore goes in the direction indicated by experiment.

Figs. 1 and 2 can be used to find by extrapolation values of the barrier $V_B$ and its position $R_B$ for an arbitrary pair. As an example we considered the case of $^{35}\text{Cl} + ^{116}\text{Sn}$ for which we found $R_B = 11$ fm and $V_B = 104$ MeV. To check this result in table 1, column 5, we show values of the fusion cross section $\sigma_F$ calculated with the classical formula

$$\sigma_F(E_{\text{c.m.}}) = \pi R_B^2 (1 - V_B / E_{\text{c.m.}})$$

(4)

to be compared with the experiment, column 6. We also give $\sigma_F$ calculated from eq. (4) for other available potentials ($R_B$ and $V_B$ are indicated). The second column contains results for the proximity potential\textsuperscript{3} for which one of the
parameters was slightly modified in Ref. 12 to obtain a better fit (column 3). In column 4 the fusion cross section with the Siwek-Wilczyńska and Wilczyński potential is given. As these authors found, their potential should be renormalized by a factor $N < 1$ for achieving a good agreement with the experiment. In our case too, a slight reduction of the attraction would be necessary.

In conclusion we consider that the small polarization effect we have obtained at $s_o > 0$ reinforces the validity of the proximity concept as introduced in Ref. 3 for the nucleus-nucleus interaction. For $s_o < 0$ we obtain potential shapes very different from those of Ref. 3. As Huizenga et al. have pointed out recently, the fusion excitation function of systems around $A_1 \sim 40$, $A_2 \sim 120$ is sensitive to the form of the nuclear potential at negative separation distances and experimental fusion results at higher energies might possibly distinguish between different potentials.

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REFERENCES


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a) \( V_B = 107.09 \) MeV , \( R_B = 10.62 \) fm ; b) \( V_B = 105.14 \) MeV , \( R_B = 10.84 \) fm

c) \( V_B = 98.79 \) MeV , \( R_B = 11.05 \) fm ; d) \( V_B = 104 \) MeV , \( R_B = 11 \) fm.
TABLE CAPTIONS

Table 1. Comparison of measured values of fusion cross section for $^{35}\text{Cl} + ^{116}\text{Sn}$ with those calculated from different potentials with eq. (4).
FIGURE CAPTIONS

Fig. 1. The barrier position $r_B = R_B/(A_1^{1/3} + A_2^{1/3})$ as a function of $Z_1Z_2/(A_1^{1/3} + A_2^{1/3})$.

Fig. 2. The nuclear potential at the barrier as a function of $r_B$.

Fig. 3. The nuclear interaction potential for $^{40}\text{Ca} + ^{40}\text{Ca}$ as a function of the separation distance $s_0$ between the surfaces. The full and dashed lines are the potential with and without polarization effect, respectively.

Fig. 4. Same as Fig. 3, but for $^{48}\text{Ca} + ^{208}\text{Pb}$.
Fig. 1

\[ r_B = \frac{Z_1 Z_2}{(A_1^{1/3} + A_2^{1/3})} \]