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THE NUCLEON-NUCLEON INTERACTION

AND

THE QUARK MODELS

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Abstract.

This talk refers to hadronic systems described in terms of quarks and gluons or flux-tubes. We discuss problems which arise in the derivation of the nucleon-nucleon interaction from one-gluon exchange. Particular attention is paid to the classification scheme of six-quark states. We propose a scheme based on molecular-orbitals and valid both for non-relativistic and relativistic quark models.

1. Introduction.

Since the nucleon is now considered to be a composite object there have been many attempts to derive the nucleon-nucleon (NN) interaction from the quark-quark interaction in a system of six quarks. The present talk refers both to non-relativistic (constituent quark) or relativistic (bag) models. A common feature of all these models is the confinement of quarks. In the non-relativistic models the confinement is achieved by the spin-independent interaction potential between quarks,[1-4] in the MIT bag[5] by appropriate boundary conditions and in the soliton bag[6] by an additional field or through the chromo-electric field.[7] Each model is successful in describing a range of properties of baryons. In details, however, there are problems in each of them.

The concept of quarks as fundamental particles has implications on understanding the nucleon-nucleon interaction as well.

In the long and medium range, on the basis of chiral symmetry, attempts have been made to generate the one-pion exchange potential. In other words one aims at deriving the pion-nucleon coupling constant. For a review, see Ref. 8.

The short range studies are concentrated on the nature of the repulsive core. The treatment is analog to molecular forces i.e. are based on the quark structure of nucleons. In the following we shall discuss only the short range aspect of nuclear forces.

By short distance we understand $R \leq 1$ fm. This is about twice the quark core radius of a nucleon. Many studies agree that a nucleon has a quark core surrounded by a pion cloud.

Most of the calculations of the nucleon-nucleon interactions at short distance have been made in the non-relativistic quark models. For a review, see Ref. 9. The non-relativistic models are appropriate for scattering through the resonating group method (RGM) but inherent van der Waals forces appear due to the two-body confinement interaction.

There are only few attempts in which relativistic models have been used. One is the work of de Tar[10] based on the MIT bag model and the Born-Oppenheimer approximation. This has been extended to the scattering problem through the P-matrix approach.[11] An attempt to solve the bag-bag scattering problem by RGM has been made in Ref. 12. More recently the soliton bag model has been used[13] to derive a local effective nucleon-nucleon interaction in the generator coordinate method (GCM). To simplify the calculations the quark-gluon exchange has been neglected.

In general the major problems to be settled are related to the dynamics of quarks and the symmetries of the six-quark states. Without loosing generality in the following we shall illustrate the approach to NN forces by using a non-relativistic model.

2. The dynamics

In semi-relativistic or non-relativistic quark-models a system of N quarks is described by a hamiltonian of the form :

$$H = T + V(r_1, r_2, \ldots r_N) + E_0 \qquad (2.1)$$

where T is the intrinsic kinetic energy, E_0 a constant to be determined from data[3] and V the total interaction between quarks. It can contain two- or N-body terms with N > 2. In the case of baryons, a three-body force has been introduced in Ref. 3 on the basis of a QCD-inspired flux-tube model. It contributes few percent to the nucleon binding energy. Six-quark forces would appear at much higher energies.[14] Therefore a good approximation to V is

$$V = \sum_{i<j}^{N} v_{ij} \qquad (2.2)$$

where the quark-quark interaction v_{ij} is the sum of a spin-independent v_{ij}^{c} and a spin-dependent v_{ij}^{hyp} terms respectively. The spin-independent term represents the color-electric field. Lattice gauge calculations for a heavy quark-antiquark pair[15] give v_{ij}^{c} as a Coulomb type + a linearly confining potential. The $SU_C(3)$ flux-tube dynamics[3] leads to the same behaviour. Taking into account the color factor v_{ij}^{c} takes the form :

$$v_{ij}^{c} = -\frac{2}{3}\frac{\alpha_s}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2}\sqrt{\sigma}|\vec{r}_i - \vec{r}_j| \qquad (2.3)$$

where α_s is the strong coupling constant and $\sqrt{\sigma}$ the string tension constant. Harvey[16] has found that such a potential can be fitted with a functional form of the type

$$v_{ij}^{c} = -\frac{2}{3}\left[A \exp\left(-\frac{r_{ij}^2}{\alpha^2}\right) + B\, r_{ij}^2 + C\right] \qquad (2.4)$$

where $-\frac{2}{3}$ represents the color factor of two quarks in an anti-symmetric state. The parameters A, B, C and α have been fitted with data extracted from N and Δ spectra. For practical purposes this is a convenient approximation in NN studies.

The spin-dependent interaction v_{ij}^{hyp} represents the magnetic field through one gluon exchange[1] and can be written as a sum of a spin-spin interaction v_{ij}^{σ} and a tensor term v_{ij}^{T}

$$v_{ij}^{hyp} = v_{ij}^{\sigma} + v_{ij}^{T} \qquad (2.5)$$

The one gluon exchange also predicts a spin-orbit force but this is usually neglected in baryon spectroscopy, as explained in Ref. 2. As in v_{ij}^{hyp} the major contribution comes from v_{ij}^{σ}. V_T can also be neglected in a first approximation. A simple form for v_{ij}^{σ} is the contact force so one can take

$$v_{ij}^{hyp} \approx v_{ij}^{\sigma} = D\, \delta(r_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j \qquad (2.6)$$

with D a parameter to be fitted from baryon spectra. In the following section we shall use

$$A = 0, \quad B = -621 \text{ MeV fm}^{-2}, \quad C = 776 \text{ MeV},$$
$$\alpha = 0.2737 \text{ fm}, \quad D = -109.5 \text{ MeV fm}^3 \qquad (2.7)$$

selected among the sets found in Ref. 17.

Finally, the kinetic energy T can be taken in a relativistic or a non-relativistic form. Harvey[16] has shown that results for the NN interaction in the Born-Oppenheimer approximation obtained with a relativistic kinematics, which is more realistic for light quarks, are very close to those given by a non-relativistic kinematics and properly chosen parameters A, B, C, D and α. For the purposes explained in the following section a non-relativistic T is satisfactory so we shall take

$$T = 6\,m\,c^2 + \frac{1}{12m}\sum_{i<j}(\vec{p}_i - \vec{p}_j) \qquad (2.8)$$

where $\vec{p}_i$ is the momentum of the quark i and for the quark mass m we shall take[17]

$$m = 362\ \text{MeV} \quad . \qquad (2.9)$$

The results given in the next section are relative to two nucleon masses where each is

$$M_N = 940\ \text{MeV} \quad . \qquad (2.10)$$

3. The classification scheme

The classification and construction of six-quark basis states is a central problem.[18] In principle the choice of the basis is arbitrary and irrelevant if sufficient states are included in the calculations. In practice one has to choose a finite set and therefore this must contains the most important ones.

To construct six-quark totally antisymmetric states such as to incorporate all degrees of freedom - space, color, isospin and spin - two classification schemes have been adopted until now. One is the SU(6) color-spin (CS) scheme[9,19] the other is the SU(4) isospin-spin (TS) scheme.[18] In the CS scheme first one couples the color singlet $[222]_C$ state and the spin state. The result is then coupled to the isospin state in order to obtain a state with the (dual) permutation symmetry requested by the orbital part of the wave function such as to have in all a totally antisymmetric state $[1^6]$. For example a T = 0, S = 1 totally antisymmetric state obtained from the orbital (O) symmetry $[42]_O$ reads :

$$\Psi^{CS}_{[42]} = \{[42]_O \times \{\{[42]_S \times [222]_C\}_{[f]} \times [33]_T\}_{[1^6]} \qquad (3.1)$$

where [f] stands for the intermediate $SU(6)_{CS}$ representation, $[42]_S$ for the S = 1 $SU(2)_S$ representation and $[33]_T$ for the T = 0 $SU(2)_T$ representation.

In the SU(4) scheme one couples the color singlet $[222]_C$ and the orbital state to a state of permutation symmetry[f"] which has to be coupled to the dual SU(4) representation $[\tilde{f}"]$ in order to give a totally antisymmetric state. For the $[42]_O$ symmetry one has

$$\Psi^{TS}_{[42]} = \{\{[42]_O \times [222]_C\}_{[f"]} \times [\tilde{f}"]\}_{[1^6]} \qquad (3.2)$$

As each classification scheme has its advantages the transformation from one to the other is now under study. In the following we shall use the scheme $SU(4)_{TS}$.

Concerning the orbital space, in most of the calculations made until now the cluster model states have been used. For a review, see Ref. 8. The two clusters, representing the separate nucleons are centered at ± 1/2 Z, where Z is the separation coordinate between two harmonic oscillator wells.

Recently,[20] we have proposed a classification scheme based on molecular orbitals instead of cluster model states. The molecular orbitals are states of definite parity, in contrast to cluster model states. They are natural to mean field or a general independent particle model (IPM). In Ref. 20 the two lowest, one even and one odd states have been considered. They were called σ and π. At $Z \to \infty$ linear combinations of them recover the cluster model states. If we denote by R and L the right and left cluster single quark wave functions we have

$$\frac{1}{\sqrt{2}}(\sigma + \pi) \xrightarrow[Z \to \infty]{} R$$

$$\frac{1}{\sqrt{2}}(\sigma - \pi) \xrightarrow[Z \to \infty]{} L \qquad (3.3)$$

At finite Z one can define molecular-type orbitals related to R and L if one takes into account the Z dependent overlap <R|L>, which tends to zero as Z goes to infinity, i.e. one has

$$\binom{g}{u} = \frac{R \pm L}{[2\,(1 \pm \langle R|L\rangle]^{\frac{1}{2}}} \quad . \tag{3.4}$$

We can then define pseudo-right and -left states r and l

$$\binom{r}{l} = 2^{\frac{1}{2}}(\sigma \pm \rho) = \frac{1}{2}\left[\frac{R+L}{(1+\langle R|L\rangle)^{\frac{1}{2}}} \pm \frac{R-L}{(1-\langle R|L\rangle)^{\frac{1}{2}}}\right] \quad . \tag{3.5}$$

The states r and l are obviously orthogonal to each other, which is a very useful property and greatly simplify the calculations. One can see that $r \to R$ and $l \to L$ when $Z \to \infty$. Otherwise they are different and can produce different results for NN interaction at short separation distance. The r,l states are useful asymptotically in the scattering problem. The σ,π states are meaningful at short separations. At Z = 0 they become s and p states, respectively. The transformation between these two representations is given in Table 1 of Ref. 20 for six-quark states of various permutation symmetries. The r,l states can be built either directly from IPM states (e.g. soliton bag) or from cluster model states as indicated above. Our ultimate aim is to use deformed soliton bag eigenstates and include the one-gluon exchange neglected in Ref. 13. But a comparative study with the cluster model states is also very useful. In Ref. 20 we have found that certain $p^n s^{6-n}$ components are absent from the wave function in the zero-separation limit when the cluster model is used while they are naturally present in a molecular basis. Starting from parity eigenfunctions or alternatively r,l states we have built up six-quark basis states in the $SU(4)_{TS}$ scheme. We found that in the (TS) = (01) or (10) sectors there are 16, in the (TS) = (00) 7 and in the (TS) = (11) 25 orthogonal states which have to be considered in NN studies. Harvey[18] advocates the use of three states in the (01), (10) or (00) sector and of six states in the (11) sector, i.e. a much smaller basis. He neglects all the SU(4) symmetries which do not represent a di-baryon. The

important achievement is the inclusion of the $[42]_O$ symmetry state for even partial waves.

In the Ref. 21 we have tried to understand which are the most important configurations to be used in NN studies based on molecular orbitals. We have diagonalized the hamiltonian of Sec. 2 in basis of various sizes built at Z = 0 according to the prescription given in Ref. 20 and in the cluster model basis of Ref. 18 with gaussians also taken at Z = 0. In Figs. 1 and 2 we reproduce results obtained for (01) and (00) sectors, respectively, as a function of the oscillator parameter beta. We reach the following conclusions, relevant for short separation distances.

1°) Configurations of the type $p^n s^{6-n}$ which are missing in the cluster model basis can be very important. At Z = 0 they bring a dramatic lowering of the energy both in the (01) or (10) and (00) sectors.

2°) Harvey's truncation of the SU(4) space is a very good approximation. The reason is that the symmetries which do not represent the di-baryon ("non-asterisked") couple to the others only through the spin-spin (and tensor, if included) interaction and this contribution is much smaller than that of the confining potential.

3°) Harvey's transformation from symmetry states to physical ($NN,\Delta\Delta$) and hidden-color (CC) states produces a very important sub-diagonalization.

4°) In both (01) and (00) sectors we find three basis states which appear to dominate the lowest eigenstate i.e. reduce the basis to the same size as in cluster model studies, but of course with basis vectors of different content.

We plan to perform a similar study based on the MIT bag model i.e. with relativistic R and L states. This would be a useful step before the deformed soliton bag eigenstates.[13] In principle the soliton bag[6] is superior to the cluster model. Also it is free[22] of color van der Waals forces.

From the pioneering studies of Liberman[23] or de Tar[10] to those performed in more extended basis[18] an important step has been achieved. It has been shown that the hidden-color states are important at short separations. The configurations which are missing in the cluster model $p^n s^{6-n}$ but natural to molecular orbitals do also lead to unphysical $r^n l^{6-n}$ $(n \neq 3)$ states. It remains to be seen how much they can improve the description of NN processes. In principle we expect a better one because the space has been enlarged although apparently we deal again with a 3 x 3 diagonalization.

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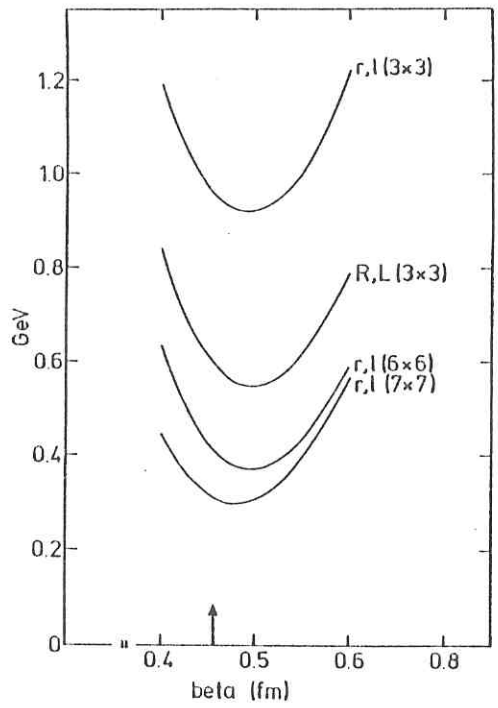

Fig. 1. The lowest eigenstate of the hamiltonian (2.1)-(2.9) in various basis for T = 0, S = 1 sector as a function of the harmonic oscillator parameter beta. The arrow indicates the equilibrium value for a nucleon.[17] (from Ref. 21)

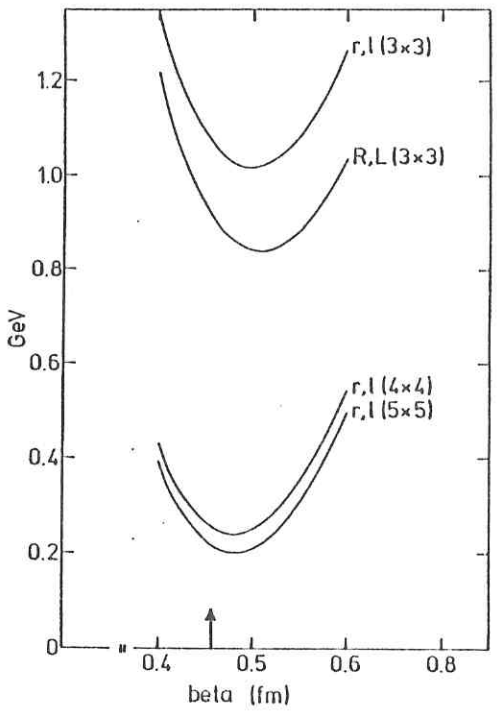

Fig. 2. Same as Fig. 1, but for T = 0, S = 0 sector. (from Ref. 21)