Detection, Localisation and Identification of Nonlinearities in Structural Dynamics

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Abstract

This paper discusses procedures for nonlinearity detection, localisation and identification in structures from time domain vibration measurements. The detection and the localisation techniques use pattern recognition tools and are based on a dissimilarity measure between the signals coming from a linear structure and the corresponding nonlinear one. The detection procedure distinguishes between linear structures and structures with a nonlinearity employing nearest neighbour techniques. The localisation procedure combines substructuring with a nonlinearity detection procedure. This technique is useful for cases of local nonlinearity, when its localisation can be of value for the consequent understanding and modelling of the structure. The identification procedure makes use of the Karhunen-Loeve transform, known also as Proper Orthogonal Decomposition (POD). It is a powerful tool for solving inverse problems in nonlinear structural dynamics. The identification procedure works on the basis of the minimisation of a difference function between the experimental and the simulated proper orthogonal modes (POM). The proposed techniques are demonstrated on a beam test case with a local damping type nonlinearity.

1. Introduction

For the purpose of dynamic analysis and design, most structures are usually approximated by a linear model. Modal testing and analysis are the most widely used linear techniques for modelling, prediction and control of the system dynamic behaviour, as well as for solving updating or damage detection problems. However, a lot of real structures exhibit nonlinear behaviour which can be caused by a number of different reasons. Nonlinear behaviour is observed even in rather simple structures like plates and beams, as a result of buckling or large deformation related effects. The nonlinear behaviour of a structure may also be due to a local (friction, joint and link flexibility, backlash and clearance, nonlinear contact) or a global nonlinearity (geometric nonlinearities, nonlinear material behaviour). The presence of nonlinearity in a system changes its behaviour, thus making the use of the linear model improper and in a lot of cases even impossible. The basic principles that apply for a linear system and which form the basis for modal analysis are not valid anymore for nonlinear systems. The superposition and the homogeneity as well as the Maxwell reciprocity principles do not apply for a nonlinear system. A nonlinear mechanical system shows a tendency to redistribute the energy of the input spectrum. This results in modulation, super- and sub-harmonics, broadband spectra in some areas. The generation of harmonics depends on the excitation. The frequency response functions are also excitation dependent, which makes impossible their further application for modal analysis. Modal models are quite inapt to predict the behaviour of nonlinear systems. Accordingly, new tools for the detection, quantification and modelling of nonlinearities in dynamical systems are necessary. A lot of effort was spent into developing methods for detecting the presence of nonlinearities in a system [10,11,15]. Some procedures rely on characteristic features for nonlinear systems, like the distortion of the FRF plots. Others suggest testing the validity of basic linear principles. Most procedures work on the basis of the comparison of the responses of the linear system and the system under test. In this study we suggest an approach that uses a dissimilarity measure between the system responses in the time

domain. It is combined with the paradigm of pattern recognition and the nearest neighbour (NN) approach. Once the detection of the nonlinearity is performed, its quantification becomes an important problem. For the case of local nonlinearities, it is rather convenient to be able to localise the nonlinearity first for the purposes of its further characterisation. In this paper we introduce a nonlinearity localisation method that works on the principle of the introduced detection procedure after substructuring. During the last years a lot of work was done on modelling the nonlinear behaviour of dynamic systems. Different tools are suggested in this direction . The use of Volterra series as a way to describe nonlinear systems is one of the most widely accepted one. Identification procedures for nonlinear dynamic systems employ different tools. Some authors suggest the use of higher order FRF's [12]. Other techniques include spectral analysis and different NARMAX models [11]. In this paper we suggest the application of the Karhunen-Loeve transform or Proper Orthogonal Decomposition (POD) for the identification of nonlinear multi-degree-of –freedom systems.

2. Test case definition

To illustrate the methods, let us consider an experimental application on an aluminium beam. The beam is mounted horizontally with a clamped end and a free end as shown in Figure 1. A nonlinear damping with Coulomb friction was added at the free end. The accelerations were measured using six accelerometers regularly spaced along the beam. An impulse force was applied by a hammer just between the two last sensors.

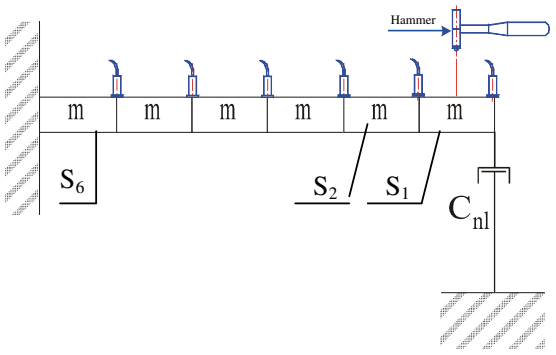

Figure 1 : Experimental beam

3. Nonlinearity detection procedure

3.1 Theory of the nonlinearity detection procedure

In a lot of cases it is necessary to first detect the presence of nonlinearity. A lot of effort was spent in this direction and a number of procedures are suggested. It is our suggestion here to define the problem as a classification one and use the nearest neighbour approach to solve it. Clearly a nonlinearity detection problem is a classification one, since the task is to distinguish between a linear and a nonlinear structure. The distinction has to be done on the basis of the response of the system. We suggest the use of the time domain acceleration measurements. Normally the vibration measurements for a structural system consist of the acceleration measurements for a number of DOF's. The ingredients for a nearest neighbour classification rule are: 1) a labelled sample database (standard samples corresponding to different classes), 2) a dissimilarity measure by which the measurements for a newly tested structure can be compared with each of the members of the labelled database. For our case of classification of a vibrating structure the database should consist of one or more time series determined by the number of measured DOF's. For the case of two classes, only the standard sample corresponding to one of the classes can be used. As a dissimilarity measure we suggest the use of the Kullback-Leibler (K-L) dissimilarity measure between time series [16, 18]. A new structure is characterised by the acceleration measurements $a_i(t)$, $i=1,2...,n$, where n is the number of measured DOF's.. Since discrete measurements are taken they form the vectors

$$\mathbf{a_i} = [a_i(t_1), a_i(t_2), ..., a_i(t_k)],$$

where k corresponds to the number of time measurements. The labelled database consists of the corresponding acceleration measurements for the linear system (its model), i.e. $\mathbf{a_i^L}$, where L stands for labelled. Thus the decision should be made on the basis of the K-L dissimilarities between $\mathbf{a_i}$ and $\mathbf{a_i^L}$.

The symmetrized K-L dissimilarity measure between two time series $a(t)$ and $a^L(t)$ is defined as:

$$I(a,a^L)=\int_{t_0}^{t_1} a(t)\log\left(\frac{a(t)}{a^L(t)}\right)dt. \\ .\int_{t_0}^{t_1} a^L(t)\log\left(\frac{a^L(t)}{a(t)}\right)dt \quad (1)$$

For the case of n acceleration measurements one will have n K-L numbers

$$I_i = I(a_i, a_i^L), \quad i = 1,2,...,n,$$

corresponding to the n DOF's where measurements are taken.

The classification rule based on the n-nearest neighbour principle for the structure S characterised by the acceleration signals $a_i^S(t)$ is given in the form [17]:

$$S \in C_L, \text{ if } D_s \le D_T \\ S \in C_N, \text{if } D_s > D_T, \quad (2)$$

where

$$D_S = \max_i I_i^S = \max_i I(a_i^S, a_i^L) \quad (3)$$

D_T is a certain threshold (which is determined as a result of a pattern recognition procedure [18]) and C_L and C_N denote the class of linear structures and the class of nonlinear structures, respectively. Thus a new structure is considered linear, or not containing nonlinearities, if its dissimilarity measure D_S, which is the maximum of all the dissimilarity measures between its acceleration signals and those of the corresponding linear structure, does not exceed a certain threshold value D_T. Accordingly a structure should be regarded as nonlinear if its dissimilarity measure exceeds the threshold value D_T.

Another possible dissimilarity measure which can be used in order to compare the acceleration signals coming from the linear structure and the one under test is the symmetrized Itakura spectral distance between two signals which is defined as

$$J(a,a^L)=\int_{\omega_0}^{\omega_1} f(\omega)/f^L(\omega)d\omega. \\ .\int_{\omega_0}^{\omega_1} f^L(\omega)/f(\omega)d\omega \quad (4)$$

where f and f^L are the spectra of the signals a and a^L, respectively. The classification rule is the same as the one introduced earlier with the K-L number (2), only for this case D_S is substituted by

$$J_S = \max_i J_i^S = \max_i J[f_i(\omega), f_i^L(\omega)] \quad (5)$$

and another threshold value J_T is introduced. The Itakura spectral distance was introduced just as another alternative for a dissimilarity measure, for the purpose of comparison.

This method, like most nonlinearity detection procedures, works on the basis of dissimilarity measures between the response of the linear structure and the structure under test. However in a lot of cases this dissimilarity can be a result of noise. All the procedures mentioned above cannot distinguish between noise and nonlinearity presence. In order to check whether the dissimilarity is due to noise effects, or to the presence of any kind of a nonlinearity in the structure, a hypothesis testing with surrogate data series can be applied [13,14].

- First the residuals series that comprise the difference between the linear and the measured time series are found
  $r_i = \left| a_i - a_i^L \right|$
- On the second step the mean residual series r_m is found.
- Surrogate series are created to preserve only the linear properties of r_m [14].
- The null hypothesis stating that r_m is linearly correlated noise is tested, using some nonlinear time series invariants and applying statistical hypothesis testing [13].
- The null hypothesis is either rejected or confirmed. When it is confirmed, the conclusion is that the dissimilarity is due to noise, and thus the structure should be considered linear.
- For the case when the null hypothesis is rejected, one should conclude that the residual series represents a nonlinear dynamic process. Hence it is derived that the structure is nonlinear.

In a number of cases, like the considered test case, it is not necessary to go through the above procedure, since there are clear indications that the dissimilarity is caused by the presence of a nonlinearity and not by noise.

3.2 Application to the test case

First the time and the frequency representations of the acceleration signals coming from the FE model of the linear structure and the tested structure are observed. Neither the differences between the corresponding signals from the linear structure and the one under test, neither the signals coming from the examined structure look random-like. In the frequency domain for the tested structure one can observe a shift in some of the frequency peaks, as well as the presence of a couple of new peaks (Fig.2). Accordingly we reject the hypothesis for the presence of noise in the tested structure.

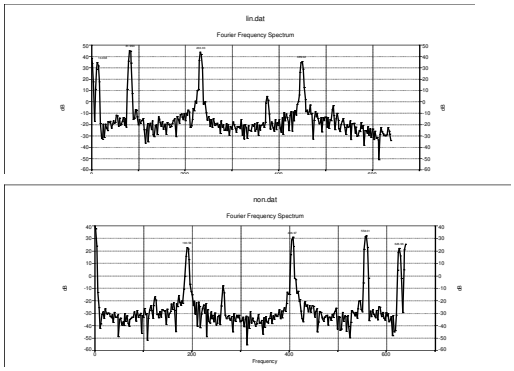

Fig.2 Fourier spectra of part of a linear signal (up) and the corresponding nonlinear one (down)

The procedure presented in the previous paragraph was applied to the introduced beam test case (Fig. 1) with both dissimilarity measures. The threshold values were obtained as a result of a n-nearest neighbour pattern recognition procedure for 2 classes [17,18]. The measurement points are as shown on Fig. 1. They are numbered starting from the free end of the beam. The following Figure 3 presents the results from the detection procedure.

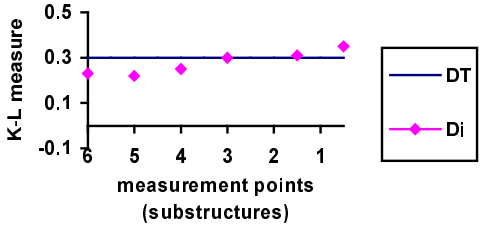

Figure 3. Detection and localisation procedure with K-L number. (The measurement points and the substructures are numbered beginning from the free end)

It is observed from Fig. 3 that all the K-L numbers, corresponding to the 6 measurement points are rather close to the threshold value D_T . However, the maximum K-L number I_1 corresponding to the first measurement point (starting from the free end) is higher than the threshold. Thus according to (2) the structure should be considered nonlinear.

A similar situation can be observed with the symmetrized Itakura distance (SID). Figure 4 shows the results from the detection procedure with the SID.

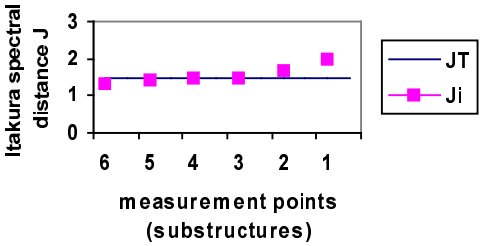

Figure 4. Detection and localisation procedure with SID. (The measurement points and the substructures are numbered beginning from the free end.)

It can be observed that the SID values for the first two measurement points are above the threshold value. The values corresponding to the rest of the measurement points are either on or below the threshold. Again according to the introduced classification rule (2) with J_S and J_T substituted for D_S and D_T, the structure should be considered as containing a nonlinearity.

Thus both procedures give correct results for the considered test case.

4. Localisation procedure

4.1 Theory of the localisation procedure

In a number of cases it is helpful for the consequent identification procedure to localise the nonlinearity in the structure under test. In order to do that we suggest in this paper an approach, which is based very much on the already introduced detection procedure. Only for the purposes of localisation it is combined with substructuring.

First the structure is divided into N substructures S_I, I=1,2,…,N. Then the presented detection procedure is applied to each substructure. For this purpose only the acceleration signals measured in DOF's from the corresponding substructure are taken into account. Any of the dissimilarity measures presented in section 2 can be applied. Then the same classification rule is employed

$$\begin{aligned} &S_I \in C_L, \text{ if } D_I \le D_T, \text{ or } J_I \le J_T \\ &S_I \in C_N, \text{if } D_I > D_T, \text{ or } J_I > J_T \end{aligned} \quad (6)$$

where

$$\begin{aligned} &D_I = \max_i I_i^I = \max_i I(a_i^I, a_i^L) \\ &\text{and } J_I = \max_i J_i^I = \max_i J(f_i^I, f_i^L), \\ &I = 1,2,...,N \\ &i = 1,2,...,n_I \end{aligned} \quad (7)$$

n_I is the number of DOF's corresponding to the substructure S_I. The classification rule (6) means that the substructure S_I is considered linear if the maximum dissimilarity between the signals D_I is less than the threshold value D_T. If on the other hand D_I exceeds D_T, then S_I should be considered nonlinear.

4.2 Application to the test case

For the purposes of the localisation procedure the cantilever beam is divided into six substructures S_i, i=1,2,…,6, which are obtained dividing the beam into 6 parts S_I with equal lengths (Fig.1).

Now the question to answer is which of the substructure(s) contain(s) a nonlinearity, that is which of the substructures can be considered nonlinear. This procedure is relevant and is expected to give meaningful results only for the case of a local nonlinearity (nonlinearities). If the nonlinearity is global, e.g. nonlinear material, or a geometric nonlinearity, then the procedure is expected to end up with indication that all the substructures are nonlinear.

The figures (Fig.3 and Fig.4) presented for the detection procedure, for this case contain the results for the localisation procedure as well. The accelerometers are positioned at the end of each substructure. Thus each substructure contains only one measurement point. Thus the dissimilarity measures for the measurement points give the dissimilarity measures for the substructures as well.

From Fig. 3 one can conclude that substructure 1, S_1, is nonlinear. Substructure 2, S_2, can be also suspected as nonlinear, since it's K-L number is very close to the threshold. All the rest of the substructures should be considered linear. Thus the K-L dissimilarity number correctly identifies the nonlinear substructures for the considered test case.

From Fig.4 one should derive that subsctructures 1 and 2, S_1 and S_2, contain anonlinearity. Substructures 3 and 4 are on the boundary between the two classes, and can be considered suspicious. The rest two substructures do not contain nonlinearities. Obviously the SID gives worse results when applied for the localisation procedure for the considered test case. It is not able to identify the nonlinear substructures properly and precisely enough.

5. Parameter Identification

5.1 The Proper Orthogonal Decomposition

The Proper Orthogonal Decomposition (POD) allows to identify a useful set of basis functions and the dimension of the subspace necessary to achieve a satisfactory approximation of the system. The POD also facilitates the resolution of the partial differential equations through their projection into a reduced-order model [2]. The definitions and formulation presented here follow closely the ones used in [4].

Let $u(x,t)$ be a random field on some domain Ω. Since the POD requires to deal with zero-mean signals, it is necessary to define $v(x,t)$ by subtracting the mean $U(x)$ from $u(x,t)$:

$$u(x,t) = U(x) + v(x,t) \quad (8)$$

These fields are sampled at finite number of time points. Then, at a fixed time t_n, the system displays a snapshot $v_n(x)$, which is a continuous function on Ω. The aim of the POD is to find the most representative structure $\phi(x)$ of the ensemble of N snapshots. This is accomplished by solving the following optimisation problem:

$$Minimise \left\{ \lambda = \sum_{n=1}^{N} (\phi(x) - v_n(x))^2 \right\} \forall x \in \Omega \quad (9)$$

It can be argued ([3] and [4]) that relation (9) is equivalent to the following integral eigenvalue problem :

$$\int_{\Omega} K(x,x')\phi(x')dx' = \lambda\phi(x') \qquad (10)$$

where the two points correlation function K is defined as :

$$K(x,x') = \frac{1}{N}\sum_{n=1}^{N} v_n(x)v_n(x') \qquad (11)$$

Equation (10) has a finite number of orthogonal solutions $\phi^i(x)$, called the proper orthogonal modes (POM) with corresponding real and positive eigenvalues λ^i. The eigenvalue with the largest magnitude is the minimum which is achieved in the minimisation problem (9). The second largest eigenvalue is the minimum of the same problem restricted to the space orthogonal to the first eigenfunction, and so forth. For the purposes of uniqueness, the eigenfunctions are normalised. Therefore the POM can be used as a basis for the decomposition of the field $v(x,t)$:

$$v(x,t) = \sum_{i=1}^{N} a^i(t)\phi^i(x) \qquad (12)$$

Moreover, by construction, the POM capture more energy than any other modes. It should also be noted that time-dependent coefficients $a^i(t)$ in (12) are uncorrelated [3]. Thus the POD can be viewed as a bi-orthogonal decomposition because of the space-time symmetry of the decomposition. For an accurate approximation of the tensor (11) it is necessary to perform a long and expensive simulation. The computation of the eigenfunctions is even more expensive. Two methods exist to solve the problem: the direct [8] and the snapshot method [4].

Discrete formulation

Suppose N linear snapshots of the acceleration v_i of size M are obtained at M locations (e.g. by measurements). The M x M covariance matrix **C** [6] is defined as :

$$C = \frac{1}{N}\sum_{i=0}^{N-1} v_i v_i^T \qquad (13)$$

Its eigensolutions (ϕ_k, λ_k) which satisfy :

$$C\phi_k = \lambda_k\phi_k, \quad k = 0,\ldots,N-1 \qquad (14)$$

with

$$\lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_{N-1} > 0$$

characterise the proper orthogonal decomposition. Each eigenfunction ϕ_k is associated with an eigenvalue λ_k. If the eigenvalues are normalised, they represent the relative energy captured by the corresponding POM. This decomposition is the best basis in term of de-correlation. The brute computation of the eigensolutions of **C** is called the direct method. However, due to the space-time symmetry property, an alternative method, called the "method of snapshots", can be employed ([4] and [6]).

Computation of the POD using SVD

The complete bi-orthogonal decomposition of the data may be obtained by use of the Singular Value Decomposition (SVD). For instance the SVD which is related to Principal Component Analysis, is used in reference [8] and [9] to compute modal metrics to solve model updating problems in an optimisation procedure. Let $x(t)$ denote a response time-history, where x is a vector containing the displacement, velocity or acceleration at M discrete locations. The discrete matrix X is formed:

$$\mathbf{X} = \begin{bmatrix} x_1(t_1) & \cdots & x_1(t_N) \\ \vdots & \ddots & \vdots \\ x_M(t_1) & \cdots & x_M(t_N) \end{bmatrix} \qquad (15)$$

Thus, each row corresponds to a time history at one location and each column corresponds to a snapshot of the system at specific time. Now the singular value decomposition of matrix **X** can be written as:

$$\mathbf{X} = \mathbf{U}\Sigma\mathbf{V}^{\mathbf{T}} \qquad (16)$$

with U an orthonormal matrix (size M x M) of eigenvectors of $\mathbf{X}\mathbf{X}^{\mathbf{T}}$ and **V** an orthonormal matrix (size N x N) of eigenvectors of $\mathbf{X}^{\mathbf{T}}\mathbf{X}$. The size of the matrix Σ is M x N but only the main diagonal has non-zero elements which are the singular values of **X**, sorted in descending order. If the matrix **X** is rank deficient, i.e. some rows (or columns) can be generated by a linear superposition of the others, a few singular values will be zero. The SVD has a lot of applications, e.g. the estimation of the rank of a matrix, the filtering of measurement noise and so forth. In this paper the aim of the SVD is to compute the POM's and the normalised basic shapes including the response time-histories [9].

5.2 POD and parameter identification

The identification of the nonlinear parameters of a structure is based on the solution of an optimisation problem, which consists in minimising the difference between the bi-orthogonal decomposition of the measured and the simulated data respectively. Let us define the objective function **F** as:

$$\mathbf{F} = \sum_i \sum_j (\Delta U_{ij})^2 + \sum_j (\Delta \Sigma_{jj})^2 + \sum_j \sum_k (\Delta V_{jk})^2 \quad (17)$$

Where $\Delta U_{ij}, \Delta \Sigma_{jj}, and\ \Delta V_{jk}$ are the differences between the matrices containing the bi-orthogonal decompositions. It must be stated that the full decomposition is not retained in the objective function. Only the terms corresponding to the higher singular values are considered, which means that we take the proper orthogonal modes that contain the greatest amount of energy in the signal. Then the objective function **F** may be minimised by standard optimisation algorithms. The term ΔV_{jk} in (17) does not represent the brute time decomposition of the data but the Fourier Transform of the time decomposition.

5.3 Experimental example

Identification of the nonlinear parameters

It is worth pointing out that before the nonlinear identification a linear test was performed in order to obtain the Young modulus.
Two different models are considered:

- A linear model is considered: in this case the variables of the optimisation are the three first modal dampings of the underlying linear system. Moreover, a linear dashpot was added at the end of the beam.
- A nonlinear model including a nonlinear dashpot modelled as a polynomial of the velocity including Coulomb friction.

As far as the optimisation process is concerned, a time period of 0.4 second is considered and the objective function is written in terms of the first three POMs. The latter contain 95% of the energy in the signal. After the optimisation, the nonlinear model improves the objective function of 8% in comparison with the linear one.
Figure 5 represents the comparison between experimental and numerical proper orthogonal decomposition. Only the POMs for the experimental test and for the nonlinear model are represented since the linear and nonlinear POM are merged. At figure 6, the displacement at the end of the experimental beam is plotted and is compared to the two models. The figure shows very slight differences between the two models and the experimental data. However, the nonlinear model leads to slight improvements and allows to identify Coulomb friction in the model.

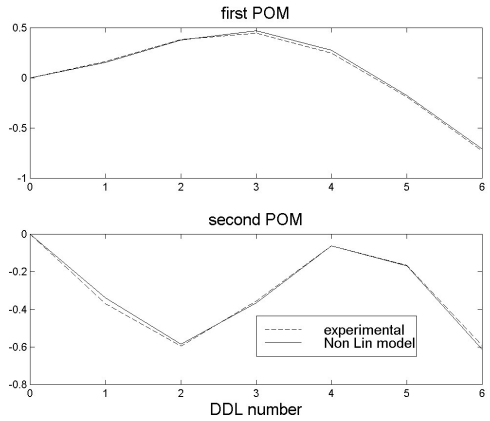

Figure 5: Comparison between the POM

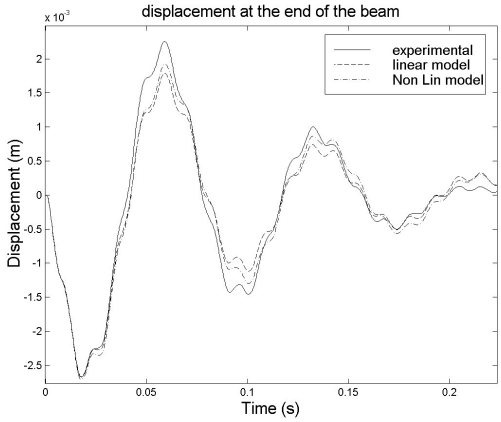

Figure 6: Comparison of the displacements

5. Conclusions

A method including three steps- nonlinearity detection, localisation and identification procedures- is introduced. The detection procedure works on the principle of comparing dissimilarities between the acceleration time series of the linear structural model and the corresponding measurements of the tested structure. Both dissimilarity measures used conclude that the structure is nonlinear. The localisation procedure uses the same dissimilarity measures that are used for the detection. The proper orthogonal decomposition allowed to identify quite efficiently the coulomb friction at the end of the experimental beam. The nonlinear model is compared to the linear one and gives only slight improvements. For

the considered test case the performance of the localisation and identification procedures is somewhat poor, which is most likely due to the weak nonlinearity introduced- the Coulomb friction at the end of the beam.

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