CONVEX APPROXIMATION METHODS FOR LARGE SCALE STRUCTURAL PROBLEMS

Pierre DUYSINX
Department of Mathematics, Technical University of Denmark, 2800 Lyngby, Denmark

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Abstract

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1 INTRODUCTION

As a result of several researches (e.g.[6, 9, 8, 12]), structural optimization problems with sizing or shape design variables can be solved efficiently by the mathematical programming approach and real-life applications can be handled. With the *homogenization method* proposed by Bendsøe and Kikuchi [2] and then further developed in several other works (see Bendsøe [1] for a review), structural optimization is now able to attack the topology design problem.

Despite the fact that the material distribution problem looks like a sizing optimization, topology problems have its own characteristics and difficulties. The discretization of the material density introduces generally between 1000 and 10000 design variables, number that was never reached before. Furthermore, solving topology problems requires often a very high number of iterations to get a stationary distribution. It is usual to spend more than 100 iterations to solve the topology problem.

Up to now, few solvers for huge optimization problems, other than simple optimality criteria, were available in topology. Thus, topology design was often restricted to formulations involving few design constraints. Generally, the topology design was formulated as a minimum problem of a stiffness criterion, like the compliance, with a bound over the volume.

This communication wants to report how we improve the solution procedure of optimization of topology huge size problem by extending the mathematical programming approach that was successful for other structural optimization problems. At first we use dual methods that are well adapted to solve the convex separable optimization problems even if the number of design variables is large. On another hand, we look at the problem of selecting the most appropriate approximations. The choice of an approximation is a compromise between precision and conservativity, between accuracy and computation effort to generate the data. After having compared available first order approximations, we wanted to further reduce the number of stages necessary to arrive to the solution. To this end, we developed and validated a new approximation procedure based on second order approximations and quasi-Newton updates preserving the diagonal structure of the estimates.

2 A DUAL SOLVER

Solution of convex subproblems of sizing and shape problems can be performed efficiently by dual methods [5, 8]. This is more than ever true with the subproblems of topology design. The primal constrained problem with a large number of design variables is replaced by a quasi unconstrained maximisation of the dual function. The dimension of the dual space is limited to the number of active constraints, which is small. The advantage of the dual formulation is real if the relationships between primal and dual variables are rigorous and inexpensive to compute. This is the case if the objective function and the constraints are linearised by *separable* and *convex* approximations.

3 CONVEX APPROXIMATIONS

3.1 FIRST ORDER CONVEX APPROXIMATIONS

The simplest first order approximation is the first order Taylor expansion. This linear approximation is efficient for the volume constraint, but its lack of convexity makes it too few reliable for structural constraints. For structural responses, it is better to turn our choice towards convex approximations. From our experiences, CONLIN [9] approximation gives rise to good results in topology. For the compliances that are self-adjoint, all the derivatives are negative and CONLIN restores the reciprocal design variables expansion that is well known to reduce the non-linearity of the structural responses. But convexity properties of CONLIN are important when treating eigenfrequencies or constraints whose first derivatives have mixed signs. The main disadvantage of CONLIN is that the approximation introduces fixed curvatures, so that the approximation might be too much or too few convex. This might give rise to a slow or unstable convergence towards the optimum. To remedy to this problem, we select MMA [12] approximation that generalises and improves CONLIN by introducing two sets of asymptotes. The choice of the moving asymptotes provides the way to modify the curvature and to fit better to the characteristics of the problem.

Nevertheless, we can conclude that both CONLIN and MMA lead to satisfactory results for topology design and improve often greatly the performances of the solution. In a lot problems, we observed that a solution was often achieved in 30 to 50 iterations depending on the difficulty of the problem and the precision of the stopping criterion. One strong advantage of CONLIN and MMA arises from the very reliable dual solvers that are used to solve the associated convex subproblems. On another hand, one major drawback of first order approximations is that we can observe a deceleration of the progression towards the optimum once the algorithm arrives in the neighbourhood of the optimum.

To accelerate the convergence rate in the final stage, one needs better approximations based on curvature information [7].

3.2 SECOND ORDER CONVEX APPROXIMATIONS

Second order are high quality approximations that are indeed more precise and that lead to faster convergence rates. Nevertheless, second order sensitivity is very onerous to compute and to store so that the overall cost of the optimization can be similar to the one of first order approximations [10]. The problem becomes quickly cumbersome and impossible to manage when the size of the problem increases.

To be able to use second order approximation schemes with large scale optimization problems, we developed a new procedure to generate an estimation of the curvature information with a small computation cost [3, 4]. As separable approximations needs only diagonal second derivatives, the idea is to built an estimation of the curvature information with a quasi-Newton update able to preserve diagonal structure of the Hessian estimates. This update scheme is derived form the general theory of quasi-Newton update with sparse Hessian estimates made by Thapa [13]. The diagonal version of the BFGS update [3, 4] that we implemented is very un-expensive even for large scale problems since it introduces only vectors manipulations. In [3], it was observed that for a given topology problem, the time spent in the diagonal BFGS update is only 3 % of the time spent in the optimizer CONLIN [6, 8] and only 0.01 % of the time needed for sensitivity analysis with a commercial finite element package.

The theoretical algorithms was adapted to the characteristics of structural optimization problems to yield quickly convergent estimates of the Hessian. This adaptation relies on the key role of the reciprocal design variables to reduce the non-linearity of the structural responses. The Hessian is updated in the space of reciprocal design variables and then converted into curvatures in terms of the direct variables to be used in the approximation. The initial guess of the Hessian is also very important. Starting in the reciprocal design space from a diagonal matrix of small terms restores the curvatures of CONLIN which is generally a good starting point.

This second order information is introduced into two well known second order approximations. The first one is a second order version of MMA proposed by Smaoui et al. [11]. The second approximation is the separable quadratic approximation suggested by Fleury [7]. Combining diagonal BFGS update with both these approximations gives very interesting results that results in important savings in terms of number of iterations and of computation time. This conclusion can be explained as follow. Firstly, the estimation of the curvature improves greatly the quality of the approximation with only the help of the accumulated first order information. Secondly, instead of ignoring the second order coupling terms, diagonal BFGS provides a way to take them into account by correction terms on the diagonal coming from the diagonal update. Due to our initial guess of the Hessian, one can observe, in the first iterations, a convergence history that is very similar to first order approximations. But after some iterations, the update procedure improves the estimation of the Hessian and one can see a real advantage in the convergence speed. Around an accumulation point satisfying the optimality conditions, we could observe a convergence speed superior to first order methods, sometimes closed from super-linear behaviour.

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