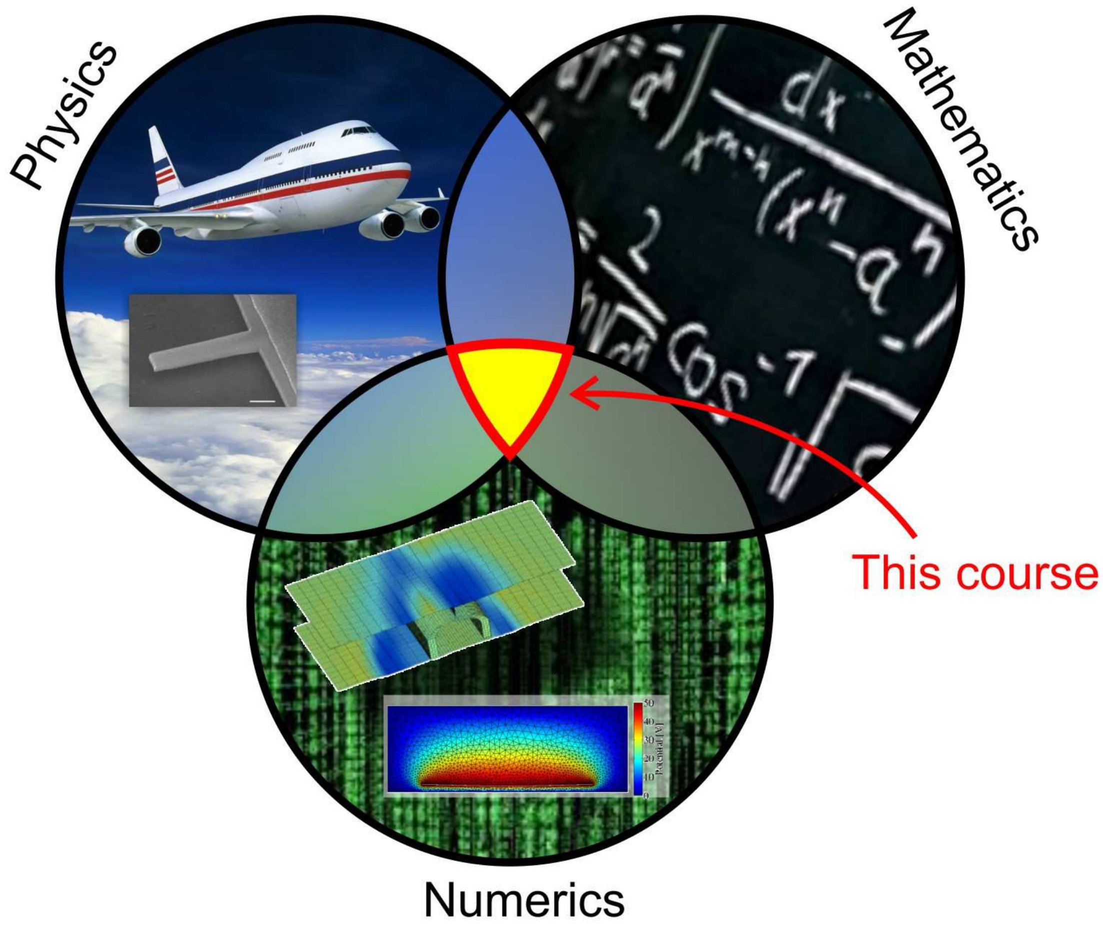
MATH0024 – Modeling with PDEs

Introduction

Maarten Arnst and Romain Boman

September 20, 2017



This course offers an understanding of the physical basis, mathematical structure, and numerical solution of different types of PDE, as well as of the relationships between these physical, mathematical, and computational perspectives.

Outline

	September 20	Introduction, notations, and review of background material Classifications of PDEs
Laplace	October 4	**Laplace equation**
	October 11	Ordinary differential equations Finite difference methods for the Laplace equation
	October 18	Variational formulation Finite element methods for the Laplace equation
	October 25	Spectral problem for the Laplace equation
	November 8	*Discussion session*
Heat	November 15	**Heat equation**
	November 22	Finite difference/finite element methods for the heat equation
	November 29	*Discussion session*
Wave	December 6	**Wave equation**
	December 13	Transport equation Finite difference/finite element methods for the transport/wave equations
	December 20	*Discussion session*

The lectures take place in B52 +2/411, 8:30–12:30, although most lectures will probably end earlier than 12:30. Exercises/review at beginning or end of class?

Course material

- Each lecture is supported by a set of slides.

- The slides are complemented by a book which **ULg library** provides free online access to:

    - P. Olver. *Introduction to partial differential equations*, Springer, 2014.

- Reference texts for students wishing to consult additional material:

    - M. Gockenbach. *Partial Differential Equations: Analytical and Numerical Methods*, SIAM, 2010.

    - R. Haberman. *Applied Partial Differential Equations with Fourier Series and Boundary Value Problems*, Pearson, 2012.

Grading

- Grading is based on regular homework and a final exam. The final grade is a weighted average of the grades obtained for the regular homework (25%) and for the final exam (75%).

  We will strive to include in the homework some questions relevant to the integrated project MATH0471 “Multiphysics integrated computational project.”

- Three homework assignments (related to Laplace, heat, and wave equations) are currently planned. Some help will be provided to students during the discussion sessions.

- The final exam will be a written exam. At the end of the written exam, students will have the opportunity to clarify any remaining issues in a discussion with the instructor as needed.

  The final exam will be closed book. Those parts of the slides that are indicated to be "review material" (a detailed list will be provided during the last lecture) will be provided to the students.

  You will be asked to reproduce one of the proofs seen in class (a detailed list will be provided during the last lecture), to solve exercises similar to the homework problems, to solve exercises to the beginning-of-class/end-of-class exercise problems, as well as to answer a number of smaller questions that will test your understanding of the material.

- Please consider this organization of the final exam as a tentative organization. Let us discuss about it around the middle of the semester in order to finalize the organization of the final exam.

Grading

List of review material that will be provided during the exam

- Notations and review of background material. Lecture 1 Part B.
- Review of ordinary differential equations (ODEs). Lecture 3.
- Review of series and function series. Lecture 5. Slides 13–20.
- Review of sampling theory. Lecture 7. Slides 21–25.

Grading

List of proofs from among which you will be asked to reproduce one at the exam

- Integral representation theorem. Lecture 2. Slides 20–21.
- Consistency, stability, and convergence of centered-in-space finite difference method. Lecture 3. Slides 35–39.
- Proofs of heat kernel being fundamental solution and of superposition formulae by using Fourier analysis. Lecture 6. Slides 11 and 18.
- Consistency, stability, and convergence of centered-in-space forward-in-time finite difference method. Lecture 7. Slides 10–15.
- Proofs of retarded Green's function being fundamental solution and of superposition formulae in 1D by using Fourier analysis. Lecture 8. Slides 19 and 24.
- Proof of Kirchhoff representation by using method of spherical means. Lecture 8. Slides 28–30.
- Proof of Poisson representation by using method of descent. Lecture 8. Slides 32–33.

- If you have questions about the material, please do not hesitate to contact us:

  - Romain Boman and Maarten Arnst (*co-titulaires*).

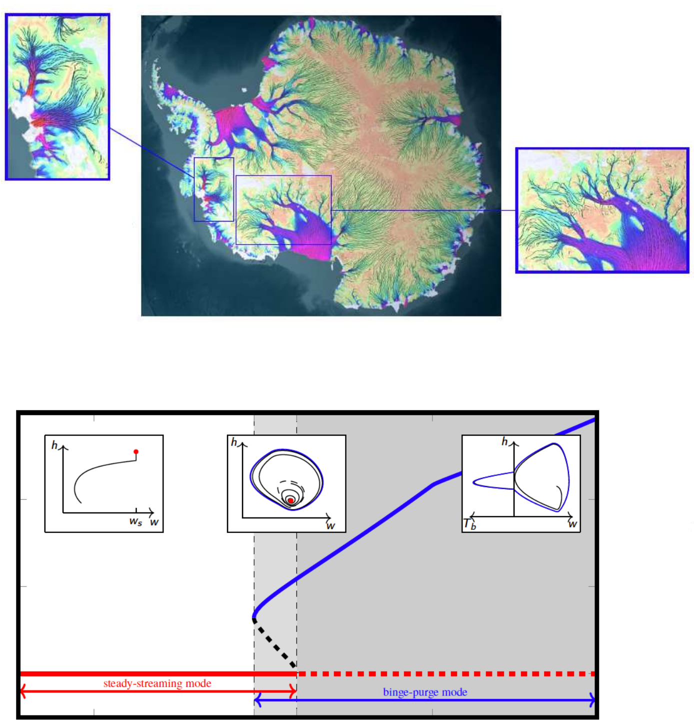
  - Preferred time for discussion: each week after class.

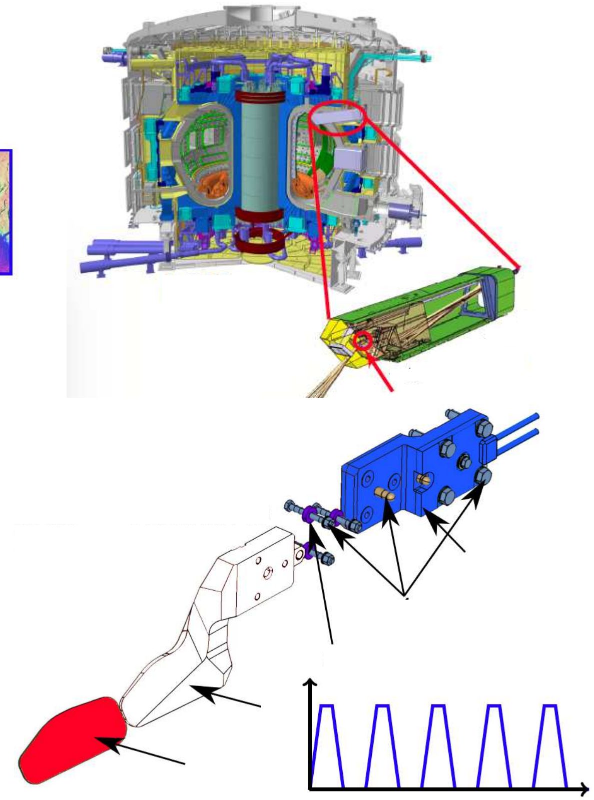
  - You are welcome to contact Romain Boman or Maarten Arnst by email to ask questions.

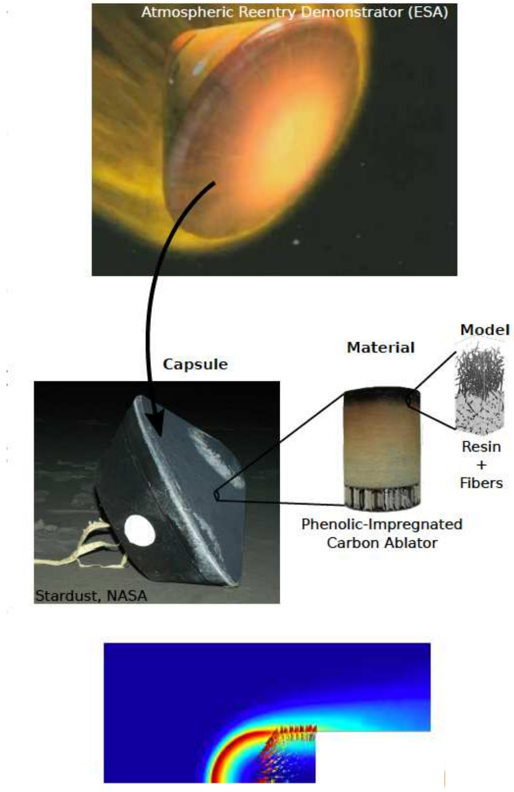
  - You are welcome to contact Romain Boman or Maarten Arnst by email to make an appointment. If you come to our office without making an appointment in advance, it might be necessary to wait a little while until a possible ongoing meeting or other activity is finished.

A bit more about the instructors

- Research interests of Maarten Arnst: Computational modeling. Stochastic modeling. Uncertainty quantification.
- Ongoing PhD. theses:







ULg & ULB
Kevin Bulthuis

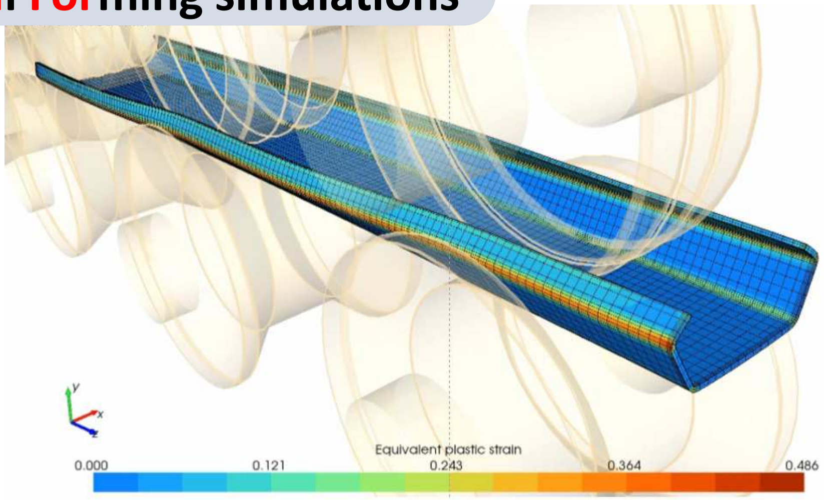
ULg & FZ Julich (GE) & Sandia NL (US)
Kim Liegeois

ULg & VKI & UCL
Joffrey Coheur

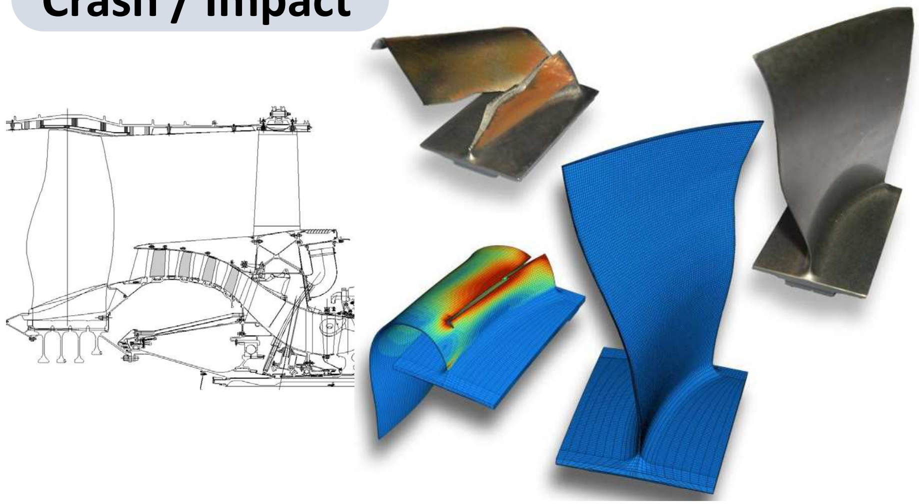
A bit more about the instructors

Research interests of Romain Boman: nonlinear solid mechanics (with Prof J.-P. Ponthot) - lead developer of Metafor - Some applications among many others:

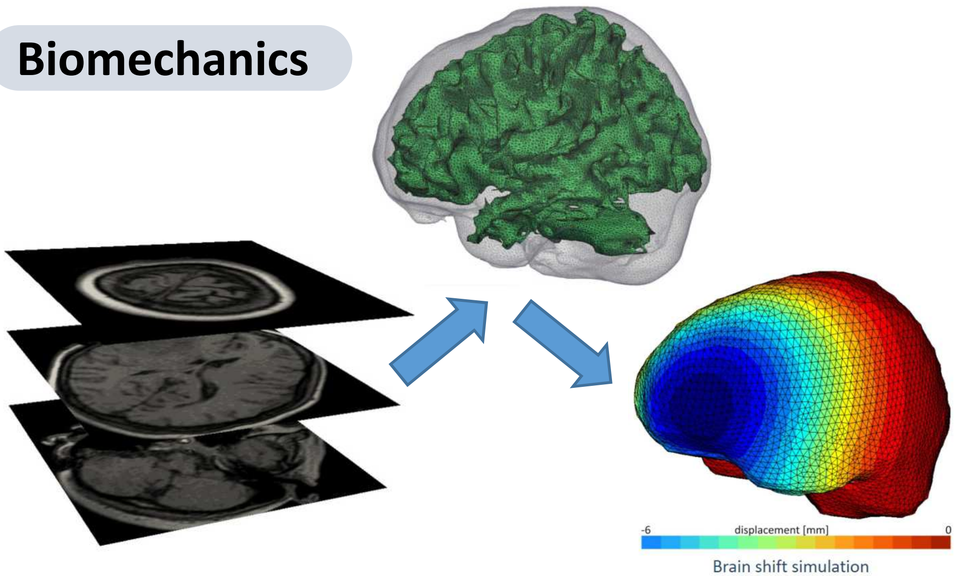
Metal Forming simulations



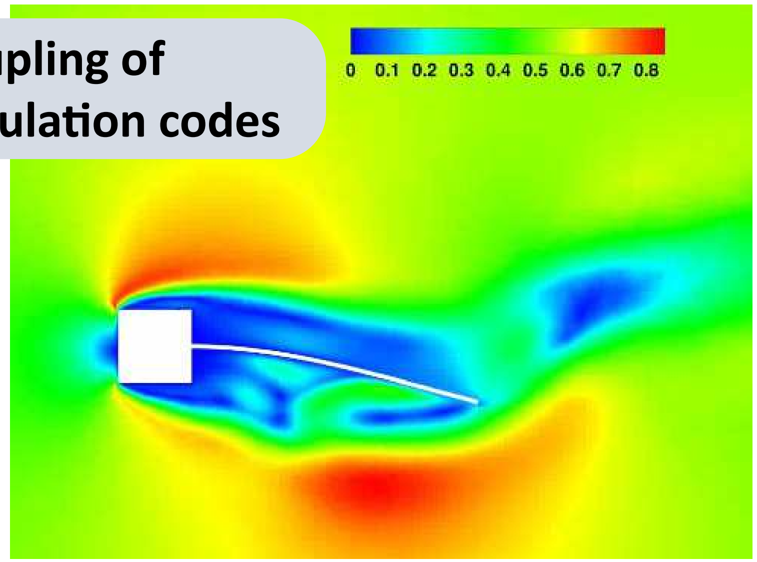
Crash / Impact



Biomechanics



Coupling of simulation codes



Contact

- Maarten Arnst
  Chargé de cours
  Aérospatiale et Mécanique
  Office: B52 0/419
  Email: maarten.arnst@ulg.ac.be

- Romain Boman
  Chef de travaux
  Aérospatiale et Mécanique
  Office: B52 2/439
  Email: r.boman@ulg.ac.be