Fatigue assessment

Thuong Van DANG

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Tanker S.S. Schenectady fractured a day after its launch in January 1941.
Concept of Fatigue

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The S-N curve

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Concept of Fatigue

➢ The word fatigue comes from the Latin verb fatigare - "to tire".

➢ Fatigue is the damage of a structural part by the initiation and gradual propagation of a crack or cracks caused by repeated applications of stress. (BS 7608-1993)

➢ The process of initiation and propagation of cracks through a structural part due to the action of fluctuating stress. (Eurocode 1993.1.9)

➢ The fatigue is defined as the deterioration of a component caused by the crack initiation or by the growth of a crack (new IIW recommendation 2008).
Fatigue failures are widely studied because it accounts for 90% of all service failures due to mechanical causes. Fatigue failures occur when metal is subjected to a repetitive or fluctuating stress and will fail at a stress much lower than its tensile strength. Fatigue failures occur without any plastic deformation (no warning). Fatigue surface appears as a smooth region, showing beach mark or origin of fatigue crack.
Factors causing fatigue failure

- **Basic factors:**
  - A maximum tensile stress of sufficiently high value.
  - A large amount of variation or fluctuation in the applied stress.
  - A sufficiently large number of cycles of the applied stress.

- **Additional factors:**
  - Stress concentration
  - Corrosion
  - Temperature
  - Overload
  - Metallurgical structure
  - Residual stress
  - Combined stress
Fatigue failure

Figure: Fatigue failure
Fatigue loading

Figure: Fatigue stress cycle
Fatigue loading

- **Stress range**: $\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}$
- **Stress amplitude**: $\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}$
- **Mean stress**: $\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}$
- **Stress ratio**: $R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$
- **Amplitude ratio**: $A = \frac{\sigma_a}{\sigma_m} = \frac{1 - R}{1 + R}$
The S-N curve

Normally data from the fatigue tests are plotted at the S-N curve. As stress $S$ versus the logarithm of the number of cycles to failure, $N$. When the curve becomes horizontal, the specimen has reached its fatigue (endurance) limit, ferrous and titanium alloys. This value is the maximum stress which can be applied over an infinite number of cycles. The fatigue limit for steel is typically 35 to 60% of the tensile strength of the material.

Fatigue strength is a term applied for nonferrous metals and alloys (Al, Cu, Mg) which do not have a fatigue limit. The fatigue strength is the stress level the material will fail at after a specified number of cycles (e.g. 10$^7$ cycles). In these cases, the S-N curve does not flatten out.

Fatigue life $N_f$, is the number of cycles that will cause failure at a constant stress level.

![Figure: Typical fatigue curves (a)](image-url)
The S-N curve

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Figure: Typical fatigue curves (b)
The S-N curve

Rotating-bending test → S-N curves

S (stress) vs. N (number of cycles to failure)

S-N curve is concerned chiefly with fatigue failure at high numbers of cycles.

- $N > 10^4$ cycles : high cycle fatigue
- $N < 10^4$ cycles : low cycle fatigue

Fatigue limit or endurance limit is normally defined at $10^7$ or $10^8$ cycles. Below this limit, the material presumably can endure an infinite number of the cycle before failure.

The nonferrous metal, i.e., aluminum, do not have the fatigue limit.
The S-N curve

Figure 1.3 – Fatigue test results of structural steel members, plotted in double logarithm scale, carried out under constant amplitude loading (TGC 10, 2006)

The fact is that the scatter of the test results is less at high ranges and larger at low stress ranges, see for example Schijve (2001). By using a logarithmic scale for both axes, the mean value of the test results for a given structural detail can be expressed, in the range between $10^4$ cycles and $651 \times 10^6$ cycles, by a straight line with the following expression:

$$m_N C \Delta \sigma = \Delta$$

(1.2)

where

- $N$: number of cycles of stress range
- $\Delta \sigma$: constant amplitude stress range
- $C$: constant representing the influence of the structural detail,
- $\Delta \sigma$: constant amplitude stress range,
- $m$: slope coefficient of the mean test results line.

The expression represents a straight line when using logarithmic scales:

$$\log_{10} N \log_{10} m = - \log_{10} \Delta$$

(1.3)

The expressions (1.2) and (1.3) can also be analytically deduced using fracture mechanics considerations (TGC 10, 2006).

The upper limit of the line (corresponding to high $\Delta \sigma$ values) corresponds to twice the ultimate static strength of the material (reverse V-notch)

Figure: Fatigue test results of structural steel members, plotted in double logarithm scale, carried out under constant amplitude loading (TGC 10, 2006)
The S-N curve

By using a logarithmic scale for both axes, the mean value of the test results for a given structural detail can be expressed, in the range between $10^4$ cycles and $5.10^6$ to $10^7$ cycles, by a straight line with the following expression:

$$N = C \Delta \sigma^{-m}$$

where:
N number of cycles of stress range $\Delta \sigma$,
C constant representing the influence of the structural detail,
$\Delta \sigma$ constant amplitude stress range,
m slope coefficient of the mean test results line.
The expression represents a straight line when using logarithmic scales:

$$\log N = \log C - m \log(\Delta \sigma)$$
The S-N curve

Figure: Fatigue classification
Miner’s rule is one of the most widely used cumulative damage models for failures caused by fatigue. It states that if there are $k$ different stress levels and the average number of cycles to failure at the $i$th stress, $S_i$, is $N_i$, then the damage fraction, $D_{tot}$, is:

$$D_{tot} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_k}{N_k} = \sum_{i=1}^{k} \frac{n_i}{N_i} = 1$$

where:
- $n_i$ is the number of cycles accumulated at stress $S_i$.
- $N_i$ is the life (in cycles) at the same over stress level $S_i$.
- $D_{tot}$ is the fraction of life consumed by exposure to the cycles at the different stress levels. In general, when the damage fraction reaches 1, failure occurs.
Fatigue crack propagation

The fracture of the mechanical components under fatigue load conditions generally distinguishes in three stages:

- Stage I: Non-propagating fatigue crack (\(\sim 0.25\text{nm/cycle}\)).
- Stage II: Stable fatigue crack propagation - widely study
- Stage III: Unstable fatigue crack propagation \(\rightarrow\) Failure

Each stage of fatigue fracture ranging from crack nucleation to catastrophic unstable fracture is controlled by different properties such as surface properties, mechanical and metallurgical properties of the components in question. Any of the factors related to the material geometry of the component and loading condition which can delay the completion of any of the above three stages of the fatigue will increase the fatigue life.

For design against fatigue failure, fracture mechanics is utilized to monitor the fatigue crack growth rate in stage II.
Fatigue crack propagation

Figure: Fatigue crack growth behavior

\[ \frac{da}{dN} = C(\Delta K)^m \]

Region I: slow crack growth region
Region II: power law region
Region III: rapid, unstable crack growth

\( K_c \) or \( K_{Ic} \) final failure

\( \Delta K, \text{ MPa m}^{1/2} \)

\( da/dN, \text{ mm/cycle} \)

Threshold \( \Delta K_{th} \)
Fatigue crack propagation

- For design against fatigue failure, fracture mechanics is utilized to monitor the fatigue crack growth rate in stage II, Paris regime.
- Stage II fatigue crack growth propagation has been widely investigated in order to determine the fatigue crack growth life from the representing stable fatigue crack growth rate.
- The linear part of the log(da/dN) vs. log(K) curve is described by Paris’ law as follows:

\[
\frac{da}{dN} = C(\Delta K)^m = C(\Delta \sigma \sqrt{\pi a Y})^m
\]

Where the fatigue crack growth rate da/dN varies with stress intensity factor range \( \Delta K \), which is a function of stress range \( \Delta \sigma \), crack size \( a \) and geometry factor \( Y \).
Fatigue crack propagation

The fatigue crack growth life \( N_f \) (stage II) can be determined by:

\[
N_f = \int_0^{N_f} dN
\]

\[
N_f = \frac{a_f^{-(m/2)+1} - a_0^{-(m/2)+1}}{(-m/2 + 1) C(\Delta\sigma)^m \pi^{m/2} Y^m}
\]

where: \( m \neq 2 \) and \( a_0 \) is an initial crack size \( a_f \) is critical crack size:

\[
a_f = \frac{1}{\pi} \left( \frac{K_{ic}}{Y \sigma_{max}} \right)^2
\]

where \( K_{ic} \) is the fracture toughness or the tenacity of material (\( K_{ic} \) is also critical stress intensity factor).
Basic concepts

Rainflow counting algorithm was developed by Japanese engineers, Tatsuo Endo and M. Matsuishi in 1968. The rainflow counting is used to determine the cycles of different mean stress and stress range present in a complex load-time history. In 1985, first ASTM Rainflow Counting standard E1049-85 is published. This standard is a compilation of acceptable procedures for cycle-counting methods employed in fatigue analysis.

Figure: Basic Fatigue Loading Parameters E 1049
Basic concepts

The most important concepts are introduced in E 1049 standard:  
*Range* is the algebraic difference between successive valley and peak loads (positive range) or between successive peak and valley loads (negative range).  
*Peak* is the derivative of the load-time history changes from a positive to a negative sign; the point of maximum load in constant amplitude loading.  
*Valley or trough* is the point at which the first derivative of the load-time history changes from a negative to a positive sign; the point of minimum load in constant amplitude loading.  
*Reversal* is the derivative of the load-time history changes sign.  
*Mean crossing or zero crossing* is the number of times that the load-time history crosses the mean-load level during a given length of the history (positive slope, negative slope, or both, as specified). Normally only crossings with positive slopes are counted.
Rainflow counting rules

Consider the process as a sequence of roofs with rain falling them. The rain-flow paths are defined according to the following rules.

- A rain flow is started at each peak and valley (trough).
- When a rain flow path started at a trough comes to a tip of the roof, the flow stops if the opposite valley is more negative than the one the flow started from.
- For a flow started at a peak it is stopped by a peak which is more positive than the one the flow started from.
- If the rain flowing down a roof intercepts flow from an earlier path, the present path is stopped.
- A new path is not started until the path under consideration is stopped.
Procedures for cycle Counting

The procedure for rainflow counting algorithm is summarized as follows:
- Step 1: Reduce the time history to a sequence of (tensile) peaks and (compression) valleys.
- Step 2: Imagine that the time history is a pagoda roof or a rigid sheet.
- Step 3: Rotate the loading history $90^\circ$ such that the time axis is vertically downward and the load time history resembles a pagoda roof.
- Step 4: Each tensile peak is imagined as a source of water that ”drips” down the pagoda.
Procedures for cycle Counting

- Step 5: Count the number of half-cycles by looking for terminations in the flow occurring when either:
  - It reaches the end of the time history;
  - It merges with a flow that started at an earlier tensile peak; or
  - It flows when an opposite tensile peak has greater magnitude.
- Step 6: Repeat step 5 for compression troughs
- Step 7: Assign a magnitude to each half-cycle equal to the stress difference between its start and termination.
- Step 8: Pair up half-cycles of identical magnitude (but opposite sense) to count the number of complete cycles. Typically, there are some residual half-cycles.
Half cycles of trough originated stress range magnitudes, \( S_i \), would be, for example, projected distances on the stress axis, (e.g., Fig. 4, [1-8], [3-3a], [5-54 etc.).

It should be noted that for \( X(t) \) sufficiently long, any trough originated half cycle will be followed by another peak originated half cycle for the same range. For simplicity, stress ranges and cycles are estimated herein by starting paths at the troughs only.

The Simulation

Because of the complexity of the rain-flow algorithm, it would be extraordinarily difficult to derive \( f_s(s) \) from a given \( W(f) \). However, Monte-Carlo methods can be used to simulate \( X(t) \) and estimate \( f_s(s) \).

A digital computer program with the capabilities shown in the flow chart Fig. 5 has been developed. Given an arbitrary \( W(f) \), a sample of \( X(t) \) is simulated using the following form (22)

\[
Z = L \left[ 2G(\omega)K^2 \right] \ln \cos(\omega K t + \phi K) (11)
\]

where \( \omega (rad/s) \) is the one sided spectral density function in terms of frequency, \( (a) = \frac{W(f)}{2\pi} \). Frequency is defined over the interval \((0, \pi)\) with partitions of length \( A\omega \).

If all \( A\omega \) are equal, \( X(t) \) will be periodic with a period of the reciprocal of the minimum frequency of the input spectral density. This problem is avoided by using random intervals for \( A\omega \). A typical simulation of \( X(t) \) from \( W(f) \) is shown in Fig. 1-a, and 6.

After \( X(t) \) is simulated for life \( T \), it is converted to point processes of peaks and troughs. A sample of \( S_i, S_i \) is obtained using the rain flow cycle counting method.

Results

Simulations of \( X(t) \) for eleven different forms of \( W(f) \) at values of \( \alpha \) ranging from 0.218 to 0.998 were performed. The \( W(f) \) used were mostly of regular and smooth form (rectangular and triangular shape), typical forms of which are shown in Fig. 6. Also the process of Fig. 1 was used. For each simulation \( S_i(i = 1, n) \) and \( n \) were recorded. The \( n \) was found to be within 5 percent of \( m<\alpha T \) in all cases. Therefore \( n \) is assumed to be deterministic and equal to \( m<\alpha T \).

\( \alpha \) and \( \alpha \) are estimated by \( S_i \) and \( ss \), respectively, where

\[
S_i = \left( \frac{2}{\pi} \right) \frac{1}{m<\alpha T} \int_0^{m<\alpha T} S(t) dt
\]

\( S_i \) and \( S_i \) are estimated by \( S_i \) and \( ss \), respectively.
The rain flow count applied to the process in Figure 11 results in:

- half cycles of trough generated stress ranges
- half cycles of peak generated stress ranges

In both cases, the stress ranges are found as the projected distances on the stress axis.

Figure 12 shows the details of the rain flow count of the process from Figure 11.

Half cycles of trough generated stress ranges:
1-8, 3-3a, 5-5a, 7-7a, 9-10, 11-12, and 13-14

Half cycles of peak generated stress ranges:
2-3, 4-5, 6-7, 10-12b, 12-12a, and 8-(9)-13

Total count:
- Full cycles: 2-3-3a, 4-5-5a, 6-7-7a, 9-10-12b(9) and 11-12-12a(11)
- Half cycles: 1-8, 8-13, and 13-14

It should be noted that for sufficiently long, it can be shown, Ref. /3/, that any trough originated half cycle will be followed by another peak originated half cycle for the same range. This is also the case for short stress histories if the stress history starts and ends at the same stress value.

Although the rain flow counting method is widely considered superior to other counting methods for fatigue calculation, a basic criticism of the method as used above has been that the fatigue damage procedure cannot account for the sequence of the stress ranges. This criticism is justified especially since fatigue tests have revealed that the sequence of the stress ranges in some cases is of importance. However, the effect of sequence has been found to even out in fatigue calculations where many time histories are considered, and currently no better procedures are available.

References:
Stress range (MPa)

Time series

-80
-60
-40
-20
0
20
40

Stress range (MPa)

Time series

4.1 4.15 4.2 4.25 4.3 4.35