Fatigue assessment

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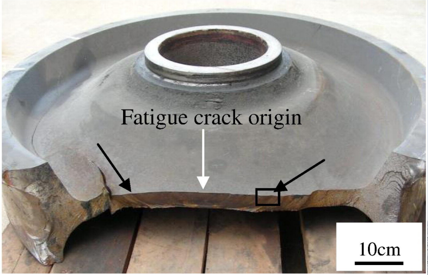



4-28-1988 After 89,090 flight cycles on a 737-200, metal fatigue lets the top go in flight.


Tanker S.S. Schenectady fractured a day after its launch in January 1941[3].

Concept of Fatigue

- The word fatigue comes from the Latin verb fatigare - "to tire".
- Fatigue is the damage of a structural part by the initiation and gradual propagation of a crack or cracks caused by repeated applications of stress. (BS 7608-1993)
- The process of initiation and propagation of cracks through a structural part due to the action of fluctuating stress. (Eurocode 1993.1.9)
- The fatigue is defined as the deterioration of a component caused by the crack initiation or by the growth of a crack (new IIW recommendation 2008).

Introduction

- Fatigue failures are widely studies because it accounts for 90% of all service failures due to mechanical causes.
- Fatigue failures occur when metal is subjected to a repetitive or fluctuating stress and will fail at a stress much lower than its tensile strength.
- Fatigue failures occur without any plastic deformation (no warning).
- Fatigue surface appears as a smooth region, showing beach mark or origin of fatigue crack.

Factors causing fatigue failure

- **Basic factors:**
  - A maximum tensile stress of sufficiently high value.
  - A large amount of variation or fluctuation in the applied stress.
  - A sufficiently large number of cycles of the applied stress.
- **Additional factors:**
  - Stress concentration
  - Corrosion
  - Temperature
  - Overload
  - Metallurgical structure
  - Residual stress
  - Combined stress

Fatigue failure

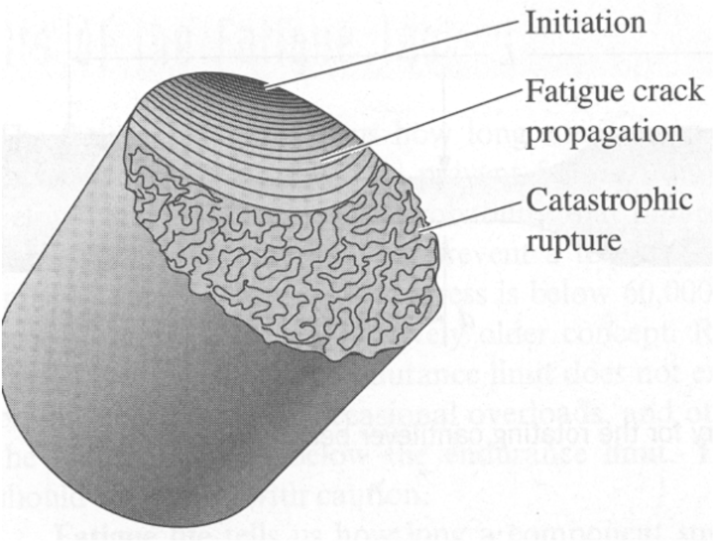


Figure: Fatigue failure

Fatigue loading

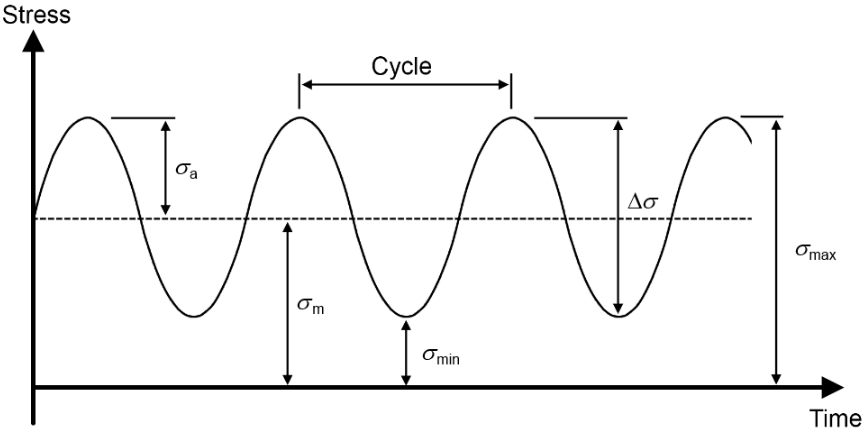
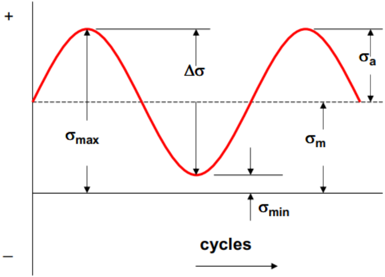


Figure: Fatigue stress cycle

Fatigue loading



- **Stress range:** $\Delta\sigma = \sigma_{max} - \sigma_{min}$
- **Stress amplitude:** $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$
- **Mean stress:** $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$
- **Stress ratio:** $R = \dfrac{\sigma_{min}}{\sigma_{max}}$
- **Amplitude ratio:** $A = \dfrac{\sigma_a}{\sigma_m} = \dfrac{1-R}{1+R}$

The S-N curve

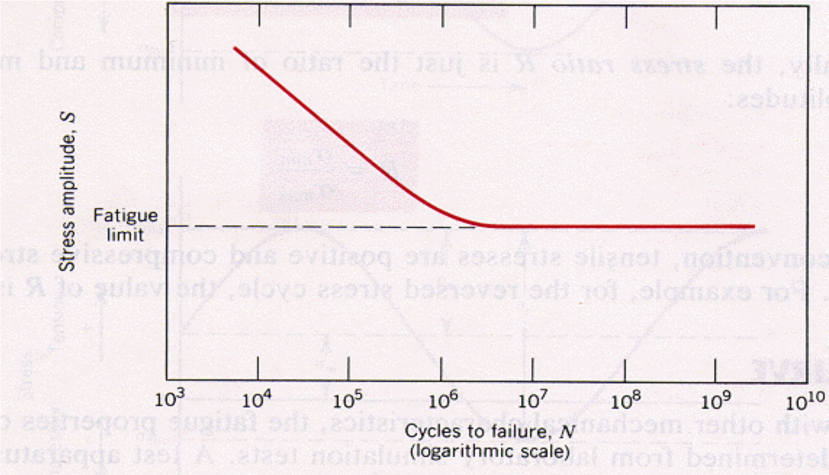


Figure: Typical fatigue curves (a)

The S-N curve

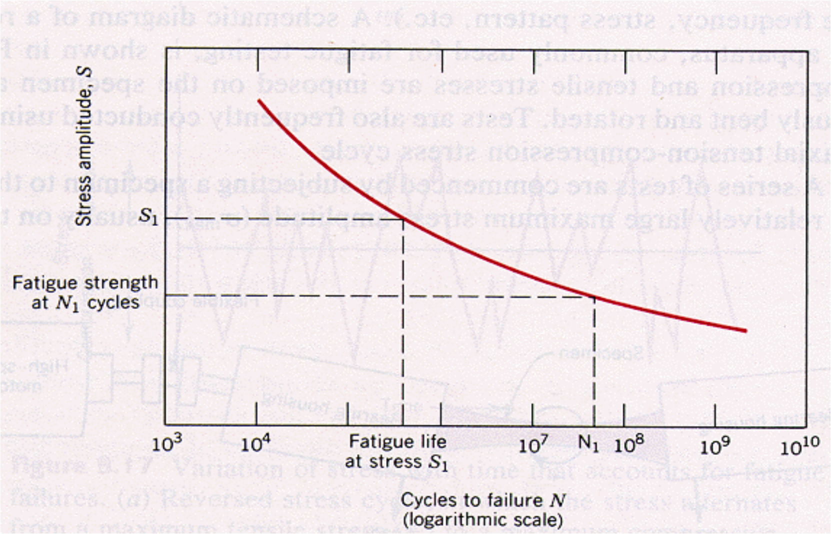
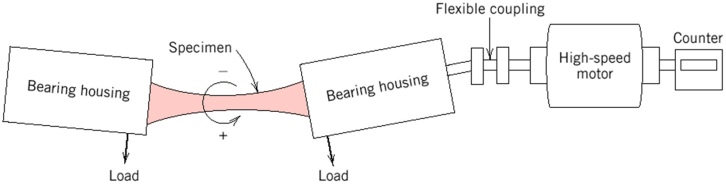


Figure: Typical fatigue curves (b)

The S-N curve

Rotating-bending test → S-N curves



S (stress) vs. N (number of cycles to failure)

S-N curve is concerned chiefly with fatigue failure at high numbers of cycles.

- $N > 10^4$ cycles : high cycle fatigue
- $N < 10^4$ cycles : low cycle fatigue

Fatigue limit or endurance limit is normally defined at 10^7 or 10^8 cycles. Below this limit, the material presumably can endure an infinite number of the cycle before failure.

The nonferrous metal, i.e., aluminum, do not have the fatigue limit.

The S-N curve

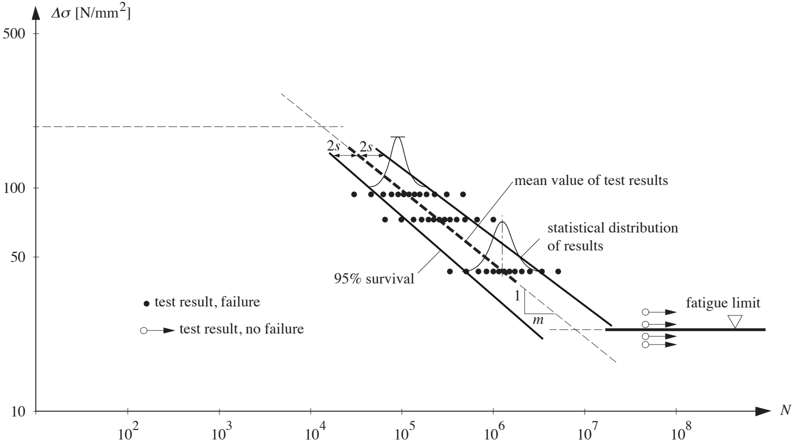


Figure: Fatigue test results of structural steel members, plotted in double logarithm scale, carried out under constant amplitude loading (TGC 10, 2006)

The S-N curve

By using a logarithmic scale for both axes, the mean value of the test results for a given structural detail can be expressed, in the range between 10^4 cycles and 5.10^6 to 10^7 cycles, by a straight line with the following expression:

$$N = C.\Delta\sigma^{-m}$$

where:
N number of cycles of stress range $\Delta\sigma$,
C constant representing the influence of the structural detail,
$\Delta\sigma$ constant amplitude stress range,
m slope coefficient of the mean test results line.
The expression represents a straight line when using logarithmic scales:

$$logN = logC - m.log(\Delta\sigma)$$

The S-N curve

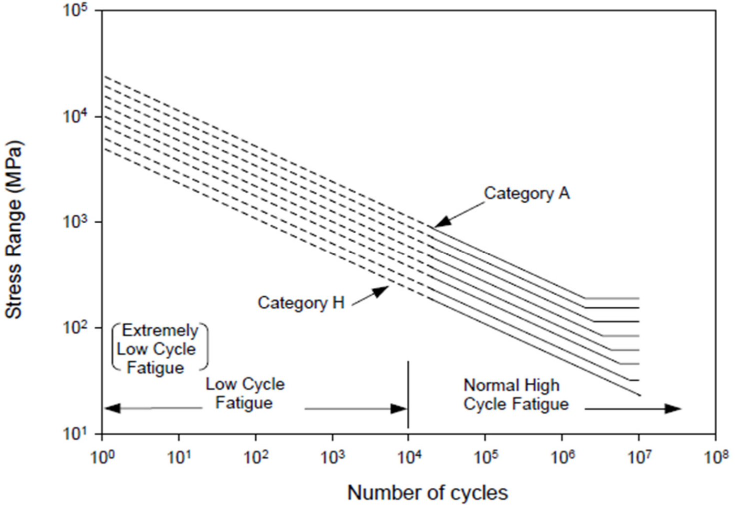


Figure: Fatigue classification

Cumulative damage

Miner′s rule is one of the most widely used cumulative damage models for failures caused by fatigue. It states that if there are k different stress levels and the average number of cycles to failure at the *i*th stress, Si, is Ni, then the damage fraction, D_{tot}, is:

$$D_{tot} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + ... + \frac{n_k}{N_k} = \sum_{i=1}^{k} \frac{n_i}{N_i} = 1$$

where:

- n_i is the number of cycles accumulated at stress S_i.
- N_i is the life (in cycles) at the same over stress level S_i.
- D_{tot} is the fraction of life consumed by exposure to the cycles at the different stress levels. In general, when the damage fraction reaches 1, failure occurs.

Fatigue crack propagation

The fracture of the mechanical components under fatigue load conditions generally distinguishes in three stages:

- Stage I: Non-propagating fatigue crack ($\sim$0.25nm/cycle).
- Stage II: Stable fatigue crack propagation - widely study
- Stage III: Unstable fatigue crack propagation $\rightarrow$ Failure

Each stage of fatigue fracture ranging from crack nucleation to catastrophic unstable fracture is controlled by different properties such as surface properties, mechanical and metallurgical properties of the components in question. Any of the factors related to the material geometry of the component and loading condition which can delay the completion of any of the above three stages of the fatigue will increase the fatigue life.
For design against fatigue failure, fracture mechanics is utilized to monitor the fatigue crack growth rate in stage II.

Fatigue crack propagation

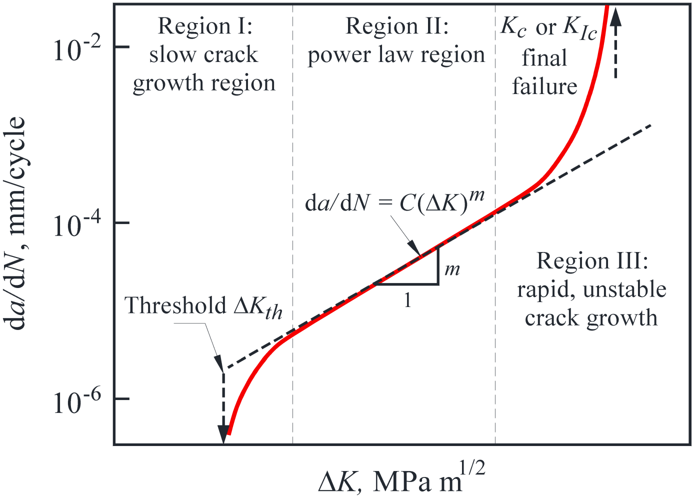


Figure: Fatigue crack growth behavior

Fatigue crack propagation

- For design against fatigue failure, fracture mechanics is utilized to monitor the fatigue crack growth rate in stage II, Paris regime.
- Stage II fatigue crack growth propagation has been widely investigated in order to determine the fatigue crack growth life from the representing stable fatigue crack growth rate.
- The linear part of the log(da/dN) vs. log(K) curve is described by Paris′ law as follows:

$$\frac{da}{dN} = C(\Delta K)^m = C(\Delta\sigma\sqrt{\pi a}Y)^m$$

Where the fatigue crack growth rate da/dN varies with stress intensity factor range ΔK, which is a function of stress range $\Delta\sigma$, crack size a and geometry factor Y.

Fatigue crack propagation

The fatigue crack growth life N_f (stage II) can be determined by:

$$N_f = \int_0^{N_f} dN$$

$$N_f = \frac{a_f^{-(m/2)+1} - a_0^{-(m/2)+1}}{(-m/2+1)C(\Delta\sigma)^m \pi^{m/2} Y^m}$$

where: m $\neq$ 2 and a_0 is an initial crack size a_f is critical crack size:

$$a_f = \frac{1}{\pi}(\frac{K_{ic}}{Y\sigma_{max}})^2$$

where K_{ic} is the fracture toughness or the tenacity of material (K_{ic} is also critical stress intensity factor).

Basic concepts

Rainflow counting algorithm was developed by Japanese engineers, Tatsuo Endo and M. Matsuishi in 1968. The rainflow counting is used to determine the cycles of different mean stress and stress range present in a complex load-time history. In 1985, first ASTM Rainflow Counting standard E1049-85 is published. This standard is a compilation of acceptable procedures for cycle-counting methods employed in fatigue analysis.

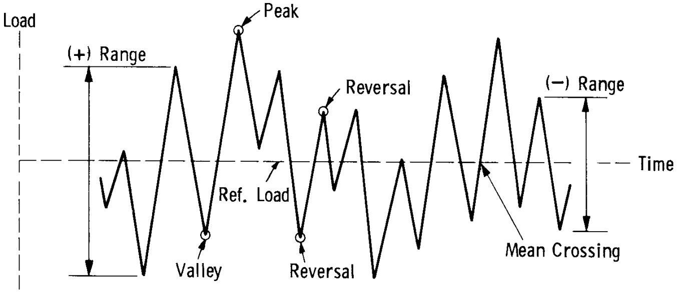


Figure: Basic Fatigue Loading Parameters E 1049

Basic concepts

The most important concepts are introduced in E 1049 standard :
Range is the algebraic difference between successive valley and peak loads (positive range) or between successive peak and valley loads (negative range).
Peak is the derivative of the load-time history changes from a positive to a negative sign; the point of maximum load in constant amplitude loading.
Valley or trough is the point at which the first derivative of the load-time history changes from a negative to a positive sign; the point of minimum load in constant amplitude loading.
Reversal is the derivative of the load-time history changes sign.
Mean crossing or zero crossing is the number of times that the load-time history crosses the mean-load level during a given length of the history (positive slope, negative slope, or both , as specified). Normally only crossings with positive slopes are counted.

Rainflow counting rules

Consider the process as a sequence of roofs with rain falling them. The rain-flow paths are defined according to the following rules.

- A rain flow is started at each peak and valley (trough).
- When a rain flow path started at a trough comes to a tip of the roof, the flow stops if the opposite valley is more negative than the one the flow started from.
- For a flow started at a peak it is stopped by a peak which is more positive than the one the flow started from.
- If the rain flowing down a roof intercepts flow from an earlier path, the present path is stopped.
- A new path is not started until the path under consideration is stopped.

Procedures for cycle Counting

The procedure for rainflow counting algorithm is summarized as follows:

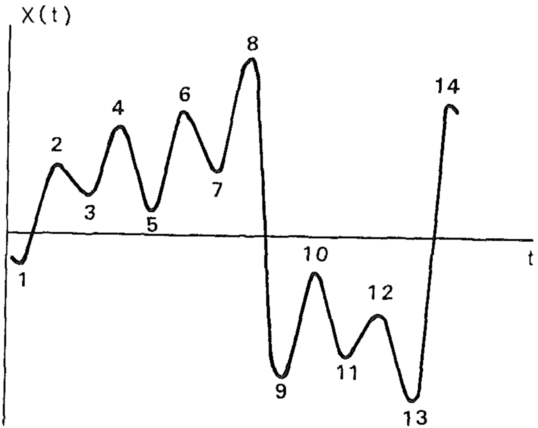
- Step 1: Reduce the time history to a sequence of (tensile) peaks and (compression) valleys.
- Step 2: Imagine that the time history is a pagoda roof or a rigid sheet.
- Step 3: Rotate the loading history 90^0 such that the time axis is vertically downward and the load time history resembles a pagoda roof.
- Step 4: Each tensile peak is imagined as a source of water that "drips" down the pagoda.

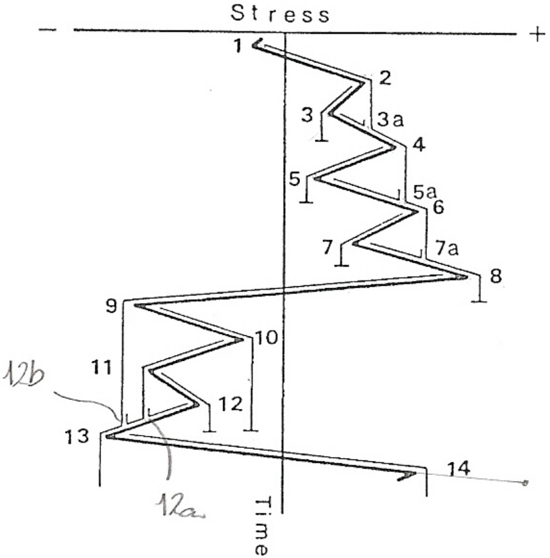
Procedures for cycle Counting

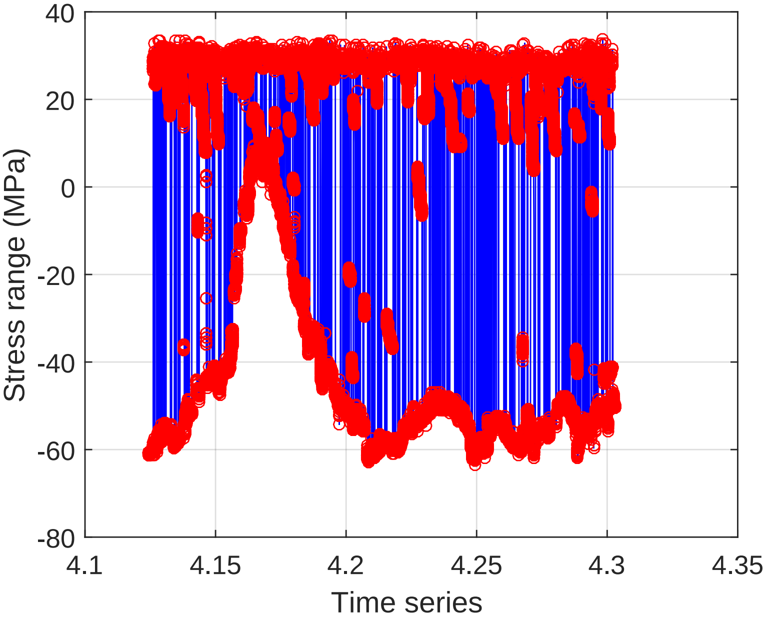
- Step 5: Count the number of half-cycles by looking for terminations in the flow occurring when either:

  - It reaches the end of the time history;
  - It merges with a flow that started at an earlier tensile peak; or
  - It flows when an opposite tensile peak has greater magnitude.

- Step 6: Repeat step 5 for compression troughs
- Step 7: Assign a magnitude to each half-cycle equal to the stress difference between its start and termination.
- Step 8: Pair up half-cycles of identical magnitude (but opposite sense) to count the number of complete cycles. Typically, there are some residual half-cycles.


X(t)
8
14
6
4
2
3
5
7
1
t
10
12
11
9
13


Stress
−
+
1
2
3
3 a
4
5
5a
6
7
7a
8
9
10
11
12b
12
13
14
12a
Time


40
20
0
-20
-40
-60
-80
Stress range (MPa)
4.1
4.15
4.2
4.25
4.3
4.35
Time series