Land Subsidence (Proceedings of the Fifth International Symposium on Land Subsidence, The Hague, October 1995). IAHS Publ. no. 234, 1995.

On the necessity to consider varying parameters in land subsidence computations

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Abstract Geotechnical scientists have long been aware that during consolidation of compressible layers, changes in porosity due to a rearrangement of the soil skeleton may lead to decrease of both the permeability and the compressibility of the porous medium. The variation of these two parameters influences strongly the further consolidation processes. These variations must be taken into account in the land subsidence models. These variable and highly interdependent geotechnical parameters introduce nonlinearities and coupling in the numerical procedure to simulate the consolidation process. A fully nonlinear approach is proposed to compute with accuracy the land subsidence due to groundwater withdrawal in loose sediments. Of course, field and laboratory data are needed to implement the additional constitutive laws describing the variations of the hydraulic conductivity and of the compressibility (coupled to the specific storage coefficient). Comparisons between computations with constant parameters and with varying parameters have been performed. The computed pore pressures are strongly affected, inducing automatically the main differences in the calculated subsidences. Moreover, it is demonstrated that with identical pore pressure distribution and initial parameters, the subsidence computed with constant parameters will be systematically overestimated when compared with those computed with varying parameters. These developments are illustrated by some linear and nonlinear computations, realized on the case study of Shanghai.

INDUCED LAND SUBSIDENCE

Young unconsolidated or semi-consolidated sediments of high porosity, laid down in alluvial or shallow marine environments, form a succession of layers which can often be characterized (from an hydrogeological point of view) as semiconfined or confined aquifer systems (Poland, 1984). They consist of silty sand and sand aquifers of high permeability (hydraulic conductivity) and low compressibility, interbedded with clayey aquitards characterized by low vertical permeability and high compressibility. According to Terzaghi (1943), the geostatic pressure or total stress (σ) that any point undergoes in the soil is considered as the result of two additional components: the fluid pore pressure and the effective stress (σ'). The global soil compressibility is a factor 20 to 1000 larger than grain compressibility, so that it would be useless to choose another principle based on, for example, Biot's theory (1956). Applying Terzaghi's principle, the lowering of

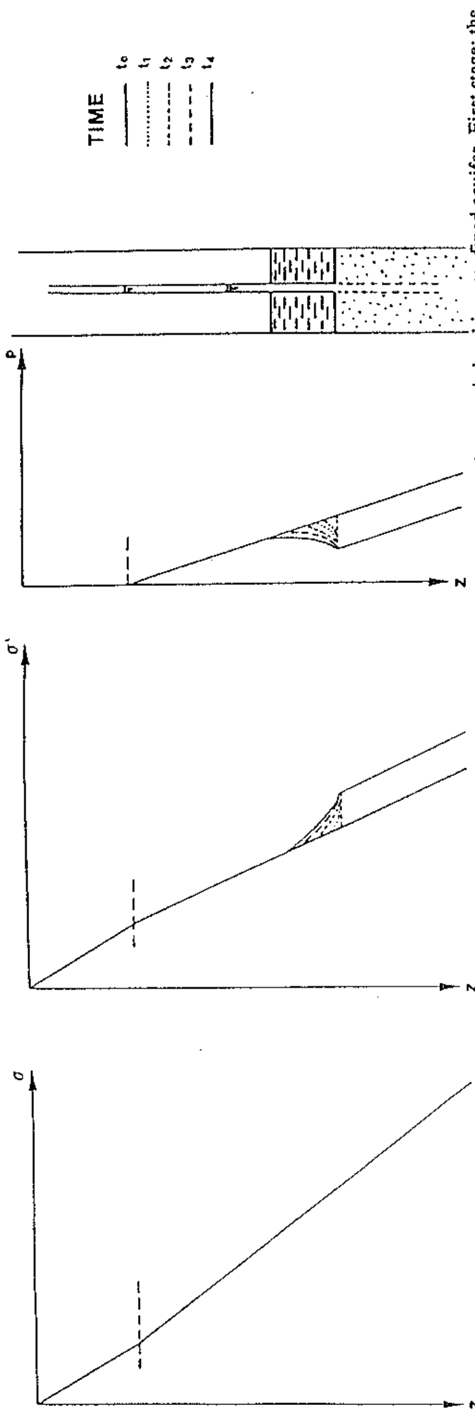

Fig. 1 Evolution of the total stress, pore pressure and effective stress as a function of depth when lowering the piezometric head in a confined aquifer. First stage: the pore pressure decrease has not yet propagated above the confining layer.

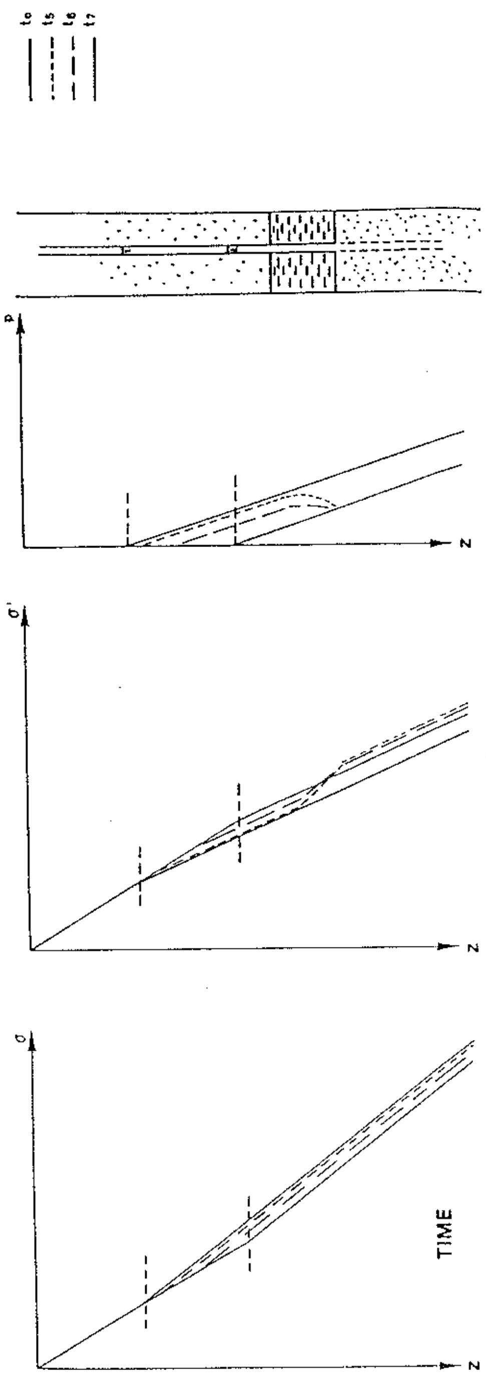

Fig. 2 Second stage: the pore pressure decrease has partially propagated.

the piezometric head in a confined aquifer (Fig. 1) induces additional effective stresses in the layers. If an initial hydrostatic pore pressure is assumed, the pore pressure is decreased in the aquifer and partially decreased in the underlying/overlying aquitards. During this stage, the total stress can be assumed constant as the pressure decrease propagation through the aquitards is very slow. Moreover, the upper layers can be saturated by the recharge. The slow propagation of the pore pressure variation in semi-permeable layers induces automatically an equivalent increase in effective stress in these compressible layers and a drained consolidation process is started. After a very long period (Fig. 2), the pore pressure decrease can reach the top of the confining layer and then, as for unconfined situations, a decrease of the thickness of saturated soils is recorded. In this case, the total stress can begin to vary (except if there is an important infiltration or recharge). There is a strong transient behaviour of the process due to the fact that the main consolidation (usually called primary consolidation) is activated by the decrease of pore pressure as long as hydrostatic equilibrium is not restored.

Physically, the structural evolution of clays during the consolidation process is dominated by the reduction of the pore dimensions, so that the whole porosity is decreasing. Many authors have observed that clay minerals tend to orient their plates orthogonally to the direction of the main applied stress developing a kind of structural anisotropy (Delage & Lefebvre, 1984; and Rieke & Chilingarian, 1974).

RHEOLOGY OF THE LOOSE SEDIMENTS

The geomechanical behaviour of the soils can be idealized in terms of rheological models. The skeleton deformation under increasing effective stress is supposed to follow elastic, plastic or visco-elastic laws or any combination of these. For example, an elasto-plastic material can be represented by a Hookean spring and a Saint-Venant resistance placed in series, a visco-elastic model describing the creep of clayey soils consists in a spring and a dashpot taken in parallel (Kelvin model). Many other models can be imagined by combination in parallel or in series of these global or individual units, trying to reproduce the real behaviour of lose soils and recently deposited sediments. Clayey soils and loose sediments have a geomechanical behaviour qualified often as nonlinear elasticity with progressive plasticity and viscosity. This particular behaviour leads, in practice, to choose models based on experimental laws rather than on combinations of theoretical models. Elasto-visco-plastic laws in one or three dimensions can be established from experimental results. Different loading steps, more or less elaborated, can be applied to samples in order to find the parameters of an experimental law. These experimental constitutive laws allow a more simple introduction of nonlinear and interaction effects of the parameters.

HYDROGEOLOGICAL AND GEOTECHNICAL PROPERTIES

For modelling purposes, space integration is realized on a Representative Elementary Volume (REV) to obtain macroscopic values for porosity and permeability (hydraulic conductivity) of this porous medium. The porosity describes the reservoir property to release a fluid quantity; the permeability describes the reservoir ability to convey the fluid flow.

The hydraulic conductivity or permeability coefficient (K) may be expressed as:

$$K = \frac{k \cdot \rho \cdot g}{\mu} \quad (1)$$

where k is the intrinsic permeability of the porous medium, ρ the density of water, g is the constant of gravity and μ the viscosity of the water. The generalized Darcy's law, in three dimensions can be written as:

$$\underline{v} = \frac{\underline{\underline{k}}}{\mu}(\underline{\nabla} p + \rho . g\ \underline{\nabla} z) = -\underline{\underline{K}}\ \underline{\nabla} h \quad (2)$$

where $\underline{v}$ is the Darcy velocity vector and $\underline{\underline{K}}$ is the permeability coefficient tensor. For silty and clayey semi-pervious formations, the permeability is often measured during consolidation tests (oedometer and triaxial tests). The values are obtained at different stages of effective stress, leading to the relation: $K = f(\sigma')$. The groundwater flow in a saturated porous medium can be expressed by the continuity equation:

$$\mathrm{div}(\rho . \underline{v}) + \rho . q = -\frac{\partial}{\partial t}(\rho . n) \quad (3)$$

where n is the porosity and q is flow rate (per volume unit) exchanged by the REV with the outside environment (positive if flow is entering into the system). The right-hand side of equation (3) characterizes the aquifer capacity to store or release a volume of water in function of the pore pressure prevailing in the formation. Seven important assumptions are needed to define the specific storage coefficient of a saturated porous medium:

- the REV concept is applied,
- isothermal conditions,
- the fluid is homogeneous, so that $\rho = \rho(p)$,
- the Darcy's velocity is a relative filtration velocity,
- the solid density (ρ_s) is constant, so that the compressibility of the solid grains is negligible,
- Terzaghi's effective stress principle holds,
- the total stress is constant.

In these conditions:

$$\frac{\partial}{\partial t}(\rho . n) = \rho . S_s . \frac{\partial h}{\partial t} \quad \text{with} \quad S_s = \rho . g . (\alpha + n\beta) \quad (4)$$

where S_s is the specific storage coefficient of a saturated porous medium, α the volume compressibility coefficient of the porous medium, and β the water compressibility coefficient. Equation (4) shows the direct coupling between the transient groundwater flow and the consolidation processes, as the specific storage coefficient (S_s) is expressed in function of the compressibility coefficients of the porous medium and water.

The specific storage coefficient is determined on basis of oedometer tests realized with the following assumptions:

- the total stress is constant (drained test),
- lateral deformations are prevented and neglected,
- uniaxial state of stress and strain.

Moreover, fluid and solid grains compressibilities are neglected in regard to α so that:

$$S_s = \rho . g \alpha \quad (5)$$

Generally, the S_s values obtained by pumping tests are larger than those obtained on samples by consolidation tests (Domenico & Mifflin, 1965). In loose sedimentary layers, it is really difficult to make such comparisons as undisturbed sampling is difficult in silty to sandy aquifer layers, and moreover the piezometric levels are hard to measure with accuracy in clayey aquitards.

PERMEABILITY AND COMPRESSIBILITY VARIATIONS

During consolidation of highly compressible clays, changes in porosity due to a rearrangement of the soil skeleton may lead to decreases in both the permeability and the compressibility of the porous medium. For example, Lambe & Whitman (1969) have presented data indicating that permeability values can change by orders of magnitude and compressibility can decrease significantly as void ratio is decreased. Neither of these variations is linear with void ratio.

Nonlinearity of the specific storage coefficient linked to the compressibility

Assuming a constant total stress and a negligible compressibility of water, the equation (5) can be used. By definition, the volumetric coefficient (α) is written:

$$\alpha = \frac{d\varepsilon_v}{d\sigma'} \quad (6)$$

where ε_v is the relative volumetric strain and σ' is the effective stress. We obtain:

$$\alpha = -\frac{dV}{V . d\sigma'} \quad \alpha = -\frac{dn}{(1-n) . d\sigma'} \quad \alpha = -\frac{de}{(1+e) . d\sigma'} \quad (7)$$

where n and e are respectively the porosity and the void ratio at the beginning of the effective stress variation ($d\sigma'$).

The oedometer tests are the common one dimensional consolidation tests. The results are usually plotted on (σ', ε_v) diagrams, allowing to determine α for each effective stress level (Fig. 3).

The compressibility coefficient for sandy to clayey materials depends on effective stress and on the effective preconsolidation stress value (σ'_p). In order to linearize

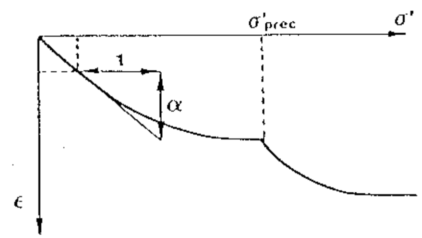

Fig. 3 Results of an oedometer test.

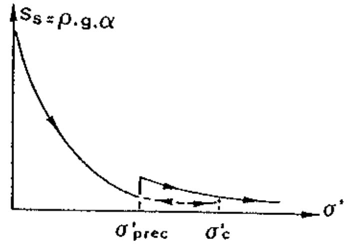

Fig. 4 Variation of the specific storage coefficient (S_s) in function of the effective stress σ'.

oedometer curves (ε_v – $\ln(\sigma')$ or (e – $\ln(\sigma')$ diagrams can be used, and the compressibility coefficient can be expressed by:

$$\alpha(\sigma') = \frac{1}{A.\sigma'} \quad \sigma' < \sigma'_p$$
$$\alpha(\sigma') = \frac{1}{C.\sigma'} \quad \sigma' \geq \sigma'_p \tag{8}$$

One can remark that the specific storage coefficient is expressed as a function of $1/\sigma'$ values (Fig. 4). A more general relation linking the void ratio (e) to the effective stress, can be proposed as follows (Feldkamp, 1989):

$$e = \frac{a}{\sigma'^b} \tag{9}$$

where a and b are experimentally determined for each material. The compressibility coefficient can then be expressed as a function of the void ratio only:

$$\alpha(e) = c.\frac{e^d}{1+e} \tag{10}$$

where c and d are experimentally determined for each material.

Nonlinearity of the permeability

To interpret porosity logs in terms of reservoir permeability, many relations linking the permeability coefficient K to total porosity or void ratio are used in reservoir engineering. However, these relations are not applicable for consolidation and subsidence computations in loose sediments as they are derived for hardened rocks. In our case, we are looking for empirical (experimentally found) relations linking permeability to void ratio or porosity, in layers characterized generally by high clay and peat contents, high compressibilities and low permeabilities. Many factors influence the permeability values:

- lithology,
- grain size,
- shapes, orientations and specific surface of the grains,
- spatial distribution of pores.

As mentioned above, the micro-structural evolution of clays during consolidation, orienting the plates more orthogonally to the direction of the vertical applied effective stress, develops an increasing structural anisotropy. This evolution increases the tortuosity of the flow channels when the groundwater flow is parallel to the vertical effective stress. This statement does not rule out the decrease in K by a decrease of the total void ratio. For practical purposes, it could be convenient to establish a relation K = f(e) in a similar form to the formalism of the (log σ', e) oedometer relations (Fig. 5):

$$e = C_{K_1} \log K + cst \quad K > K_p$$
$$e = C_{K_2} \log K + cst \quad K \leq K_p \tag{11}$$

where K_p is the permeability coefficient corresponding to the effective preconsolidation stress (σ'_p), C_{K_1} and C_{K_2} are defined respectively as the elastic and plastic rate of K-variation during the consolidation. The constant being determined experimentally, the relation could be generalized:

$$K = \frac{C}{\sigma'^a} \tag{12}$$

However it could be difficult to determine α or C_{K_1} and C_{K_2} in practice. Many other relations can be experimentally fitted to the test results (Terzaghi, 1943, Rieke & Chilingarian, 1974; Barends, 1990; Safai & Pinder, 1980; Lambe & Whitman, 1969). For example, Nishida & Nakagawa (1970) have presented an equation linking K to e, taking into account the plasticity index (Ip) as an additional parameter. This last relation has been generalized and applied successfully for the computation of the subsidence in the Shanghai area (Dassargues *et al.*, 1991) with the form:

$$K = e^{a.e+b} \quad \text{where } a = \frac{2.3}{c.Ip+d} \tag{13}$$

b, c, d are experimentally determined. Therefore, many relations are known, more or less well adapted to each studied case. It is important to choose a relation and to fit the parameters, constants or exponents using the maximum of available data.

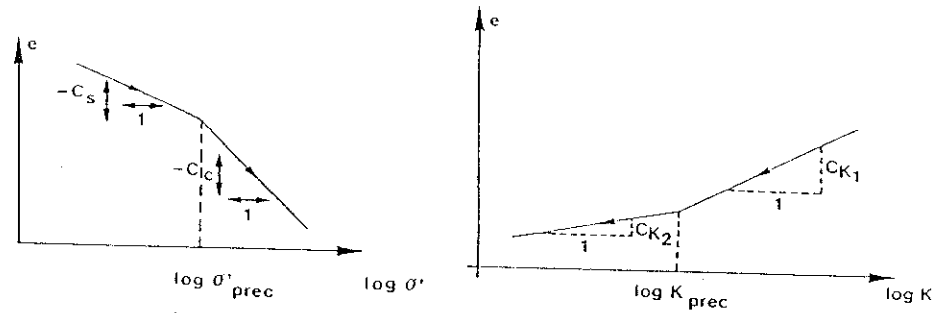

Fig. 5 (log K, e) diagram very similar to the (log σ' – e) diagram describing the oedometer test results.

INFLUENCE ON COMPUTED CONSOLIDATIONS

Rudolph & Frind (1991) have shown that for a pore pressure variation imposed at the bottom of a clayey column, it takes more time to reach permanent flow conditions with varying parameters than with constant parameters. These conclusions are always verified if K and S_s values are taken identical at the beginning of both computations. Differences in the calculated pore pressure spatial distributions induce automatically (by the Terzaghi principle) the main differences in calculated subsidences. Moreover, if pore pressures are taken rigorously identical, the subsidence computed by the simulation with constant parameters will be systematically overestimated when compared to the subsidence calculated with varying parameters (if the initial parameters are taken identical). From equations (5) and (6), the strain of the porous medium can be expressed on a time step Δt by:

$$\int_{\epsilon(t)}^{\epsilon(t+\Delta t)} d\epsilon = \int_{\sigma'(t)}^{\sigma'(t+\Delta t)} \frac{S_s}{\rho . g} d\sigma' \quad (14)$$

In the case of constant parameters (K and S_s), equation (14) becomes:

$$\int_{\epsilon(t)}^{\epsilon(t+\Delta t)} d\epsilon = \frac{S_s}{\rho . g} \int_{\sigma'(t)}^{\sigma'(t+\Delta t)} d\sigma' \quad (15)$$

On the contrary, in the case where S_s may vary as a function of σ', equation (14) becomes:

$$\int_{\epsilon(t)}^{\epsilon(t+\Delta t)} d\epsilon = \frac{1}{\rho . g} \int_{\sigma'(t)}^{\sigma'(t+\Delta t)} S_s(\sigma') d\sigma' \quad (16)$$

Replacing S_s by its value in equation (8), and after integration, the right-hand sides of equations (15) and (16) are respectively:

$$\frac{1}{\sigma'_{imp}}\left(\sigma'(t+\Delta t) - \sigma'(t)\right); \quad \ln\left[\frac{\sigma'(t+\Delta t)}{\sigma'(t)}\right]$$

where σ'_{imp} is a constant implicitly chosen when working with a constant value of S_s (in each layer). At the first time step of the computation, the first value of S_s for the non linear simulation is: $S_s = 1/(C.\sigma'_{imp}(t))$ because the initial value of S_s is the same for both simulations. Since $\exp((a - c)/c) \geq a/c$ if $a \geq c$, we can write:

$$\frac{\sigma'(t+\Delta t)}{\sigma'_{imp}(t)} - 1 \geq \ln\left[\frac{\sigma'(t+\Delta t)}{\sigma'_{imp}(t)}\right] \quad \text{with } \sigma'(t+\Delta t) \geq \sigma'_{imp}(t) \quad (17)$$

For the next time step of the simulation, we can write:

$$\frac{1}{\sigma'_{imp}(t)}\left[\sigma'(t+\Delta t) - \sigma'(t)\right] \geq \ln\left[\frac{\sigma'(t+\Delta t)}{\sigma'(t)}\right] \quad \text{with } \sigma'(t+\Delta t) \geq \sigma'(t) \geq \sigma'_{imp}(t) \quad (18)$$

because $\exp((a - b)/c) \geq a/b$ if $a \geq b \geq c$. These inequalities (17) and (18) (Dassargues, 1991) prove that if the initial parameters are taken identical, the subsidence computed by the simulation with constant parameters will be systematically overestimated when compared to the subsidence calculated with varying parameters.

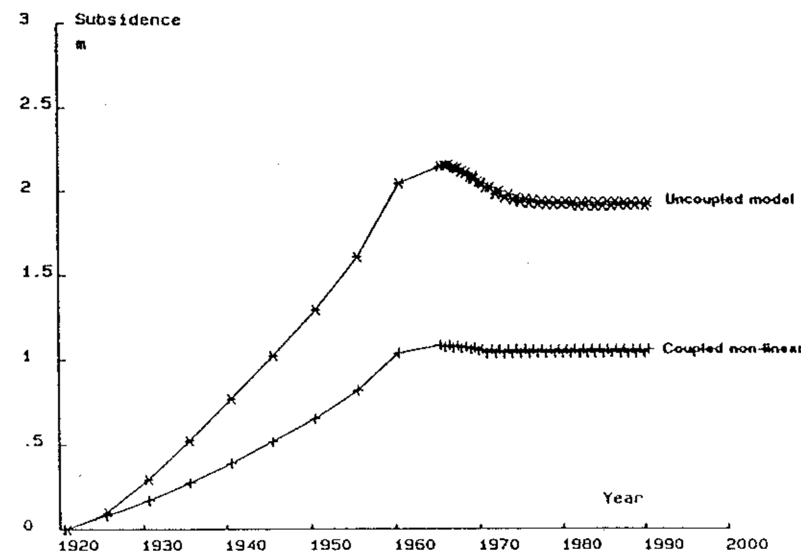

Fig. 6 Total computed subsidence since 1920 for one column of the case study of Shanghai; the computation neglecting the variation of K and S_s during the consolidation process leads to an overestimation of the subsidence of nearly 100% at this place (from Dassargues, 1991).

CONCLUSIONS AND EXAMPLE

For the case study of Shanghai, coupled and nonlinear models, taking into account the variation of the permeability and of the compressibility, have been applied. An example of results is given in Fig. 6 showing how inaccurate a model can be, when the variation of parameters in flow-compaction computations is neglected. For more details about this case history, see Dassargues *et al.* (1991) about the preparation of the hydrogeological and geotechnical data, and Dassargues & Li (1991) for a summary of the computational aspects. Moreover the Bulletin of the International Association of Engineering Geology (IAEG) has published recently a group of papers describing the whole study and the detailed data set used for the simulations: Dassargues & Zhang (1992), Schroeder *et al.* (1992), Dassargues *et al.* (1993a,b). Using the Finite Element Method (FEM), the computations are based on a detailed three dimensional flow model of the whole area. This flow model has been coupled to 32 nonlinear one dimensional flow-compaction models, located where accurate measured data were available (32 boreholes). Careful calibrations of both hydrogeological and geotechnical parameters have been made in the 32 detailed flow-compaction models. Then, future subsidence may be computed until the year 2000, with global pumping = 1.3 x recharge in the aquifers.

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