University of Liège
Faculty of Applied Sciences
Aerospace and Mechanical Department

Contributions to handle maximum size constraints in density-based topology optimization

ECCOMAS Young Investigators Conference 2017

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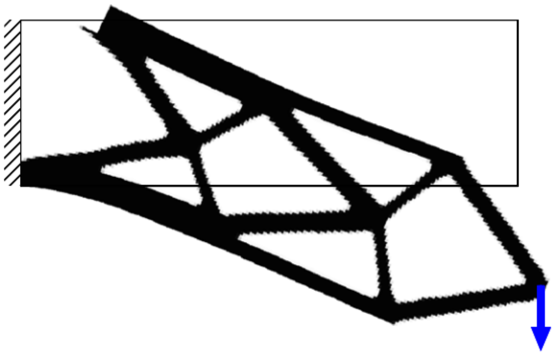
Maximum size in the design process

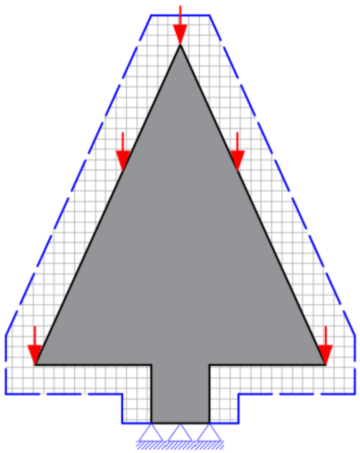
Indirect desired properties



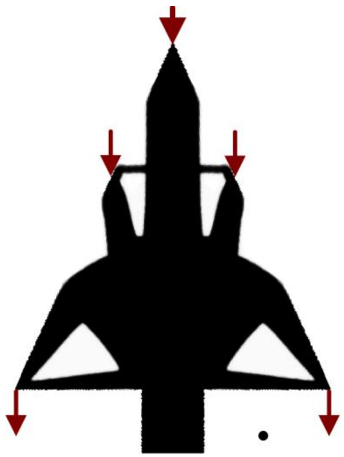
Optimized Cantilever beam under local damage scenarios , **Jansen et al (2014).**

Cantilever beam **with maximum size and minimum gap** control.

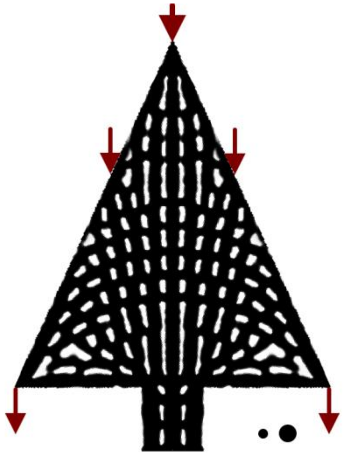
Aesthetic reasons



Design domain

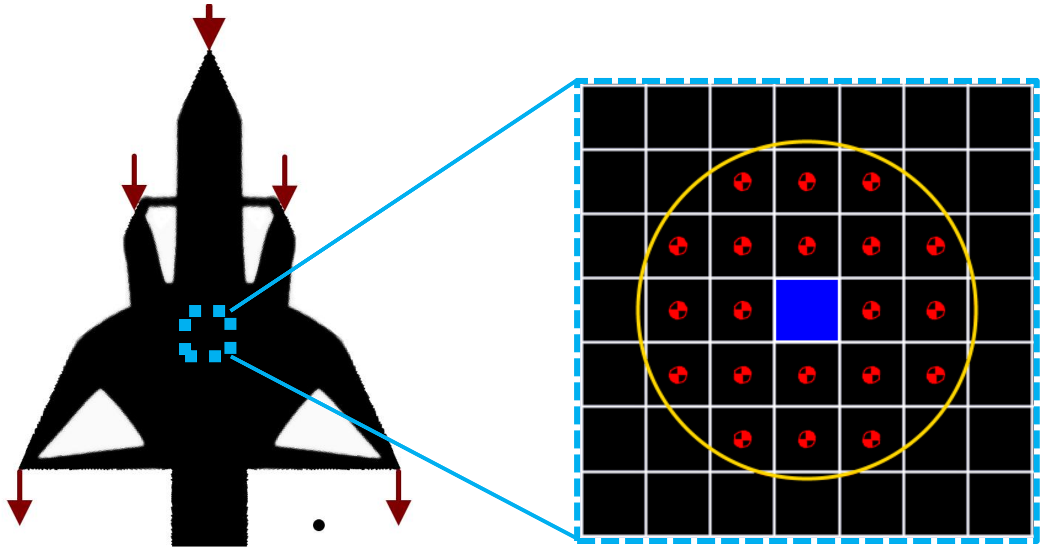


Minimum size



Maximum size

Guest's approach (2009)

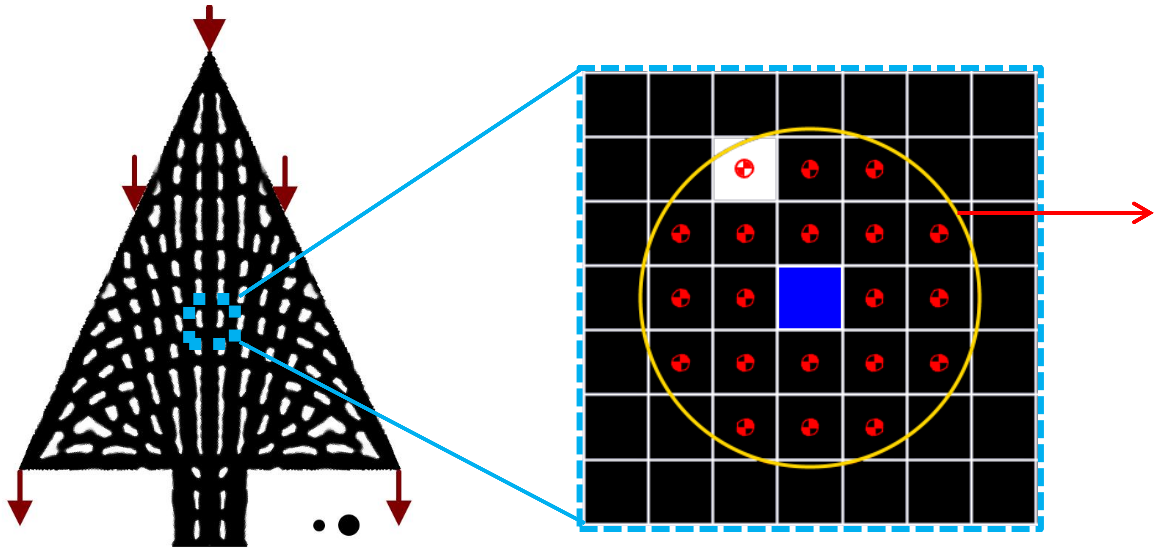


Constraint:

$$\boxed{Amount\ of\ voids\ \geq \varepsilon}$$

Guest (2009)

Guest *JK, Imposing maximum length scale in topology optimization, Struct Multidisc Optim, 37, 463-473,* ***2009****.*



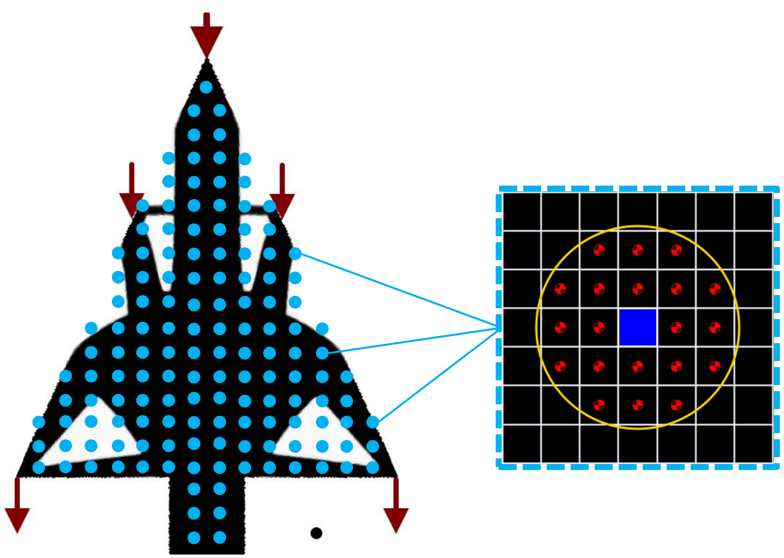
$$Amount\ of\ voids\ = 4.8\%$$

Condition satisfied if $\varepsilon = 3\%$

Maximum size formulation

Local Constraint :

$$g_e = \varepsilon - \frac{\displaystyle\sum_{k \in \Omega_e} v_k \, (1 - \tilde{\rho}_k)^q}{V_{\Omega_e}} \leq 0$$



Compliance minimization with maximum size constraints

$$\begin{aligned} \min \quad & \boldsymbol{f}^\intercal \, \boldsymbol{u} \\ \text{s.t.}: \quad & \mathbf{K}(\rho) \, \boldsymbol{u} = \boldsymbol{f} \\ & \sum_e v_e \, \tilde{\rho}_e \leq V^* \\ & \left. \begin{aligned} g_1 &\leq 0 \\ g_2 &\leq 0 \\ g_3 &\leq 0 \\ &\vdots \\ g_N &\leq 0 \end{aligned} \right\} Max.\ size \\ & 0 \leq \rho \leq 1 \end{aligned}$$

Compliance minimization with maximum size constraints

Do not change during the optimization

$$\varepsilon - \frac{\sum_{k \in \boldsymbol{\Omega_e}} \boldsymbol{v_k} \, (1 - \tilde{\rho}_k)^q}{\boldsymbol{V_{\Omega_e}}} \leq 0$$

$$\begin{aligned} \mathbf{min} \quad & \boldsymbol{f}^{\intercal} \, \boldsymbol{u} \\ \mathbf{s.t.:} \quad & \mathbf{K}(\rho) \, \boldsymbol{u} = \boldsymbol{f} \\ & \sum_e v_e \, \tilde{\rho}_e \leq V^* \\ & \boxed{\varepsilon - \mathbf{D}_{\mathrm{II}} \, \boldsymbol{\delta} \leq 0} \\ & 0 \leq \rho \leq 1 \end{aligned}$$

<u>**Matrix** with neighbors information</u>

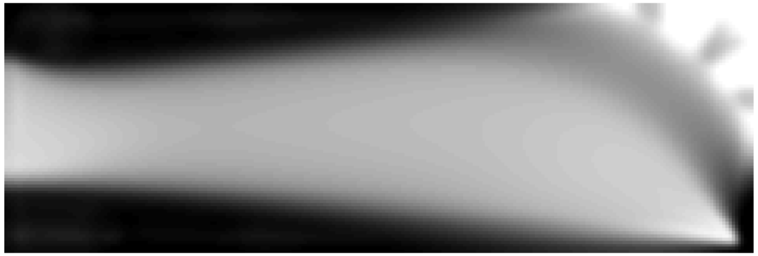
$$\mathrm{D}_{\mathrm{II}(e,j)} = \begin{cases} \frac{v_j}{V_{\Omega^e}} & if \quad x_j \in \Omega^e \\ 0 & if \quad x_j \notin \Omega^e \end{cases}$$

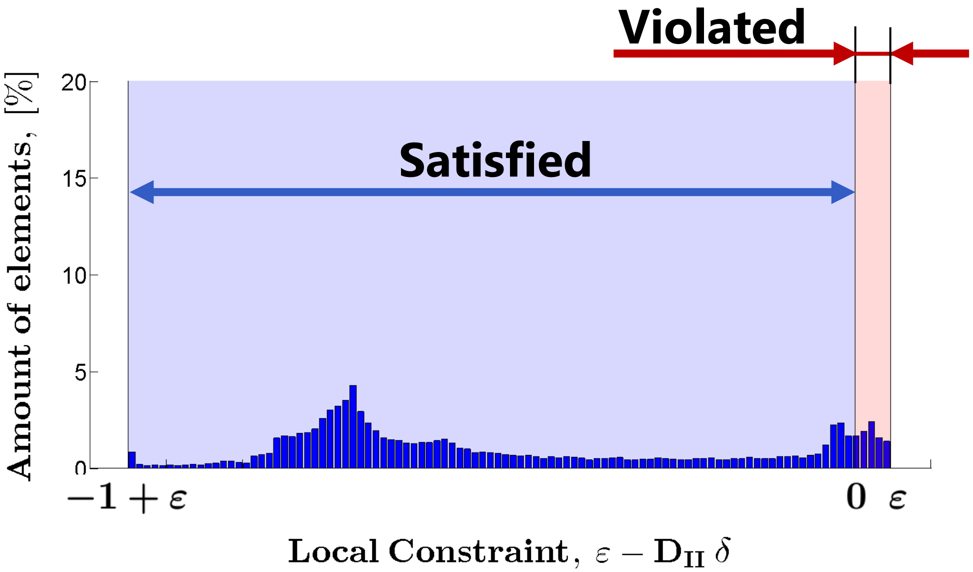
<u>**Voids** vector with penalization</u>

$$\delta_e = (1 - \tilde{\rho}_e)^q$$

Constraints distribution and aggregation

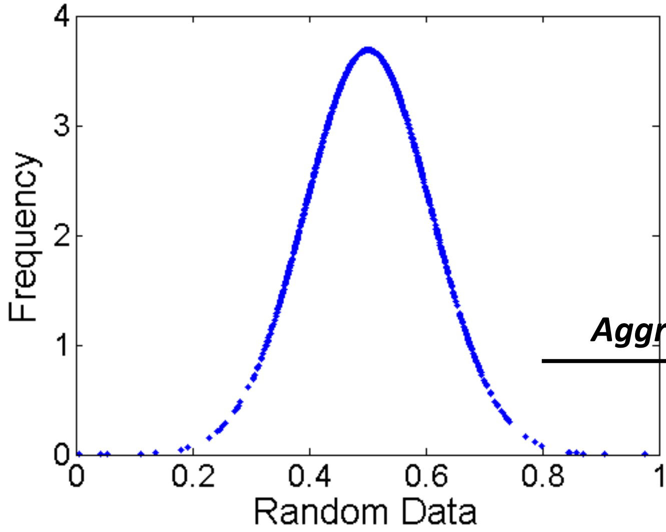
Iteration of the MBB beam with **maximum size** constraints



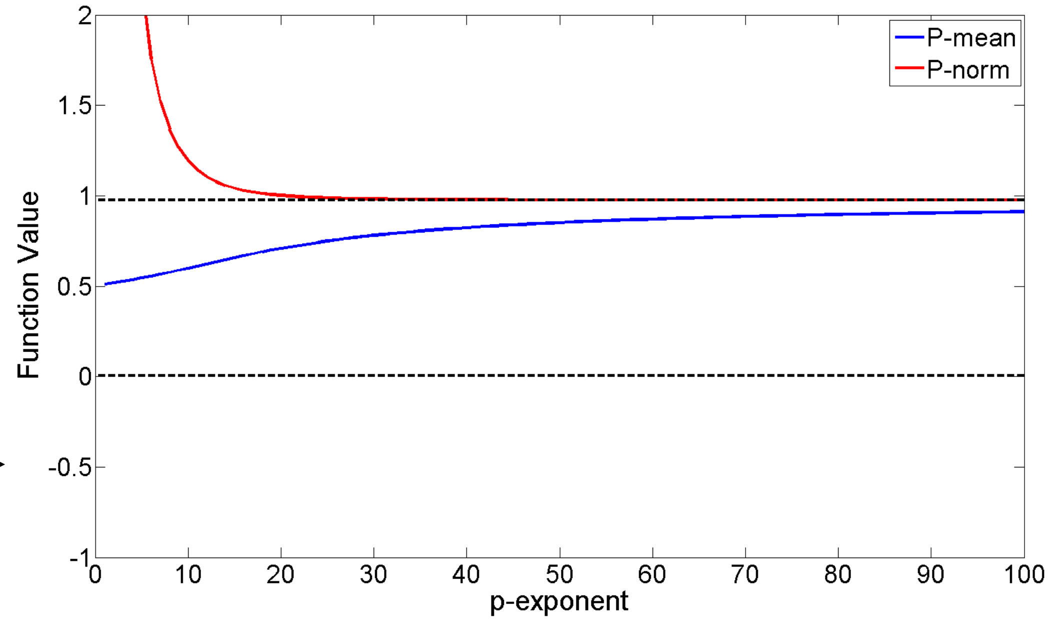


Aggregation functions: p-mean & p-norm

$$P_m = \left(\frac{1}{N} \sum_{e=1}^{N} (s^e)^p \right)^{\frac{1}{p}} \qquad P_n = \left(\sum_{e=1}^{N} (s^e)^p \right)^{\frac{1}{p}}$$



Aggregation



Aggregation functions: p-mean & p-norm

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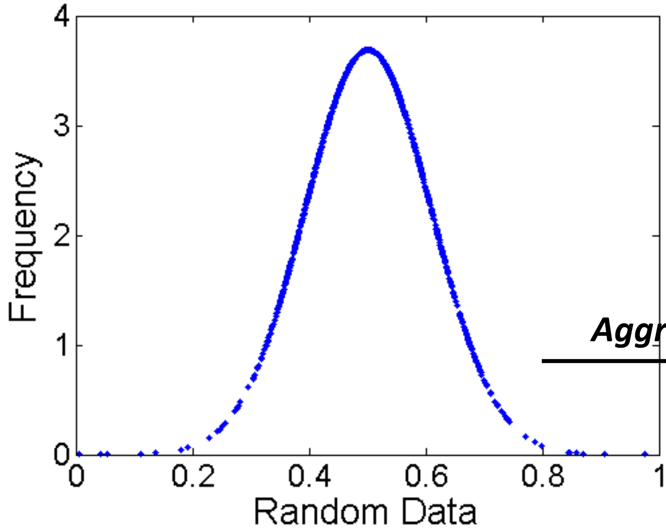
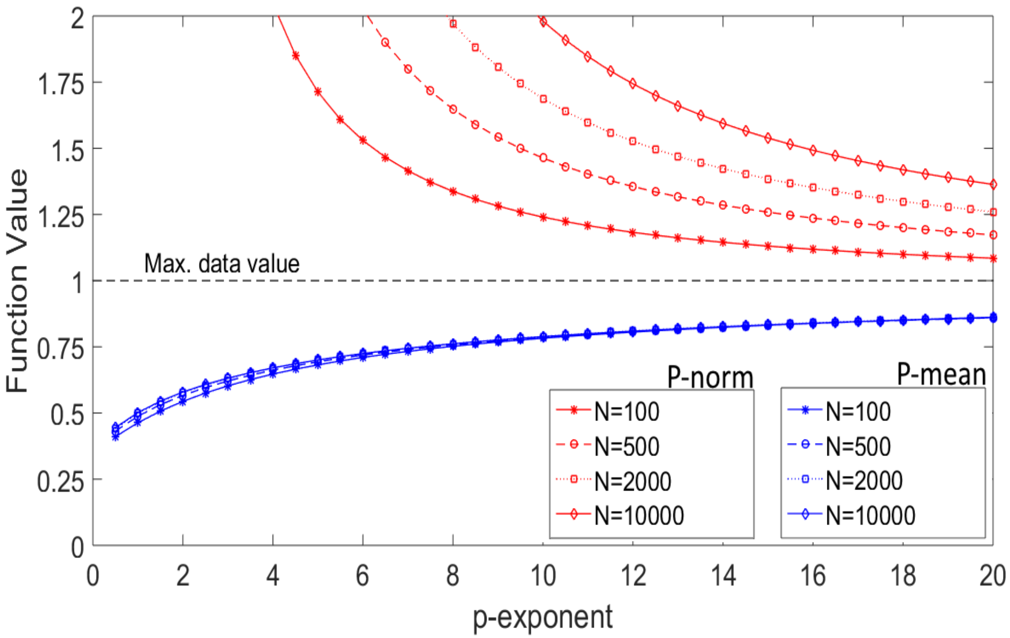
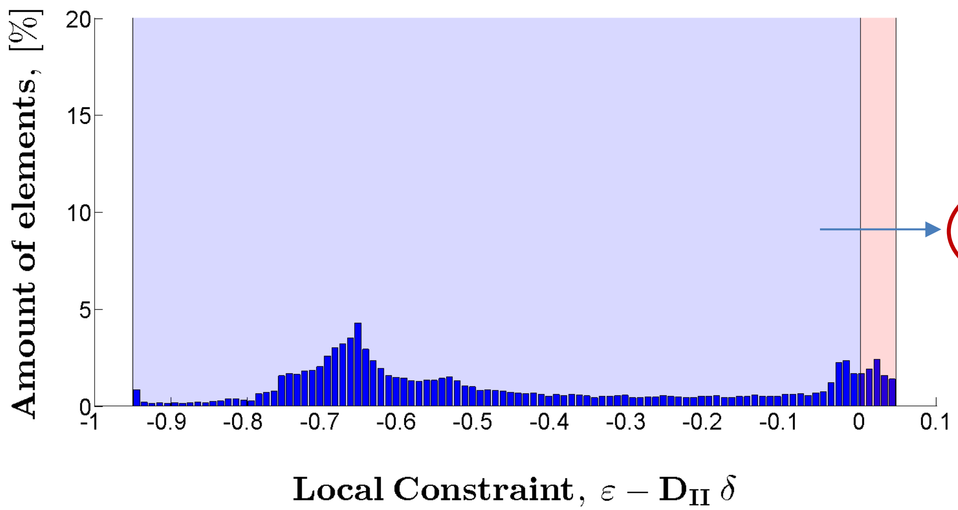


Aggregation



Aggregation of maximum size constraints



$$s_i = \max(g_i, 0) \longrightarrow P_n = \left(\sum_{e=1}^{N} (s^e)^p\right)^{\frac{1}{p}}$$

$$\begin{aligned} \min \quad & \boldsymbol{f}^\intercal \boldsymbol{u} \\ \text{s.t.}: \quad & \mathbf{K}(\rho)\, \boldsymbol{u} = \boldsymbol{f} \\ & \sum_e v_e\, \tilde{\rho}_e \le V^* \\ & G_{ms} \le 0 \\ & 0 \le \rho \le 1 \end{aligned}$$

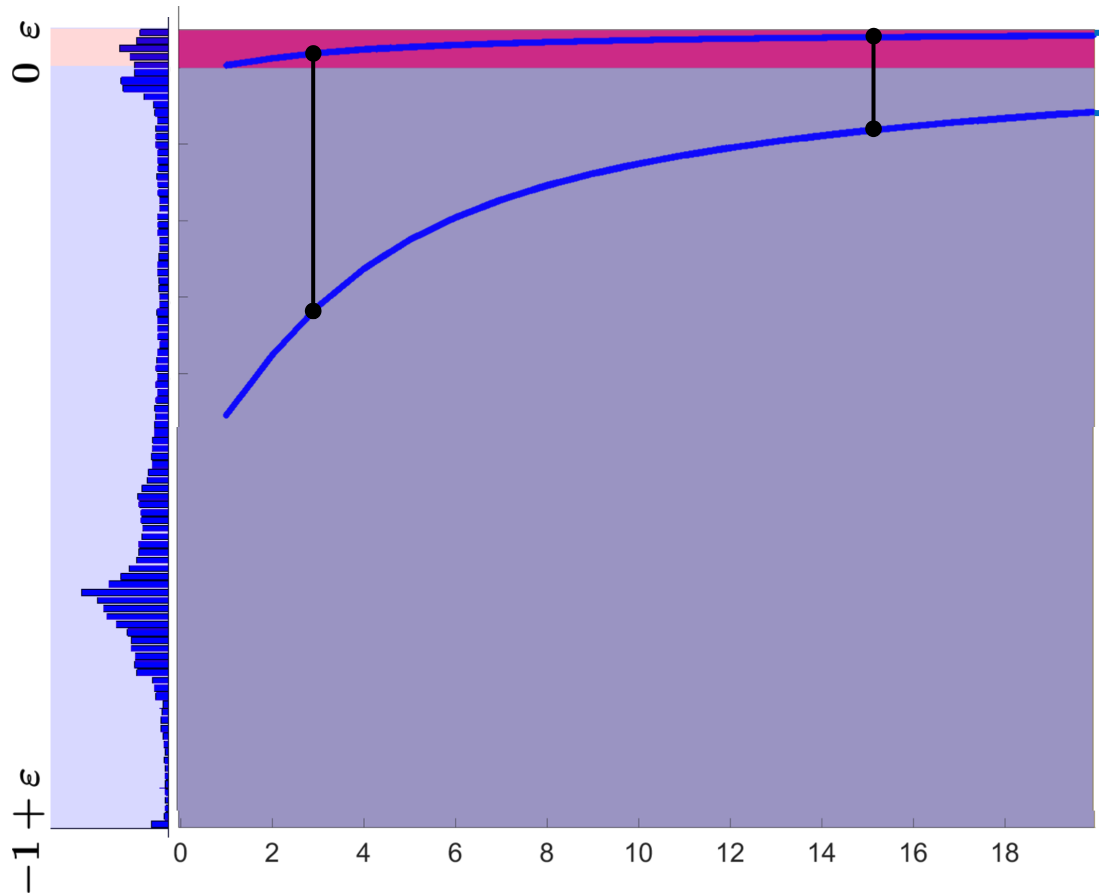
$$\boldsymbol{\rho} \xrightarrow{\text{Filter}} \check{\boldsymbol{\rho}} \xrightarrow{\text{Projec.}} \tilde{\boldsymbol{\rho}} \xrightarrow{\text{Voids vector}} \boldsymbol{\delta} \xrightarrow{\text{Const.}} \boldsymbol{g} \xrightarrow{\text{Feature}} \boldsymbol{s} \xrightarrow{\text{Aggreg.}} A_gg \longrightarrow G_{ms} \le 0$$

$$\frac{dG_{ms}}{d\boldsymbol{\rho}} = \frac{d\check{\boldsymbol{\rho}}}{d\boldsymbol{\rho}}\,\frac{d\tilde{\boldsymbol{\rho}}}{d\check{\boldsymbol{\rho}}}\,\frac{d\boldsymbol{\delta}}{d\tilde{\boldsymbol{\rho}}}\,\frac{d\boldsymbol{g}}{d\boldsymbol{\delta}}\,\frac{d\boldsymbol{s}}{d\boldsymbol{g}}\,\frac{dAgg}{d\boldsymbol{s}}\,\frac{dG_{ms}}{dA_gg}$$

Aggregation of maximum size constraints

Modification of the sensitivities to improve convergence

P-mean curves



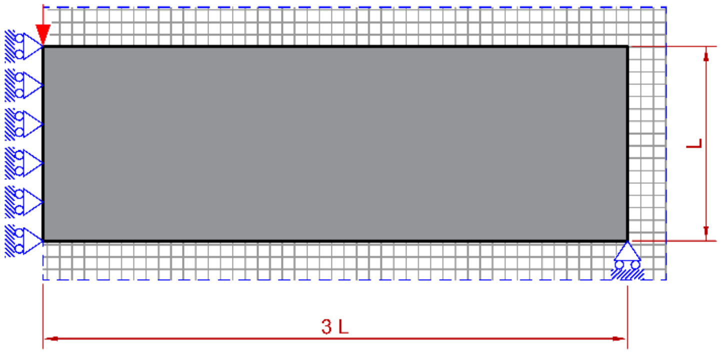
$$\boldsymbol{s}_1 = \max(\boldsymbol{g}\ , 0)$$

$$\boldsymbol{s}_2 = \boldsymbol{g}$$

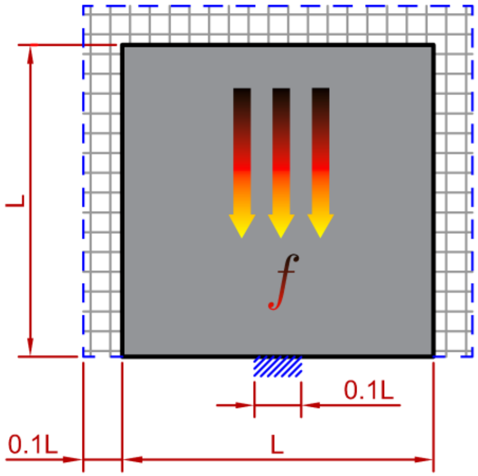
$$\frac{ds_1}{d\boldsymbol{\rho}} = \begin{Bmatrix} {ds_1}/{d\rho_1} \\ {ds_1}/{d\rho_2} \\ 0 \\ 0 \\ {ds_1}/{d\rho_5} \\ \vdots \\ {ds_1}/{d\rho_N} \end{Bmatrix} \qquad 0 \rightarrow \frac{\partial f(s_1, s_2)}{\partial \rho_3}$$

Addressed problems in linear elasticity

Compliance Minimization

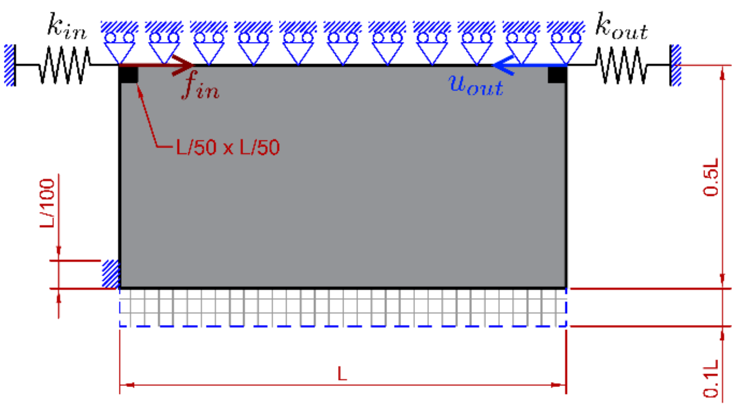


Heat conduction problem

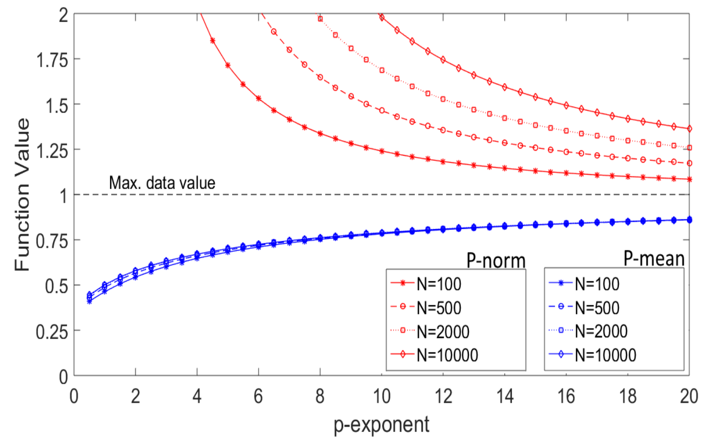


$$
\begin{aligned}
\min_{\rho} \quad & \boldsymbol{t}^{\intercal}\, \boldsymbol{u} \\
s.t.: \quad & \mathbf{K}(\rho)\, \boldsymbol{u} = \boldsymbol{f} \\
& \sum_i v_i\, \bar{\rho}_i \leq V^* \\
& G_{ms} \leq 0 \\
& 0 \leq \rho \leq 1
\end{aligned}
$$

Compliant Mechanism



MBB-beam with different mesh sizes



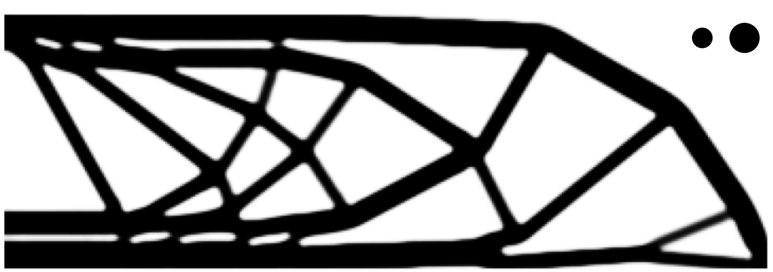
P-norm

P-mean

$C = 57.7$

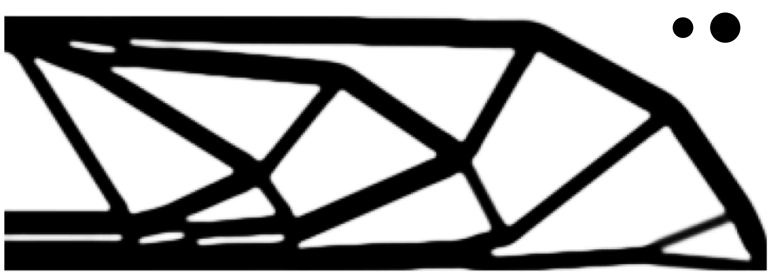
6144 Elements

$C = 56.4$



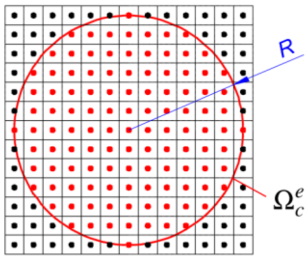
$C = 57.6$

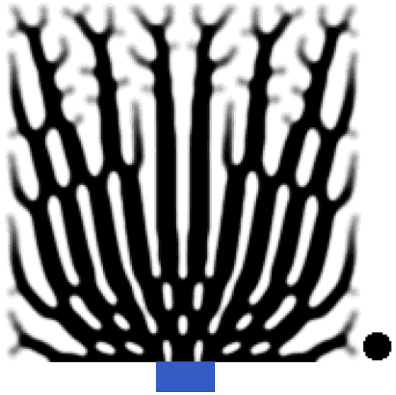
55296 Elements

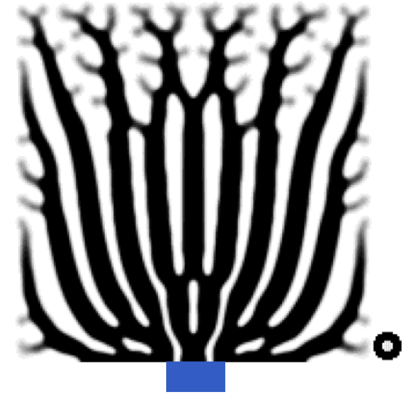


New test region

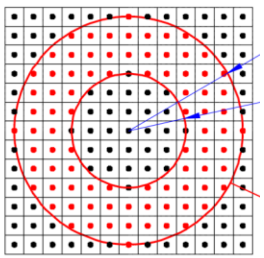
Circular region

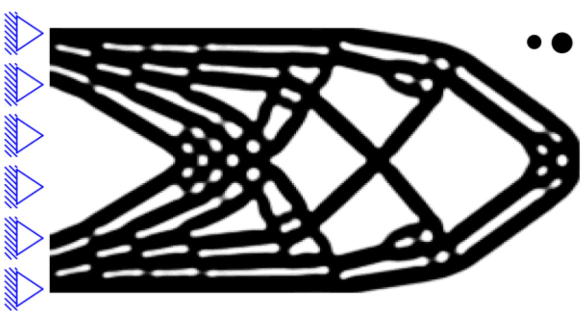


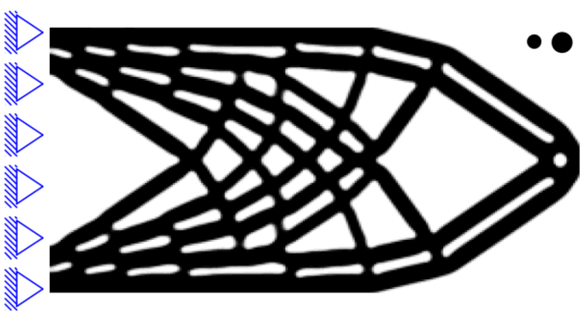




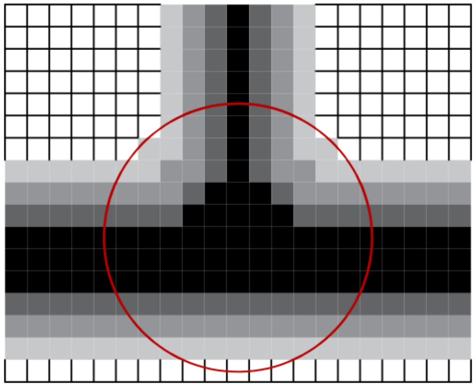
Annular region



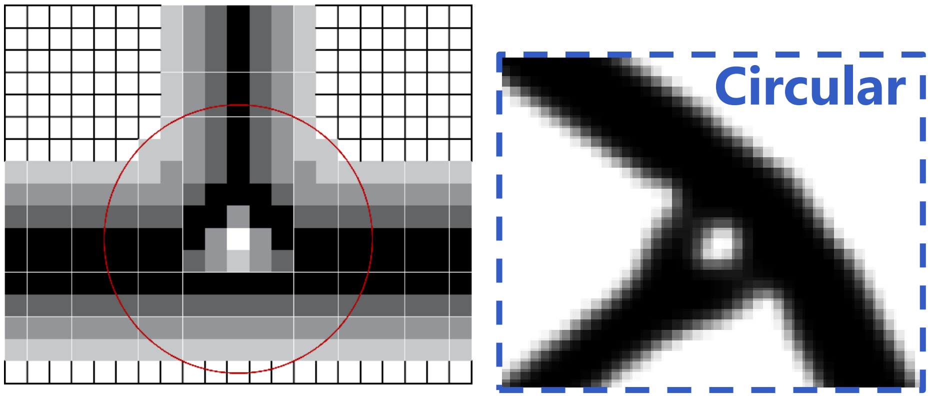


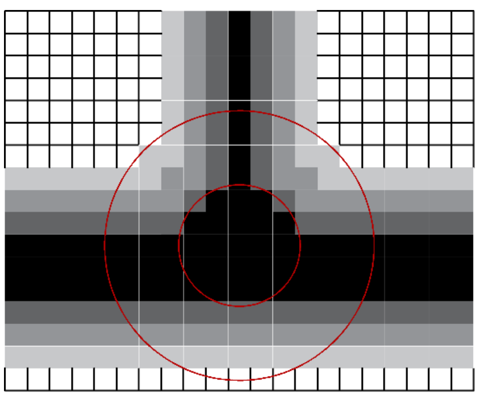


New test region

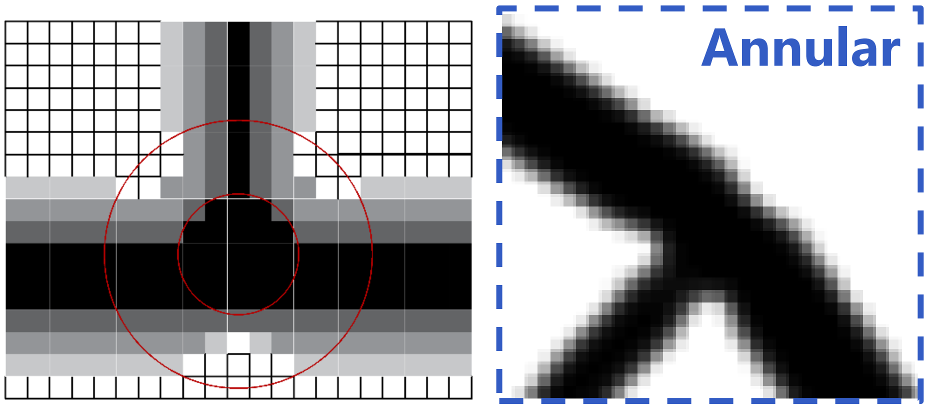


Circular





Annular



Compliance minimization with maximum size

$$
\begin{aligned}
\min \quad & \boldsymbol{f}^{\intercal}\, \boldsymbol{u} \\
\text{s.t.} : \quad & \mathbf{K}(\rho)\, \boldsymbol{u} = \boldsymbol{f} \\
& \sum_{e} v_e\, \tilde{\rho}_e \leq V^* \\
& {\color{red} G_{ms}} \leq 0 \\
& 0 \leq \rho \leq 1
\end{aligned}
$$

SIMP

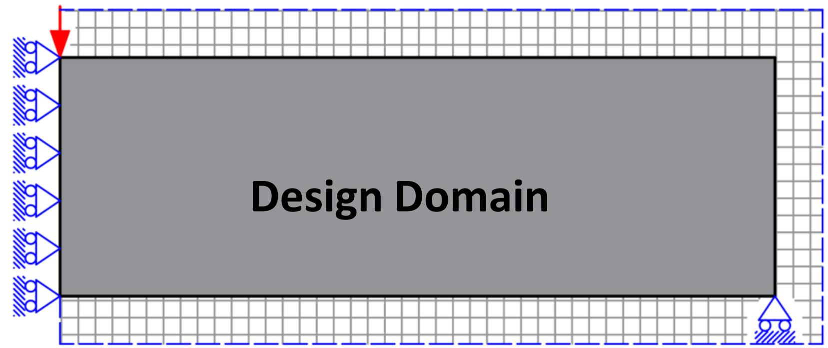
$$\mathbf{K}_e = (E_{min} + \tilde{\rho}_e^{\eta}(E_0 - E_{min}))\mathbf{K}_0$$

Continuation procedure :

$$\eta = 1 : 0.25 : 3.5$$

$$\beta = 2^{\eta} - 1$$

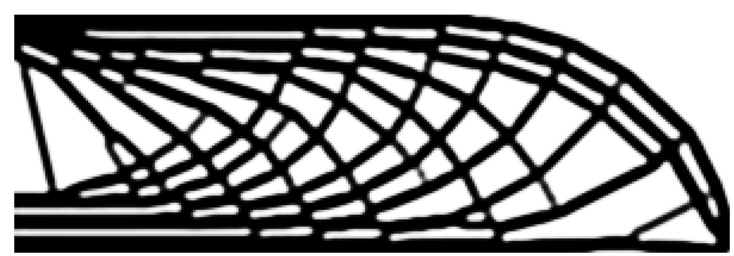
$$iter = 20_{iter} \times 11_{steps} = 220$$



Minimum Size

Time/Iter = Ref.

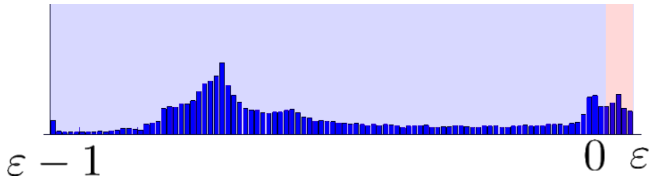
Maximum size

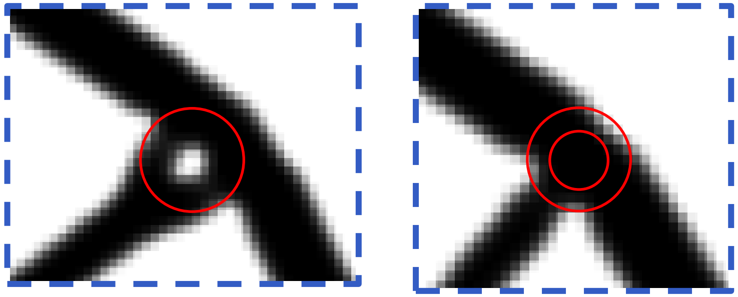


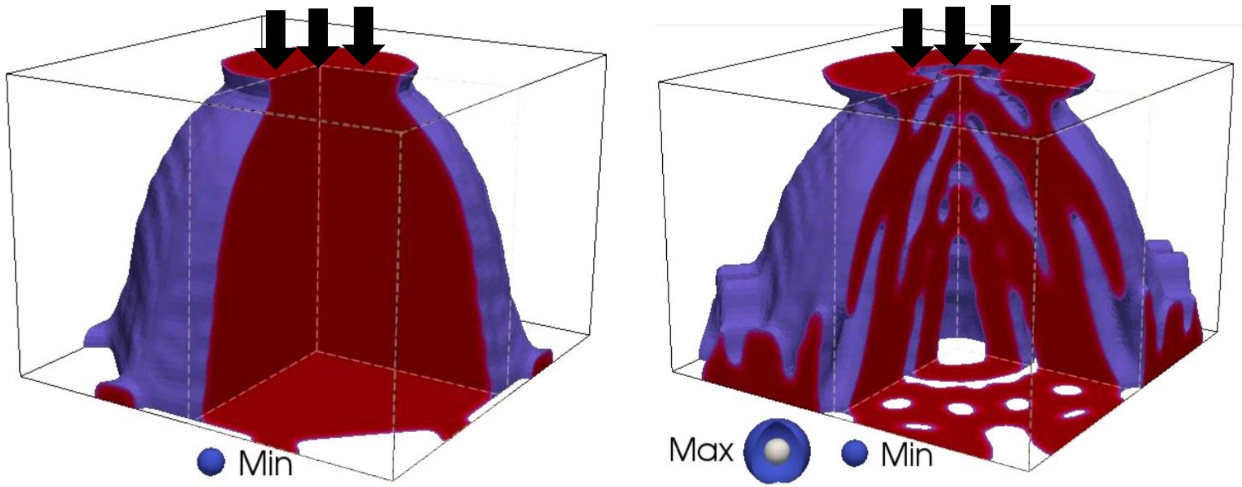
Time/Iter = +12%

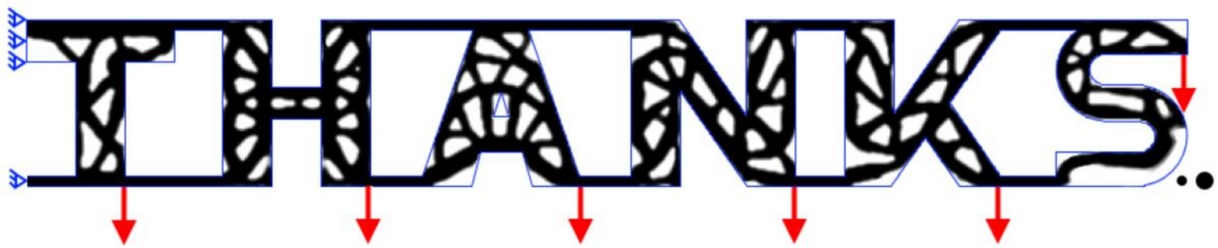
Concluding remarks

- Maximum size constraints where aggregated using **p-mean** and **p-norm** functions.
- With p-norm is not possible to get mesh-indenpent solutions.
- The **ring-shaped** region reduces the amount of holes in the design
- The aggregation reduces the computation time compared to a highly constrained problem and allows to solve 3D problems in a reasonable time.









/ Maximum size

Acknowledgement

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