Factoring Characteristics into Returns:

A Clinical Approach to Fama-French Portfolio Decomposition

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Abstract

This paper thoroughly analyzes competing construction methods for factoring characteristics into returns. We show the importance of ensuring a proper diversification of the factor's portfolio constituents for producing relevant and unbiased risk factors or benchmark portfolios. This is an important issue to be solved for asset pricing and performance models defined as a function of characteristics. As a practical case, the paper works on the design of size and value spread portfolios *à la* Fama-French. This quasi-clinical investigation examines three methodological choices that have an impact on portfolio diversification: the (in)dependence and the (a)symmetry of the stock sorting procedure, and the sorting breakpoints. A sequential and symmetric sort of stocks into long and short portfolios conditioned on control variables produces unbiased factors. Our results are stronger when whole firm samples are used to define breakpoints and are also robust to the inclusion of a third dimension in the multiple sorting.

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1. Introduction

The empirical pricing literature contains a variety of multifactor models that attempt to explain security returns[1]. These models use stock "attributes" such as market capitalization, book-to-market or even return statistics such as returns' levels of skewness and excess kurtosis (Fama and French 1993, Carhart 1997; Agarwal et al. 2009, Fama and French 2015, Hou et al. 2015b; Lambert and Hübner 2013). The empirical implementation of these fundamental multifactor models faces the common challenge of the construction of mimicking or hedge portfolios that capture the marginal returns associated with a unit of exposure to each attribute. This challenge is important, as Kogan and Tian (2015) show that the performance of multifactor models is highly sensitive to factor construction methodology. Our paper is the first dedicated to a comprehensive review of factor construction methods and their implications for factoring characteristics into returns. In this way, we contribute to the recent debate on the difficulty in inferring independent information about average returns (Green et al. 2017). We also contribute to the literature covering the empirical implementation of asset pricing models (Cochrane 2011) or benchmark portfolios (Chen et al. 2009) as a function of characteristics.

For more than two decades, the mimicking portfolios for size and book-to-market risks developed by Fama and French (1993) (commonly augmented with the momentum factor by Carhart (1997) computed using a similar method) have been the predominant standard in constructing fundamental risk factors[2]. These authors consider two independent methods of sorting US stocks (on market capitalization and book-to-market) and construct six value-weighted two-dimensional portfolios at the intersections of the rankings. The *size* factor measures the return differential between the averages of small-cap and big-cap portfolios, whereas the *book-to-market* factor measures the return differential between the averages of value and

[1] According to Connor (1995), the fundamental approach to pricing equity returns has outperformed other approaches.

[2] One can perform a Fama and Macbeth (1973) type of regression on the risk fundamentals to extract unit-beta portfolios (see, for instance, Back et al. 2013). One can also construct portfolios by aggregating assets according to their correlations with the fundamentals (Balduzzi and Robotti 2010).

growth portfolios. To break stocks into portfolios, the authors use New York Stock Exchange (NYSE) breakpoints to balance market capitalization among portfolios.

Important but thus far overlooked features when sorting stocks into portfolios are the effects of the use of NYSE breakpoints, the effects of an asymmetric sort on multiple characteristics and the relevance of an independent sort when characteristics are correlated. How do these methodological choices affect the factors to be priced? Answering this question is crucial for several reasons. First, Daniel and Titman (1997)[3] show that *characteristics* rather than *covariations* with pervasive factors drive the co-movements among value and growth stocks. Subsequent research has moreover outlined the implications of their study for performance evaluation (e.g. Chan et al. 2009). Second, there is an inflation of discovered factors estimated through ad hoc adjustments to the original Fama and French (1993) methodology. For instance, Stambaugh and Yuan (2017) replace NYSE breakpoints by whole sample breakpoints when building their mispricing factors. Similarly, Hou *et al.* (2015b) also recompute financial anomalies using whole sample breakpoints. Other examples concern the dependence of the sort (e.g. Liew and Vassalou 2000, Chen et al. 2009, see infra). With Fama and French (1993) as the established benchmark, we study the various options for constructing risk factors using this framework over a very long period (51 years). We compare sequential to independent, symmetric versus asymmetric sorting procedures when forming hedge portfolios and explore their implications for factoring characteristics into returns. More precisely, we examine the differences between the following two ways of implementing a sequential sort: (i) conditioning first on the variable to be priced and then on the control variables ("post-conditioning") and (ii) conditioning first on the control variables ("preconditioning"). We also consider extending portfolio sorting to a third attribute contrary to the current literature which only focuses on two attributes at a time.

Assuming that sorting stocks on characteristics consists of a sort on expected returns, Cochrane (2011) formally shows the effect of portfolio sorts on the Sharpe ratio of spread portfolios and on its distance to the Sharpe of the underlying risk factor. The distance inversely depends on the number of stocks into the

[3] Winner of the 1997 Smith-Breeden Prize.

short and long legs of the spread: a lower number of stocks will reduce the portfolio spread's Sharpe ratio and *t*-statistic by introducing idiosyncratic risk. We show that a sequential (dependent) and symmetric multistep sort together with sorting breakpoints defined on the whole sample allows the number of stocks in each portfolio to be maximized while increasing the characteristics spread in extreme portfolios. The consequence is that the Sharpe ratio of the spread portfolios more closely matches the one of the underlying risk factor. We demonstrate the biases introduced by unequal repartition of stocks into portfolios when building risk factors under the framework of an independent sort (See Section 5).

With the Fama-French (henceforth, FF) standard approach as a benchmark, our paper corresponds to the adaptation of a "clinical study" in finance investigating (i) the correlation bias in stocks attributes (ii) the implications of a dependent sort and (iii) the joint implications of other sorting features.

Correlation bias in stock attributes. Daniel et al. (1997), Daniel and Titman (1998), and Wermers (2004) favor a characteristic-based approach to study the performance of US equity funds over traditional regression-based analyses. Chan et al. (2009) emphasize that regression-based analyses fail at valuing the performance of passive portfolios because of the correlation bias in stock attributes driving stock allocation into portfolios. In other words, the intrinsic correlation between size and value characteristics, if not controlled by the factor construction, can not be properly controlled for in a regression. For instance, abnormal returns of a small-value portfolio might be underestimated when small growth stocks outperform: the small-value portfolios would load heavily on the size factor inflated by the outperformance small growth stocks (without appropriate controls for building the size factor). Hence, the regression-based analysis does not completely solve for the intrinsic correlation between factors through orthogonalization.

Dependent sort. The comprehensive study of Chan et al. (2009) unambiguously outlines the advantages of attribute-matched portfolios rather than regression-based analyses for assessing portfolio performance. They also justify the use of a sequential sort over an independent one by identifying adequate size and value benchmarks. The authors also prove the practical relevance of a sequential sort through its similarity with the construction mechanism of renowned benchmark indices widely used by institutional investors such as Russell, Standard and Poor's and Wilshire. However, whereas their study focuses on the quality of benchmarks produced with alternative stock classification procedures, it does not investigate the properties of the methodological choices underlying the sorting procedure.

Several other examples exist of the use of a conditional sorting procedure, especially when data are scarce or in international studies (Daniel et al. 1997, Daniel and Titman 1998, Ang et al. 2006, Novy-Marx 2013). Agarwal et al. (2009) sort hedge funds into portfolios using the same sequential approach with the objective of estimating higher moment risk factors. In international asset pricing, Liew and Vassalou (2000) adapt it with a triple conditional sort to compute size, value and momentum factors for various countries.

Performing a dependent sort poses several challenges that have not been investigated such as the ordering of the sort or the joint effect of alternative portfolio construction choices.

Other sorting features. The dimensionality of the sort and the definition of the breakpoints are diverse. Our results are stronger when using whole sample breakpoints and triple sort, which ensure the same diversification (in terms of number of firms) across the characteristic-sorted portfolios forming the long and short legs of the factor. We also check whether our framework can accommodate a third attribute. As a practical example, we investigate momentum effects when pricing size and value factors. Recent papers show the effect of news events on stock returns (Savor and Wilson 2016) and a return clustering effect for market anomalies around news events (Bowles et al. 2017, Engelberg et al. 2018). Sorting on momentum might constitute a control for the release of news. Our results are robust to the inclusion of this additional control.

To conclude, the preconditioning approach uncovers the independent contribution (uncorrelated with the controls) of the priced variable. This better identification of the size and value effects on the US market has implications in factor investing and asset pricing which will be investigated in future research.

The rest of the paper is organized as follows. Section 2 provides the Fama and French (1993) factor construction background and describes the dataset of US stocks used to perform the clinical exercise. Section 3 analyzes the implications of sorting stocks using a sequential versus an independent sorting method. Section 4 considers alternatives when sorting stocks into portfolios. In the various sorts, we exclude changes related to the frequency of rebalancing and keep the portfolios fixed to annual rebalancing so that only methodological changes are captured. Section 5 presents the theoretical framework to measure the bias introduced by an independent, asymmetric sorting framework using NYSE breakpoints and builds an empirical exercise. Section 6 introduces a multidimensional procedure as a generalization for constructing risk factors. Section 7 discusses the implications of the different methods of constructing risk factors. Section 8 concludes the paper.

2. Reproducing the Fama and French Standard Method (1993)

Pricing anomalies related to size (Banz 1981), value (Basu 1983), and momentum (Jegadeesh and Titman 1993) effects on the US stock market have been documented since the early 1980s. These effects, which were initially related to mispricing in the Capital Asset Pricing Model, have been widely recognized as pricing factors ever since the influential work of Fama and French (1993).

The Fama and French (1993) three-factor model and its extension to momentum by Carhart (1997) have become the benchmark of empirical asset pricing. Using a dataset from the Center for Research in Security Prices (CRSP), Fama and French consider two independent methods of scaling US stocks, including an annual two-way sort on market equity and an annual three-way sort on book-to-market according to NYSE breakpoints (quantiles). Next, they construct six value-weighted (two-dimensional) portfolios at the intersections of the annual rankings (performed each June of year y according to the fundamentals displayed in December of year y-1). The size or SMB factor (Small minus Big) measures the return differential between the average small-cap and the average big-cap portfolios, whereas the book-to-market or HML factor (High minus Low) measures the return differential between the average value and the average growth portfolios. The resulting so-called "Fama-French three-factor model" has become a core version of empirical asset pricing models.

Since the purpose of this paper is to build a framework that allows for a robust comparison with the original Fama and French approach considered as a standard, we strictly follow their stock selection methodology to construct our risk factors. The period ranges from July 1963 (as in Fama and French 1993) to December 2014 and comprises all NYSE, AMEX, and NASDAQ stocks collected from the merger between the CRSP and COMPUSTAT databases. The analysis covers 618 monthly observations. The market risk premium corresponds to the value-weighted return on all US stocks minus the one-month T-Bill rate from Ibbotson Associates (from Ken French's website). We consider stocks that fully match the following lists of filtering criteria: a CRSP share code (SHRCD) of 10 or 11 at the beginning of month *t*; an exchange code (EXCHCD) of 1, 2 or 3, available shares (SHROUT) and price (PRC) data at the beginning of month *t*; available return (RET) data for month *t*; at least two years of listing on COMPUSTAT to avoid survival bias (Fama and French 1993) and a positive book-equity value at the end of December of year *y*-1. Thus, our sample varies over time. For instance, from 5,612 stocks available as of December 2014, our conditions restrict the usable sample to 3,335 stocks (for 2014).

As in Fama and French (1993), we define the book value of equity as stockholders' equity reported by COMPUSTAT (SEQ) plus balance-sheet deferred taxes and the investment tax credit (TXDITC). If available, we decrease this amount by the book value of preferred stock (PSTK). If the book value of stockholders' equity (SEQ) plus the balance-sheet deferred taxes and investment tax credit (TXDITC) is not available, we use the firm's total assets (AT) minus total liabilities (LT).

Book-to-market is the ratio between book common equity for the fiscal year ending in calendar year *t*-1 and the market equity of December *t*-1. Market equity is defined as the price (PRC) of the stock times the number of shares outstanding (SHROUT) at the end of June *y* to construct the size factor and at the end of December of year *y*-1 to construct the value factor.

Carhart (1997) completes the Fama and French three-factor model by computing a momentum (i.e., a *t*-2 until *t*-12 cumulative prior-return) or UMD (Up minus Down) factor that reflects the return differential between the highest and the lowest prior-return portfolios.

3. Portfolio Construction Using a Sequential or an Independent Sort

The Fama and French (1993)'s independent 2x3 sort performed on a firm's size and value characteristics is compared to a sequential sort on the same characteristics in Figure 1. An independent sort slices the stock universe according to the size and value characteristics (Panel A). A sequential sort proceeds in two steps to form the book-to-market/market capitalization portfolios. It can either start with the book-to-market (Panel B: first line) or with the market capitalization (Panel B: second line).

Unlike the independent sort, the sequential sort adjusts the breakpoints in the second sorting step, considering the correlation among the size and value characteristics.[4]

Figure 2 compares the stock allocation into portfolios under a sequential/dependent and an independent sort. Panels A and C (resp. B and D) illustrate the case of an independent (resp. dependent) sort on negatively correlated characteristics such as book-to-market and market capitalization. Panels A and B depict a situation in which the two fundamentals are correlated at -30%, whereas Panels C and D consider a perfect negative correlation (-100%) between the two characteristics on which the sort is performed.

The figure shows that the high level of correlation produces imbalanced portfolios under an independent framework. The figures also illustrate how the adjustments of the breakpoints under the sequential sort allow for the even split of stocks into portfolios. When the characteristics are perfectly correlated, an independent sort would even produce empty portfolios, as shown in Panel C. The consequences on the Sharpe ratio and *t*-statistic of mimicking portfolios will be investigated in Section 5.

Figure 1 Independent vs Sequential Sorting: 2x3 Portfolios

The figure schematizes the Fama and French (1993) independent sorting (Panel A) and the sequential sorting (Panel B) procedures to construct the 2x3 size/value portfolios. The circle represents the US stock universe.

[4] For comparative purposes, we assume breakpoints are defined according to NYSE stocks, as in Fama and French (1993). We relax this assumption later in the paper.

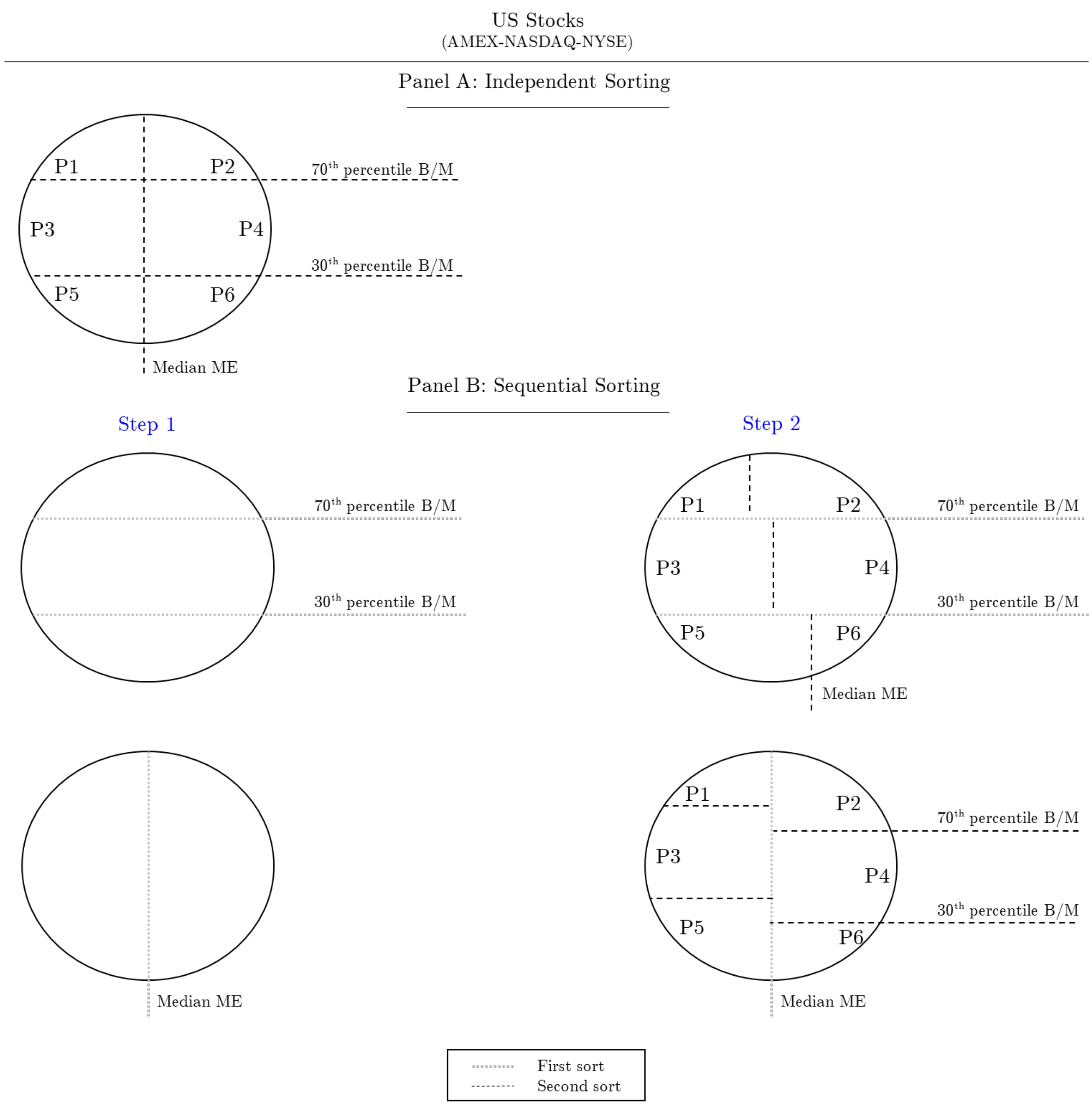
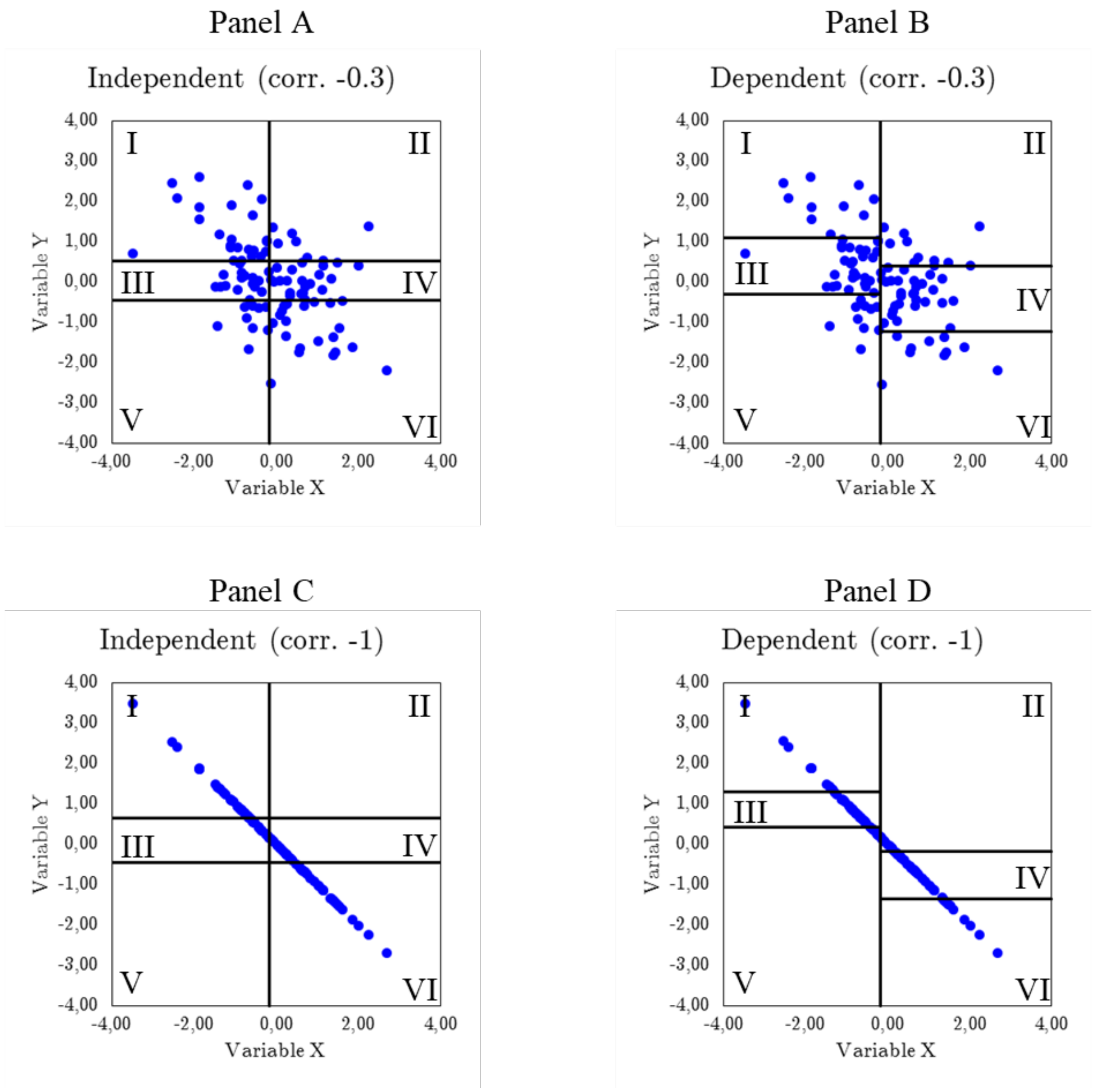


Figure 2 Independent vs Sequential Sorting: Allocation into Portfolios[5]

The figures illustrate the allocation of 100 stocks sorted across six portfolios on variables *x* and *y*. Panel A (Panel B) shows the allocation according to an independent (dependent) sort when the correlation between the characteristics *x* and *y* is -30%. Panel C (Panel D) shows the allocation according to an independent (dependent) sort when the correlation between the characteristics *x* and *y* is a perfectly negative (-100%).

[5] We thank Nick Baltas for suggesting this analysis.



The sequential sort can be performed using a preconditioning either on the control variables or on the characteristics to be priced (i.e., post-conditioning on the control variables). The two procedures do not capture the same pricing effects. Before empirically analyzing the statistical properties of the two different procedures, we describe in the next subsections their construction methodologies with illustrative examples.

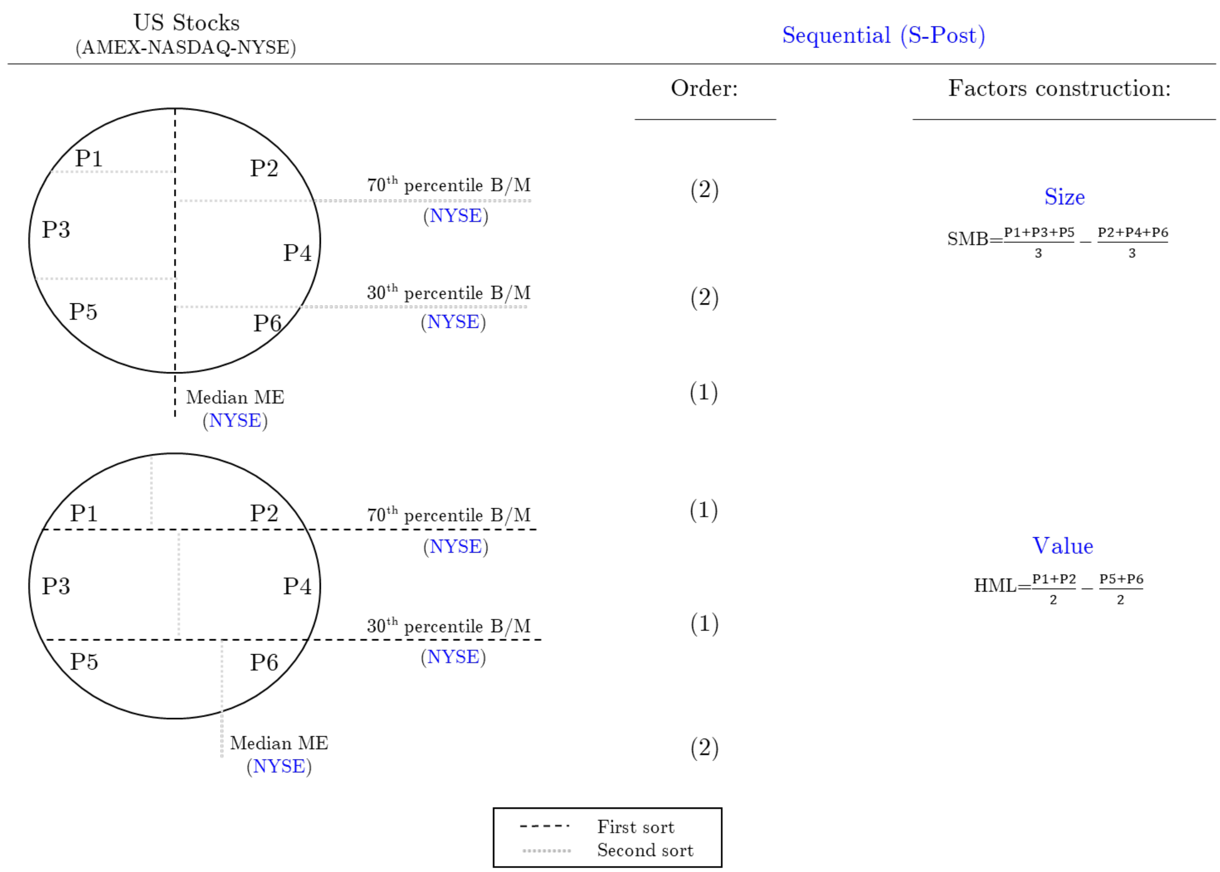
3.1. Post-conditioning on the control variables

Under the framework "Sequential-Post" (hereafter S-Post), we construct the size and value factors by *post-conditioning* the sort on the control variable[6]. Figure 3 illustrates the procedure for the size and value factors. This methodology equally averages the three components of the small portfolios (small/low (i.e., P5), small/mid (i.e., P3) and small/high (i.e., P1)) with P1+P3+P5 making up a small portfolio made on a one-sort procedure and of the large portfolios (i.e., (P2+P4+6)/3) before computing the spread between the averaged small- and large-size portfolios (see the first line of Figure 3). Therefore, the method is similar to the Fama and French (1993) approach, except that conditional sorts lead to a different stock allocation into B/M portfolios and avoids empty portfolios in case of high correlation among the sorting attributes. The sequential sort considers the correlation between the sorting characteristics and adjusts the breakpoints of the second step of the sorting procedure. Consequently, the weight of each firm within the small or large portfolios is modified with respect to Fama and French.

Figure 3 Sequential Sorting (2x3): Post-Conditioning Method

The figure schematizes the sequential sorting procedure by post-conditioning on the control variable. In other words, the illustration shows the construction of the size premium (SMB) by first sorting stocks according to their market capitalization (priced variable) and then sequentially sorting stocks according to their book-to-market ratio (control variable). The value premium (HML) is formed by first allocating stocks into portfolios according to their book-to-market ratio (priced variable) and then sequentially sorting stocks for their market capitalization (control variable).

[6] We thank Vikas Agarwal for suggesting this approach.



3.2. Preconditioning on the control variables

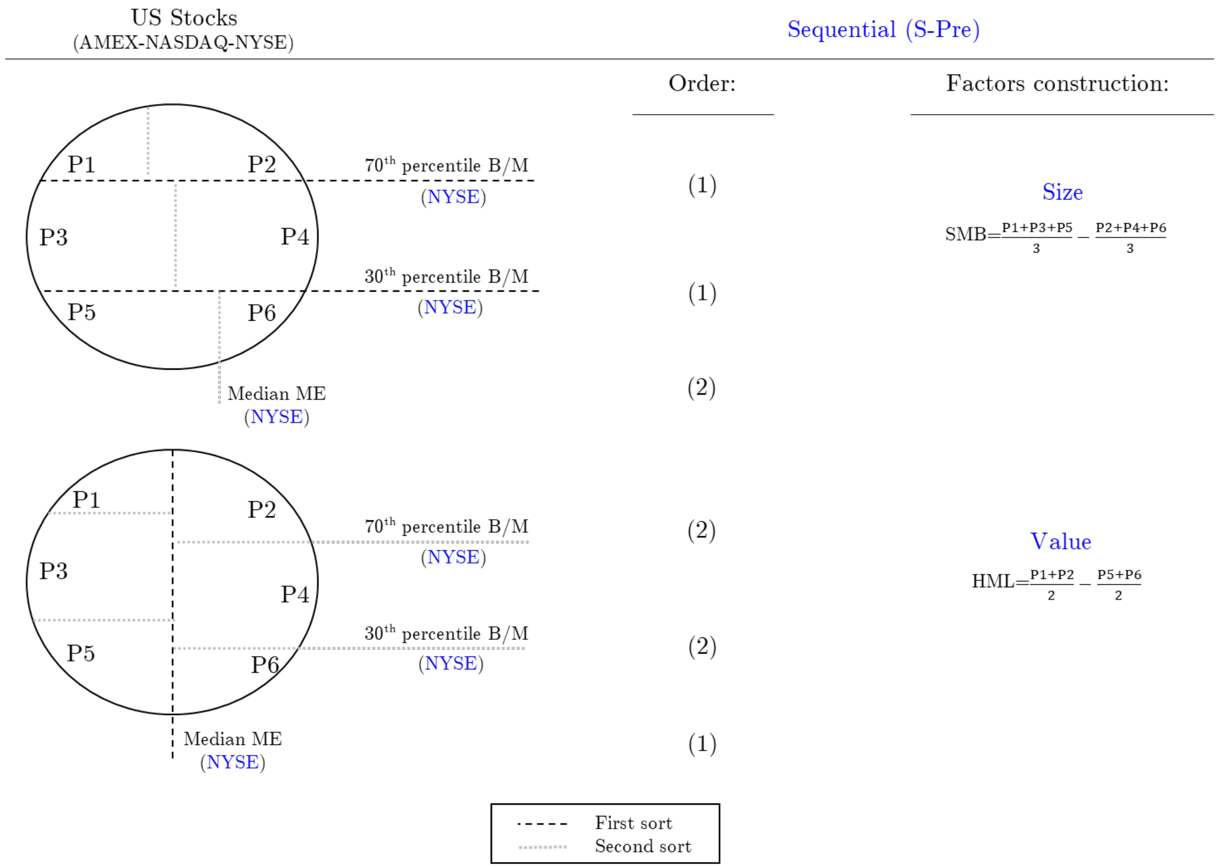
We construct the size and value factors under the "Sequential-Pre" (hereafter S-Pre) framework by *preconditioning* the sorting procedure on the control variable and ending the sort with the variable to be priced, such as in Lambert and Hübner (2013). Figure 4 illustrates the procedure for the size and value factors under the S-Pre framework. To build the size factor (see the first line of Figure 4), this methodology equally averages portfolios P1, P3 and P5 and separately averages P2, P4 and P6 to form a small and a large portfolio. It then takes the spread of the small and large portfolios to compute the size factor. The exercise is similar for the value factor and illustrated in the second line of Figure 4. The methodology equally averages portfolios P1 and P2 and separately averages P5 and P6 to form a value and a growth portfolio. The value factor is the spread between the value and growth portfolios.

In S-Post (as well as in FF), the procedure first rebuilds the small and large portfolios by rebalancing the components of each portfolio across the three levels of the control variable, including book-to-market.

For example, P1+P3+P5 corresponds to a single-sort small portfolio and P2+P4+P6 corresponds to a single-sort large portfolio. However, in S-Pre, the sum of these portfolios does not comprise either a single-sorted small portfolio or a single-sorted large portfolio. This alternative approach to performing a sequential sort does more than simply rebalancing the stock weight into the portfolios to form the spread. The S-Pre construction first builds return spreads within each subsample formed on the control variable (the sum of these subsamples making up the control universe), and then it aggregates these spreads into a single factor. This approach relies on the evidence that there is a tilt and concentration of small stocks in the value portfolios and that the value effect is strongest among small caps stocks. For instance, in constructing the HML factor, the S-Pre construction reduces the tilt by putting the same weight on the HML spreads from both control samples (small and large). However, the S-Post and FF procedures perform an adjusted single-sort spread where the weights (but not the allocation) of stocks are adjusted for the correlation between the pricing and the control variables. This result constitutes the main difference between the frameworks. Unlike the S-Post approach, the S-Pre approach ensures that the risk factor is an equally weighted average of the spreads for each level of control. S-Post does not ensure this because (P1-P2), (P3-P6), or (P5-P6), which form the three size return spreads, do not have the same average levels of book-to-market. Under the S-Pre framework and using a 2x3 framework, the SMB spread in the low B/M portfolio exactly counts for a 1/3 weight. This result does not hold for HML spreads because of the non-symmetrical sorting. Symmetrical sorting will be tested later in the paper.

Figure 4 Sequential Sorting (2x3): Preconditioning Method

The figure schematizes the sequential sorting procedure by using preconditioning on the control variable. The illustration shows the construction of the size premium (SMB) using preconditioning on the book-to-market ratio (control variable) and then sorting for the market capitalization (priced variable) and the value premium (HML) using preconditioning on the market capitalization (control variable) and then sorting for the book-to-market ratio (priced variable).



3.3. Pricing effects of the alternative methodologies

To analyze the pricing effects induced by the sorting methodologies, we perform "spanning regressions" as defined in Fama and French (2015, 2017) and in Novy-Marx (2015). The factors constructed under different configurations (dependent/independent sort) are regressed on Fama and French and Carhart factors to understand their return drivers.

$$\mathrm{y}_t = \alpha + \beta' X_t + \varepsilon_t, \tag{1}$$

where y_t is the factors constructed under different configurations (dependent/independent sort), X_t is the matrix of risk factors, β' is the vector of parameter estimates for the risk factors, and ε_t is the error terms. The results are displayed in Table 1.

Table 1 Spanning Regressions: 2x3 Portfolios

The table reports the spanning regression results for the alternatives size and value factors. T-statistics of the estimation parameters are in parentheses. The significance of the parameter estimates is reported as performed. *, **, and *** indicate statistical significance at the 0.1, 0.05 and 0.01 levels, respectively. The denomination "*S-Post*" refers to a post-conditioning on the control variable, whereas "*S-Pre*" refers to a preconditioning on the control variable. For instance, $HML_{S\text{-}Post}$ represents the value factor when stocks are first sorted by their book-to-market ratio (variable to be priced) and then by their market equity, whereas $HML_{S\text{-}Pre}$ represents the value factor when stocks are first sorted by the market equity (control variable) and then by their book-to-market equity (priced variable). The period used to perform the regressions ranges from July 1963 to December 2014.

Spanning regressions on *SMB* and *HML factors using a 2x3 approach*				
	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$
Alpha	0.036	0.007	0.016	0.017
	(1.25)	(0.24)	(1.01)	(0.89)
RM_{ff}	-0.022***	0.01	0.000	-0.016***
	(-3.30)	(1.61)	(-0.12)	(-3.43)
SMB_{ff}	0.268***	0.034***	1.005***	0.936***
	(28.61)	(3.82)	(199.62)	(145.79)
HML_{ff}	0.961***	0.935***	0.186***	-0.011
	(92.63)	(95.05)	(33.35)	(-1.57)
UMD	0.005	-0.015**	-0.007**	-0.006
	(0.68)	(-2.36)	(-2.00)	(-1.29)
R^2	94.06%	94.43%	98.59%	97.49%

The Fama and French (1993) methodology is used as a benchmark both for comparison and for understanding the drivers of the methodological changes. Premia defined under a sequential sort (either using a preconditioning or post-conditioning on the control variable) both significantly load on their Fama-French equivalent factors. However, they adjust the factors defined under the independent sort as shown by the significant loadings on other factors inside the spanning regression. We however find differences among the two ways of implementing a sequential sort.

In the S-Post framework, the value factor is the result of an adjusted HML spread that puts equal weights on so-defined[7] small and large stocks. Contrary to the independent sort, the correlation between the characteristics does not affect the sorting since the second sort is made within each first sort. The outcome of this construction method is a very low (but positive) correlation between the $SMB_{S\text{-}Post}$ and $HML_{S\text{-}Post}$ factors, leading to 4% compared to -23% under a classical independent sort. However, by reducing the large weight put on large caps with respect to the independent framework, the S-Post procedure a slight tilts of the value factor toward a small-value premium. This result is confirmed in Table 1, which shows that the $HML_{S\text{-}Post}$ is long the Fama and French size factor. Moreover, the $HML_{S\text{-}Post}$ is independent from a momentum strategy and slightly reduces the impact of market conditions when pricing book-to-market effects. We observe a similar effect for the newly defined size premium. To conclude, the use of a sequential sorting approach that preconditions on the variable to be priced modifies the underlying risk drivers of the factors compared to the original Fama and French framework. Similar to Fama and French, it adjusts the weight of the control variable in each of the portfolios constituting the spread. However, unlike Fama and French, it considers the negative correlation between market capitalization and book-to-market characteristics. It readjusts the breakpoints to consider the correlation among the sorting characteristics. This approach controls for the reversal component of the HML factor (lower market capitalization leading to higher book-to-market).

In the "Sequential-Pre" framework, the $SMB_{S\text{-}Pre}$ factor does not display any exposure to the HML_{FF}. The previous analyses showed that the SMB factor defined under the S-Pre framework is not a simple adjusted spread but is the average small size spread across the three levels of book-to-market. The S-Pre framework goes one step further than S-Post. The construction method builds the factors for each level of the control variable and then aggregates them within the whole control universe. Consequently, it completely diversifies the external factors. However, the $HML_{S\text{-}Pre}$, with regard to its Fama and French equivalent, adds exposure to the SMB_{FF} factor, *ceteris paribus*. Again, using a 2x3 framework, the value-

[7] Please note that the sample of small and large stocks differs under the Pre- and Post-Sequential framework.

growth spread does not account for the medium B/M portfolio; therefore, it cannot eliminate the effects of the control variable. This issue will be solved later in the paper when considering a triple (symmetric) sort. Moreover, the $SMB_{S\text{-}Pre}$ reduces the return related to the size effect for market conditions (i.e., the market factor) contrary to both the independent and the post-conditioning sequential frameworks. The next section further investigates the consequences of the two frameworks when considering alternative methodological choices. Simply moving from an independent to a dependent sorting ensures better stocks allocation into portfolio but does not significantly alter the size and value factors as shown by the insignificant alphas from the spanning regressions under both sequential frameworks. In Section 4, we estimate the joint effects with alternative choices for sorting out stocks.

4. Alternative Choices for Sorting Out Stocks

This section investigates the effect of other alternative methodological choices when factoring characteristics into returns, including the definition of breakpoints and multiple sorts. The breakpoints used as a scale to allocate stocks into level-portfolios can be defined either using the whole sample (i.e., using all firms and *all names*) or using only the firms from the NYSE. For sake of simplicity, we refer to "name" and NYSE breakpoints, respectively.

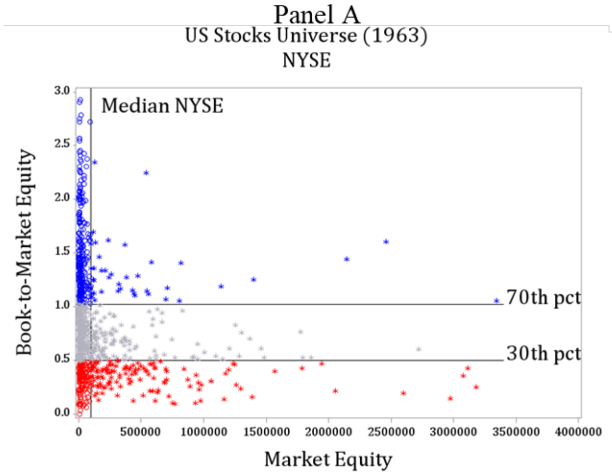
4.1. Name breakpoints versus NYSE breakpoints

The traditional 2x3 independent sort of Fama and French (1993) is performed using NYSE breakpoints. Figure 5 shows that the breakpoints used for book-to-market characteristics are almost unchanged across the sample period (1963, 1994, 2001 and 2014). However, breakpoints for market capitalizations vary widely under changing market conditions. The NYSE size breakpoints increase in favorable market conditions, which induce a market effect in the Fama and French (1993) size premium and a consequent reversal in the HML effect. Sorting stocks according to the breakpoints defined on the entire sample introduces relatively resilient allocation keys into portfolios. Note that under this construction, NASDAQ stocks are largely represented in the small-cap portfolios and represent the main risk dynamics of this sub-portfolio.

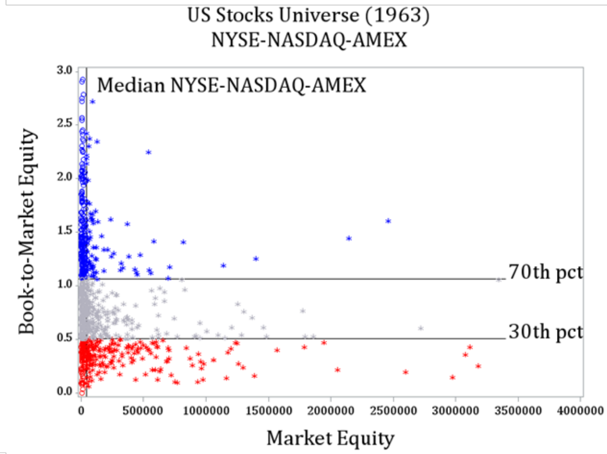
Figure 5 NYSE vs Name Breakpoints

Figures A to H report the stratification of the US stock universe among the FF 2x3 characteristic-sorted portfolios on size (small and big) and book-to-market (low, medium and high). The panels on the left use the NYSE breakpoints, whereas the panels on the right use the whole sample to estimate the breakpoints. Results are reported for the years 1963, 1994, 2001, and 2014. The x-axis refers to the market equity and the y-axis to the book-to-market equity. For better clarity of the breakpoints, outliers are not reported, the x-axis is capped between 0 and $4,000 billion, and the y-axis is truncated between 0 and 3. The exercise could also be performed without truncating axes on a log-scale, and this would lead to equivalent interpretations.

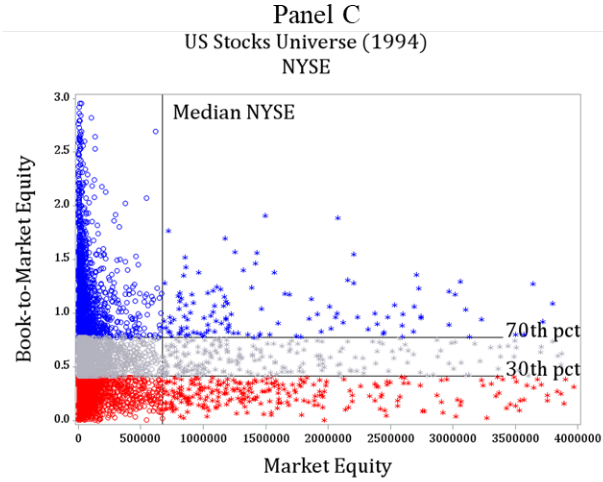
Panel A



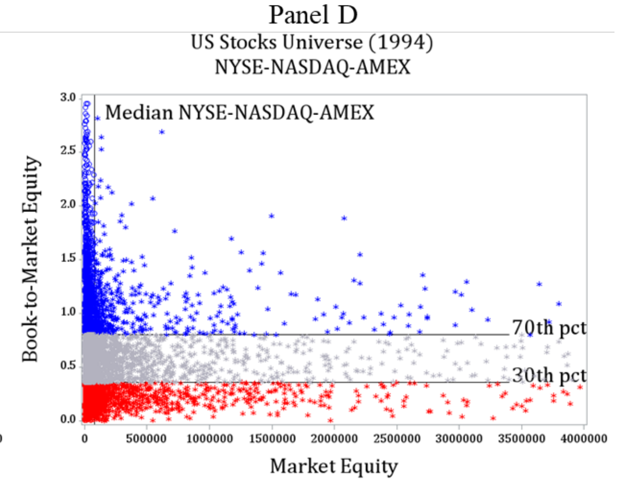
Panel B

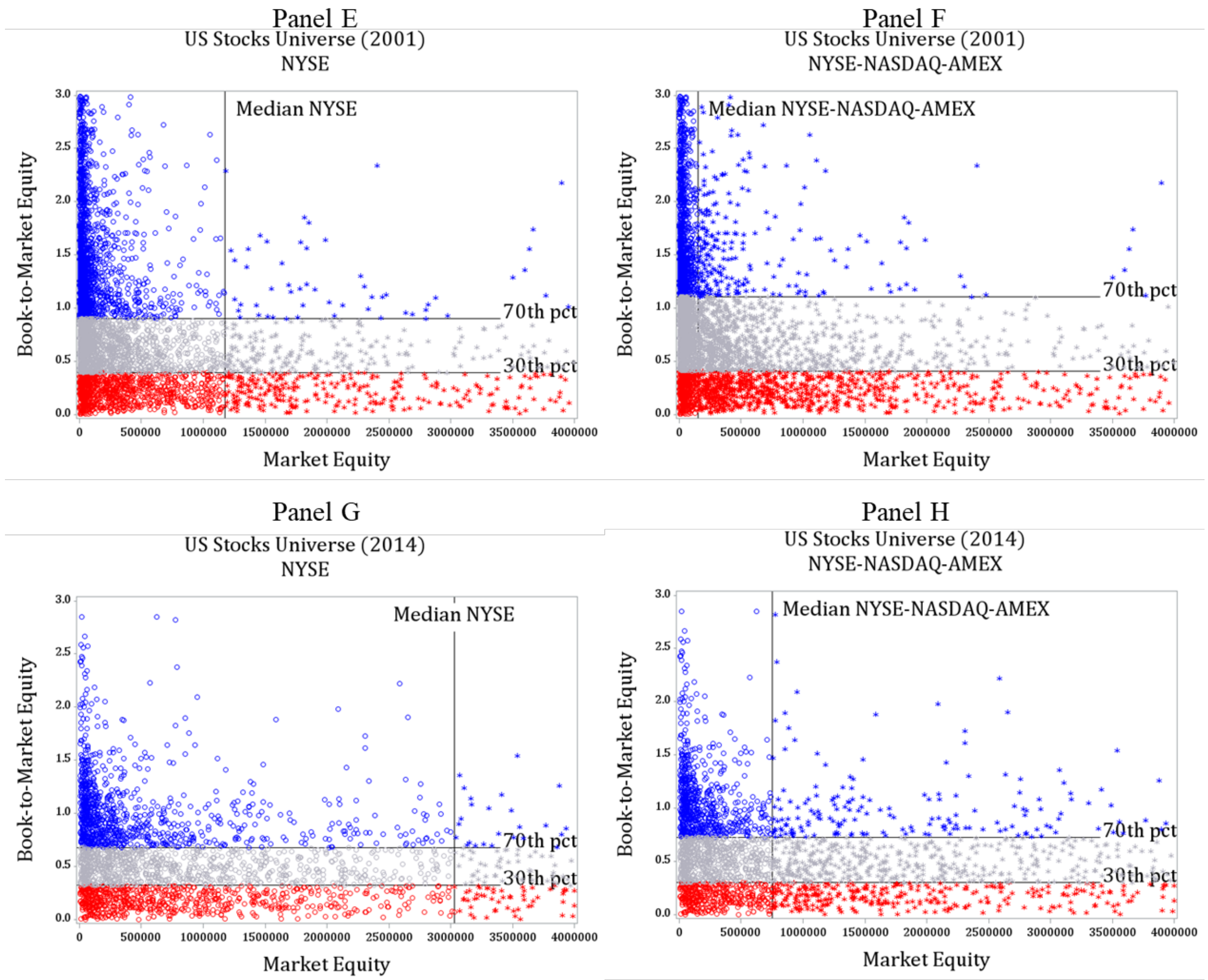


Panel C



Panel D





Panels A-C-E-G (resp. Panels B-D-F-H) display the yearly values of the NYSE (resp. name) breakpoints for market capitalization and book-to-market under a 2x3 independent sorting of stocks. Panels G and H illustrate the momentum/market effect induced in the portfolios sorted using the NYSE breakpoints. To be included in a large-cap portfolio, a given stock needs to be above the threshold defined by the current market conditions. The definition of large caps is much more stable across time using whole sample breakpoints.

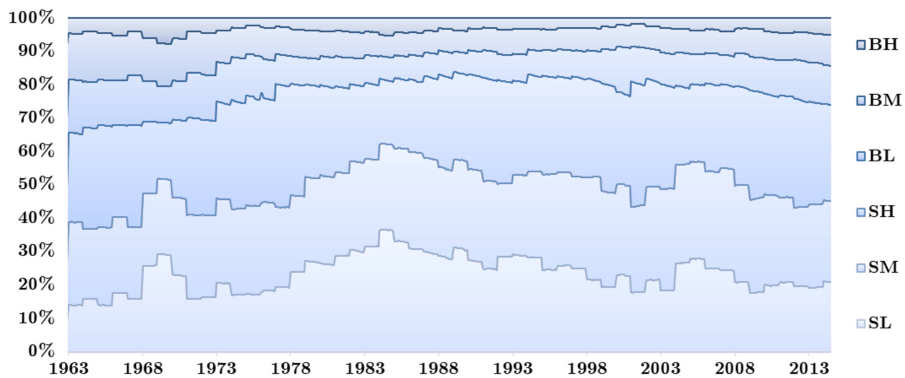
4.2. Name breakpoints versus NYSE breakpoints under a sequential framework

One frequently cited reason for using NYSE breakpoints is that it places more stocks in the low size portfolios, with the objective of capturing a higher percentage of the small capitalization universe in that portfolio. A whole sample approach takes another perspective by having an exogenous definition of a small stock and a classification independent from current market conditions, which might induce various levels of capitalization across portfolios. More specifically, a NYSE framework seeks a balance between the different portfolios (small and large portfolios) based on the total market capitalization included in each portfolio. However, combining the use of whole sample breakpoints with a sequential framework seeks to create a balance between portfolios based on the number of stocks. Consequently, a 2x3 independent sort will induce an imbalance in the number of stocks in portfolios to counter the capitalization effect, while the use of whole sample breakpoints under a sequential sorting would create an imbalance in market capitalization but the same repartition in terms of number of stocks. We visually illustrate the stock repartition for an independent sort (Panel A) and a sequential sort (Panel B) in Figure 6.

Figure 6 Relative Stock Distribution among the 2x3 Characteristics Portfolios

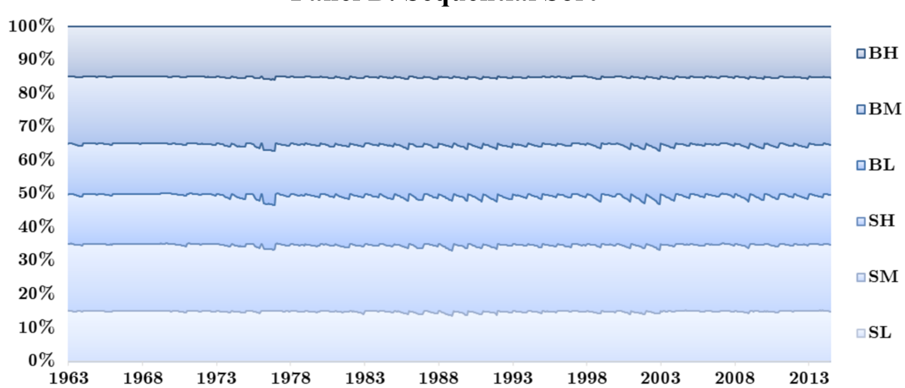
The figure displays the stock repartition (in %) for 2x3 characteristic-sorted portfolios on size (small and big) and book-to-market (low, medium and high) from July 1963 to December 2014. We also report under the figure the mean, standard deviation (SD), minimum and maximum of stocks found in each portfolio. Panel A illustrates the results for the independent (Fama and French 1993) methodology with the NYSE breakpoints. Panel B illustrates the results for the sequential methodology in which the breakpoints are based on the whole sample (NYSE-NASDAQ-AMEX).

Panel A: Independent sort



	SL	SM	SH	BL	BM	BH
Mean	804	892	945	321	261	120
SD	350	364	387	109	56	36
Min	122	206	232	136	123	40
Max	1410	1763	1924	711	387	185

Panel B: Sequential Sort

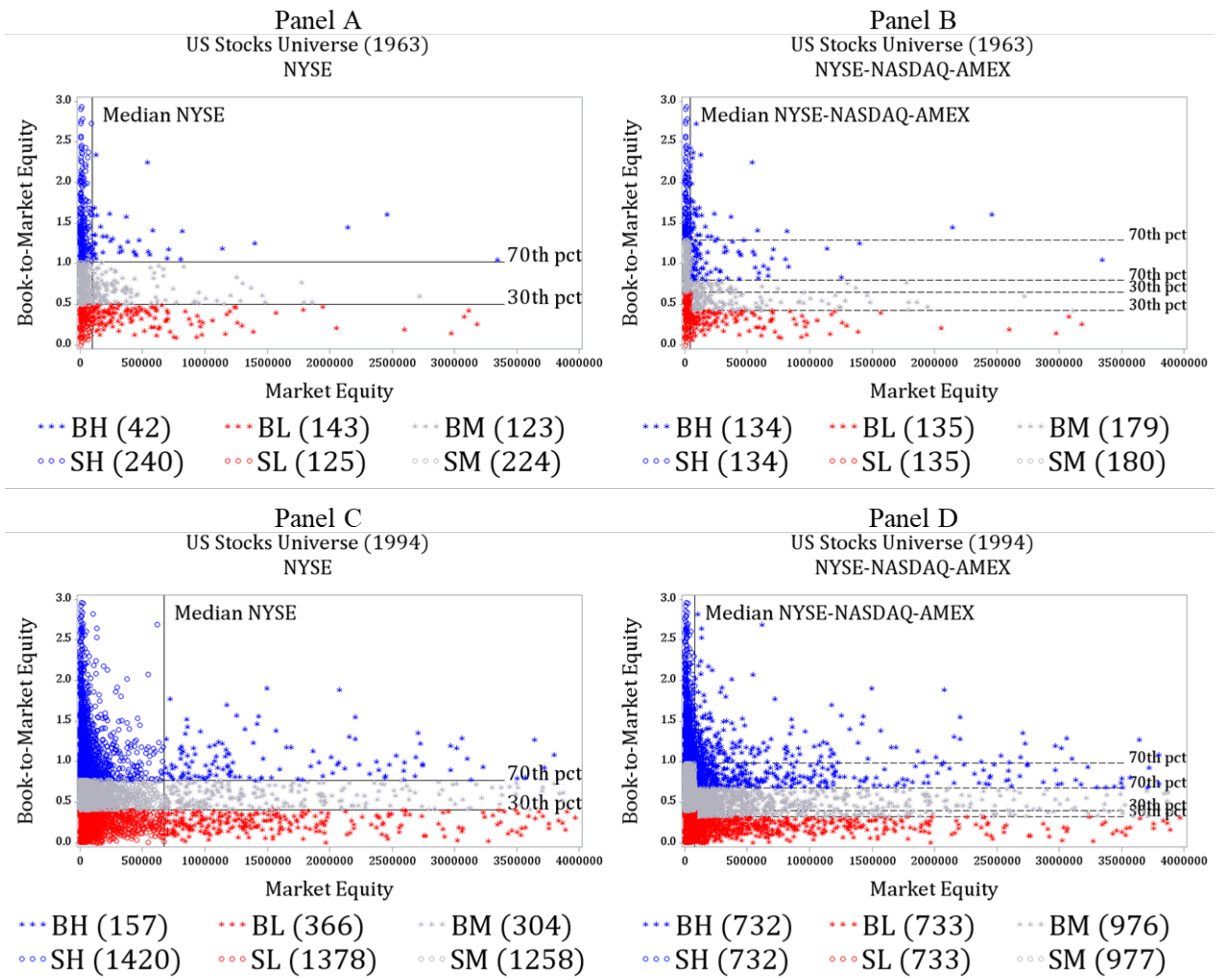


	SL	SM	SH	BL	BM	BH
Mean	496	663	489	510	678	508
SD	177	238	175	184	244	182
Min	131	173	128	133	179	134
Max	864	1151	863	864	1151	863

In other words, by applying the method of size and book-to-market dimensions, many stocks fall into the small-value corner (as noted by Cremers et al. 2012) under an independent 2x3 sorting. This classification bias might have practical unpleasant consequences. Chan et al. (2009, p. 4579) indicate that, for instance, "*many of the stocks that a large-value manager would hold in practice are classified as large-growth stocks under an independent sort procedure*". They also note that this effect is more pronounced at the end of the 1990s. Figure 7 illustrates the effects of breakpoints and sorting procedure on the allocation of stocks into portfolio. At the beginning of July 1994, under an independent sort, the number of stocks in large-value portfolio is 157 but 366 in the large-growth portfolio, supporting Chan et al.'s observation. However, we do not observe such a discrepancy under a dependent framework with 732 stocks in the large-value portfolio and 733 in the large-growth portfolio.

Figure 7 Relative Stock Distribution under Independent vs Dependent Sort

Figures A to H report the stratification of the US stock universe according to the sorting definition. The panel on the left uses the NYSE breakpoints for the traditional FF 2x3 characteristic-sorted portfolios on size (small and big) and book-to-market (low, medium and high), whereas the panel on the right uses the whole sample to estimate the breakpoints with the dependent sort on the same characteristics. We report in parentheses the number of stocks falling in the portfolios. The results are reported for the years 1963, 1994, 2001, and 2014. The x-axis refers to the market equity and the y-axis to the book-to-market equity. For better clarity of the breakpoints, outliers are not reported, the x-axis is capped between 0 and \$4,000 billion, and the y-axis is truncated between 0 and 3. The exercise could also be performed without truncating axes on a log-scale, and this would lead to equivalent interpretations.



Spanning tests on sequential factors (under S-Post and S-Pre frameworks) defined using name (i.e., whole sample, i.e., using all firms, all names) and NYSE breakpoints are displayed in Table 2.

Table 2 Spanning Regressions: 2x3 Portfolios and Breakpoints Definition

The table reports the spanning regression results for the alternative size and value factors. T-statistics of the estimation parameters are in parentheses. The significance of the parameter estimates is reported as performed. *, **, and *** indicate statistical significance at the 0.1, 0.05 and 0.01 levels, respectively. Breakpoints are defined according to either the NYSE (left panel) or the whole sample breakpoints (right panel). The denomination "*S-Post*" refers to a post-conditioning on the control variable, whereas "*S-Pre*" refers to a preconditioning on the control variable. For instance, $HML_{S\text{-}Post}$ represents the value factor when stocks are first sorted by their book-to-market ratio (variable to be priced) and then by their market equity. $HML_{S\text{-}Pre}$ represents the value factor when stocks are first sorted by the market equity (control variable) and then by the book-to-market equity. The same process applies for the size premium. The period used to perform the regressions ranges from July 1963 to December 2014.

	Spanning regressions on *SMB factor using a 2x3 approach*			
	NYSE breakpoints		*Name breakpoints*	
	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$
Alpha	0.016	0.017	-0.025	-0.018
	(-1.01)	(-0.89)	(-0.36)	(-0.28)
RM_{ff}	0.000	-0.016***	-0.083***	-0.106***
	(-0.12)	(-3.43)	(-5.14)	(-6.95)
SMB_{ff}	1.005***	0.936***	1.264***	1.176***
	(-199.62)	(-145.79)	(-56.91)	(-55.55)
HML_{ff}	0.186***	-0.011	0.123***	-0.102***
	(-33.35)	(-1.57)	(-4.99)	(-4.37)
UMD	-0.007**	-0.006	-0.002	-0.011
	(-2.00)	(-1.29)	(-0.14)	(-0.76)
R^2	98.59%	97.49%	84.66%	84.73%

	Spanning regressions on *HML factor using a 2x3 approach*			
	NYSE breakpoints		*Name breakpoints*	
	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$
Alpha	0.036	0.007	0.332***	0.197***
	(-1.25)	(-0.24)	(-6.2)	(-4.07)
RM_{ff}	-0.022***	0.01	-0.088***	-0.007
	(-3.30)	(-1.61)	(-6.94)	(-0.64)
SMB_{ff}	0.268***	0.034***	0.114***	-0.054***
	(-28.61)	(-3.82)	(-6.52)	(-3.41)
HML_{ff}	0.961***	0.935***	0.894***	0.929***
	(-92.63)	(-95.05)	(-46.18)	(-53.37)
UMD	0.005	-0.015**	0.015	-0.014
	(-0.68)	(-2.36)	(-1.2)	(-1.25)
R^2	94.06%	94.43%	81.08%	85.10%

Table 2 shows that when defining the factors using name breakpoints, the SMB and HML premia are independent from the momentum effect under both sequential frameworks. The sequential framework that ends with the dimension to be priced together with the name breakpoint induces a downward adjustment of both the SMB and the HML factors with regard to the original factors to cure for cross-sectional effects.

This evidence was not found in Table 1, in which a small positive coefficient was found for the HML factor. We are comfortable concluding that (unlike the S-Post approach) the S-Pre sequential framework along with the whole sample breakpoints reduces both factors for the outperformance of small value stocks.

The HML factors defined under a sequential approach (S-Post or S-Pre) using the whole sample breakpoints add information to the traditional 2x3 independent framework using the NYSE breakpoints (the alphas of the regressions are significant at the usual significance levels). Preconditioning on the pricing factor (i.e., post-conditioning on the control variable) induces a tilt of the value factor toward small-value stocks, as evidenced by the significant positive exposure of the HML factor to SMB. The second sequential procedure, which consists of performing a conditional sort on the variable to be priced as the last step, produces only a tiny (and even negative) exposure to the size factor (from 11.4% to -5.4%) after controlling for the other source of risk. The sequential pre-conditioning factors both hedge the risk related to size within the HML factor and vice versa through a downward adjustment. Name breakpoints ensure that the long and short legs have the same level of diversification in terms of numbers of firms. The sorting procedure underlying $HML_{S\text{-}Post}$ adds exposure to the size factor as the factor is formed putting equal weight on size controls (i.e., small and large portfolios within the book-to-market portfolios). The logic underlying the $HML_{S\text{-}Pre}$ factor is different. The premium adjusts exposure to size by shorting small stocks (reducing the weight on these stocks), as shown in Table 2. The pre-conditioning framework also eliminates the effects of market conditions on the size premium as shown by the significant negative loadings of $SMB_{S\text{-}Pre}$ on the market portfolio.

4.3. Name/NYSE breakpoints and 3x3 portfolios

We now consider a 3x3 sorting procedure that puts equal weight on each risk dimension. Spanning tests on sequential factors (under S-Post and S-Pre) defined using name/NYSE breakpoints and 3x3 multiple sorting are displayed in Table 3 for comparison.

Table 3 Spanning Regressions: 3x3 Portfolios and Breakpoints Definition

The table reports the spanning regression results for the alternatives size and value factors. T-statistics of the estimation parameters are in parentheses. The significance of the parameter estimates is reported as performed. *, **, and *** indicate statistical significance at the 0.1, 0.05 and 0.01 levels, respectively. Breakpoints are defined according to either the NYSE (left panel) or the whole sample breakpoints (right panel). The denomination "*S-Post*" refers to a preconditioning on the characteristics to be priced, whereas "*S-Pre*" refers to a preconditioning on the control variable. For instance, $HML_{S\text{-}Post}$ represents the value factor when stocks are first sorted by their book-to-market ratio (variable to be priced) and then by their market equity. $HML_{S\text{-}Pre}$ represents the value factor when stocks are first sorted by the market equity (control variable) and then by the book-to-market equity. The same process applies for the size premium. The period used to perform the regressions ranges from July 1963 to December 2014.

Panel A - Spanning regressions on *SMB factors*

	2x3 and name breakpoints		*3x3 and NYSE breakpoints*		*3x3 and name breakpoints*	
	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$
Alpha	-0.025	-0.018	0.004	-0.003	0.089	0.015
	(-0.36)	(-0.28)	(0.09)	(-0.08)	(0.79)	(0.13)
RM_{ff}	-0.083***	-0.106***	-0.011	-0.038***	-0.173***	-0.167***
	(-5.14)	(-6.95)	(-1.47)	(-5.76)	(-6.55)	(-6.53)
SMB_{ff}	1.264***	1.176***	1.234***	1.166***	1.351***	1.345***
	(56.91)	(55.55)	(114.86)	(126.71)	(36.94)	(38.12)
HML_{ff}	0.123***	-0.102***	0.261***	-0.042***	0.157***	-0.106***
	(4.99)	(-4.37)	(22.01)	(-4.12)	(3.89)	(-2.71)
UMD	-0.002	-0.011	-0.005	-0.008	-0.021	-0.039
	(-0.14)	(-0.76)	(-0.59)	(-1.22)	(-0.81)	(-1.56)
R^2	84.66%	84.73%	95.82%	96.70%	69.26%	71.87%

Panel B - Spanning regression on *HML factors*						
	2x3 and name breakpoints		*3x3 and NYSE breakpoints*		*3x3 and name breakpoints*	
	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$
Alpha	0.332***	0.197***	0.072**	0.043	0.402***	0.262***
	(6.20)	(4.07)	(2.53)	(1.43)	(7.32)	(5.46)
RM_{ff}	-0.088***	-0.007	-0.042***	-0.006	-0.095***	-0.034***
	(-6.94)	(-0.64)	(-6.28)	(-0.85)	(-7.31)	(-3.02)
SMB_{ff}	0.114***	-0.054***	0.256***	-0.041***	0.073***	-0.104***
	(6.52)	(-3.41)	(27.86)	(-4.23)	(4.09)	(-6.64)
HML_{ff}	0.894***	0.929***	0.954***	0.9***	0.883***	0.852***
	(46.18)	(53.37)	(94.04)	(83.55)	(44.52)	(49.21)
UMD	0.015	-0.014	0.007	-0.017**	0.016	0.01
	(1.20)	(-1.25)	(1.09)	(-2.43)	(1.22)	(0.90)
R^2	81.08%	85.10%	94.31%	93.29%	80.35%	83.85%

The joint effects of sequential and name breakpoints on the HML is illustrated in Table 3. Pairing these methodological choices produce a HML factor which deviates significantly from the Fama-French original factor as shown by the significant positive alphas. The table allows us to compare NYSE breakpoints with name breakpoints for a 3x3 sort using information. A 3x3 sort using name breakpoints decreases the previously identified size tilt of the conditional HML factor after preconditioning on the book-to-market ($HML_{S\text{-}Post}$). The HML sequential factor preconditioning on control variables ($HML_{S\text{-}Pre}$) adjusts for the momentum effect caused by the NYSE breakpoints as shown by the negative and significant loading on the UMD factor. Sequential factors defined under *S-Pre*, name breakpoints and symmetric adjusts the size and value factors for their exposure towards the market factor and the other factors. This makes the $HML_{S\text{-}Pre}$ factor defined as the Fama-French original factors with two downward corrections, one for market effects and a second for the size effect, which lead to an abnormal return not priced under the original framework. The same downward adjustments occur for the *S-Pre* size factor without however creating abnormal performance. Comparing the R-squared and alphas of the regressions, we can conclude that using name breakpoints instead of NYSE breakpoints significantly affects the composition of risk factors.

5. Portfolio Diversification and the Effect on Factor Construction Biases

In this section, we build on the framework of Cochrane (2011, Appendix B Asset Pricing as a Function of Characteristics, p. 1097). Under the framework of Daniel and Titman (1997), Cochrane (2011) posits the following relationship between characteristics and expected returns:

$$E\left(R^i - R^j\right) = b\left(C_i - C_j\right) \tag{2}$$

where *C* stands for the characteristics (i.e., size or book-to-market) of portfolio *i* or *j*.

Assuming that the sort on characteristics corresponds to an expected return sort with an underlying common risk factor *f*, Cochrane (2011) writes the variance of the spread portfolio as

$$\sigma^2\left(R^i - R^j\right) = (\beta^i - \beta^j)^2\sigma(f)^2 + 2\frac{\sigma_\varepsilon^2}{N} = \frac{b^2}{E(f)^2}(C^i - C^j)^2\sigma(f)^2 + 2\frac{\sigma_\varepsilon^2}{N} \tag{3}$$

where β stands for the exposure of the portfolio *i* or *j* to the common factor *f*, N is the number of stocks within the spread portfolios, and σ_ε^2 is the idiosyncratic variance of the individual stocks composing the spread portfolios.

We extend this equation with subscripts *i* and *j* for the number of stocks into respectively the spread's long and short leg of the spread portfolios because they might differ.

$$\sigma^2\left(R^i - R^j\right) = (\beta^i - \beta^j)^2\sigma(f)^2 + \frac{\sigma_\varepsilon^2}{N^i} + \frac{\sigma_\varepsilon^2}{N^j} = \frac{b^2}{E(f)^2}(C^i - C^j)^2\sigma(f)^2 + \frac{\sigma_\varepsilon^2}{N^i} + \frac{\sigma_\varepsilon^2}{N^j} \tag{4}$$

The Sharpe ratio of the spread portfolio return can now be defined as,

$$\frac{E\left(R^i - R^j\right)}{\sigma\left(R^i - R^j\right)} = \frac{E(f)}{\sigma(f)}\frac{b\left(C_i - C_j\right)}{\sqrt{b^2\left(C_i - C_j\right)^2 + \frac{\sigma_\varepsilon^2}{N^i}\frac{E(f)^2}{\sigma(f)^2} + \frac{\sigma_\varepsilon^2}{N^j}\frac{E(f)^2}{\sigma(f)^2}}} \tag{5}$$

Defining the SMB factor spread as $SMB = {}^1/_3\left[(R^{SL} - R^{BL}) + (R^{SM} - R^{BM}) + (R^{SH} - R^{BH})\right]$, and if we assume, for the sake of simplicity, that the portfolio spreads for different levels of book-to-market are perfectly correlated (the risk premium related to size should be the same across the control variables), we can generalize the formula by writing,:

$$\frac{E(SMB)}{\sigma(SMB)} = \frac{E(f)}{\sigma(f)} \frac{\sum_{i=L,M,H} b(C_{Si} - C_{Bi})}{\sum_{i=L,M,H} \sqrt{b^2(C_{Si} - C_{Bi})^2 + \frac{\sigma_\varepsilon^2}{N^{Si}} \frac{E(f)^2}{\sigma(f)^2} + \frac{\sigma_\varepsilon^2}{N^{Bi}} \frac{E(f)^2}{\sigma(f)^2}}} \tag{6}$$

where S and B denote "Small" and "Big", respectively, among the three levels of controls (L, M, H) and *f* is the underlying common risk factor.

From equation (6), if follows that the Sharpe and *t*-statistics of the spread portfolio will be closer to the one of the true common factor for large characteristics spread, all else being equal. The problem consists in ensuring a proper diversification of the portfolios as a finer sort might diminish the number of stocks into portfolios.

We illustrate this theoretical framework in case of a 2-dimensional sort with perfectly negatively correlated variables. We consider a first sort on market capitalization (ME) (each year in June) and a second sort on the inverse of the market capitalization, i.e., - ME. The aim of the exercise is to show that sorting on two characteristics with a perfect negative correlation should deliver the same pricing effect for an SMB factor and an HML factor unless the sorting methodology produces undesirable effects. We construct an SMB factor with the market capitalization (ME) and an HML factor with the inverse of the market capitalization (-ME).

We first display in Table 4 the number of stocks that falls into the portfolios for the different sorting configurations, i.e., the choice of the sort (independent or dependent), the sort scaling (2x3 or 3x3) and the definition used for the breakpoints (NYSE or all names).

Table 4 Stock Distribution among Portfolios sorted on Correlated Characteristics

Panels A to H report the stock distribution for the size-sorted portfolios. This consists in a double sort: the scale on the first sort on ME ranges from S to B (Small (S), Medium (M), Big (B)), and the scale of the second sort on –ME from L to H (Low (L), Medium (M), High (H)). We report the mean, standard deviation (SD), minimum and maximum of stocks in each portfolio. The sample period is July 1963 to December 2014.

Panel A: Independent - 2x3 - NYSE

	SL	**SM**	**SH**	**BL**	**BM**	**BH**
Mean	0	416	2226	394	309	0
SD	0	131	907	98	85	0
Min	0	133	439	182	124	0
Max	0	696	4085	638	525	0

Panel B: Independent - 2x3 - Name

	SL	**SM**	**SH**	**BL**	**BM**	**BH**
Mean	0	671	977	1021	674	0
SD	0	241	349	367	243	0
Min	0	177	255	269	177	0
Max	0	1152	1726	1727	1151	0

Panel C: Independent - 3x3 - NYSE

	SL	**SM**	**SH**	**BL**	**BM**	**BH**	**ML**	**MM**	**MH**
Mean	0	1	2226	393	0	0	1	724	0
SD	0	0	907	98	0	0	0	214	0
Min	0	1	439	181	0	0	1	256	0
Max	0	2	4085	637	0	0	1	1220	0

Panel D: Independent - 3x3 - Name

	SL	**SM**	**SH**	**BL**	**BM**	**BH**	**ML**	**MM**	**MH**
Mean	0	1	977	1021	0	0	1	1345	0
SD	0	0	349	367	0	0	0	483	0
Min	0	1	255	268	0	0	1	353	0
Max	0	1	1726	1726	0	0	1	2302	0

Panel E: Dependent - 2x3 - NYSE

Portfolios (first sort on the variable used for the HML factor, i.e., - ME)						
	LS	**MS**	**HS**	**LB**	**MB**	**HB**
Mean	196	362	1097	197	363	1129
SD	49	107	447	49	107	461
Min	92	129	217	90	128	222
Max	319	611	2043	319	610	2042
Portfolios (first sort on the variable used for the SMB factor, i.e., ME)						
	SL	**BL**	**SM**	**BM**	**SH**	**BH**
Mean	806	213	1066	281	770	209
SD	315	55	417	72	301	54
Min	177	92	230	123	165	91
Max	1414	349	1885	466	1413	348

Panel F: Dependent - 2x3 - Name									
Portfolios (first sort on the variable used for the HML factor, i.e., - ME)									
	LS	**MS**	**HL**	**LB**	**MB**	**HB**			
Mean	509	671	477	513	675	500			
SD	182	241	170	184	243	180			
Min	135	176	127	134	178	128			
Max	864	1152	863	863	1151	863			
Portfolios (first sort on the variable used for the SMB factor, i.e., ME)									
	SL	**BL**	**SM**	**BM**	**SH**	**BH**			
Mean	504	513	668	677	477	506			
SD	180	184	240	244	170	182			
Min	133	134	173	178	126	134			
Max	864	864	1151	1151	863	863			
Panel G: Dependent - 3x3 - NYSE									
Portfolios (first sort on the variable used for the HML factor, i.e., - ME)									
	LS	**MS**	**HS**	**LB**	**MB**	**HB**	**LM**	**MM**	**HM**
Mean	127	266	1489	111	178	245	156	280	492
SD	33	88	612	26	48	95	40	82	221
Min	57	81	262	53	76	66	72	100	111
Max	201	445	2821	183	313	440	254	463	927
Portfolios (first sort on the variable used for the SMB factor, i.e., ME)									
	SL	**ML**	**BL**	**SM**	**MM**	**BM**	**SH**	**MH**	**BH**
Mean	245	179	112	492	280	156	1489	266	126
SD	95	48	26	220	82	40	613	88	33
Min	66	77	54	113	100	71	261	80	56
Max	441	314	183	920	463	254	2825	444	200
Panel H: Dependent - 3x3 - Name									
Portfolios (first sort on the variable used for the HML factor, i.e., - ME)									
	LS	**MS**	**HS**	**LM**	**MM**	**HM**	**LB**	**MB**	**HB**
Mean	305	402	280	408	538	396	309	405	300
SD	109	145	100	146	193	143	111	146	108
Min	81	105	76	108	143	102	80	106	77
Max	519	691	518	690	922	691	518	690	517
Portfolios (first sort on the variable used for the SMB factor, i.e., ME)									
	SL	**ML**	**BL**	**SH**	**MH**	**BH**	**SM**	**MM**	**BM**
Mean	302	406	309	280	402	304	397	538	407
SD	108	146	111	100	145	109	143	193	146
Min	78	107	80	76	104	80	102	143	108
Max	519	691	518	518	690	517	690	922	691

The table shows that an independent 2x3 sorting produces a strong imbalance in the number of stocks into portfolio, i.e. from 0 to 2226 stocks using NYSE breakpoints and from 0 to 1021 stocks using name breakpoints. The problem worsens under a symmetric sort 3x3. A dependent sort with name breakpoints produces a much better diversified portfolio across the different configurations: the best diversification is achieved under a symmetric sort.

Under Cochrane (2011)'s framework, this problem in stock allocation into portfolio could affect the risk premium's performance. To measure the bias it induces, we construct different size premia using either the first or the second sort and compare their descriptive statistics and correlation. By construction, they should display similar descriptive statistics and should be perfectly correlated.

Table 5 Measuring Bias in Factor Construction Method

Panels A to H displays the size premia under the different configurations of factor construction. We construct a "Small minus Big" (SMB) factor with the market capitalization (ME) and a "High minus Low" (HML) factor with the inverse of the market capitalization (-ME). We report the mean, standard deviation (SD), sum, minimum, maximum, and *t*-statistics of the factors returns. We also display the correlation matrix between the SMB and HML factors. The sample period for this exercise is July 1963 to December 2014.

Factor construction	# Obs	Mean	SD	Sum	Min	Max	*t*-stat	Correlation Matrix	
				Panel A: Independent - 2x3 - NYSE					
SMB	618	0.136	1.651	83.954	-10.004	9.998	2.045	1.0	
HML	618	0.164	1.962	101.336	-9.331	12.650	2.078	0.98041***	1.0
				Panel B: Independent - 2x3 - Name					
SMB	618	0.151	2.102	93.178	-8.786	14.983	1.783	1.0	
HML	618	0.204	2.411	126.171	-8.586	14.941	2.105	0.97861***	1.0
				Panel C: Independent - 3x3 - NYSE					
SMB	618	0.400	4.105	247.040	-16.863	19.502	2.421	1.0	
HML	618	-0.244	3.248	-150.934	-18.457	16.306	-1.869	-0.0184	1.0
				Panel D: Independent - 3x3 - Name					
SMB	618	0.705	7.601	435.411	-21.778	77.421	2.304	1.0	
HML	618	-0.097	4.112	-59.800	-26.556	16.624	-0.585	0.0579	1.0
				Panel E: Dependent - 2x3 - NYSE					
SMB	618	0.138	1.264	85.005	-4.122	7.008	2.706	1.0	
HML	618	0.193	2.351	119.105	-7.380	11.086	2.038	0.88938***	1.0
				Panel F: Dependent - 2x3 - Name					
SMB	618	0.117	1.367	72.300	-6.708	8.798	2.128	1.0	
HML	618	0.231	2.360	142.796	-6.815	11.168	2.434	0.8237***	1.0
				Panel G: Dependent - 3x3 - NYSE					
SMB	618	0.104	1.476	63.960	-6.986	9.285	1.743	1.0	
HML	618	0.104	1.478	64.238	-7.090	9.274	1.748	0.9993***	1.0
				Panel H: Dependent - 3x3 - Name					
SMB	618	0.190	1.765	117.177	-7.679	11.161	2.670	1.0	
HML	618	0.193	1.763	119.366	-7.685	11.091	2.724	0.99953***	1.0

Although correlation between the two size premia produced under the 2x3 independent NYSE framework appears to be very high (98%), the premia's statistics (i.e. mean return and volatility) strongly differ (Panel A, Table 5). However the standard method fails when we try to extend it to name breakpoints or to a symmetric double sorting inducing a finer stratification of US stocks into portfolios. Per construction, the pairs of factors should indeed price similarly the size effect. This is not achieved as shown by the correlation coefficients for these extensions of the original framework. The 3x3 independent sort (Panel C, Table 5) induces an imbalance stratification of stocks into portfolios (Panel C, Table 4). The independent framework fails to price the size effects when extended to a symmetric sort. This illustrates the biases introduced by an independent sorting on correlated characteristics.

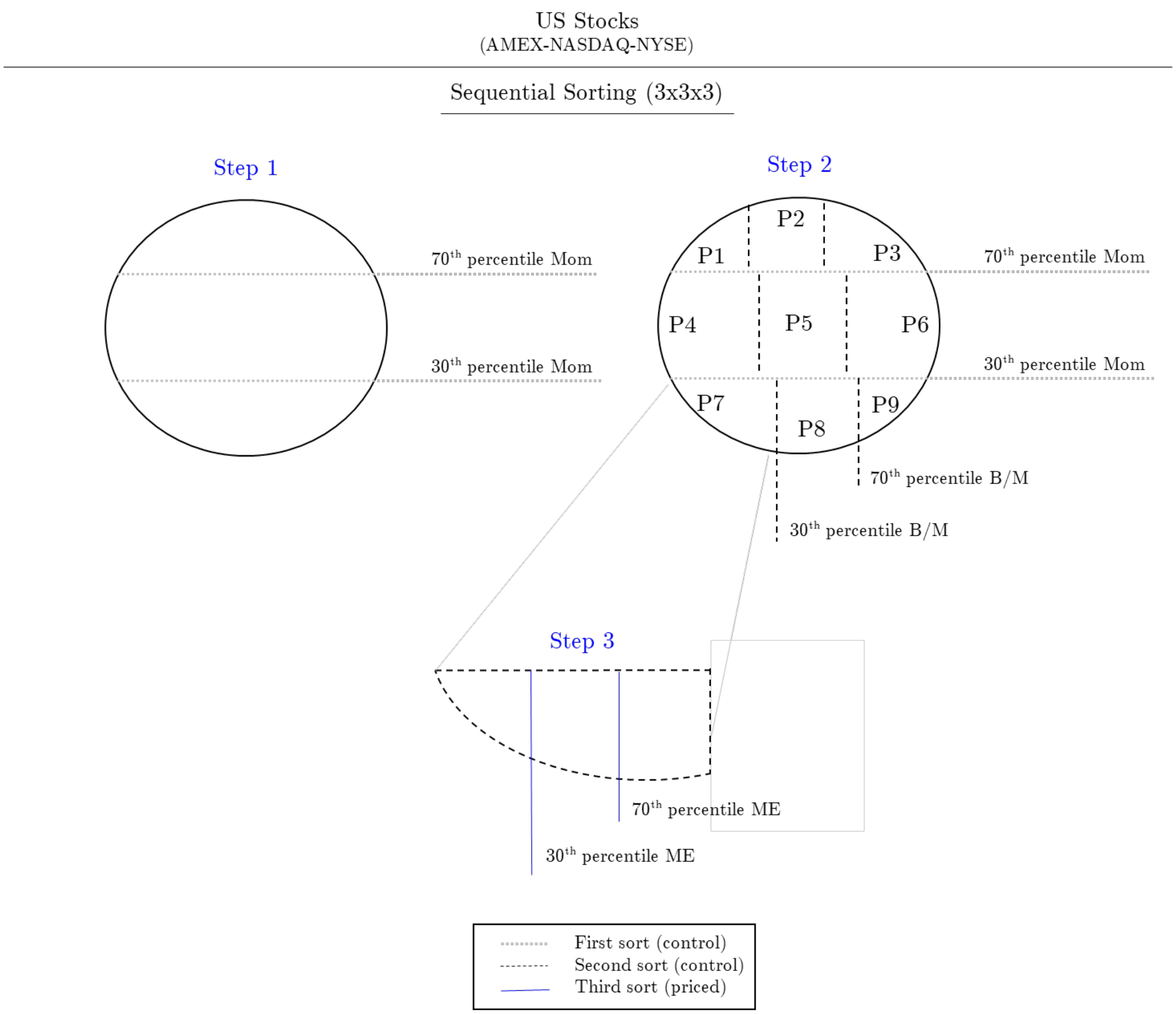
Under a dependent, symmetric (3x3) framework, the twin premia's correlation is close to 100% with very similar descriptive statistics, as per definition. This demonstrates that the dependent symmetric framework can be applied to highly correlated characteristics without introducing measurement biases. Besides, the *t*-statistics are the highest for the dependent 3x3 name breakpoint configuration (Panel H, Table 5). This evidence should be linked with the results of Table 4, Panel H which shows that this sorting methodology ensures a maximization of diversification across the constituting portfolios. Under the framework of Cochrane (2011) described by equation (5), a high level of diversification within portfolios, i.e. high values for N^{SI} and N^{BI}, indeed decreases the distance between the Sharpe or *t*-statistics of the benchmark portfolio (or spread portfolio) and those of the underlying risk factor (Panel H, Tables 4 and 5).

6. Triple Sort on 3x3x3 Portfolios

This section extends the method of a conditional sorting procedure using a triple sort on name breakpoints. It can be viewed as an extension of the approach to two control variables and one pricing factor. We consider three risk dimensions (size, value and momentum) with preconditioning on momentum to control for the business cycle, earnings surprise and profitability shocks. For illustrative purposes, Figure 8 displays the formation of the large momentum-value-size portfolio first by sorting by momentum, then by book-to-market (control variables), and finally by size (priced variable). The US stock universe would be composed of 27 portfolios used to reconstruct one single factor (either size or value).

Figure 8 Sequential Sorting (3x3x3): Preconditioning Method

The figure illustrates the three-dimensional sequential sorting procedure by preconditioning on the control variables. The illustration shows the construction of the size premium (SMB) by preconditioning on the momentum, then the book-to-market ratio (control variable) and finally the sorting on the market capitalization (priced variable).



Under such a framework (i.e., "S-Pre", 3x3x3 and name breakpoints), the size factor is made of the outperformance of stocks with low market capitalization over large market caps among the control subportfolios[8]. This practice ensures that the stocks with high B/M due to tiny market caps do not drive up the HML premium. As in Fama and French (1993, p. 12), we refer to these stocks as "fallen angels" as a reference to “big firms with low stock prices”. Moreover, a stock whose own characteristics remain unchanged may move to another B/M classification even if the full B/M cross-section does not change in a year. This movement could occur if the stock returns follow an upward trend that would inflate its market value and wrongly affect its B/M ratio. An independent sorting would miss this information and incorrectly conclude a low B/M. Such flexibility in stock migration is certainly a core element of the S-Pre procedure, since it advocates that the classification for one of the priced variables (e.g., book-to-market) should not be affected by the controls (e.g., market equity).

The first conclusion that could be drawn from Table 6 is that working in higher dimensions produces factors that deviate further from the FF factors (based on the R-squared). Similar to the previous sequential sorts, the risk factors defined in a sequential 3x3x3 with whole sample breakpoints are not affected by a momentum effect, unlike the FF factors. A size premium defined under a sequential framework is between 1.2 and 1.4 times stronger than the independent 2x3 size premium. Preconditioning or post-conditioning significantly alters the premium’s definition. Post-conditioning on the pricing variable induces a positive relationship between the size and value effects (value stocks tend to be small caps), but a negative effect when first conditioning on the control variables. This contrasted relationship is even stronger under the 3x3x3 sort in which the SMB factor becomes independent from the value effect under the S-Pre framework. The sequential value premium when preconditioning on control variables is also less dependent on market conditions. All these results can be found in Table 6.

[8] A book-to-market ratio (second sort) of 0.5 may put a stock in the high B/M portfolio in one momentum-size portfolio (first sort), in the medium B/M in another, and in the low B/M in a third depending on the cross-sectional variation into the subportfolios.

Table 6 Spanning Regressions: *N*x*N and N*x*N*x*N* Portfolios on Name Breakpoints

The table reports the spanning regression results for the alternative size and value factors. T-statistics of the estimation parameters are in parentheses. The significance of the parameter estimates is reported as performed. *, **, and *** indicate statistical significance at the 0.1, 0.05 and 0.01 levels, respectively. Breakpoints are defined according to the whole sample breakpoints. The denomination "*S-Post*" refers to a post-conditioning on the control variable(s), whereas "*S-Pre*" refers to a preconditioning on the control variable(s). The period used to perform the regressions ranges from July 1963 to December 2014.

Panel A—Spanning regressions on *SMB factors using name breakpoints*						
	2x3		*3x3*		*3x3x3*	
	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$	$SMB_{S\text{-}Post}$	$SMB_{S\text{-}Pre}$
Alpha	-0.025	-0.018	0.089	0.015	0.096	0.075
	(-0.36)	(-0.28)	(0.79)	(0.13)	(0.82)	(0.71)
RM_{ff}	-0.083***	-0.106***	-0.173***	-0.167***	-0.175***	-0.131***
	(-5.14)	(-6.95)	(-6.55)	(-6.53)	(-6.31)	(-5.27)
SMB_{ff}	1.264***	1.176***	1.351***	1.345***	1.329***	1.216***
	(56.91)	(55.55)	(36.94)	(38.12)	(34.62)	(35.38)
HML_{ff}	0.123***	-0.102***	0.157***	-0.106***	0.168***	-0.058
	(4.99)	(-4.37)	(3.89)	(-2.71)	(3.95)	(-1.53)
UMD	-0.002	-0.011	-0.021	-0.039	-0.037	0.005
	(-0.14)	(-0.76)	(-0.81)	(-1.56)	(-1.36)	(0.22)
R^2	84.66%	84.73%	69.26%	71.87%	66.40%	68.65%
Panel B—Spanning regression on *HML factors using name breakpoints*						
	2x3		*3x3*		*3x3x3*	
	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$	$HML_{S\text{-}Post}$	$HML_{S\text{-}Pre}$
Alpha	0.332***	0.197***	0.402***	0.262***	0.371***	0.237***
	(6.20)	(4.07)	(7.32)	(5.46)	(6.42)	(4.79)
RM_{ff}	-0.088***	-0.007	-0.095***	-0.034***	-0.092***	-0.022*
	(-6.94)	(-0.64)	(-7.31)	(-3.02)	(-6.74)	(-1.90)
SMB_{ff}	0.114***	-0.054***	0.073***	-0.104***	0.103***	-0.081***
	(6.52)	(-3.41)	(4.09)	(-6.64)	(5.47)	(-5.04)
HML_{ff}	0.894***	0.929***	0.883***	0.852***	0.881***	0.73***
	(46.18)	(53.37)	(44.52)	(49.21)	(42.23)	(40.97)
UMD	0.015	-0.014	0.016	0.01	0.014	0.001
	(1.20)	(-1.25)	(1.22)	(0.90)	(1.07)	(0.10)
R^2	81.08%	85.10%	80.35%	83.85%	78.37%	77.99%

Table 7 uses all previous factor configurations (including the general case of a 3x3x3) for an illustrative pricing exercise on the 10x10 book-to-market/market capitalization portfolios[9]. We perform either a 3- or a 4-factor model and analyze the cross-section of alphas through (i) the percentage of portfolios for which the alphas are significant at the 10% level, (ii) the average absolute alphas and t-stat, and (iii) the average adjusted R-square. The table shows that the best pricing is achieved for the sequential 3x3x3 defined using whole sample breakpoints as shown by the lowest percentage of significant portfolios achieved in this category.

Table 7 Pricing Errors on 10x10 Size and Value Portfolios

This table exhibits specification errors (α) for the 100 portfolios sorted on size (market equity) and value (book-to-market equity) made available on Ken French's website. The results are reported for the size and value factors based on three construction methodologies: an independent sort (Fama and French 1993), a dependent sort (S-Post) with preconditioning on the variable to be priced and a dependent sort (S-Pre) with preconditioning on the control variable(s). The results are also displayed according to the definition of the breakpoints used to construct the factors (NYSE or whole sample). In the first column, we report the number of significant specification errors (alpha). In the second column, we report their average absolute alpha. In the third column, we report their average absolute t-statistics. Finally, in the fourth column, we report the average R-square of the spanning regressions. In Panel A, we use a 3-factor model composed of the excess market return (MKT-Rf), size (SMB) and value (HML). In Panel B, we use a 4-factor model composed of the 3-factor model and the momentum factor (UMD) from the Ken French library. The sample period ranges from July 1963 to December 2014. The threshold of significance for the intercept estimations is set to 10%.

[9] The analysis of the pricing power of newly defined factors is part of our research agenda.

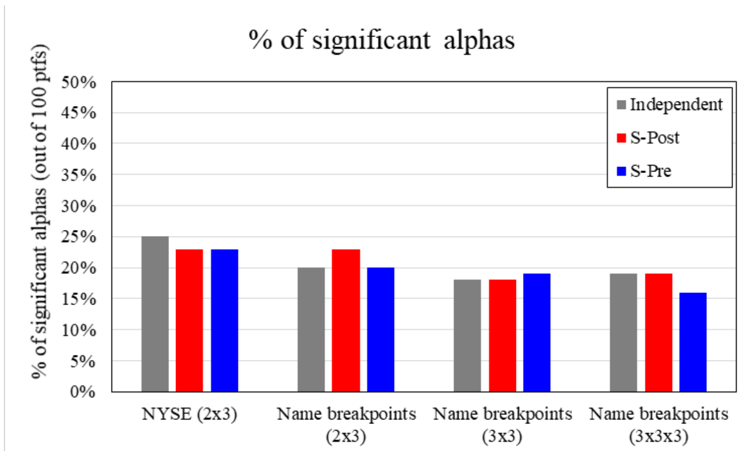
	Spanning test on size and value 10x10 portfolios							
	Panel A: 3-Factor model				Panel B: 4-Factor model			
	# of significant alphas	Average Abs. Alpha	Average Abs. t-stat	Average R^2	# of significant alphas	Average Abs. Alpha	Average Abs. t-stat	Average R^2
	NYSE (2x3)				NYSE (2x3)			
Independent	28%	0.13	1.175	80.16%	25%	0.12	1.044	80.31%
S-Post	26%	0.13	1.152	79.59%	23%	0.12	1.044	79.76%
S-Pre	29%	0.13	1.183	79.79%	23%	0.12	1.059	79.93%
	NYSE (3x3)				NYSE (3x3)			
Independent	25%	0.13	1.156	79.97%	21%	0.11	1.019	80.14%
S-Post	25%	0.13	1.153	79.57%	21%	0.12	1.069	79.75%
S-Pre	27%	0.13	1.191	79.88%	26%	0.12	1.057	80.02%
	Name breakpoints (2x3)				Name breakpoints (2x3)			
Independent	24%	0.14	1.190	78.73%	20%	0.11	0.969	78.90%
S-Post	29%	0.15	1.216	77.79%	23%	0.13	1.081	78.01%
S-Pre	24%	0.13	1.175	78.62%	20%	0.11	0.985	78.76%
	Name breakpoints (3x3)				Name breakpoints (3x3)			
Independent	22%	0.15	1.186	76.90%	18%	0.11	0.932	77.11%
S-Post	27%	0.16	1.257	76.42%	18%	0.13	1.073	76.63%
S-Pre	26%	0.14	1.159	76.99%	19%	0.12	0.955	77.16%
	Name breakpoints (3x3x3)				Name breakpoints (3x3x3)			
Independent	26%	0.15	1.160	76.34%	19%	0.11	0.908	76.58%
S-Post	28%	0.15	1.200	76.12%	19%	0.13	1.037	76.32%
S-Pre	21%	0.14	1.132	76.62%	16%	0.11	0.885	76.81%

Table 7 shows that the need for a momentum factor to price the 10x10 portfolios is more important (comparing the number of significant alphas from Panel A to Panel B) when using name breakpoints and the sequential procedures, since these construction features lead to factors exempted from a momentum effect.

When controlling for momentum (Panel B), the improvement in pricing power is almost observed at each methodological step (S-Pre, name breakpoints, and three-dimension sorting). The illustration is provided in Figure 9.

Figure 9 Percentage of Significant Alphas under Each Framework

The figure presents the percentage of portfolios remaining with a significant intercept at a 90% confidence interval. The results are from Panel B of Table 5, and the regression model is the Carhart 4-factor model composed of the excess market return (MKT-Rf), size (SMB), value (HML) and the momentum factor (UMD) from the Ken French library. The results are presented for the independent (gray), S-Post (red), and S-Pre (blue) sorting methodologies. The sample period ranges from July 1963 to December 2014.



7. Discussion

The paper examines the pricing effect of a set of methodological choices when building empirical mimicking portfolios that account for the traditional Fama-French-Carhart risk factors. The properties of the different factors must be analyzed in the framework of Cochrane (2011) with regard to portfolio diversification.

From the decomposition of the empirical choices for constructing the hedge portfolios, we obtain the following key findings.

- Using an independent versus a sequential approach leads to different pricing effects. Both sets of sequential factors adjust the original factors for external factors (See Section 3);
- Using whole sample breakpoints allows for resilient breakpoints across time and avoids a momentum/market effect within the size factor (See Section 4);

- In the case of a sequential sort, preconditioning or post-conditioning on the control variable(s) affects the meaning of the factor. Post-conditioning will simply reallocate weights among the controls, whereas preconditioning performs an effective filter. The consequence illustrated in this paper for the size and book-to-market effects is that the preconditioning sorting approach produces a value factor that is almost not driven by the small-value effect. Indeed, using name breakpoints, the $HML_{S\text{-}Pre}$ factor tends to short the size factor to hedge for size effects, whereas $HML_{S\text{-}Post}$ takes a long position with respect to size-related risks (See Section 3); and
- Multiple sorting on correlated characteristics will produce biased factors under an independent framework due to an unequal stock allocation into spread's portfolio constituents. However, a sequential and symmetric multiple sort on characteristics using name breakpoints produces robust risk factors by ensuring a proper diversification of the portfolios constituting the long and short legs of the spread (See Sections 5 and 6).

Section 6 shows that the best pricing performance is achieved when adding a multiple sorting procedure. The sequential framework that preconditions on control variables answers the question, "how profitable is the part of SMB (resp. HML) that is orthogonal to the other factors?" To answer this question, the sequential methodology orthogonalizes the rankings that underlie the sort of stocks into portfolios. It directly indicates whether the return variation related to the third risk criterion still exists after controlling for two other risk dimensions. These portfolios can be used as a proxy for long/short risk factors (without the biases mentioned in the introduction and in Section 5) because the factors will not over-represent one control category, will ensure a proper diversification of the spread portfolios, and will maximize the characteristics spread. This ensures, following Cochrane (2011), that the benchmark factor converges to the unobservable underlying risk factor.

8. Conclusion

In this paper, we claim that naive portfolio sorts can lead to the definition of biased fundamental risk factors. In turn, these shortcomings will affect the quality of performance evaluation and portfolio benchmarking. We show that portfolio sorting choices affect the definition of the priced factor. The criteria should therefore be carefully selected before producing risk factors. To date, despite the critical importance of this challenge, no study has comprehensively analyzed the consequences of alternative spread portfolio constructions and the biases introduced by an unequal distribution of stocks into portfolios constituting risk factors. This analysis has always been approached as a byproduct of empirical studies. With our purely methodological paper, we aim to fill this gap in the literature with both an empirical and a theoretical framework to factoring characteristics into returns.

Given the intrinsic correlation between fundamental pricing characteristics, the sequential sorting procedure is a powerful tool that adjusts the effect of external factors. Using name breakpoints reduces the momentum effect within the size factor and the reversal effect of the value factor. Together with a dependent and symmetric sort, it ensures a proper diversification of the long and short legs of the spread portfolio while maximizing the characteristics spread. Following Cochrane (2011), this minimizes the distance between the constructed benchmark and the true underlying factor. The results are robust for multidimensional sorts.

A distinction should be made though for two types of dependent sorting. Post-conditioning on the control variables reduces the weights of external effects on the spread. By preconditioning the sort on the control variable(s) however, the effects of external factors are almost eliminated by the procedure. Our results indicate that the latter sequencing approach has better statistical and economic properties than the former. From a trading perspective, this means that the factors represent the returns of a pure strategy that hedges against the external correlated risks by shortening for instance the size factor to capture a pure value premium.

Naturally, the evidence presented here is limited to the Fama-French-Carhart set of original factors, but those factors are very influential in the empirical asset pricing literature.

Beyond the original size-value-momentum four-factor model, our article paves the way for the systematic use of a (preferably preconditioning on the control variable(s)) sequential approach for the construction of spread portfolios mimicking multidimensional risk factors. The empirical asset pricing literature has witnessed a multiplication of K-factor models rooted in the Fama-French tradition, such as the extended 5-factor model (Fama and French 2016), the q-factor model (Hou *et al.* 2015a), and the recent mispricing factors (Stambaugh and Yuan 2017). What are the implications of their construction procedures for their explanatory powers? Could a more accurate portfolio construction process lead to greater parsimony in the design of factor models, or are some of the attributes proposed in the literature genuinely associated with identifiable risk factors? Our methodological discussion could answer these important questions. These research directions occupy a prominent position in our future research agenda.

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