



Diametral dimensions and some applications to spaces S^ν

Dissertation presented by

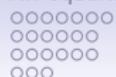
Loïc DEMEULENAERE

for the degree of Doctor in Sciences

Université de Liège – Faculty of Sciences – *Mathematics* Research Unit

Advisor: Françoise BASTIN

Liège, May 15, 2018



Introduction

An equality between two diametral dimensions

Some definitions and the open question

Main results

Two other tools

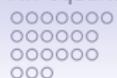
In non-metrizable spaces

Study of spaces S^ν

Definition and properties

The concave case

The locally p -convex case



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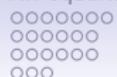
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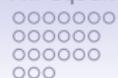
Spaces S^ν

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- Approximation of spectrum of singularities?
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Spaces S^ν

- **Origin:** *Multifractal Analysis (spectrum of singularities)*.
- Approximation of spectrum of singularities?
 \rightsquigarrow *(Sequence) spaces S^ν* (Jaffard, 2004).
- **Topological study of spaces S^ν :** metrizable topological vector spaces (t.v.s.), separable, complete, locally pseudoconvex, **Schwartz, non-nuclear** (Aubry, Bastin, Dispa, Esser, Jaffard).

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Spaces S^ν and diametral dimension

- **Idea:** determining the *diametral dimension* Δ of spaces S^ν (Aubry, Bastin, 2010).

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Spaces S^ν and diametral dimension

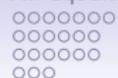
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 - ▶ Diametral dimension for non-locally p -convex spaces S^ν ?
 - ▶ Isomorphic?

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Diametral dimensions

- *Another* diametral dimension Δ_b ... (Mityagin, 1961)



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$$\Delta = \Delta_b$$

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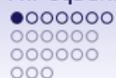
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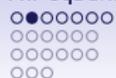
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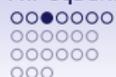
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Kolmogorov's diameters

Let E be a vector space, $V, U \subseteq E$, $V \subseteq \mu U$ ($\mu > 0$).



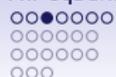
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The n^{th} Kolmogorov's diameter of V with respect to U is

$$\delta_n(V, U) := \inf \{ \delta > 0 : \exists L \subseteq E, \dim(L) \leq n, \text{ s.t. } V \subseteq \delta U + L \}.$$



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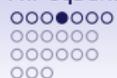
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Important property

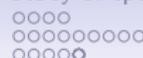
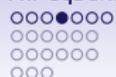
If U is absolutely convex and absorbing, V is precompact with respect to U iff

$$\delta_n(V, U) \rightarrow 0 \text{ if } n \rightarrow \infty.$$



The “first” diametral dimension

Let E be a topological vector space (t.v.s.) and \mathcal{U} be a basis of 0-nghbs.



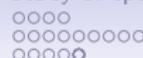
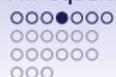
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The *diametral dimension* of E is

$$\Delta(E) := \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U}, \exists V \in \mathcal{U} \text{ s.t. } (\xi_n \delta_n(V, U))_{n \in \mathbb{N}_0} \in c_0 \right\}.$$



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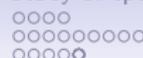
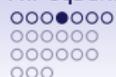
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Δ is a topological invariant: if $E \cong F$, then $\Delta(E) = \Delta(F)$.



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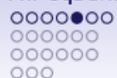
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NB: $c_0 \subseteq \Delta(E)$.



Topological characterizations

Theorem

If E is a l.c.s., TFAE:

- E is Schwartz;
- $l_{\infty} \subseteq \Delta(E)$;
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Topological characterizations

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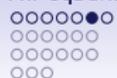
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If E is a l.c.s., TFAE:

- E is nuclear;
- $\forall p > 0, ((n+1)^p)_{n \in \mathbb{N}_0} \in \Delta(E)$;
- $\exists p > 0$ s.t. $((n+1)^p)_{n \in \mathbb{N}_0} \in \Delta(E)$.

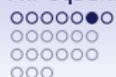


The “second” diametral dimension

Definition

If E is a t.v.s. and \mathcal{U} is basis of 0-nghbs,

$$\Delta_b(E) := \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U}, \forall B \text{ bounded, } (\xi_n \delta_n(B, U))_{n \in \mathbb{N}_0} \in c_0 \right\}.$$



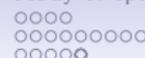
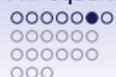
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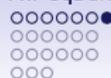
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Conjecture (Mityagin)

If E is Fréchet,

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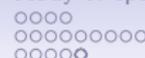
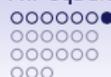


Topological Characterization

Theorem

If E is a l.c.s., TFAE:

- Every bounded set of E is precompact (“ E is *pseudo-Montel*”);
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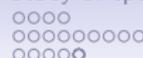
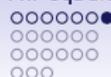
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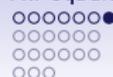
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- If E is pseudo-Montel, but not Schwartz, $\Delta(E) = c_0 \subsetneq \Delta_b(E)$.



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New conjecture

If E is Fréchet-Schwartz,

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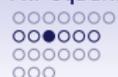
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A first approach

Idea

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We fix a metrizable l.c.s. E , with $\mathcal{U} = (U_k)_{k \in \mathbb{N}_0}$ a decreasing basis of absolutely convex 0-nghbs.



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E has the *property of large bounded sets* if, $\forall m \in \mathbb{N}_0$,
 $\forall (r_k)_{k \geq m} \subseteq (0, \infty)$, $\exists M \geq m, \exists B$ bounded set with

$$\delta_n(B, U_M) \geq \inf_{k \geq m} (r_k \delta_n(U_k, U_m)) \quad (n \in \mathbb{N}_0).$$



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Theorem (Bastin, L.D., 2017)

If $\Delta(E) = \Delta^\infty(E)$ and if E has the property of large bounded sets, then

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Positive examples: Regular spaces, smooth sequence spaces



A generalization

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Spaces with $\Delta(E) = \Delta^\infty(E)$

- *Köthe-Schwartz sequence spaces* (Bastin, L.D., 2017)



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But no definitive conclusion...



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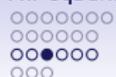
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Δ -stability



Δ -stability

Definition

A space E is Δ -stable if

$$\Delta(E) \subseteq \Delta(E) * \Delta(E),$$

where, for $x, y \in \mathbb{C}^{\mathbb{N}_0}$, $x * y = x_0, y_0, x_1, y_1, x_2, y_2, \dots$



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Proposition

1. If E, F are Δ -stable,

$$\Delta(E \times F) = \Delta(E) \cap \Delta(F)$$

2. If $\Delta(E) \subseteq \Delta(F)$ and if E is Δ -stable,

$$\Delta(E \times F) = \Delta(E)$$



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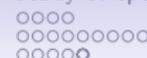
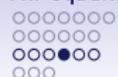
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Warning!

There exist non- Δ -stable Köthe-Schwartz sequence spaces!

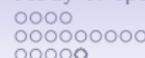
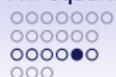


Prominent bounded sets

Definition (Terzioğlu, 2013)

A bounded set B in E is *prominent* if

$$\Delta(E) = \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U}, (\xi_n \delta_n(B, U))_{n \in \mathbb{N}_0} \in c_0 \right\}.$$



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If E has a prominent bounded set, $\Delta(E) = \Delta_b(E)$.



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There exist Köthe spaces **with prominent bounded sets** (smooth sequence spaces of finite type)...

But also **without any prominent set** (smooth sequence spaces of infinite type)!



Prominent bounded sets and property $(\overline{\Omega})$

Definition

A Fréchet space E , with a fundamental system of increasing seminorms $(\|\cdot\|_k)_{k \in \mathbb{N}_0}$ has the *property* $(\overline{\Omega})$ if

$$\forall m \exists k \forall j \exists C > 0 : (\|x'\|_k^*)^2 \leq C \|x'\|_m^* \|x'\|_j^* \quad \forall x' \in E'$$

where $\|x'\|_k^* := \sup\{|x'(x)| : \|x\|_k \leq 1\}$.



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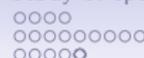
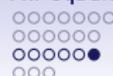
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$(\overline{\Omega})$ implies the existence of prominent sets in

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Warning! $H(\mathbb{D}) \times H(\mathbb{C})$ has prominent sets but not $(\overline{\Omega})$! (L.D., Frerick, Wengenroth, 2016)



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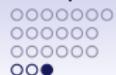
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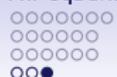


A difference between diametral dimensions?

Idea

Topological properties to have

$$\Delta(E) \subsetneq \mathbb{C}^{\mathbb{N}_0} = \Delta_b(E)?$$



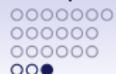
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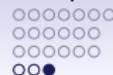
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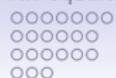
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In metrizable spaces?

If E is metrizable,

$$\Delta(E) = \mathbb{C}^{\mathbb{N}_0} \text{ or } \Delta_b(E) = \mathbb{C}^{\mathbb{N}_0} \iff E \text{ is a subspace of } \omega \dots$$



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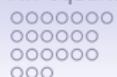
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Notations

Sequences indexed by a binary tree:

- $\Lambda := \{(j, k) \in \mathbb{N}_0^2 : k \leq 2^{j-1}\}$;
- $\Omega := \mathbb{C}^\Lambda$;
- Notations of elements of Ω : “ $\vec{c} \in \Omega$ ”.



Notations

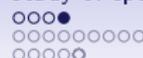
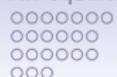
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Definition

A map $\nu : \mathbb{R} \rightarrow \{-\infty\} \cup [0, 1]$ is an *admissible profile* if it is increasing, right-continuous, and s.t.

$$\alpha_{\min} := \inf\{\alpha \in \mathbb{R} : \nu(\alpha) \geq 0\} \in \mathbb{R}.$$



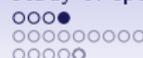
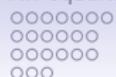
Spaces S^ν

Definition

$\vec{c} \in S^\nu$ if

$$\forall \alpha > 0, \forall \varepsilon > 0, \forall C > 0, \exists J \in \mathbb{N}_0 : \#E_j(C, \alpha)(\vec{c}) \leq 2^{(\nu(\alpha) + \varepsilon)j} \quad \forall j \geq J$$

where $E_j(C, \alpha)(\vec{c}) := \{k : |c_{j,k}| \geq C2^{-\alpha j}\}$.



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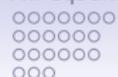
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Natural metric topology

T.v.s., separable, complete, locally pseudo-convex or locally p -convex, Schwartz, non-nuclear.



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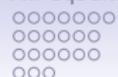
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Theorem (Aubry, Bastin, 2010)

If S^ν is locally p -convex, then

$$\Delta(S^\nu) = \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall s > 0, (\xi_n(n+1)^{-s})_{n \in \mathbb{N}_0} \in c_0 \right\}.$$



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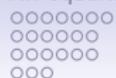
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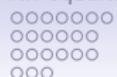
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When ν is concave...

Theorem

If ν is concave, $\varepsilon_m \rightarrow 0^+$, and $(p_n)_{n \in \mathbb{N}_0}$ is dense in $(0, \infty)$,

$$S^\nu = \bigcap_{m \in \mathbb{N}_0} \bigcap_{n \in \mathbb{N}_0} b_{p_m, \infty}^{\eta(p_n)/p_n - \varepsilon_m}$$



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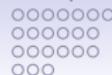
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where

- $\eta : p > 0 \mapsto \inf_{\alpha \geq \alpha_{\min}} \{ \alpha p - \nu(\alpha) + 1 \}$
- $b_{p, \infty}^s$ is the (pseudo)normed space

$$\left\{ \vec{c} \in \Omega : \|\vec{c}\|_{b_{p, \infty}^s} := \sup_{j \in \mathbb{N}_0} \left[2^{\left(s - \frac{1}{p}\right)j} \left(\sum_{k=0}^{2^j - 1} |c_{j,k}|^p \right)^{1/p} \right] < \infty \right\}.$$



When ν is concave...

Topology of S^ν defined by

$$P_m^{(I)} : \vec{c} \in S^\nu \mapsto \sup_{i \in I} \|\vec{c}\|_{b_{p_i, \infty}^{p'_i - \varepsilon_m}}$$

with $m \in \mathbb{N}_0$, $I \subseteq \mathbb{N}_0$, $\#I < \infty$, and $p'_i := \eta(p_i)/p_i$.



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To simplify: $\varepsilon_m = 1/(m+1)$, $\{p_n : n \in \mathbb{N}_0\} = \mathbb{Q} \cap (0, \infty) =: \mathbb{Q}^+$.



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$$\Delta(S^\nu) = \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall s > 0, (\xi_n(n+1)^{-s})_{n \in \mathbb{N}_0} \in c_0 \right\}.$$



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Purpose

$$\Delta(S^\nu) \subseteq \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall s > 0, (\xi_n(n+1)^{-s})_{n \in \mathbb{N}_0} \in c_0 \right\} ?$$



When ν is concave...

Topology of S^ν defined by

$$P_m^{(I)} : \vec{c} \in S^\nu \mapsto \sup_{i \in I} \|\vec{c}\|_{b_{p_i, \infty}^{p'_i - \varepsilon_m}}$$

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Lower-bounds for Kolmogorov's diameters in spaces S^ν ?



Idea

Tikhomirov's Theorem

Let E be a normed space, U be its closed unit ball, and B be a bounded set. If $P : E \rightarrow E$ is a projection with $\|P\| \leq 1$ and $\dim(P(E)) = n + 1$ and if $\delta > 0$ is s.t.

$$\delta U \cap P(E) \subseteq B,$$

then $\delta_n(B, U) \geq \delta$.



Idea

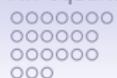
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$$\leadsto \|\cdot\|_B \leq \frac{1}{\delta} \|\cdot\|_U \quad \text{on } P(E).$$



The index sets I_ε

Lemma

$$\lim_{p \rightarrow 0^+} \left(\frac{\eta(p)}{p} \right) = \alpha_{max} := \inf\{\alpha \in \mathbb{R} : \nu(\alpha) = 1\}.$$

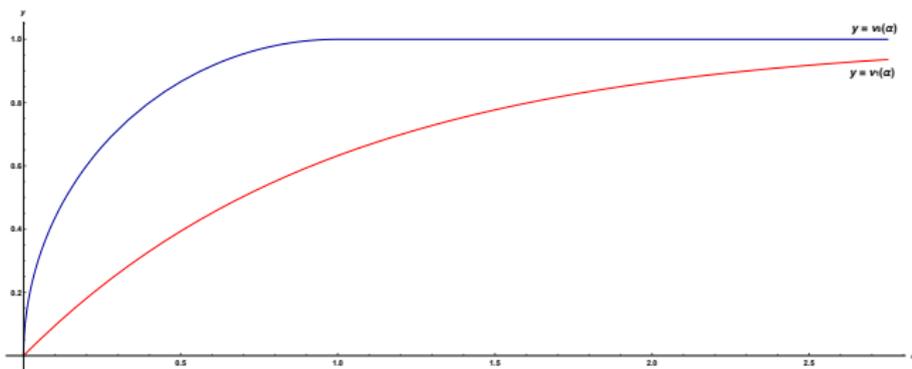


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$\alpha_{\max} < \infty$ or $\alpha_{\max} = \infty$?

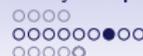
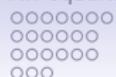




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Construction (L.D., 2017)

If $\alpha_{\max} < \infty$ and if $\varepsilon \in \mathbb{Q}^+$.



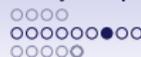
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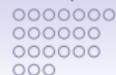
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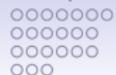
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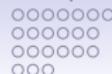


Properties of the sets I_ε

Proposition

For all $n \in \mathbb{N}_0$, $\varepsilon \in \mathbb{Q}^+$, $m \in \mathbb{N}_0$, and $\vec{c} \in S^\nu$,

$$\|\vec{c}\|_{b_{pn, \infty}^{p'_n - \varepsilon m}} \leq \sup_{i \in I_\varepsilon} \sup_{j \in \mathbb{N}_0} \left[2 \left(p'_i - \frac{1}{p_i} + \varepsilon - \varepsilon m \right) j \left(\sum_{k=0}^{2^j - 1} |c_{j,k}|^{p_i} \right)^{1/p_i} \right].$$



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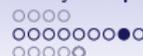
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Corollary

For all $m, k \in \mathbb{N}_0$, with $k \geq m$, $\varepsilon \in \mathbb{Q}^+$, $J \subseteq \mathbb{N}_0$ s.t. $J \supseteq I_\varepsilon$,

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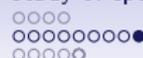
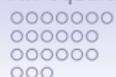
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\rightsquigarrow formula of $\Delta(S^\nu)$ + extension to some locally pseudoconvex spaces S^ν !



Property $(\overline{\Omega})$



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Theorem

If E is Fréchet space and if $(U_k)_{k \in \mathbb{N}_0}$ a basis of 0-neighbourhoods, then E verifies $(\overline{\Omega})$ iff

$$\forall m \in \mathbb{N}_0 \exists k \in \mathbb{N}_0 \forall j \in \mathbb{N}_0 \exists C > 0 : U_k \subseteq rU_j + \frac{C}{r}U_m \quad \forall r > 0.$$



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Theorem (L.D., 2017)

If ν is concave and if $\alpha_{\max} < \infty$, then S^ν verifies (Ω_{id}) .



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Some definitions and the open question

Main results

Two other tools

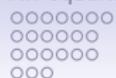
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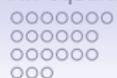
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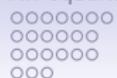
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When S^ν is locally p -convex...

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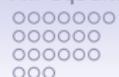
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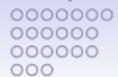
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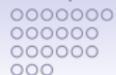
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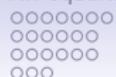


Thank you for your attention!



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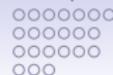
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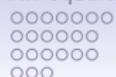
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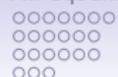
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