

# Modelling of three-phase four-wire low-voltage cables taking into account the neutral connection to the earth

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## Introduction

Local energy communities (LECs) usually occur at the level of low-voltage distribution networks, which are inherently unbalanced due to single-phase household appliances and distributed generation. To simulate and optimise the behaviour of an LEC, the three phases and neutral must be modelled explicitly. This paper aims at numerically assessing the influence of the modelling of the earth and the connection between the neutral and the earth, in terms of voltages and currents.

## Electrical line modelling

The electrical lines are composed of four conductors (three phases and one neutral), modelled with an impedance matrix, which corresponds to the resistance, self- and mutual impedance of the conductors, computed using PowerFactory, by entering the physical constants of the conductors (material, section, etc.), and using Carson's equations to update the impedance matrix to take into consideration the earth return path.

## Test cases

1. No buses are grounded, except the neutral point of the distribution transformer which is perfectly grounded.
2. The neutral of specific buses is grounded through a 0.5-Ω impedance.
3. The neutral of specific buses is perfectly grounded.
4. The neutral of all buses is perfectly grounded.

## Hypotheses of Carson's equations

- They are originally designed for overhead lines with widely spaced conductors.
- They use image theory to compute the earth return path, and thus assume the latter is parallel to the conductors.

Thus, they are not necessarily applicable to LV distribution system, especially if they present a complex topology and a proximity to underground pipes.

## Test system

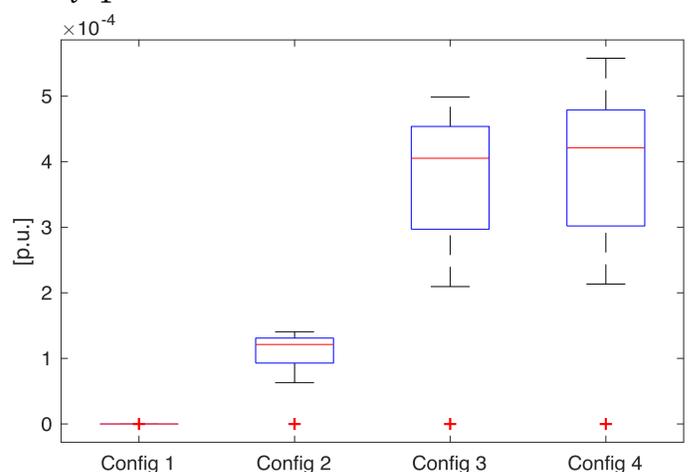
Existing Belgian LV distribution network

- composed of one underground feeder with a star configuration 400V/230V,
- supplying 19 houses, which are all equipped with a smart meter measuring the mean voltage, current, and active and reactive power every minute for each phase.

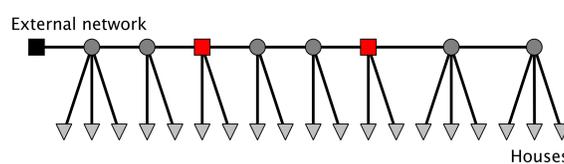
## Results – Modelling the earth return path

Simulations have been performed for each network configuration, using and without using Carson's equations to represent the earth return path. The difference between the simulation results from the two earth modelling techniques is assessed with the mean absolute difference (MeanAD):

$$\frac{1}{n} \sum_{i=1}^n \frac{|V_i - V_i'|}{230} \quad \text{where } V_i \text{ and } V_i' \text{ are two voltage time series, and } n \text{ is their length.}$$



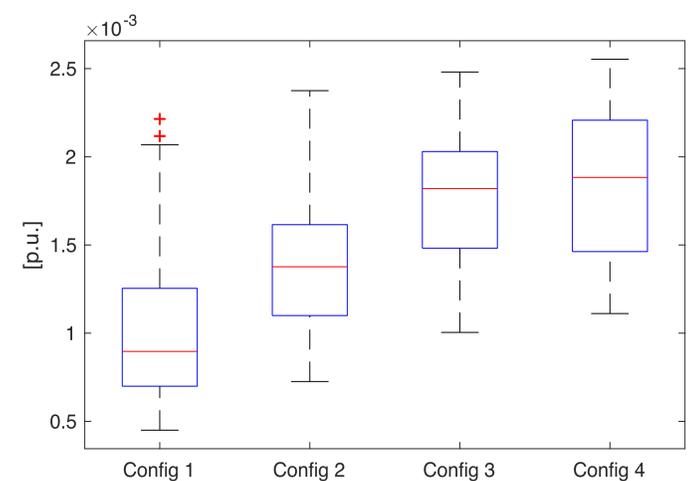
Boxplot of the mean absolute differences between the phase-to-neutral voltages when the impedance of the earth is not modelled, and when Carson's equations are used.



**Graphic representation of the test network**  
The actual configuration of the network is the first one (ungrounded neutral). In configurations 2 and 3, the neutral is assumed to be grounded at the buses marked with a red square

## Results – Comparison between simulations and measurements

The figure below shows the error between the phase-to-neutral voltage measurements and the voltages simulated in the exact same conditions (i.e. same network topology, same cables, same consumption and production in the houses), when the earth is not modelled with Carson's equations.



Boxplot of mean absolute errors between the phase-to-neutral voltages that are simulated and measured.

## Conclusion

Carson's equations should be used with caution as their hypotheses may not hold in LV networks. For the sake of simplicity, we advise one to consider the earth as a single electrical point in the network to which all the grounded neutral points are connected, with or without a grounding impedance. Indeed, the simulations show that the explicit modelling of the earth using Carson's equations has a moderate effect on the simulation results. In particular, it creates differences in the simulations that are around ten-times smaller than the errors between simulations and measurements.