Stochastic Modeling of Uncertainties in Fast Essential Antarctic Ice Sheet Models

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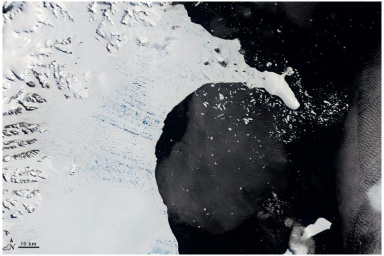
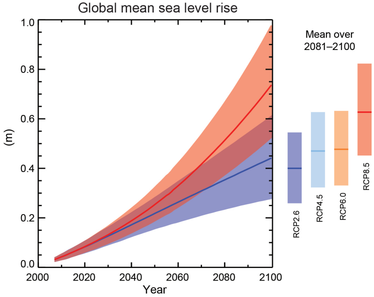
Motivation

- Predicting Antarctica's contribution to future sea-level rise in a warming world (∼200 million people at risk in coastal regions).
- Understanding and identifying the physical processes, feedbacks and instability mechanisms that govern Antarctica's response to climate changes.
- Robust policy response strategies to tackle climate changes should rely on integrated risk and uncertainty assessment in climate change projections [IPCC, 2013].



Collapse of Larsen B ice shelf [Nasa]



Projected sea-level rise [IPCC]

Outline

(1) Motivation

(2) Ice-sheet modeling

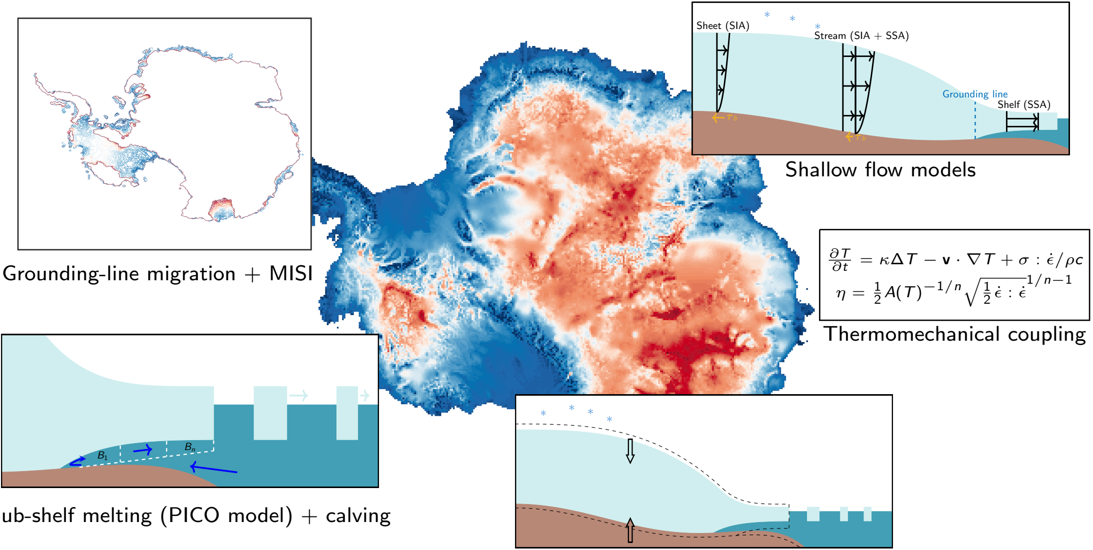
(3) UQ for ice-sheet models

(4) Application: the f.ETISh ice-sheet model

- Methodology
- Results

(5) Conclusion

The f.ETISh model: overview



Shallow flow models

$$\frac{\partial T}{\partial t} = \kappa \Delta T - \mathbf{v} \cdot \nabla T + \sigma : \dot{\epsilon}/\rho c$$

$$\eta = \frac{1}{2} A(T)^{-1/n} \sqrt{\frac{1}{2} \dot{\epsilon} : \dot{\epsilon}}^{\,1/n-1}$$

Thermomechanical coupling

Grounding-line migration + MISI

Sub-shelf melting (PICO model) + calving

Isostatic bedrock adjustment

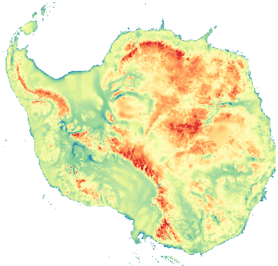
Numerical ice-sheet models

- High-fidelity ice-sheet models:
  - Solve the Stokes equations or high-order ice flow models;
  - Capable of simulating ice flow with high accuracy at high resolution ($\sim$100 m);
  - Relevant for simulations on regional scales and multidecadal periods.
- Essential ice-sheet models (ISMs):
  - Based on shallow-ice approximations of the Stokes equations;
  - Focus on the essential mechanisms (e.g. MISI) and feedbacks of ice-sheet flow (through appropriate parameterizations);
  - Can simulate large ice sheets at low resolution ($\sim$10 km) on millennial time scales;
  - Computationally tractable for large ensemble analysis;
  - Computationally tractable for integration into Earth system models.

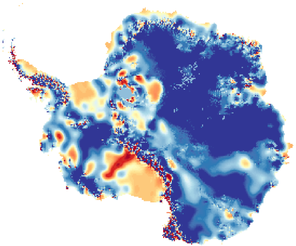
This talk: UQ of multicentennial Antarctica's response with essential ISMs.

Predicting Antarctica's response with f.ETISh

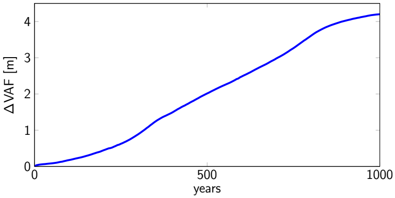
- Input data: ice thickness, bedrock topography, snow accumulation, geothermal heat flux, calving rate, bedrock relaxation time,. . .
- Computation:
  1. (1) Initialization: Identification of the basal friction coefficient to match present-day conditions;
  2. (2) Forward run over several centuries under climate change conditions (outputs: volume above floatation (VAF) + grounding-line position).



Bedrock topography [Fretwell, 2013]



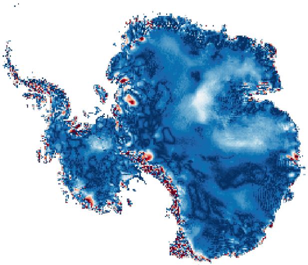
Optimized basal friction coefficient



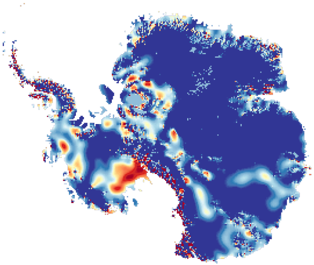
Projected sea-level rise

Model initialization: Data assimilation of ice-sheet geometry

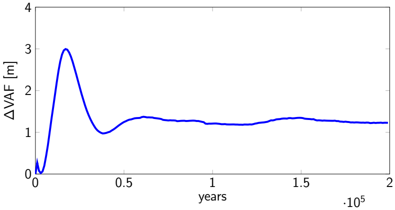
- Basal sliding is a pivotal process governing ice-sheet motion. However, the friction coefficient can not be determined directly ⇒ Need for efficient calibration methods.
- Algorithm [Pollard, 2012]:
  1. Solve continuity equation + flow equations till equilibrium (with fixed grounding line);
  2. Adjust basal friction coefficient to match present-day surface elevation;
  3. Repeat 1. & 2. till convergence is reached (fixed-point iteration).



$|h_s - h_s^{\mathrm{obs}}|$



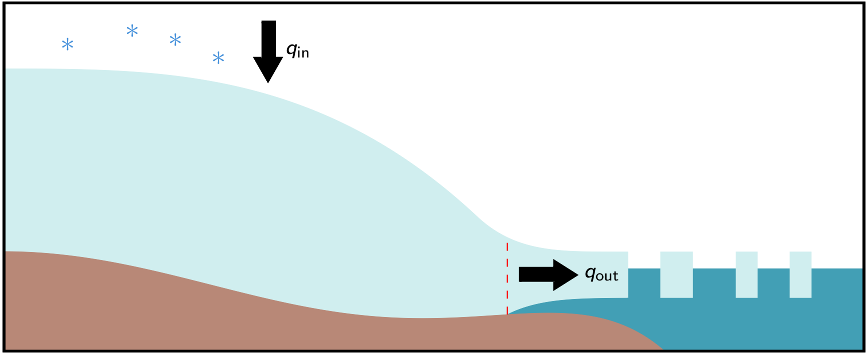
Optimized basal friction coefficient



Convergence visualization

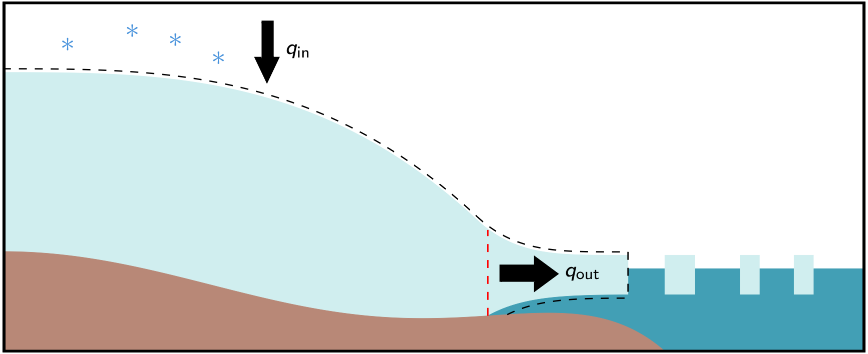
Marine ice sheet instability mechanism

- Step 1: Steady state on an upward sloping bed ($q_{\text{in}} = q_{\text{out}}$).



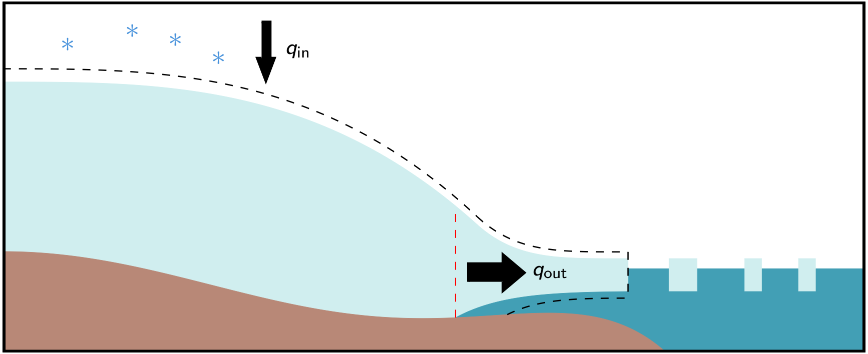
Marine ice sheet instability mechanism

- Step 2: Initiation of grounding-line retreat ($q_{\text{in}} < q_{\text{out}}$).



Marine ice sheet instability mechanism

- Step 3: Self-sustained grounding-line retreat ($q_{\text{in}} \ll q_{\text{out}}$).



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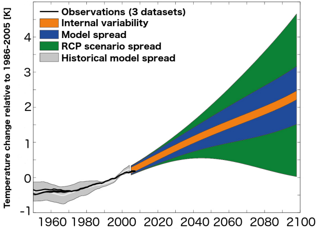
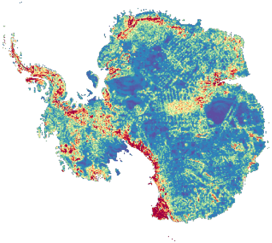
Uncertainties in ice-sheet models

- Intrinsic variability/uncertainty in the climate system + Noisy data:
  - Climate forcing: atmospheric (natural and anthropogenic) and oceanic forcings;
  - Present-day configuration: bedrock topography, geothermal flux, ocean temperature, . . . ;
  - Basal friction condition.
- Modeling errors:
  - Choice of models for ice rheology, basal friction, ice dynamics, bedrock response, sub-shelf melting, . . . ;
  - Initialization (formulation, numerical approximation, noisy observations);
  - Parameterizations of complex processes (with free parameters);
  - Numerical errors (discretization, numerical noise);
- Parametric uncertainty in physical models (e.g. Glen's exponent) and parameterizations.



Uncertainty in global mean temperature [IPCC, 2013]



Uncertainty in bedrock topography [Fretwell, 2013]

Challenges about UQ in ice-sheet models

- Characterization of uncertainties:
  - Publicly available observational datasets [Rignot, 2011; Fretwell, 2013; An, 2015];
  - Spatially nonhomogeneous fields (identification);
  - Schematic representation of uncertainties: RCP scenarios, sliding laws;
  - Correction factors in parameterizations (based on expert assessment).
- Propagation of uncertainties:
  - Spatially nonhomogeneous responses (propagation, representation, visualization);
  - Global (ΔVAF) vs local (surface elevation, grounding-line position) quantities of interest;
  - Complex dynamics: strong nonlinearities, multiphysics coupling, instability mechanisms, feedbacks, tipping points, multi-scale processes, strong interactions with the Earth system.
- Implementation:
  - Computational cost:
    - High computational cost for high-fidelity ISMs prohibits their use for UQ analysis;
    - Essential ISMs allow to generate large numbers of samples for UQ analysis (1 simulation over 1000 yrs with 20 km resolution $\sim$10 hours with f.ETISh).

UQ in ice-sheet models: Review

- Initialization methods:
  - Spin-up methods [Golledge, 2015];
  - Assimilation of observed surface velocity [Morlighem, 2010; Petra, 2012];
  - Assimilation of observed surface elevation [Pollard, 2012];
  - Bayesian inverse methods [Isaac, 2015].
- Ensemble modeling: Run the model with different parameter values to span the entire range of model outputs [Bindschadler, 2013; Pollard, 2016].
- Gaussian process modeling: Build a Gaussian process emulator to reduce the computational cost + ensemble modeling [McNeall, 2013; Pollard, 2016].
- Sensitivity analysis:
  - Adjoint-based methods [Heimbach, 2009];
  - Sampling methods [Larour, 2012];
  - Local reliability methods [Larour, 2012]

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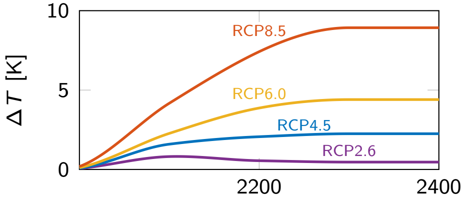
UQ Methodology: Characterization of input uncertainties

- Spatially nonhomogeneous fields are replaced by global input parameters.
- Uncertain climate forcings: Representative scenarios relevant for policymakers.
- Poorly constrained parameters:
  - Extremal and nominal cases:
    - Lower computational cost;
    - Consistent with practice for friction [Ritz, 2015];
    - OK for weakly nonlinear models.
  - Stochastic modeling (random variables):
    - Higher computational cost;
    - Span the entire range of input parameters and model outputs (with associated pdf);
    - OK for nonlinear models;
    - Expert assessment of intervals (uniform) or hyperparameters (Gaussian).



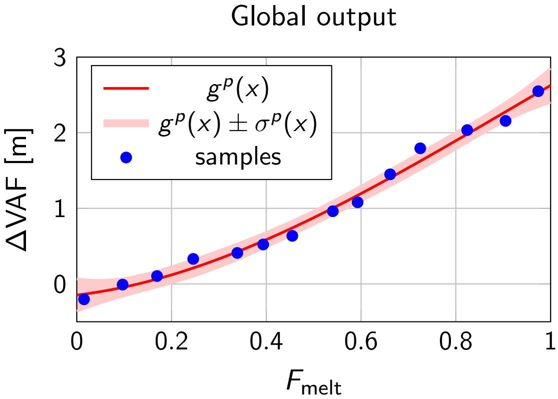
Parameter	min	nominal	max
m	1	2	3

Parameter	Distribution
F_{calv}	$\mathcal{U}[0.5, 1.5]$
F_{melt}	$\mathcal{U}[0.1, 0.8]$
E_{shelf}	$\mathcal{U}[0.2, 1]$
τ_w^{EAIS}	$\mathcal{U}[1000, 3000]$ yrs
τ_w^{WAIS}	$\mathcal{U}[1000, 5000]$ yrs

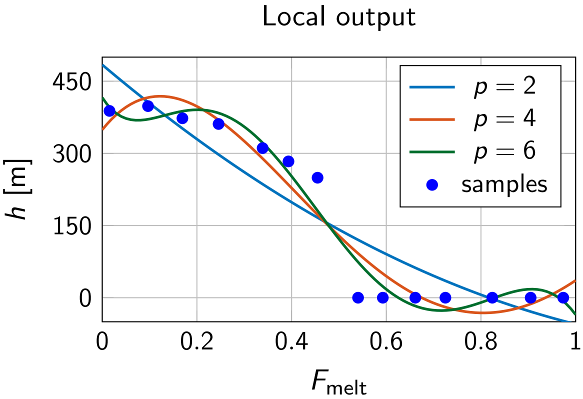
UQ Methodology: Propagation of uncertainties

- Spatially nonhomogeneous responses (propagation, representation, visualization):
  - Global outputs (reduction) (e.g. ΔVAF for the Antarctic ice sheet):
    - Global (large-scale) outputs smooth out local (small-scale) non-smooth responses.
    - Stochastic expansions (through regression or Bayesian-based regression [Sargsyan, 2017] to accomodate noisy data and occasional faults) or Gaussian metamodeling (surrogate models);
    - Sensitivity analysis: Sobol indices, HSIC indices,. . .
  - Local outputs (ice thickness, grounding-line position):
    - Potentially highly nonlinear (non-smooth) outputs (especially where MISI can occur);
    - Monte-Carlo sampling (or similar);
    - Confidence region for excursion sets and contours (grounded ice, grounding-line position).
  - Regional outputs (partial reduction) (e.g. ΔVAF for major Antarctic basins):
    - Output regularity depends on the size and position (marine or grounded) of the region;
    - Weakly nonlinear outputs: see global outputs;
    - Highly nonlinear outputs: see local outputs.

Stochastic expansion: Comparison of global and local outputs



Smooth response with noisy data



Response with abrupt change

Confidence regions for excursion sets

- Sea-level rise depends on grounded ice ⇒ Need to quantify grounded-ice retreat.
- Determine a confidence region with probability level $1-\alpha$ where height above floatation is above zero (HAF>0):
  - Marginal set:

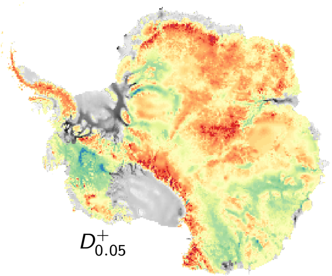
$$D_\alpha^+ = \{\mathbf{x} : P(\mathrm{HAF}(\mathbf{x}) > 0) \geqslant 1-\alpha\}\,.$$

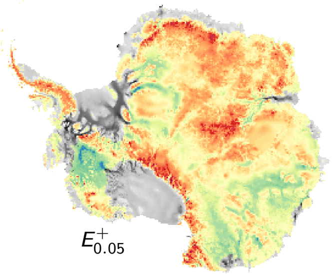
  - Excursion set:

$$E_\alpha^+ = \arg\max_D \left\{|D| : P(D \subseteq A^+(\mathrm{HAF})) \geqslant 1-\alpha\right\}$$

where

$$A^+(\mathrm{HAF}) = \{\mathbf{x} : \mathrm{HAF}(\mathbf{x}) > 0\}\,.$$



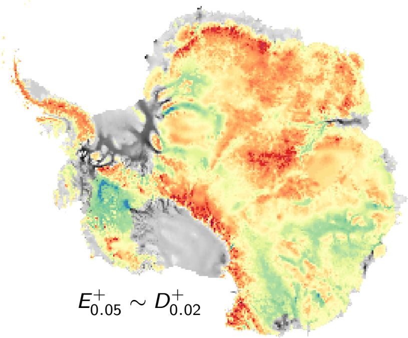


Algorithm for excursion sets

- Algorithm [Bolin, 2015]: Build a parametric family of sets and select the set that gives the best approximation for E_{α}^{+} .

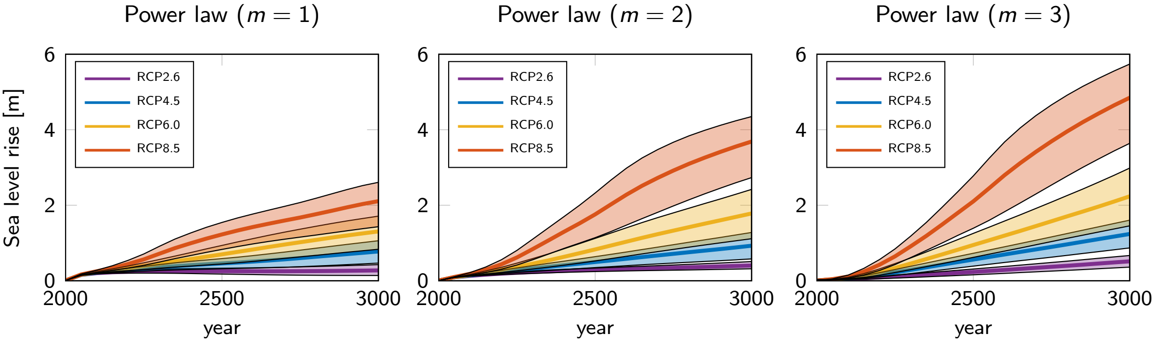
Algorithm 1: Calculate excursion sets

```
Data: Monte-Carlo realizations
Result: Excursion set
Initialization: Choose a parametric family $D(\rho)$ such
    that $D(\rho_1) \subseteq D(\rho_2)$ if $\rho_1 < \rho_2$;
while $P(D(\rho_i) \subseteq A^{+}(HAF)) \geqslant 1 - \alpha$ do
    $\rho_i \rightarrow \rho_{i+1}$, $\rho_{i+1} > \rho_i$.
end
$E_{\alpha}^{+}$ is given by the last set $D(\rho_i)$ with
    $P(D(\rho_i) \subseteq A^{+}(\mathrm{HAF})) \geqslant 1 - \alpha$.
```



- Easy family: Chose D_{ρ}^{+} for $D(\rho)$.

VAF projections with 33%-66% quantiles



$\mu_Y = 0.30, 0.77, 1.23, 1.89$ [m]

$\sigma_Y = 0.33, 0.60, 0.82, 1.10$ [m]

$\sigma_Y/\mu_Y = 1.07, 0.77, 0.67, 0.58$

$\mu_Y = 0.45, 0.98, 1.80, 3.32$ [m]

$\sigma_Y = 0.24, 0.66, 1.15, 1.51$ [m]

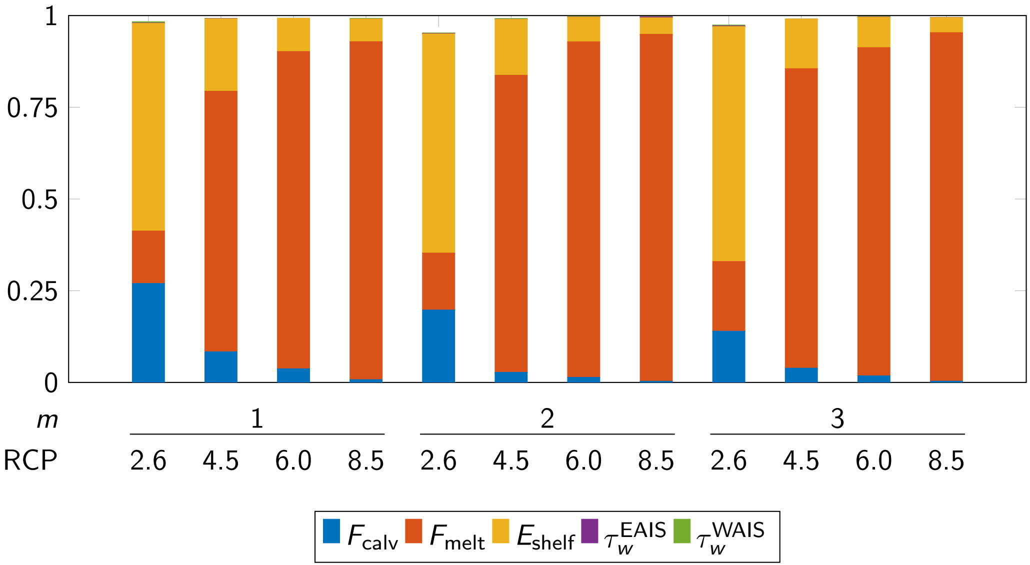
$\sigma_Y/\mu_Y = 0.54, 0.68, 0.64, 0.46$

$\mu_Y = 0.53, 1.26, 2.28, 4.47$ [m]

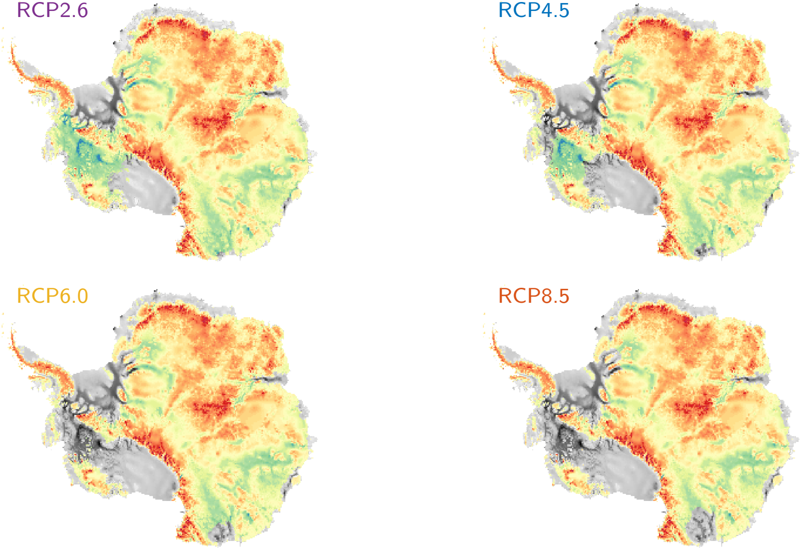
$\sigma_Y = 0.34, 0.73, 1.39, 1.93$ [m]

$\sigma_Y/\mu_Y = 0.64, 0.58, 0.61, 0.43$

Sensitivity analysis: Sobol indices ($t = 1000$ yrs)



$E^{+}_{0.05}$ under nominal sliding conditions



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Conclusion

- Essential ice-sheet models:
  - Focus on essential mechanisms (MISI, ocean interaction, shallow flow,...);
  - Can be integrated in global Earth system;
  - Allows to generate large numbers of samples for UQ analysis.
- UQ for ice-sheet models:
  - Characterisation of uncertainties: spatially nonhomogenous fields, representative scenarios, extreme and nominal cases, stochastic modeling.
  - Propagation of uncertainties:
    - Global outputs: MC sampling, surrogate models, sensitivity analysis;
    - Local outputs: MC sampling, confidence region.
- Future perspectives:
  - Stability analysis under stochastic perturbations;
  - Gain deeper insight into the interactions of input parameters and their influence on ice-sheet response.

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FREEDOM TO RESEARCH