An optimized domain decomposition method between interior and exterior domains for harmonic, penetrable and inhomogeneous electromagnetic scattering problems

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Keywords: Harmonic electromagnetic scattering, FEM/BEM coupling, Domain decomposition

1 Introduction

This ongoing work focuses on the numerical resolution of three-dimensional harmonic electromagnetic scattering problems for which the scatterer is dielectric and inhomogeneous. Denoting by Ω_- the scatterer, Ω_+ the exterior domain, Γ the surface of the scatterer, $\mathbf{n}$ the outward-pointing unit normal vector to Ω_- and $(\mathbf{E}_\pm;\mathbf{H}_\pm)$ the electromagnetic fields within $\Omega_\pm$, such problems read:

$$\begin{array}{l}\mathbf{rot}\,\mathbf{E}_\pm - ik_\pm Z_\pm \mathbf{H}_\pm = \mathbf{0}\\ \mathbf{rot}\,\mathbf{H}_\pm + ik_\pm Z_\pm^{-1}\mathbf{E}_\pm = \mathbf{0}\end{array} \text{ in } \Omega_\pm \quad , \quad \begin{array}{l}\mathbf{E}_- \wedge \mathbf{n} = \mathbf{E}_+ \wedge \mathbf{n} + \mathbf{E}_i \wedge \mathbf{n}\\ \mathbf{H}_- \wedge \mathbf{n} = \mathbf{H}_+ \wedge \mathbf{n} + \mathbf{H}_i \wedge \mathbf{n}\end{array} \text{ on } \Gamma. \tag{1}$$

The interior wave number, k_-, and impedance, Z_-, could be functions on Ω_-, $(\mathbf{E}_i;\mathbf{H}_i)$ is the incident wave and the scattered field $(\mathbf{E}_+;\mathbf{H}_+)$ should satisfy the Silver-Müller radiation condition. A standard approach to solve (1) consists in combining integral equations for the exterior domain and a variational formulation for the interior domain resulting in a strong coupling of the finite (FEM) and boundary (BEM) element methods. However, there is a drawback to this approach. Implementing the strong FEM/BEM coupling by combining two pre-existing solvers, a FEM solver for arbitrary interior problems and a BEM solver for arbitrary exterior problems, will most-likely require serious modification of the pre-existing solvers' source codes. We introduce a weak FEM/BEM coupling, based on domain decomposition (DD), allowing to couple the pre-existing solvers with minimal implementation effort.

2 The weak FEM/BEM coupling

The weak FEM/BEM coupling is an equivalent reformulation of the transmission problem (1):

$$(\mathbf{Id} - \mathbf{S}_\pi)\begin{pmatrix}\mathbf{g}_-\\ \mathbf{g}_+\end{pmatrix} = \begin{pmatrix}\mathbf{H}_i \wedge \mathbf{n} + \mathbf{T}_-(\mathbf{E}_i \wedge \mathbf{n})\\ -\mathbf{H}_i \wedge \mathbf{n} + \mathbf{T}_+(\mathbf{E}_i \wedge \mathbf{n})\end{pmatrix} \quad , \quad \begin{array}{c}\mathbf{S}_\pi = \begin{pmatrix}0 & \mathbf{S}_+\\ \mathbf{S}_- & 0\end{pmatrix}\\ \mathbf{S}_\pm = \mathbf{Id} \pm (\mathbf{T}_+ + \mathbf{T}_-)\mathbf{R}_\pm\end{array} \quad , \quad \mathbf{g}_\pm = (\mathbf{H}_\pm \wedge \mathbf{n}) \mp \mathbf{T}_\pm(\mathbf{E}_\pm \wedge \mathbf{n}).$$

The operators $\mathbf{T}_\pm$ transfer information between Ω_- and Ω_+ in a DD-like manner, $\mathbf{g}_\pm$ should be understood in terms of traces on Γ and $\mathbf{R}_\pm$ are the continuous analogs of the pre-existing solvers:

$$\mathbf{R}_\pm \mathbf{g} = \tilde{\mathbf{E}}_\pm \wedge \mathbf{n} \quad , \quad \begin{array}{l}\mathbf{rot}\,\tilde{\mathbf{E}}_\pm - ik_\pm Z_\pm \tilde{\mathbf{H}}_\pm = \mathbf{0}\\ \mathbf{rot}\,\tilde{\mathbf{H}}_\pm + ik_\pm Z_\pm^{-1}\tilde{\mathbf{E}}_\pm = \mathbf{0}\end{array} \text{ in } \Omega_\pm \quad , \quad (\tilde{\mathbf{H}}_\pm \wedge \mathbf{n}) \mp \mathbf{T}_\pm(\tilde{\mathbf{E}}_\pm \wedge \mathbf{n}) = \mathbf{g} \text{ on } \Gamma.$$

The weak FEM/BEM coupling should be solved iteratively, typically using the GMRES method. Choosing $\mathbf{T}_\pm$ as proper Padé approximations of approximate Magnetic-to-Electric operators [1]:

$$\frac{1}{Z_\pm}\left(\mathbf{Id} + \frac{\mathbf{\Delta}_\Gamma}{\tilde{k}_\pm^2}\right)^{-\frac{1}{2}}\left(\mathbf{Id} - \frac{1}{\tilde{k}_\pm^2}\mathbf{rot}_\Gamma \mathrm{rot}_\Gamma\right)(\cdot \wedge \mathbf{n}) \quad , \quad \tilde{k}_\pm = k_\pm + i\epsilon_\pm,$$

the regularizing functions $\epsilon_\pm$ being strictly positive, ensures a good GMRES convergence.

References

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