A well-conditioned weak coupling between interior and exterior domains for harmonic electromagnetic scattering

Boris Caudron[1,*], **Xavier Antoine**[1], **Christophe Geuzaine**[2]

[1]Institut Elie Cartan de Lorraine, Université de Lorraine, UMR CNRS 7502, France

[2]University of Liège, Department of Electrical Engineering and Computer Science, Belgium

[*]Email: boris.caudron@univ-lorraine.fr

Abstract

A new weak coupling between the boundary element method and the finite element method for solving harmonic electromagnetic scattering problems is introduced. This method is inspired by domain decomposition techniques. Using suitable approximations of Magnetic-to-Electric operators to build proper transmission conditions allows for good convergence properties.

Keywords: time-harmonic electromagnetic scattering, BEM/FEM coupling, domain decomposition methods, microlocal operators

1 Introduction

The aim of this ongoing work is to numerically solve time-harmonic electromagnetic (EM) scattering problems for which the scatterer is dielectric and inhomogeneous. A standard approach consists in combining integral equations for the exterior domain and a weak formulation for the interior domain resulting in a formulation coupling the boundary element method (BEM) and the finite element method (FEM). A drawback of this strong coupling is that it is not possible to easily combine two pre-existing solvers, one FEM solver for interior domains and one BEM solver for exterior domains, to construct a global solver for the original problem. We present here a weak BEM/FEM coupling, based on a domain decomposition approach, allowing a simple construction of such a solver.

2 The EM scattering problem

We denote by Ω_- the scatterer, Ω_+ the exterior domain, Γ the surface of the scatterer and $\mathbf{n}$ the outward-pointing unit normal vector to Ω_-. The scattered field and the one within the scatterer satisfy in $\Omega_\pm$:

$$\mathbf{rot}\,\mathbf{E}_\pm - ik_\pm Z_\pm \mathbf{H}_\pm = \mathbf{0}, \tag{1}$$

$$\mathbf{rot}\,\mathbf{H}_\pm + ik_\pm Z_\pm^{-1} \mathbf{E}_\pm = \mathbf{0}, \tag{2}$$

where $k_\pm$ and $Z_\pm$ are respectively the wave numbers and the impedances of the problem. Note that the scattered field, as any exterior field, should also satisfy the Silver-Müller radiation condition. Finally, the interface conditions on Γ are:

$$\mathbf{E}_- \wedge \mathbf{n} = \mathbf{E}_+ \wedge \mathbf{n} + \mathbf{E}_i \wedge \mathbf{n}, \tag{3}$$

$$\mathbf{H}_- \wedge \mathbf{n} = \mathbf{H}_+ \wedge \mathbf{n} + \mathbf{H}_i \wedge \mathbf{n}, \tag{4}$$

where $(\mathbf{E}_i; \mathbf{H}_i)$ is an incident plane wave.

3 Reformulating the problem

Introducing two operators $\mathbf{T}_\pm$, refered to as the transmission operators, problem (1)-(4) can be equivalently recast as:

$$(\mathbf{Id} - \mathbf{S}_\pi) \begin{pmatrix} \mathbf{g}_- \\ \mathbf{g}_+ \end{pmatrix} = \mathbf{B}, \tag{5}$$

where the right-hand side $\mathbf{B}$ is defined through $(\mathbf{E}_i; \mathbf{H}_i)$ and $\mathbf{T}_\pm$, and:

$$\mathbf{S}_\pi = \begin{pmatrix} 0 & \mathbf{S}_+ \\ \mathbf{S}_- & 0 \end{pmatrix},$$

$$\mathbf{S}_\pm = \mathbf{Id} \pm (\mathbf{T}_+ + \mathbf{T}_-)\mathbf{R}_\pm,$$

The functions $\mathbf{g}_\pm$ should be understood in terms of traces on Γ:

$$\mathbf{g}_\pm = (\mathbf{H}_\pm \wedge \mathbf{n}) \mp \mathbf{T}_\pm(\mathbf{E}_\pm \wedge \mathbf{n}).$$

Finally, $\mathbf{R}_\pm$ are resolution operators defined as:

$$\mathbf{R}_\pm \mathbf{g} = \tilde{\mathbf{E}}_\pm \wedge \mathbf{n},$$

where $(\tilde{\mathbf{E}}_\pm; \tilde{\mathbf{H}}_\pm)$ are the solutions of the following boundary-value problems:

$$\mathbf{rot}\,\tilde{\mathbf{E}}_\pm - ik_\pm Z_\pm \tilde{\mathbf{H}}_\pm = \mathbf{0} \text{ in } \Omega_\pm,$$

$$\mathbf{rot}\,\tilde{\mathbf{H}}_\pm + ik_\pm Z_\pm^{-1} \tilde{\mathbf{E}}_\pm = \mathbf{0} \text{ in } \Omega_\pm,$$

$$(\tilde{\mathbf{H}}_\pm \wedge \mathbf{n}) \mp \mathbf{T}_\pm(\tilde{\mathbf{E}}_\pm \wedge \mathbf{n}) = \mathbf{g} \text{ on } \Gamma.$$

The resolution operators $\mathbf{R}_\pm$ should be understood as the pre-existing FEM solver for the interior domain and BEM solver for the exterior domain, as mentioned in the introduction.

4 Solving the reformulation

To solve (5), the GMRES method is employed. As a result, the solvers for the interior and exterior domains are repeatedly and independently called. For this reason, the weak coupling can be interpreted in a domain decomposition method (DDM) framework. Moreover, solving (5) using a fixed point method is equivalent to a standard Jacobi-type DDM method:

$$\begin{aligned}\mathbf{rot}\,\mathbf{E}_{\pm}^{n+1} - ik_{\pm}Z_{\pm}\mathbf{H}_{\pm}^{n+1} &= \mathbf{0} \text{ in } \Omega_{\pm},\\ \mathbf{rot}\,\mathbf{H}_{\pm}^{n+1} + ik_{\pm}Z_{\pm}^{-1}\mathbf{E}_{\pm}^{n+1} &= \mathbf{0} \text{ in } \Omega_{\pm},\end{aligned}$$

the interface conditions on Γ being:

$$\begin{gathered}(\mathbf{H}_{\pm}^{n+1} \wedge \mathbf{n}) \mp \mathbf{T}_{\pm}(\mathbf{E}_{\pm}^{n+1} \wedge \mathbf{n}) = (\mathbf{H}_{\mp}^{n} \wedge \mathbf{n})\\ \mp\mathbf{T}_{\pm}(\mathbf{E}_{\mp}^{n} \wedge \mathbf{n}) \mp \mathbf{H}_i \wedge \mathbf{n} + \mathbf{T}_{\pm}\mathbf{E}_i \wedge \mathbf{n}.\end{gathered}$$

5 Choosing the transmission operators

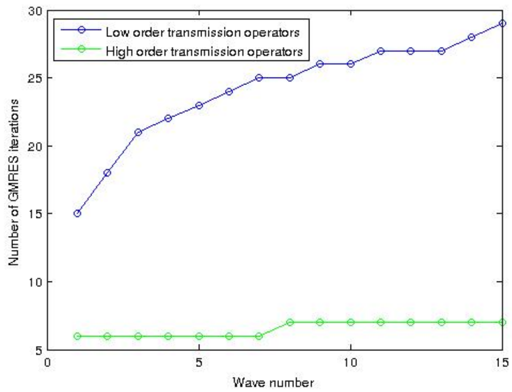
The choices for $\mathbf{T}_{\pm}$ impact the convergence of the GMRES method. Choosing proper Magnetic-to-Electric (MtE) operators as transmission operators ensures that $\mathbf{S}_{\pi}$ vanishes rendering (5) trivial to solve. It is therefore natural to use approximations of MtE operators as transmission operators. Accurate approximations typically involve the principal part of the MtE operators:

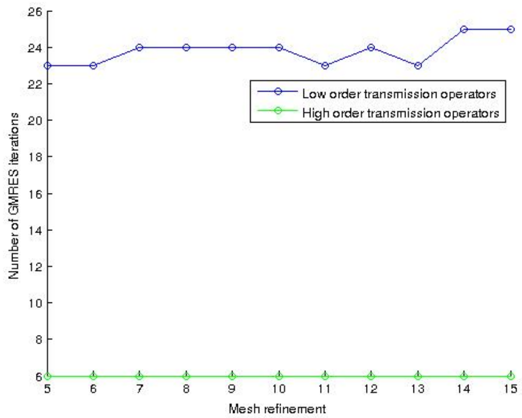
$$\frac{1}{Z_{\pm}}\left(\mathbf{Id} + \frac{\mathbf{\Delta}_{\Gamma}}{\tilde{k}_{\pm}^2}\right)^{-\frac{1}{2}}\left(\mathbf{Id} - \frac{1}{\tilde{k}_{\pm}^2}\mathbf{rot}_{\Gamma}\mathrm{rot}_{\Gamma}\right)(\cdot \wedge \mathbf{n}),$$

where $\tilde{k}_{\pm} = k_{\pm} + i\epsilon_{\pm}$, $\epsilon_{\pm} > 0$. Although nonlocal, the above operators can be localized using complex Padé approximants. Such transmission operators have already been successfully used for DDM, resulting in fast GMRES with a convergence only slightly dependent on the mesh refinement and wave number [1]. For the weak coupling, some numerical/analytical tests have been conducted in the case of the sphere, for which the eigenvalues of all the involved operators are known analytically, suggest that it should also be the case for the GMRES convergence of (5).

6 Numerical results

To test formulation (5), we consider a toy problem: a unit transparent sphere (meaning that $k_- = k_+$ and $Z_- = Z_+$) solved by coupling two BEM solutions, i.e. a BEM/BEM DDM. The GMRES-convergence is promising as shown below (the GMRES tolerance is fixed to 10^{-4}). In this case, high order transmission operators refer to principal parts of MtE operators with Padé approximation whereas low order transmission operators refer to a simple impedance operator (zeroth order Taylor expansion): $\frac{1}{Z_{\pm}}(\cdot \wedge \mathbf{n})$. The mesh parameters h are defined as fractions of the wave length λ: $h = \frac{\lambda}{l} = \frac{2\pi}{kl}$. For the first figure, $l = 5$. For the second one, the wave number is equal to 5. In addition, we will explain how to construct well-conditioned and efficient iterative BEM solvers for the interior and exterior problems.





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References

[1] M. El Bouajaji, B. Thierry, X. Antoine, C. Geuzaine, A Quasi-Optimal Domain Decomposition Algorithm for the Time-Harmonic Maxwell's Equations, *Journal of Computational Physics* **294 (1)** (2015), pp. 38–57.