# Multiple outputs operational modal identification of time-varying systems

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# The global framework of this research is the modal identification of structures

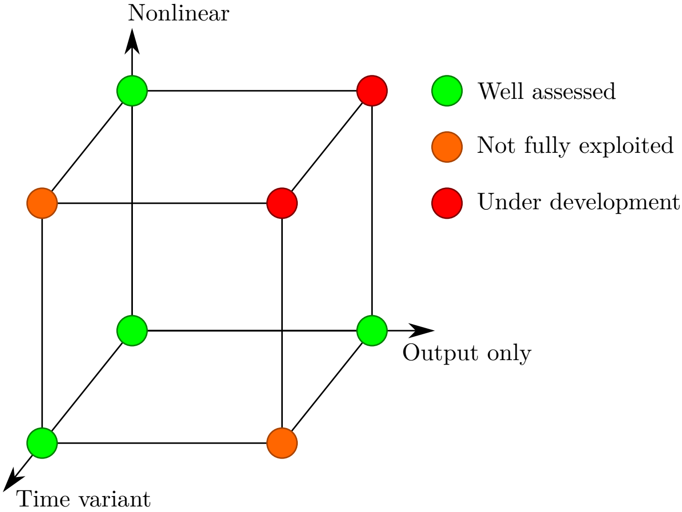
It is a pretty large field of structural analysis

It can be decomposed in:

- Experimental or **Operational** Modal Analysis
- **Linear** or Nonlinear Systems
- Time Invariant or **Time-Varying**
- Single or **Multiple Outputs**

- And any of their combinations...

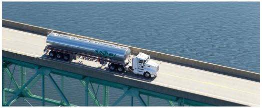
# The global framework of this research is the modal identification of structures



# Why time-varying behaviour can occur ?

Several possible origins :

- Structural changes



- Operating conditions

- Damage occurrence

# MDOF linear time-varying mechanical systems are considered

$$\boldsymbol{M}(t)\,\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(t)\,\dot{\boldsymbol{y}}(t) + \boldsymbol{K}(t)\,\boldsymbol{y}(t) = \boldsymbol{f}(t)$$

The dynamics of such systems is characterized by :

- Non-stationary time series
- Instantaneous modal properties
  - Frequencies : $\omega_r(t)$
  - Damping ratio's : $\zeta_r(t)$
  - Modal deformations : $\boldsymbol{v}_r(t)$

# Outline of the presentation

Several identification methods are proposed in the thesis

The presentation is organized as follows:

Non-parametric approach

- Presentation of the experimental setup

Combined parametric and non-parametric approach

Fully parametric approaches

Applications to more complex cases

# Some classical signal processing techniques can deal with nonstationary signals

Time-frequency representations:

- Short-time Fourier transform
- Wavelet analysis
- Wigner-Ville distribution

Signal decomposition methods:

- Hilbert-Huang Transform (HHT)
- Hilbert Vibration Decomposition (HVD)

# The Hilbert Transform

The Hilbert transform $\mathcal{H}$ of a signal $x(t)$ is the convolution product of this signal with the impulse response $h(t) = \frac{1}{\pi\, t}$

$$\begin{aligned}\tilde{x}(t) &= \mathcal{H}(x(t)) = (h(t) * x(t)) \\ &= \text{p.v.} \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau \\ &= \frac{1}{\pi}\, \text{p.v.} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau}\, d\tau\end{aligned}$$

It is a particular transform that remains in the same domain as the original signal

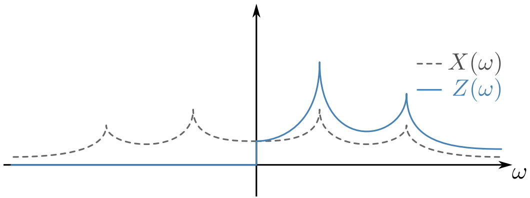
It corresponds to a phase shift of $\frac{\pi}{2}$ of the signal

# The Hilbert transform is used to build the complex analytic form of a signal

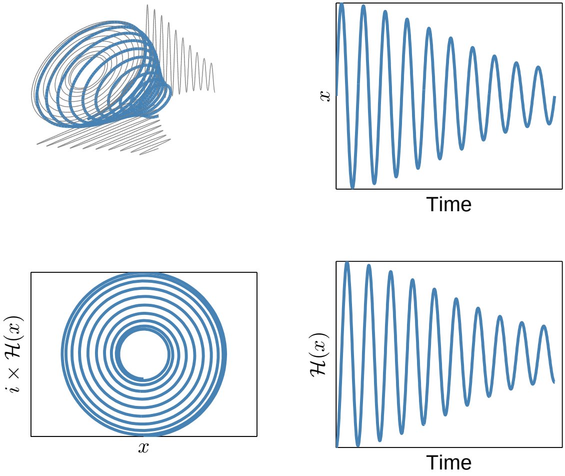
The **analytic signal** $z$ is built as

$$\begin{aligned} z(t) &= x(t) + i\,\mathcal{H}(x(t)) \\ &= A(t)\,e^{i\phi(t)} \end{aligned}$$

In the frequency domain, the analytic signal becomes a one-sided signal



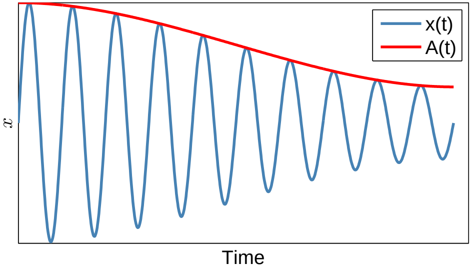
# The analytic signal can be seen as a rotating phasor in the complex plane



# It is suitable to find the envelope of the signal

The instantaneous envelope of the signal is given by the absolute value of the analytic signal

$$A(t) = |z(t)|$$



# It also gives information about the instantaneous phase

The instantaneous phase angle of the signal is given by the argument of the analytic signal

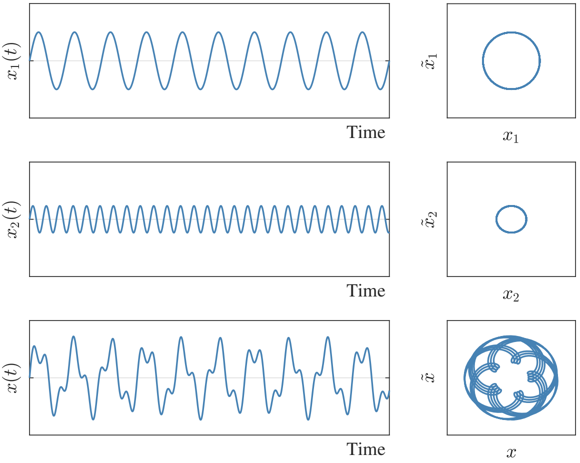
$$\phi(t) = \angle z(t)$$

The time derivative of the phase angle gives the instantaneous frequency

$$\omega(t) = \frac{\mathrm{d}\phi(t)}{\mathrm{d}t}$$

# The Hilbert transform is powerful but only meaningful for monocomponent signals

Let us consider a 2-component signal mixture



# Signal decomposition methods

In order to get meaningful instantaneous frequencies, the signals have to be decomposed.

Two good candidates exist:

The Empirical Mode Decomposition method

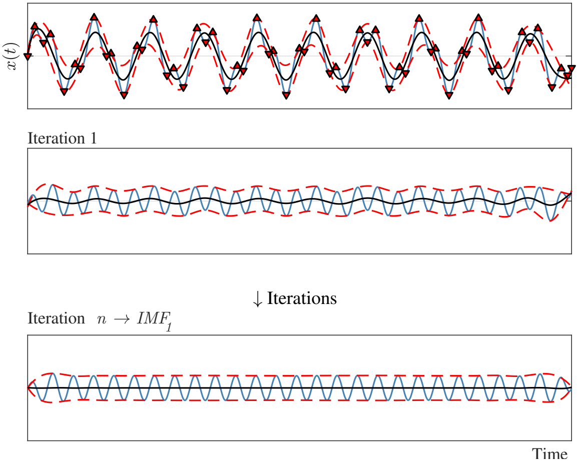
- Successive extraction of the highest instantaneous frequency component

The Hilbert Vibration Decomposition Method

- Successive extraction of the highest instantaneous amplitude component

# The Empirical Mode Decomposition sifting process

The EMD method iteratively removes the mean of the upper and lower envelopes computed by spline fitting



# The Hilbert Vibration Decomposition sifting process

$x(t)$

↓

**Analytic signal**
$z(t) = x(t) + i\,\mathcal{H}(x(t))$

↓

**Frequency extraction**
$\omega(t) = \frac{d\phi(t)}{dt} = \frac{d\angle z(t)}{dt}$

↓

**Lowpass filtering**
$\omega(t) \rightarrow \omega_k(t)$

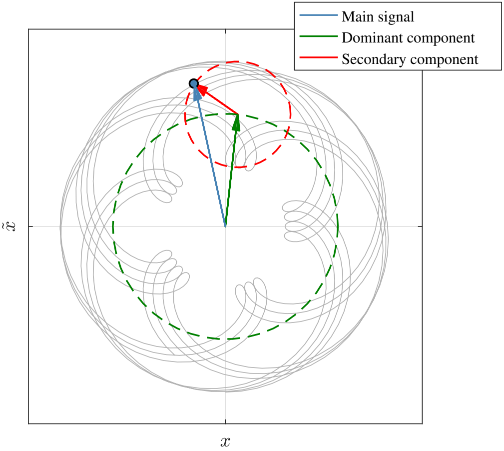
↓

**Synchronous demodulation**
$x_k(t)$

↓

**Sifting process**
$x(t) := x(t) - x_k(t)$

(loop back to Analytic signal)



# The HVD method in that scheme has some drawbacks

It is applicable to single channel measurement
The application on multiple channels has to be done in parallel

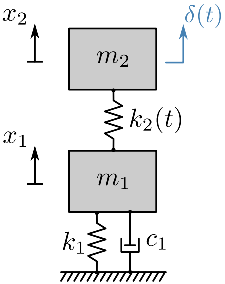
In a multivariate case, all the modes have to be excited at each time instant on all the channels

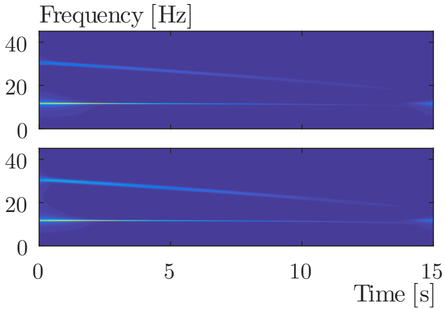
The method will always follow the instantaneous dominant mode

# Example: a simple 2–DoF time–variant system

System properties:

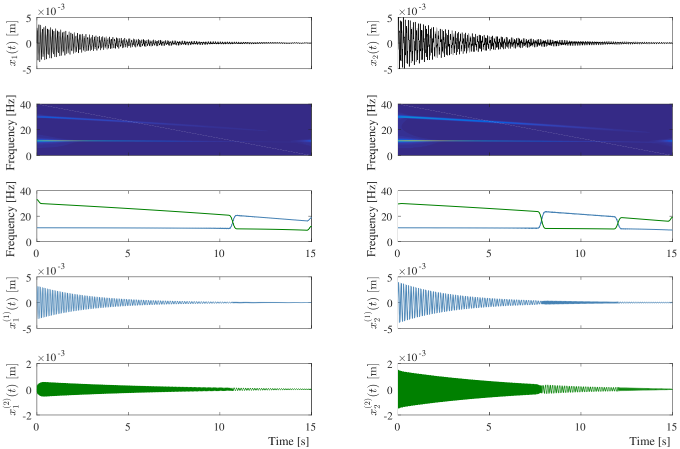
- $m_1 = 3$ [kg]
- $m_2 = 1$ [kg]
- $k_1 = 20000$ [N/m]
- $c_1 = 3$ [N.s/m]
- $k_2 = 25000 \searrow 5000$ [N/m]



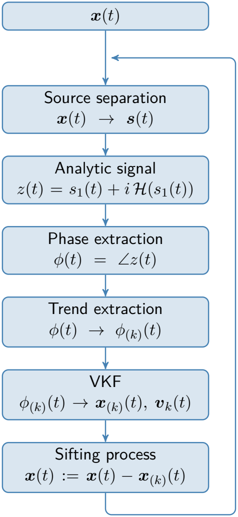



# Application of the HVD method on each channel

A simple application of the HVD method on each channel leads to mode mixing



# In the case of multiple channel measurements, a source separation step is introduced in the algorithm



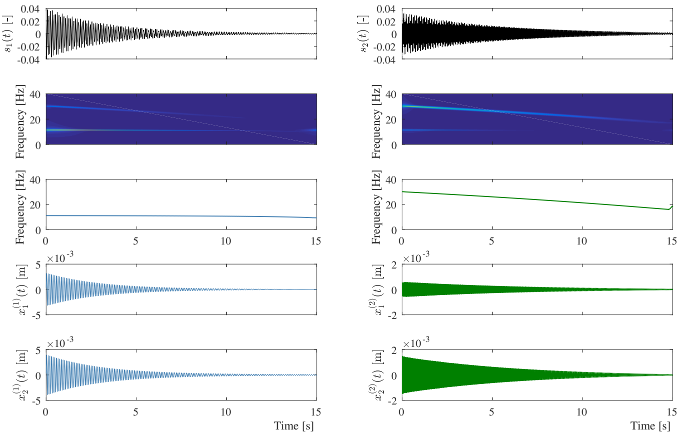

The sources are used as references to get the instantaneous frequencies

A trend extraction method computes the phase of the dominant mode

A Vold-Kalman filter (VKF) is used for component extraction

# Introducing a source separation method can help to avoid the mode mixing phenomenon

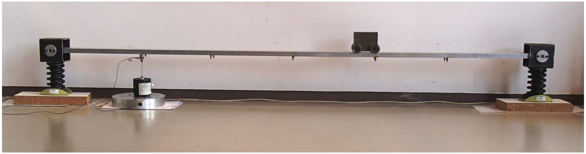
A Blind Source Separation method (BSS) separates a set of signals in a set of uncorrelated or independent sources



# The experimental setup

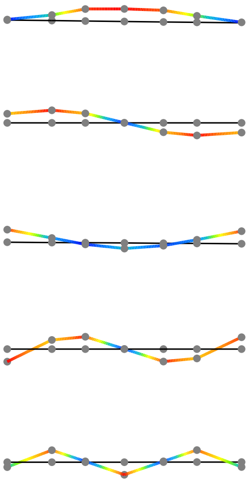
The experimental setup is an aluminum beam with a moving mass.

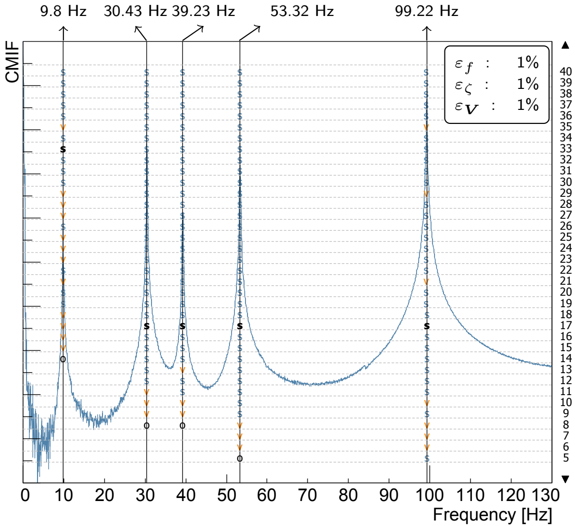
The whole system is supported by springs and excited by a shaker.



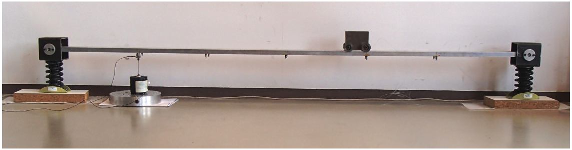
- $2.1$ meter long and $8 \times 2$ cm for the cross section
- $9$ kg for the beam and $\approx 3.5$ kg for the moving mass (ratio of $38.6$ %)
- The excitation and measurements are performed with a Siemens LMS system

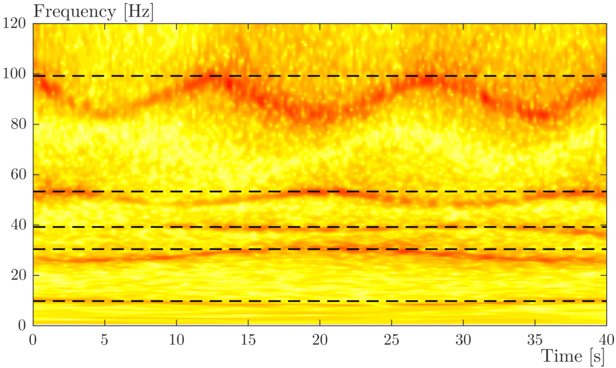
# Time invariant modal identification of the beam subsystem



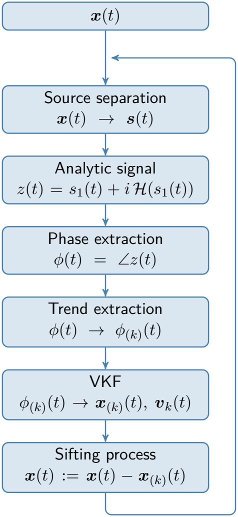


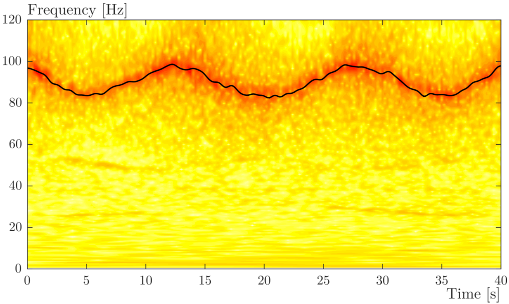
# Time-varying dynamics of the system

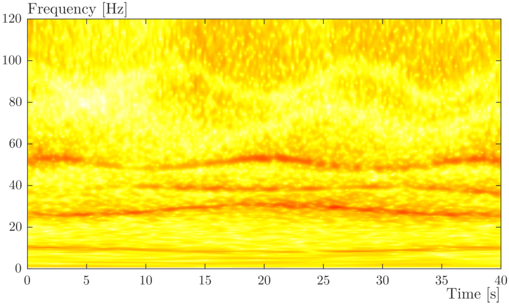




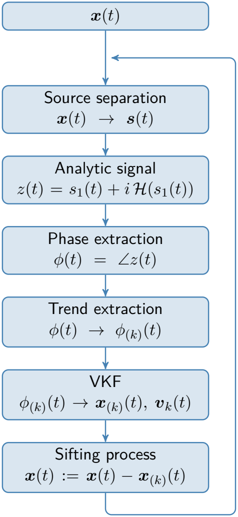
# The sifting process and the benefit of the source separation

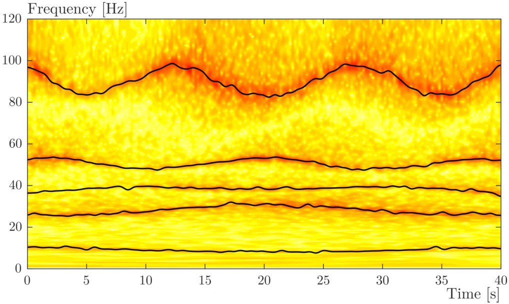


# All the modes are extracted after few iterations

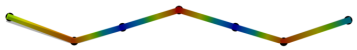


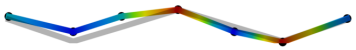
# Finally the mode shapes can be retrieved in the VKF process

As an example the fifth mode is represented

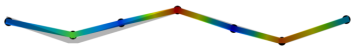
The inertia effect of the mass is visible when it passes at the nodes of vibration



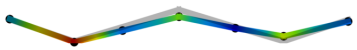
(a) $t = 0$ s.



(b) $t = 5$ s.

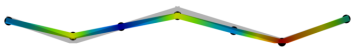


(c) $t = 10$ s.

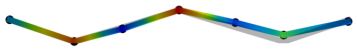


(d) $t = 15$ s.

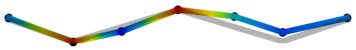
(e) $t = 20$ s.



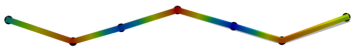
(f) $t = 25$ s.



(g) $t = 30$ s.



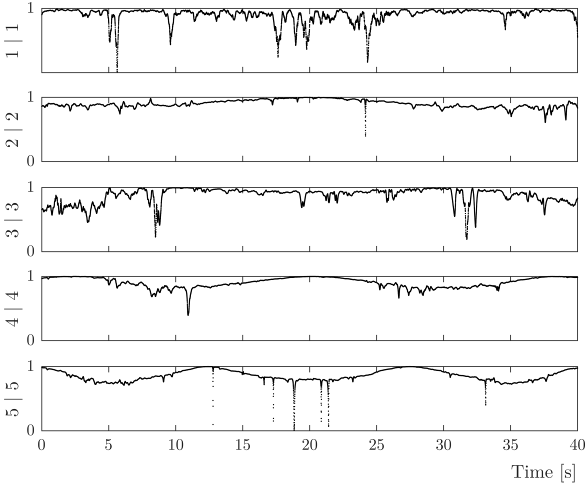
(h) $t = 35$ s.



(i) $t = 40$ s.

# Finally the mode shapes can be recovered in the VKF process

The modal assurance Criterion can be calculated with respect to the LTI mode shapes



# Outline of the presentation

Several identification methods are proposed in the thesis

The presentation is organized as follows:

Non parametric approach

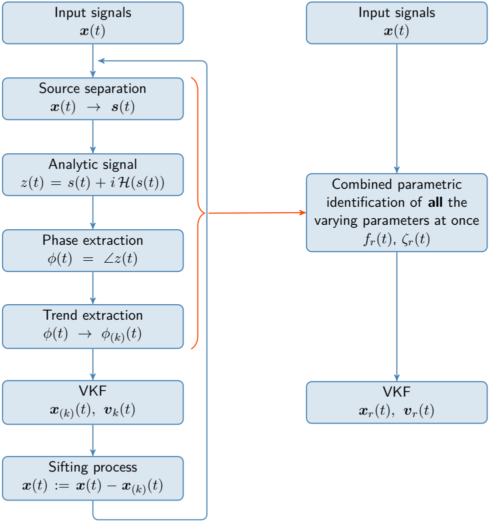
- Presentation of the experimental setup

Combined parametric and non-parametric approach

Fully parametric approaches

Applications to more complex cases

# Let us introduce some parametric modelling in our identification process



# A parametric model is applied to our measurements

It is chosen here to work with AutoRegressive Moving-Average (ARMA) models

$$y[t]+a_1y[t-1]+\cdots+a_{n_a}y[t-n_a] = e[t]+b_1e[t-1]+\cdots+b_{n_b}e[t-n_a]$$

In which:

- $y[t]$ is the data sequence
- $e[t]$ is the innovation sequence

In the $z$-domain, one has

$$Y[z] = \frac{B(z,\boldsymbol{\theta})}{A(z,\boldsymbol{\theta})}\,E[z] = H(z,\boldsymbol{\theta})\,E[z]$$

with the polynomials

$$\begin{aligned} A(z,\boldsymbol{\theta}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a} \\ B(z,\boldsymbol{\theta}) &= 1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b} \end{aligned}$$

# The model identification is performed by the Prediction Error Method (PEM)

Defining the predictor $\hat{y}[t, \boldsymbol{\theta}]$ with the model parameters, the prediction error is given by

$$e[t, \boldsymbol{\theta}] = y[t] - \hat{y}[t, \boldsymbol{\theta}]$$

A common way to identify the model parameters is to rely on the minimization of a scalar cost function. A usual choice is to minimize the sum of squared errors

$$\begin{aligned} V(\boldsymbol{\theta}) &= \frac{1}{2N} \sum_{t=1}^{N} \left(e[t, \boldsymbol{\theta}]\right)^2 \\ &= \frac{1}{2N} \sum_{t=1}^{N} \left(y[t] - \hat{y}[t, \boldsymbol{\theta}]\right)^2 \end{aligned}$$

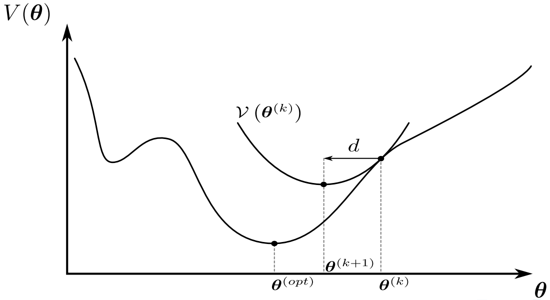
# The model identification is performed by the Prediction Error Method (PEM)

The minimization is straightforward in a pure AR case

- Linear least square problem

The minimization is more complex once a MA part is considered

- Nonlinear least square problem
- 2 Stages Least Squares or iterative optimization



# How to adapt the previous model to our case?

The previously described identification process identifies scalar time invariant systems

- How to take the time dependence into account?
- How to adapt it to multiple measurements at once?

Further

- How is a good model structure chosen? (Principle of parcimony)
- How are the physical poles selected in an overparameterized case?

# Modelling of time-dependent processes

To take the time variation into account, the model parameters are let free to vary

$$
\begin{aligned}
H(z, \boldsymbol{\theta}[t]) &= \frac{B(z, \boldsymbol{\theta}[t])}{A(z, \boldsymbol{\theta}[t])} \\
&= \frac{1 + b_1[t]z + b_2[t]z^{-2} + \cdots + b_{n_b}[t]z^{-n_b}}{1 + a_1[t]z + a_2[t]z^{-2} + \cdots + a_{n_a}[t]z^{-n_a}}
\end{aligned}
$$

This is the **frozen-time** approach

The **frozen-poles** are computed as the roots of the denominator

To model the variation of the parameters, the **basis functions** approach is chosen

$$
\theta_i[t] = \sum_{j=1}^{k} \theta_{ij} f_j[t]
$$

# Multiple measurements are managed by a common denominator modelling

Because the poles of a dynamic system are **global properties**, they are common on each channel
Conversely, the zeros are local to each channel

$$
\begin{aligned}
x_1[t] + \cdots + a_{n_a}[t]\, x_1[t-n_a] &= e_1[t] + \cdots + b^1_{n_b}[t]\, e_1[t-n_b] \\
x_2[t] + \cdots + a_{n_a}[t]\, x_2[t-n_a] &= e_2[t] + \cdots + b^2_{n_b}[t]\, e_2[t-n_b] \\
&\vdots \\
\underbrace{x_{n_o}[t] + \cdots + a_{n_a}[t]\, x_{n_o}[t-n_a]}_{\text{Common AR modelling}} &= \underbrace{e_{n_o}[t] + \cdots + b^{n_o}_{n_b}[t]\, e_{n_o}[t-n_b]}_{\text{Individual MA modelling}}
\end{aligned}
$$

The cost function is adapted to all the prediction errors

$$V(\boldsymbol{\theta}) = \sum_o \frac{1}{2N} \sum_t e^o[t, \boldsymbol{\theta}]^2$$

# The model structure is chosen based on information criteria

Commonly used criteria are

The Akaike's Final Prediction Error (FPE):

$$FPE = \frac{1 + \frac{d_{\mathcal{M}}}{N}}{1 - \frac{d_{\mathcal{M}}}{N}} V(\boldsymbol{\theta}^*_{\mathcal{M}})$$

The Akaike's Information Criterion (AIC):

$$AIC = \ln V(\boldsymbol{\theta}^*_{\mathcal{M}}) + \frac{2\, d_{\mathcal{M}}}{N}.$$

The Bayesian Information Criterion (BIC):

$$BIC = \ln V(\boldsymbol{\theta}^*_{\mathcal{M}}) + d_{\mathcal{M}} \frac{\ln N}{N}.$$

# Selection of the physical poles

Because some overparameterization may be required, some spurious poles can appear

The selection process is pretty simple and relies on the fact that the physical poles have usually a low damping ratio when compared to the spurious ones

The idea is to retain the $p$ poles having the closest trajectories to the unit circle

# Application to the experimental setup

The procedure followed for the identification is the following one

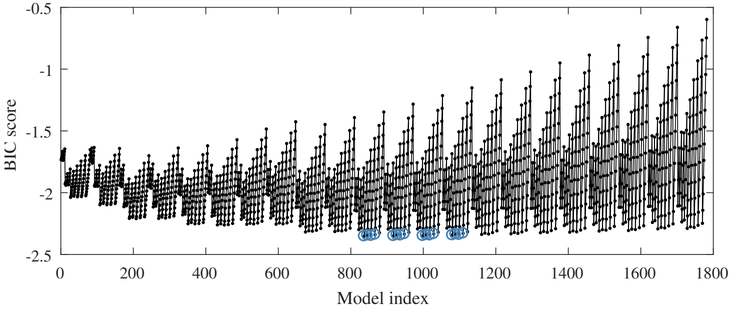
- A large batch of fast 2SLS identifications to determine good model structure candidates
- A refined identification using the nonlinear optimization process
- The selection of the best model and extraction of the mode shapes with the non-parametric VKF

For all the identifications, Chebyshev polynomials are used as basis functions

# Batch of fast preliminary identifications

The batch of 2SLS analyses sweeps a large number of model orders and sizes of the bases of functions

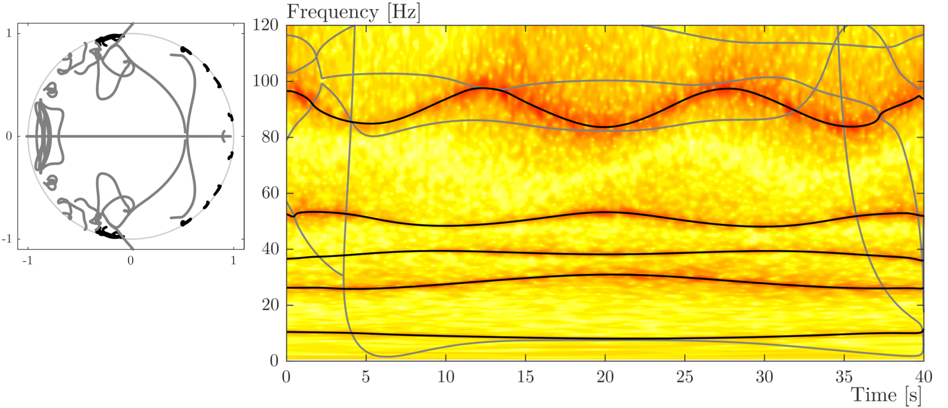
Usually, the BIC is more severe on the model complexity than the other two criteria



# Accurate identification of the time-varying beam

Finally, the ARMA[22,21](9,9) is chosen

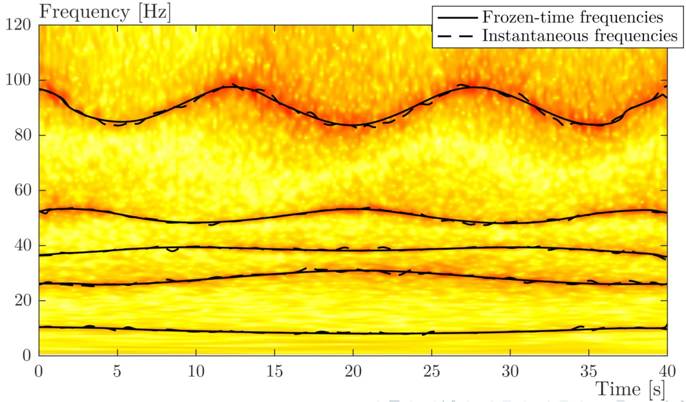
Five physical poles are selected



# Comparison between the frozen and the instantaneous frequencies

The two sets of results are in agreement

This also validates the frozen-time assumption



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# Fully parametric approaches

In this part, the system is identified with parametric modelling only

We deal with multivariate modelling

- Multivariate ARMA models
- Multivariate State-Space modal models

The mode shapes are now identified together with the poles

The consequence is that we have now to identify matrix coefficients

# Time-varying multivariate AutoRegressive Moving-Average (ARMAV) model

Multivariate models are more complete than univariate ones

The time-varying ARMAV model is simply the vector counterpart of the scalar ARMA model

$$\boldsymbol{M}(t)\,\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(t)\,\dot{\boldsymbol{y}}(t) + \boldsymbol{K}(t)\,\boldsymbol{y}(t) = \boldsymbol{f}(t)$$

The same basis function approach applies to the matrix coefficients

$$\boldsymbol{y}[t] + \sum_{i=1}^{n_a}\sum_{k=1}^{r_A} \boldsymbol{A}_{i,k}\, f_k[t]\, \boldsymbol{y}[t-i] = \boldsymbol{e}[t] + \sum_{j=1}^{n_b}\sum_{k=1}^{r_B} \boldsymbol{B}_{j,k}\, f_k[t]\, \boldsymbol{e}[t-j].$$

# The time-varying ARMAV model may be equivalently cast in a State-Space form

The model can be written in an innovation state-space model

$$\begin{aligned} \boldsymbol{x}[t+1] &= \boldsymbol{F}[t]\,\boldsymbol{x}[t] + \boldsymbol{K}[t]\,\boldsymbol{e}[t] \\ \boldsymbol{y}[t] &= \boldsymbol{C}\,\boldsymbol{x}[t] + \boldsymbol{e}[t] \end{aligned}$$

With

$$\boldsymbol{F}[t] = \begin{bmatrix} -\boldsymbol{A}_1[t] & \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ -\boldsymbol{A}_2[t] & \boldsymbol{0} & \boldsymbol{I} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{A}_{n_a-1}[t] & \vdots & \vdots & & \boldsymbol{I} \\ -\boldsymbol{A}_{n_a}[t] & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix} \qquad \boldsymbol{K}[t] = \begin{bmatrix} \boldsymbol{B}_1[t] - \boldsymbol{A}_1[t] \\ \boldsymbol{B}_2[t] - \boldsymbol{A}_2[t] \\ \vdots \\ \boldsymbol{B}_n[t] - \boldsymbol{A}_n[t] \end{bmatrix}$$

and

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix}$$

# Computation of the frozen-modal parameters

The matrix

$$\boldsymbol{F}[t] = \begin{bmatrix} -\boldsymbol{A}_1[t] & \boldsymbol{I} & \mathbf{0} & \cdots & \mathbf{0} \\ -\boldsymbol{A}_2[t] & \mathbf{0} & \boldsymbol{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{A}_{n_a-1}[t] & \vdots & \vdots & & \boldsymbol{I} \\ -\boldsymbol{A}_{n_a}[t] & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}$$

is the **state-transition matrix** of the SS model.

It is also the **companion matrix** of the AR matrix polynomial. Its eigenvalues/vectors decomposition at time $t$ provides the frozen-poles and mode shapes

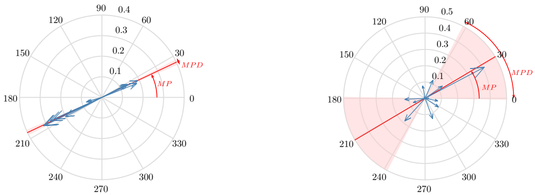
# Experimental identification of the time-varying beam

The identification process is similar to the univariate case

- Large batch of fast 2SLS identifications
- Refined identification by nonlinear optimization
- The same least squares cost function is used

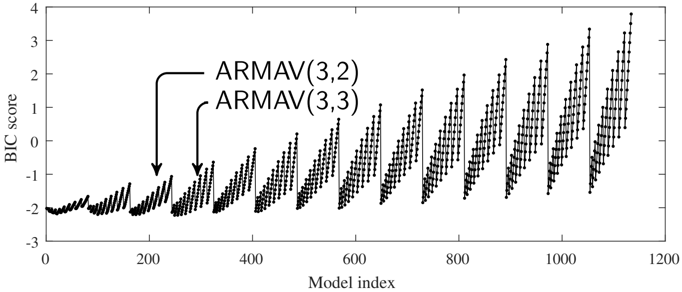
$$V(\boldsymbol{\theta}) = \frac{1}{2N} \sum_{t=1}^{N} \boldsymbol{e}[t, \boldsymbol{\theta}]^T \, \boldsymbol{e}[t, \boldsymbol{\theta}]$$

- The selection of the physical modes now also uses the **mean phase deviation** of the mode shapes



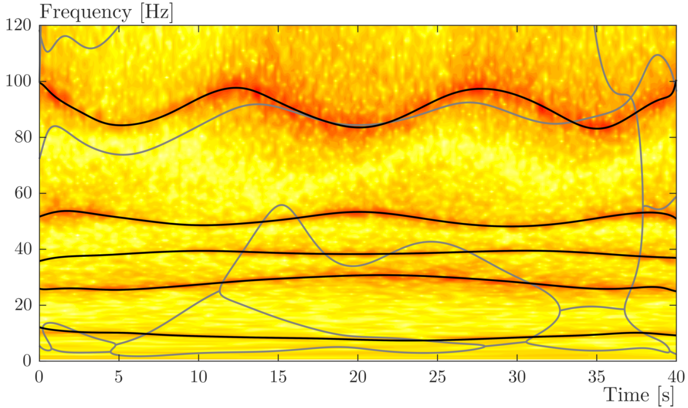
# The batch of 2SLS indentifications gives some clue on the potentially good model structures

The families of ARMAV(3,2) or ARMAV(3,3) seem to contain good model candidates



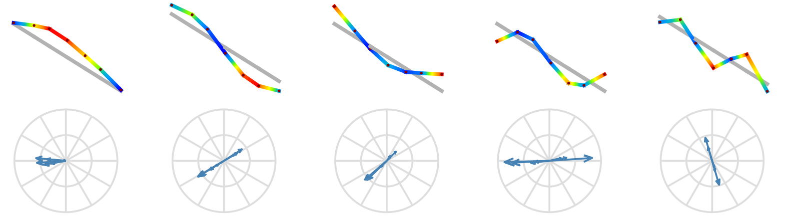
# Refined identification with the nonlinear optimization process

A more precise identification with the iterative optimization scheme reveals that the ARMAV(3,3)[9,1] is the best one



# Example of obtained mode shapes

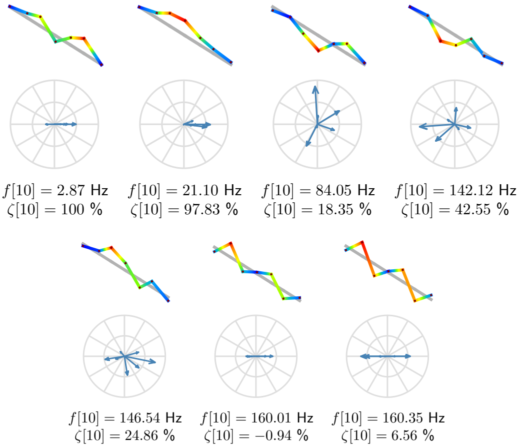
At $t = 10$ s, the physical mode shapes are the following ones



| | | | | |
|---|---|---|---|---|
| $f_1[10] = 8.95$ Hz | $f_2[10] = 27.39$ Hz | $f_3[10] = 39.42$ Hz | $f_4[10] = 48.60$ Hz | $f_5[10] = 93.87$ Hz |
| $\zeta_1[10] = 5.32$ % | $\zeta_2[10] = 1.35$ % | $\zeta_3[10] = 0.037$ % | $\zeta_4[10] = 2.25$ % | $\zeta_5[10] = -0.21$ % |

# Example of obtained mode shapes

At the same time the identified spurious modes show either a higher dispersion in the complex plane or they are simply purely real.



# Alternative modelling for the identification
## Parameterization in the modal domain

Starting with the following ***innovation state-space model***:

$$\begin{cases} \boldsymbol{x}[t+1] &= \boldsymbol{F}[t]\,\boldsymbol{x}[t] + \boldsymbol{K}[t]\,\boldsymbol{e}[t] \\ \boldsymbol{y}[t] &= \boldsymbol{C}[t]\,\boldsymbol{x}[t] + \boldsymbol{e}[t] \end{cases}$$

we can transform it into a ***modal form***

$$\begin{cases} \boldsymbol{\eta}[t+1] &= \boldsymbol{A}[t]\,\boldsymbol{\eta}[t] + \boldsymbol{\Psi}[t]\,\boldsymbol{e}[t] \\ \boldsymbol{y}[t] &= \boldsymbol{\Phi}[t]\,\boldsymbol{\eta}[t] + \boldsymbol{e}[t] \end{cases}$$

with

$$\begin{aligned} \boldsymbol{A}[t] &= \boldsymbol{V}[t]^{-1}\,\boldsymbol{F}[t]\,\boldsymbol{V}[t], \\ \boldsymbol{\eta}[t] &= \boldsymbol{V}[t]^{-1}\,\boldsymbol{x}[t], \\ \boldsymbol{\Phi}[t] &= \boldsymbol{C}[t]\,\boldsymbol{V}[t], \\ \boldsymbol{\Psi}[t] &= \boldsymbol{V}[t]^{-1}\,\boldsymbol{K}[t]. \end{aligned}$$

## Alternative modelling for the identification
## Parameterization in the modal domain

To avoid treating complex values, all the parameters are separated into their real and imaginary parts.

The modal decoupling is still valid.

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 & & & \\ & \boldsymbol{A}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{A}_n \end{bmatrix} \qquad \begin{aligned} \boldsymbol{\Phi} &= \begin{bmatrix} \boldsymbol{\Phi}_1^{\mathcal{R}} & \boldsymbol{\Phi}_1^{\mathcal{I}} & \boldsymbol{\Phi}_2^{\mathcal{R}} & \boldsymbol{\Phi}_2^{\mathcal{I}} & \cdots \end{bmatrix} \\ \boldsymbol{\Psi}^T &= \begin{bmatrix} \boldsymbol{\Psi}_1^{\mathcal{R}} & \boldsymbol{\Psi}_1^{\mathcal{I}} & \boldsymbol{\Psi}_2^{\mathcal{R}} & \boldsymbol{\Psi}_2^{\mathcal{I}} & \cdots \end{bmatrix} \end{aligned}$$

with

$$\boldsymbol{A}_i = \begin{bmatrix} a_i & b_i \\ -b_i & a_i \end{bmatrix}$$

All the coefficients of these matrices are stacked in a parameters vector $\boldsymbol{\theta}$

# Alternative modelling for the identification
## Parameterization in the modal domain

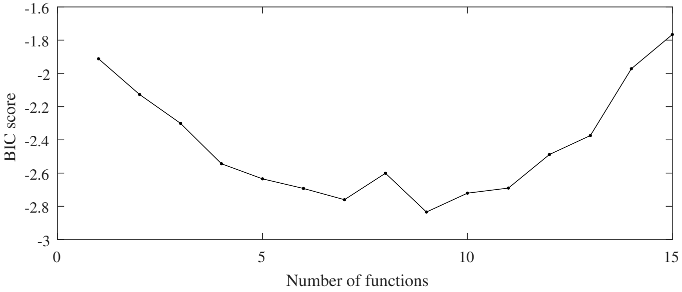
In theory, the ARMAV model is more parsimonious

But this kind of modelling offers some advantages

- The model parameters have now a physical meaning
- No more eigenvalue decompositions
- The model order is easily fixed
- It can be initialized by approximate LTI modal results
- The optimization process can be guided by the modal decoupling (graduated optimization)

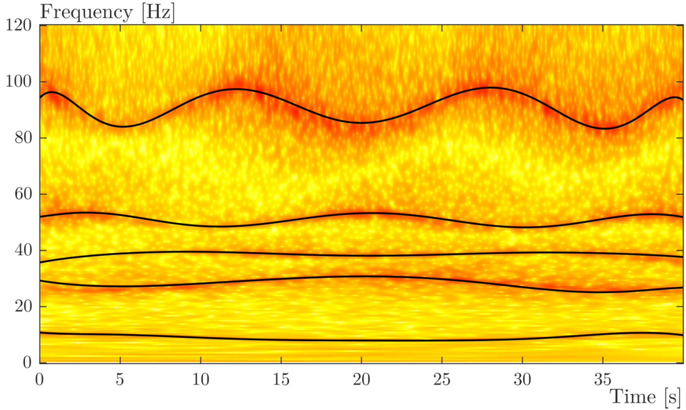
# Identification of the time-varying beam with the modal SS model

Only the size of the basis of functions needs to be determined



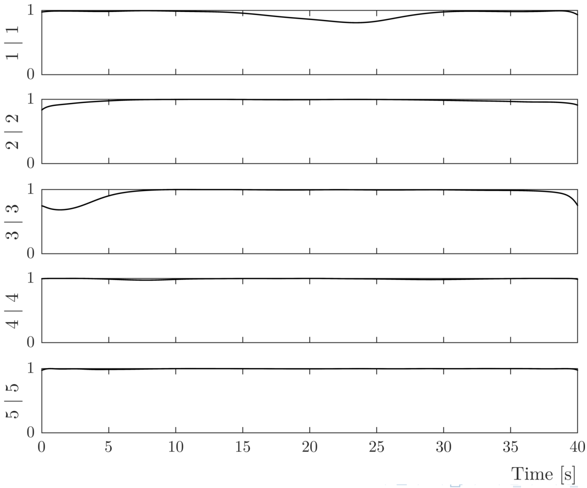
# Identification of the time-varying beam with the modal SS model

The model with 9 Chebyshev polynomials gives the best results
No spurious mode is introduced



# Comparison with the results obtained with the ARMAV model

The modal correlation is quite good between the two sets of varying mode shapes



# Outline of the presentation

Several identification methods are proposed in the thesis

The presentation is organized as follows:

Non parametric approach

- Presentation of the experimental setup

Combined parametric and non-parametric approach

Fully parametric approaches

Applications to more complex cases

# Extended applications

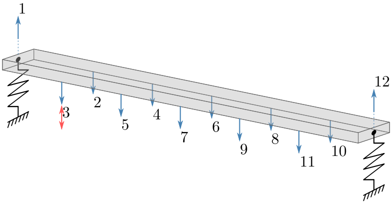
The purpose of this section is to test the proposed method on more complex problems

The time-varying beam is kept as example but extended with

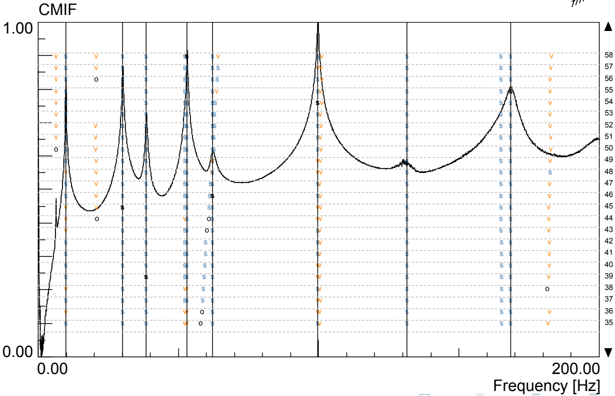
- An increased frequency range
- More acquisition channels
- Knowledge of additional information (position of the mass)

Application for monitoring purposes

# Extended experimental setup

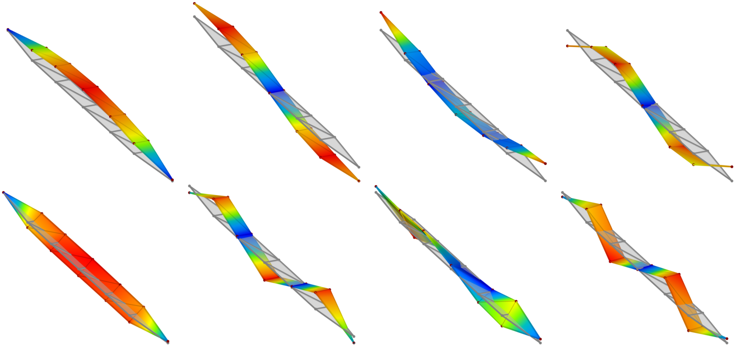



| | $f_r$ [Hz] | $\zeta_r$ [%] |
|---|---|---|
| 1 | 9.86 | 0.32 |
| 2 | 30.12 | 0.52 |
| 3 | 38.6 | 0.65 |
| 4 | 53.14 | 0.28 |
| 5 | 62.17 | 1.57 |
| 6 | 99.70 | 0.28 |
| 7 | 131.57 | 2.039 |
| 8 | 168.60 | 0.99 |



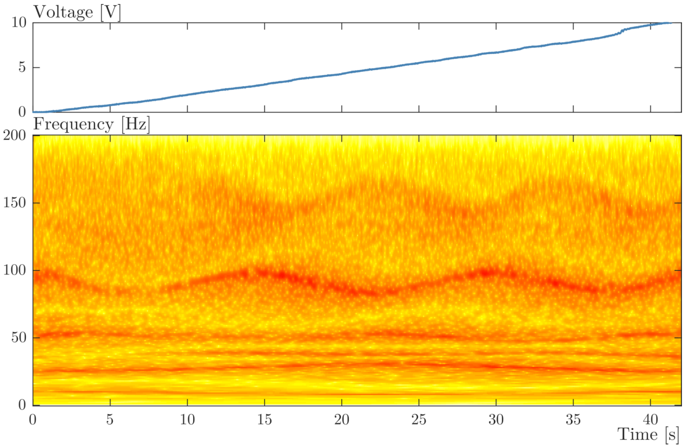
# Extended experimental setup

We have to deal with one additional bending mode and two rotation/torsion modes



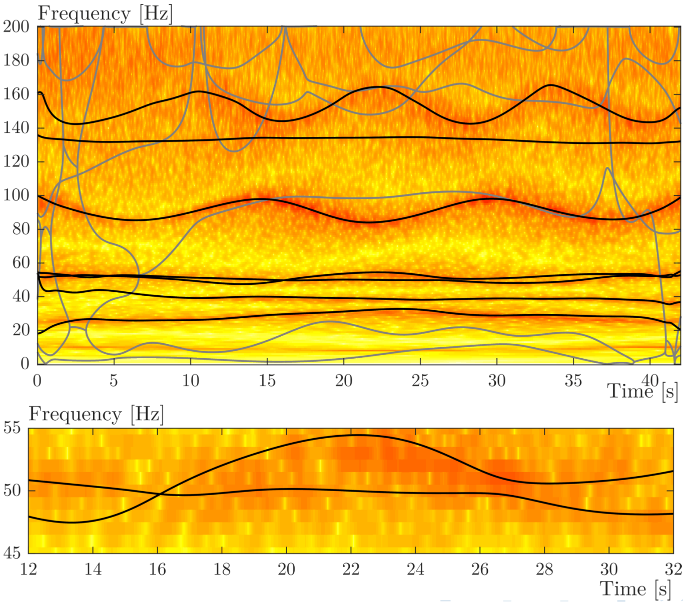
# Identification with the ARMAV and SS models

First, the mass is pulled with an approximately constant speed



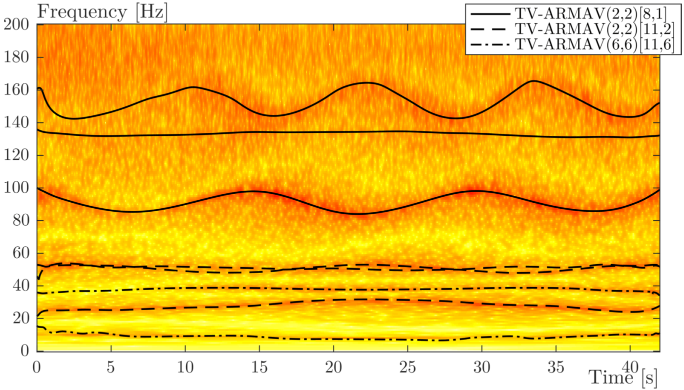
# Identification with the time-varying ARMAV model

ARMAV(2,2)[8,1]



# Identification with the time-varying ARMAV model

The full identification is then performed by mixing the results of several model structures



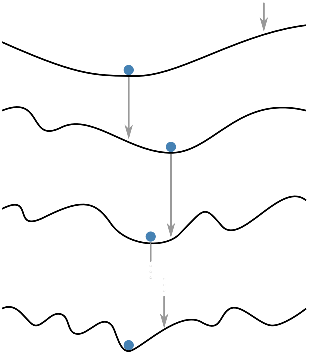
# Identification with the time-varying ARMAV model

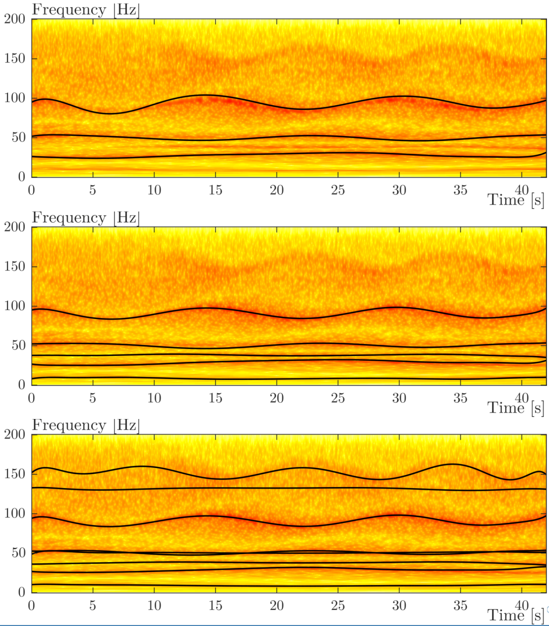
Some general observations :

- The number of model parameters drastically increased
- Idem for the complexity of the optimization process
- The selection of a good model structure is difficult
- No single model structure was able to identify all the modes

# Identification with the time-varying State-Space modal model

12 Chebyshev polynomials are used

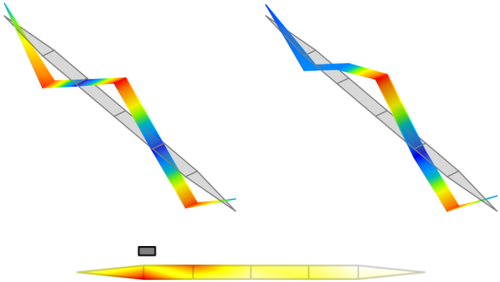




# Application for monitoring purposes

The goal of this part is to locate the modification of the system based on the identification results

The $COMAC$ is first used



# Application for monitoring purposes

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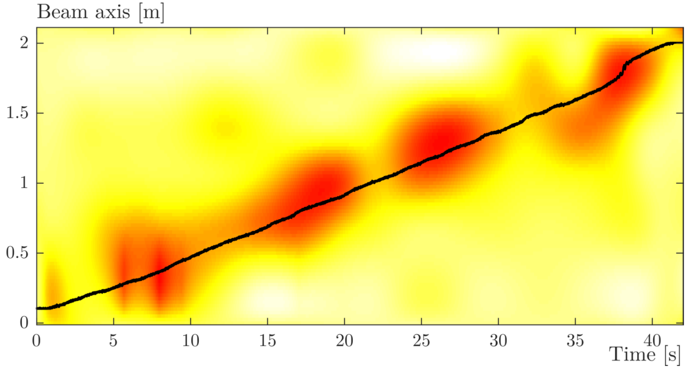
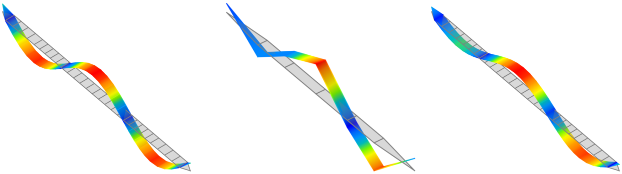


# Application for monitoring purposes

Another possibility is to rely on a reference finite element model and reduction/expansion methods

Discrepancies in elementary potential or kinematic energies are considered as criteria

$$E_j^M = \sum_{i=1}^{N_m} \left(\boldsymbol{X}^{(j)} - \boldsymbol{Z}^{(j)}\right)^T \boldsymbol{M}^{(j)} \left(\boldsymbol{X}^{(j)} - \boldsymbol{Z}^{(j)}\right)$$



# Application for monitoring purrposes

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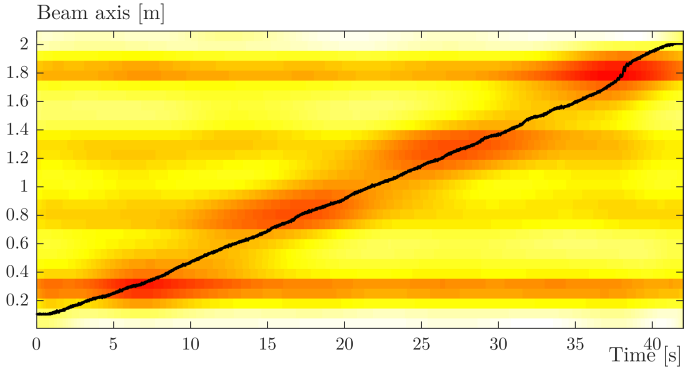
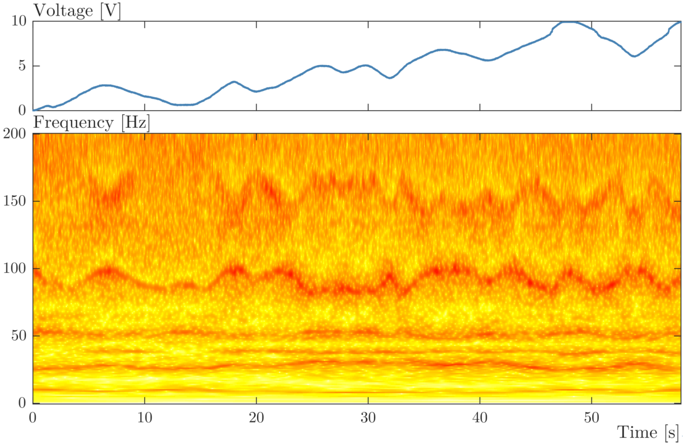


# Application with additional information

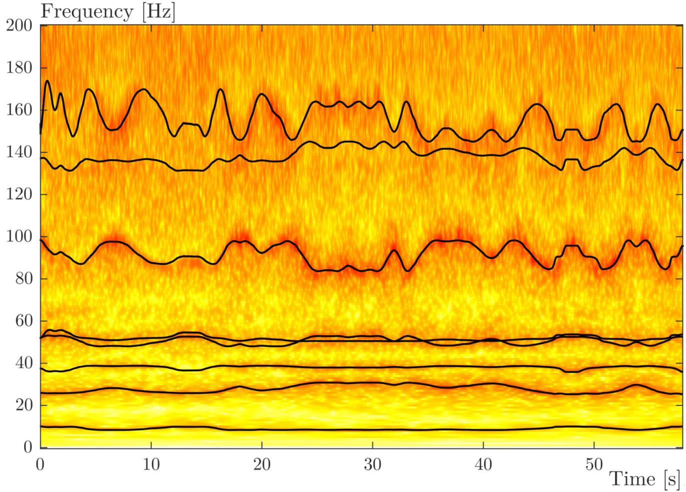
We considered general time-varying systems

But how can we manage some knowledge about a varying parameter?



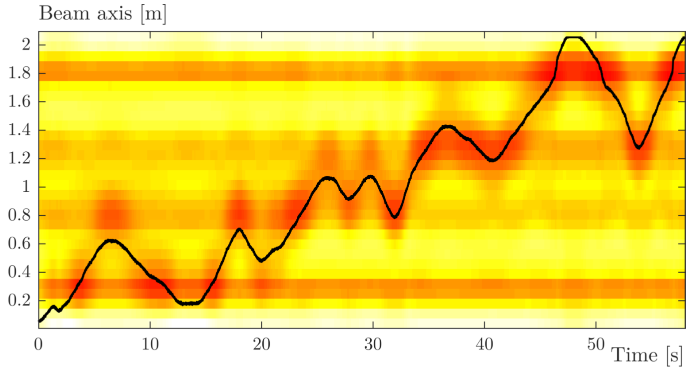
# Application with additional information

Identification with the modal State-Space model
12 position-based Chebychev polynomials



# Application with additional information

The mass tracking remains pretty accurate



# Concluding remarks

Time-varying mechanical systems were considered

Focus on MDOF methods and operational conditions

Several methods were proposed

- Non parametric
- Univariate parametric model
- Multivariate parametric models

All the methods were experimentally tested

Thank you for your attention