INVESTIGATING LUNAR TOPOGRAPHIC PROPERTIES AT DIFFERENT SPATIAL SCALES USING DATA FROM THE LUNAR ORBITER LASER ALTIMETER AND THE WAVELET LEADERS METHOD

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1. Introduction

- Various approaches exist to study the roughness of planetary bodies. The most commonly used focus on characterizing the roughness spatially by deriving statistics [e.g., 1-5] or characterizing the roughness in frequency space, such as those using Fourier transforms.
- Wavelet-based analyses are multifractal analyses used to characterize the surface roughness both spatially and in frequency. They have been rarely used in a planetary science context so far. The Mexican Hat Wavelet Transform (MHWT) has been used to characterize the roughness of Mars in 1D using topographic profiles [6], while the Wavelet Leaders Method (WLM) has been used to characterize the roughness of Mars in 1D and in 2D using gridded topographic data [7]. Results from the 1D roughness characterization of Mars using the WLM are in agreement with the results using the MHWT and statistical approaches [e.g., 1-2], while the 2D roughness characterization of Mars was the first of its kind.
- The Wavelet Leaders Method (WLM) involves the computation of wavelet coefficients at multiple scales and the propagation of the maximum of the absolute value of the wavelet coefficients in a pixel neighborhood, providing a more numerically stable computation of the pointwise Hölder regularity than other wavelet-based methods.

2. Objectives

2.1 Main objective:

- Use and refine the WLM [7] to study the roughness of the Moon both spatially and in frequency in 1D, using latitudinal and longitudinal profiles from the Lunar Orbiter Laser Altimeter gridded topographic data.

2.2 Secondary objectives:

- Determine the different scaling regimes present in each longitudinal or latitudinal data profile (*i.e.*, at which scales or spatial resolution changes in what governs topographic processes occur),
- Determine whether the data at a given latitude or longitude is monofractal or multifractal,
- Determine the value of the Hölder exponent for each latitudinal and longitudinal profile.

3. Data

- We used topographic data from LOLA that has been gridded and projected into a simple cylindrical projection (PDS3, V1.05) at 1024 ppd (or ~30 m/pixel), which is the highest spatial resolution currently available for the whole Moon. We downloaded individual tiles of 15° in latitude by 30° in longitude to obtain data for the whole globe. We then analyzed each of the 184,320 lines (latitudinal roughness) and 368,640 columns (longitudinal roughness) of data.
- The WLM uses data of size 2^x as input, so we downsampled each line of data (368,640 pixels) to 2^{18} (262,144) pixels, and each column of data (184,320 pixels) to 2^{17} (131,072) pixels. This corresponds to a spatial resolution of 728 ppd or ~41 m/pixel.
- Each latitudinal and longitudinal profile is compared to a theoretical 3rd order Daubechies wavelet (Fig. 1).

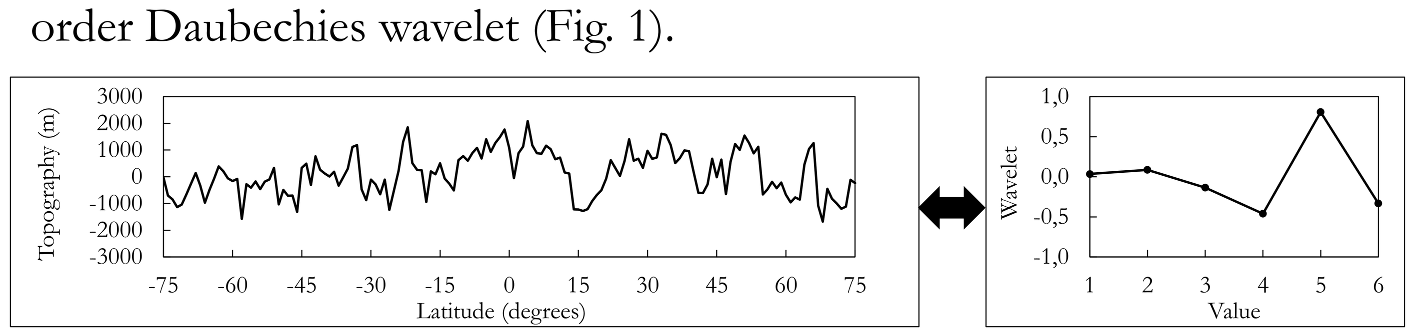


Figure 1. Each longitudinal topographic profile is compared to a theoretical 3rd order Daubechies wavelet.

4. Method

- The wavelet components at various spatial scales for each vector of data are first calculated. The topographic signal of pixel *i* and its neighbors at scale *j* (2^x pixels) is compared to a 3rd order Daubechies wavelet, yielding two components that each have 2^{x-1} pixels: the wavelet and the scaling coefficients. The former contain the high-frequency information (analogous to detrended topographic data) and is set aside for subsequent analysis. The latter contain the low-frequency information (analogous to the topographic data itself) which is used as input for the subsequent comparison between the "topographic" data and the theoretical wavelet at scale *j*+1. This process is done iteratively until there are 2^0 pixels left (Fig. 2).

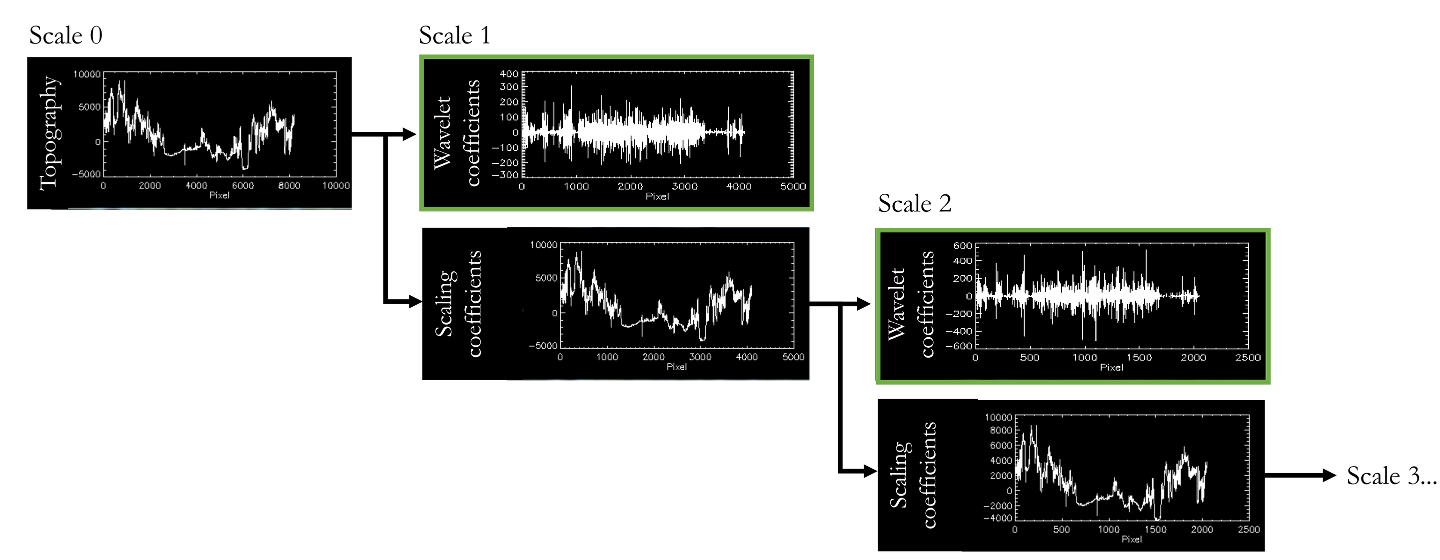


Figure 2. Comparing the topographic signal of pixel *i* and its neighbors at scale *j* (2^x pixels) to a 3rd order Daubechies wavelet yields wavelet coefficients and the scaling coefficients at each scale until there are 2^0 pixels left.

- The wavelet "leaders" at each scale are then identified. To do so, the wavelet coefficients obtained at each scale are compared using a dyadic cube; the maximum absolute value of the wavelet coefficient for pixel *i*-1 to *i*+1 at scale *j* and all finer scales is the wavelet leader for pixel *i* at scale *j* (Fig. 3).

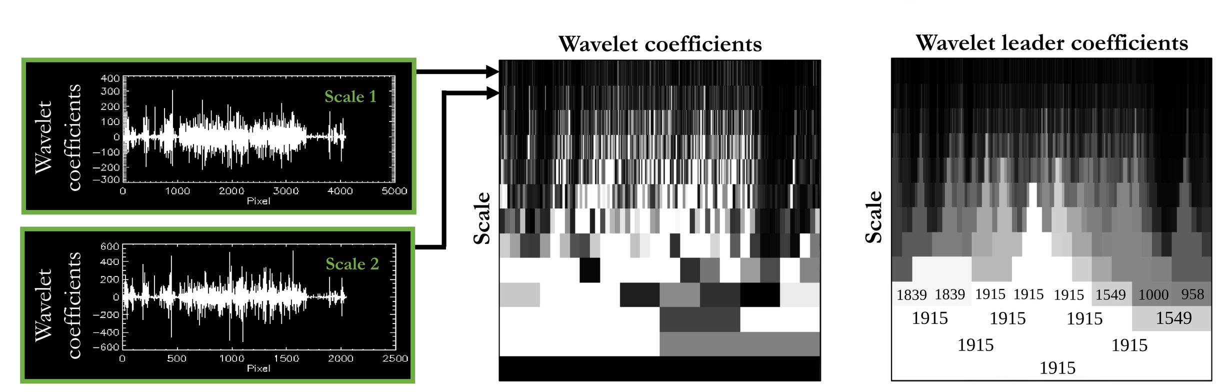


Figure 3. Calculating the wavelet leader coefficients based on the wavelet coefficients obtained at all spatial scales for a given line of topographic data.

- The wavelet leaders are used to identify the different scaling regimes in a vector of data via the structure function *S* (Eq 1) where *j* is the scale, λ is the dyadic cube, d_λ is the wavelet leader for that dyadic cube, and *q* is the order of the structure function. This can be done by (1) plotting $\log_2 S(j,q)$ versus *j* (Fig. 4), and (2) identifying the absolute value of the curvature on that plot, where the highest curvature value(s) represent the likeliest scale break(s).
- The scaling function η is used to determine if the scaling regimes have a mono- or a multifractal behavior (Eq 2). This is done by (1) plotting $\eta(q)$ versus *q* (for *q* = -2 to 2) for each scaling regime, and (2) calculating the correlation *r* between η and its regression (Fig. 5). The data has a monofractal nature if *r* is ~1. Here we considered the data monofractal if *r* > 0.98, and multifractal if *r* ≤ 0.98. If the data is monofractal, the slope of $\eta(q)$ versus *q* coincides with the Hölder exponent and characterizes its irregularity. If the data is multifractal, the slope gives the dominant Hölder exponent but does not fully represent the fractal properties of the signal.

$$S(j,q) = 2^{-j} \sum_{\lambda} d_\lambda^q \qquad (Eq\ 1)$$

$$\eta(q) = \lim_{j \to +\infty} \frac{\log S(j,q)}{\log 2^{-j}} \qquad (Eq\ 2)$$

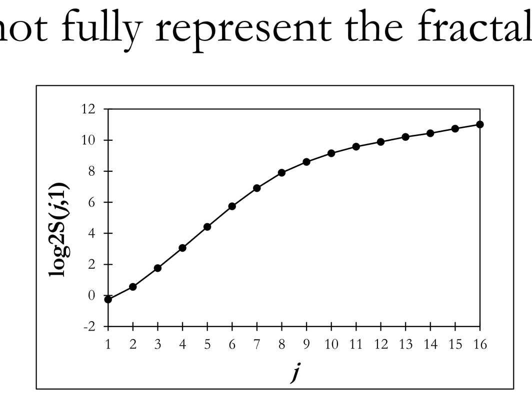


Figure 4. Plot of $\log_2 S(j,q)$ versus *j* (where *q*=1) for a given line of topographic data (at 50°N) used to identify the different scaling regimes occurring at that latitude.

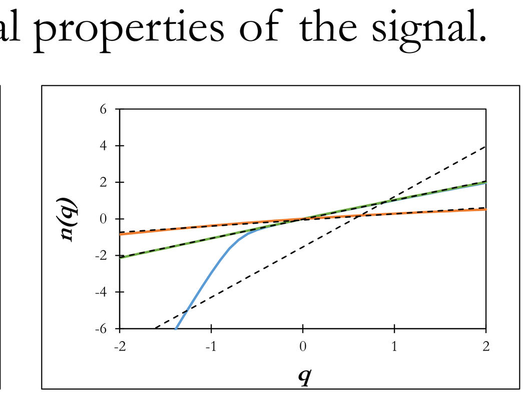


Figure 5. Plot of $\eta(q)$ versus *q* for the three scaling regimes (solid lines) identified at 50°N and their corresponding best linear fit (dashed lines).

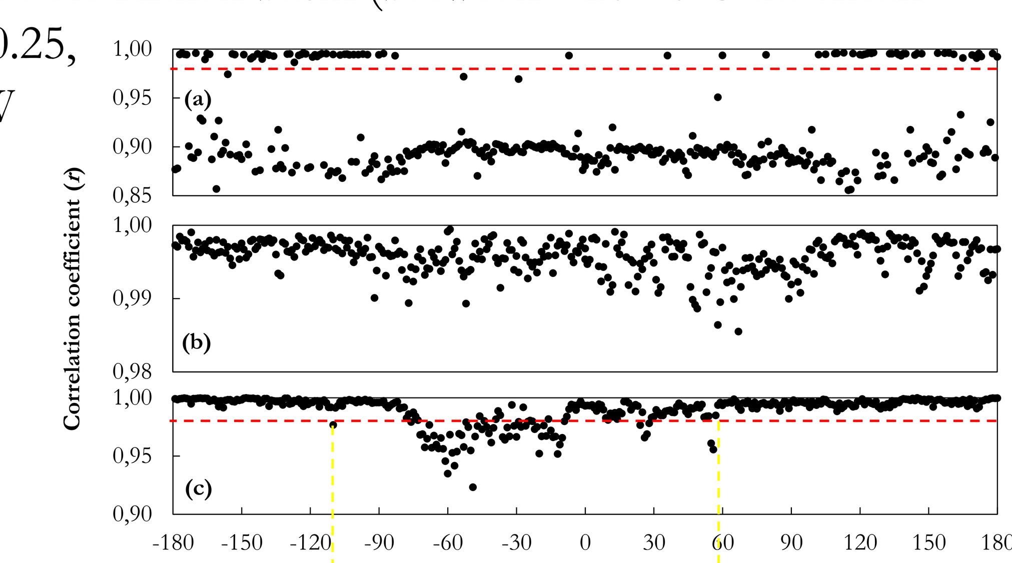
5. Results

5.1 Scaling regimes

- Scale breaks occur most often at *j* = 3, 10, and 15, corresponding to spatial resolutions of 659 m/pixel, 84 km/pixel and 2,700 km/pixel.
- They indicate that three scaling regimes are generally present at the discrete scales investigated here: *j* = 1-3 (165-659 m/pixel), *j* = 4-10 (1-84 km/pixel), and *j* = 11-15 (169-2,700 km/pixel). The scale breaks identified at *j* = 3 and 15 are still under investigation as data from fewer scales were involved in the computation of the curvature.
- The smallest scaling regime is consistent with [5] who found that within the baselines they investigated (~17 m to ~2.7 km), competing surface processes mostly occurs near 1 km.
- We hypothesize that the intermediate scaling regime is characterized by the formation of simple and complex craters, whereas the largest scaling by the formation of impact basins up to the largest on the Moon, the South Pole-Aitken basin.

5.2 Fractal behavior and value of the Hölder exponent

- The smallest scaling regime has a multifractal behavior (*r* ≤ 0.98) at all latitudes and most longitudes. A monofractal behavior (*r* > 0.98) is seen occasionally on the far side (Fig. 6).
- The intermediate scaling regime has a monofractal behavior at all latitudes and longitudes (Fig. 6).
- The largest scaling regime has a multifractal behavior in the maria (between ~25°S-65°N the mean Hölder in latitude is 0.34, between ~110°W-58 °E the mean Hölder in longitude is 0.42), and a monofractal behavior in the South Pole-Aitken basin (between ~25-75°S the mean Hölder in latitude is 0.25, between ~180-120°W and 120-180°E the mean Hölder in longitude is 0.24) (Fig. 6).



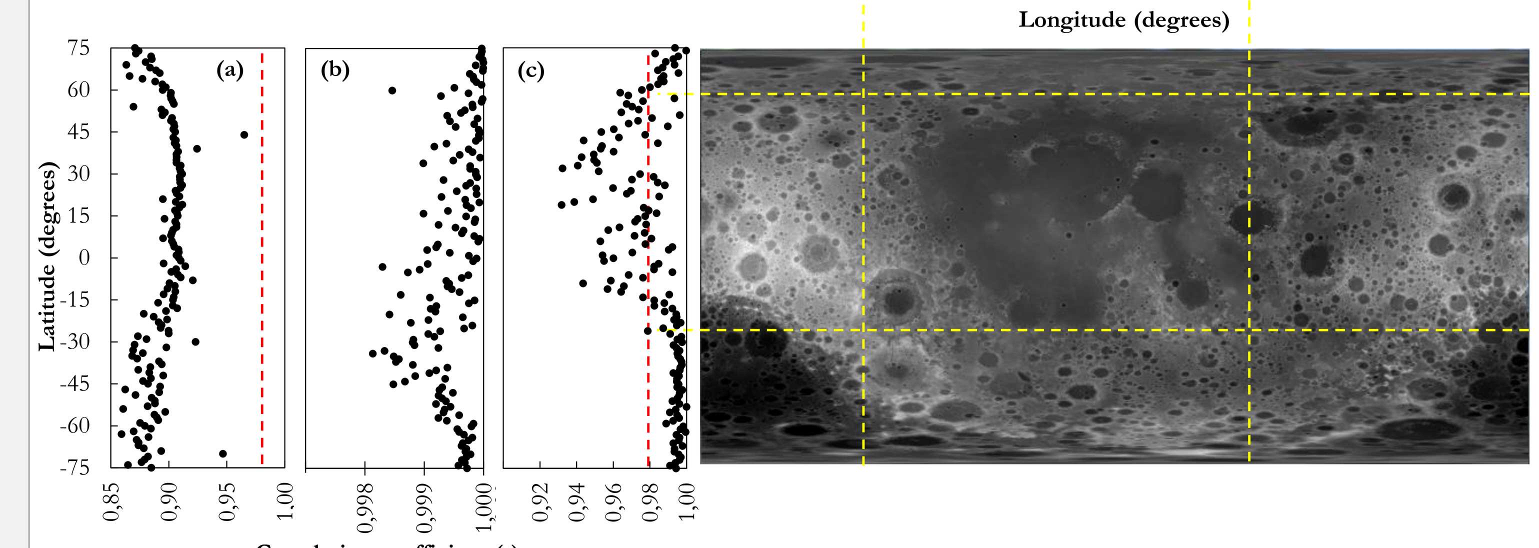


Figure 6. Determining the fractal behavior of each scaling regime in latitude and in longitude. The scaling regimes are: (a) 165-659 m/pixel, (b) 1-84 km/pixel, and (c) 169-2,700 km/pixel. Correlation coefficients lower than 0.98 suggest a multifractal behavior, and higher than 0.98 suggest a monofractal behavior. The maria exhibits a multifractal behavior in (c)

6. Upcoming work

The characterization of the 2D roughness (each pixel) will be undertaken next, which will allow a more precise characterization of the surface roughness especially by studying the difference between highlands and maria.

References: [1] Kreslavsky M. and Head J. (2000) *JGR, 105*, 26695-26711. [2] Orosei R. et al. (2003) *JGR*, *108*, 8023. [3] Aharonson O. and Schorghofer N. (2006) *JGR*, *111*, E11007. [4] Yokota Y. et al. (2008) *LPSC 39*, abstract #1921. [5] Rosenburg M. A. et al. (2011) *JGR*, *116*, E02001. [6] Malamud B. and Turcotte D. (2001) *JGR*, *108*, 17497-17504. [7] Deliège A. et al. (2017), *PSS*, *136*, 46-58.