Stochastic study of unidirectional composites: from micro-structural statistical information to multi-scale analyzes

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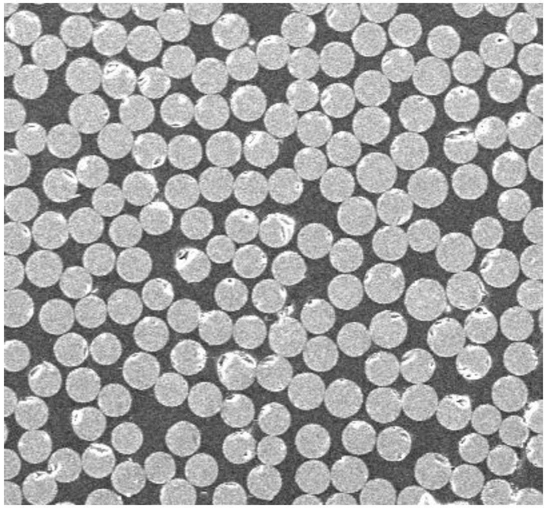

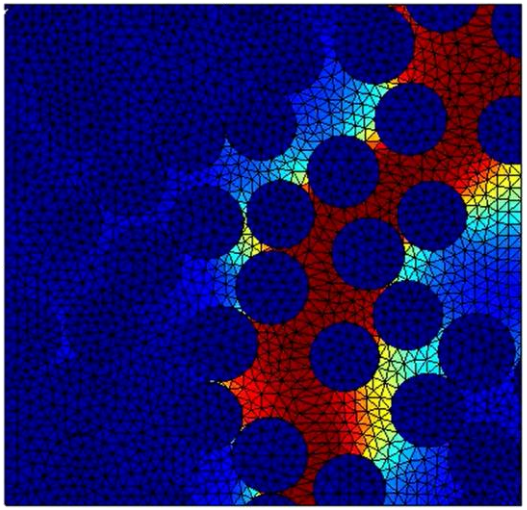

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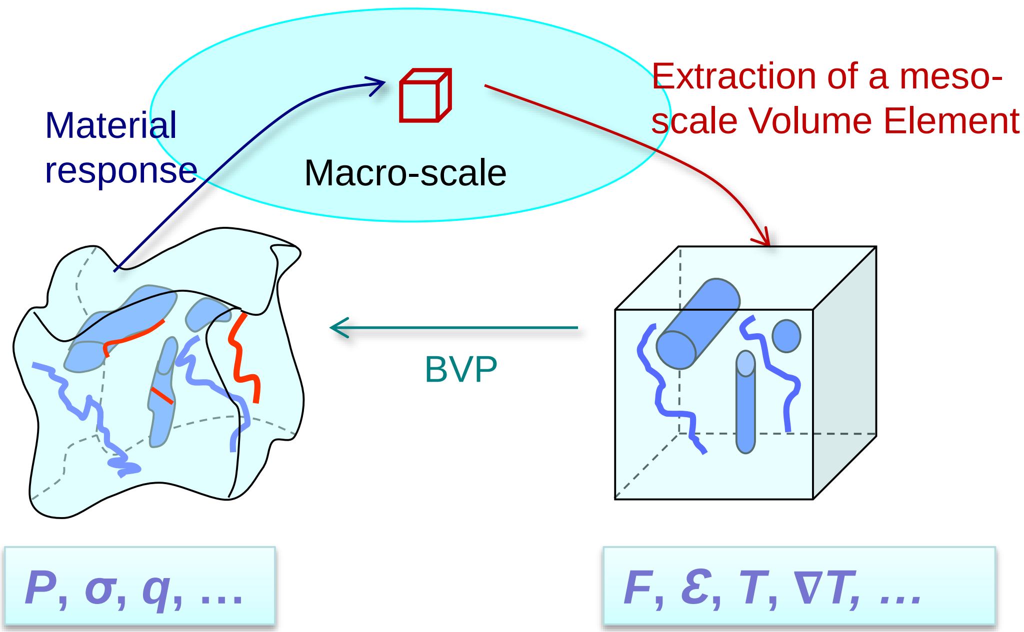
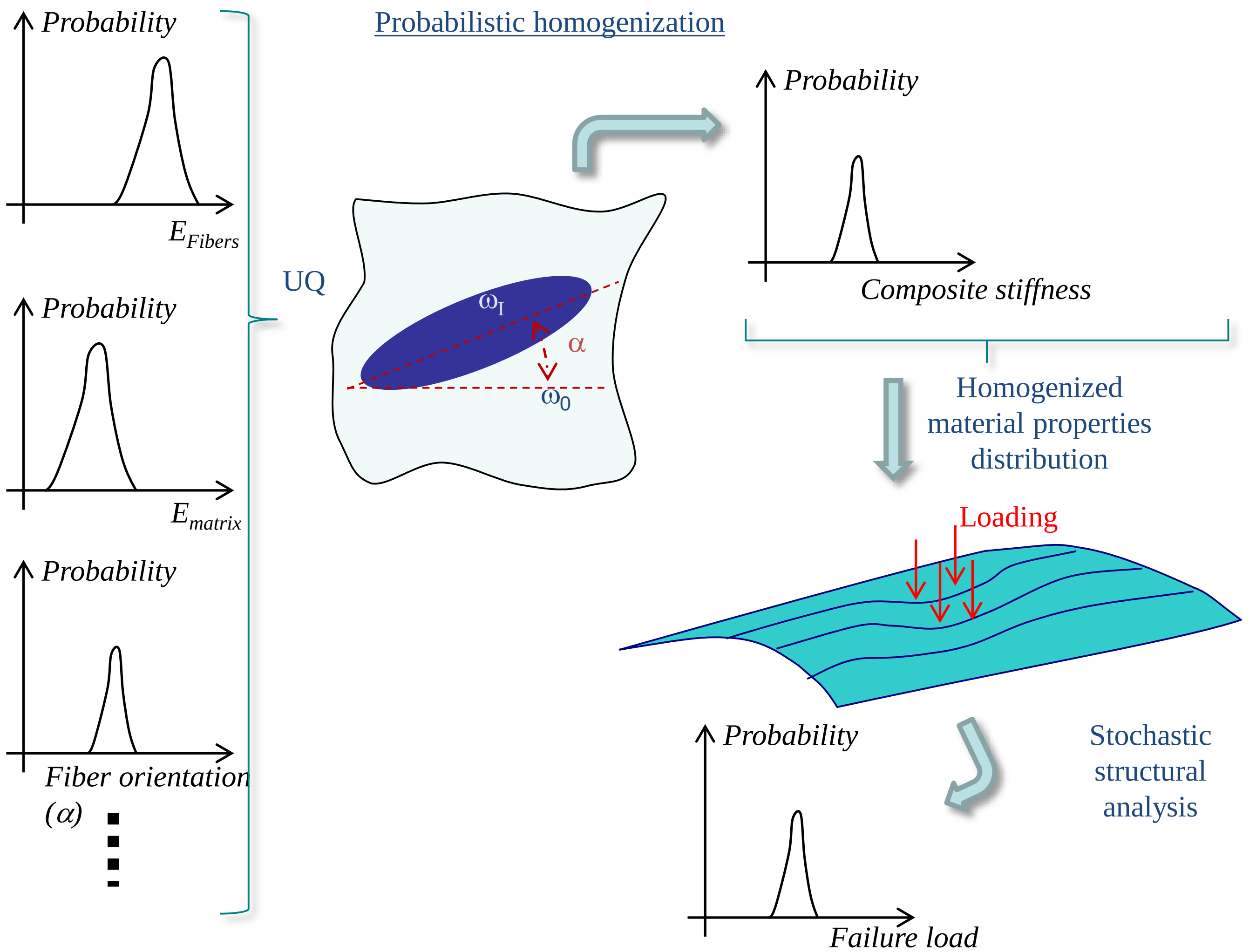
Multi-scale modelling

- Two-scale modelling
  - One way: homogenization
  - 2 problems are solved (concurrently)
    - The macro-scale problem
    - The meso-scale problem (on a meso-scale Volume Element)



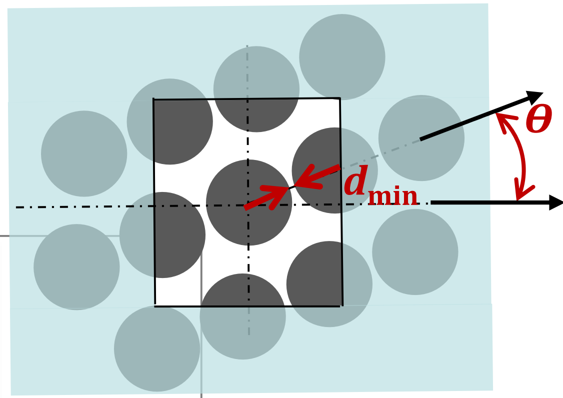
The problem

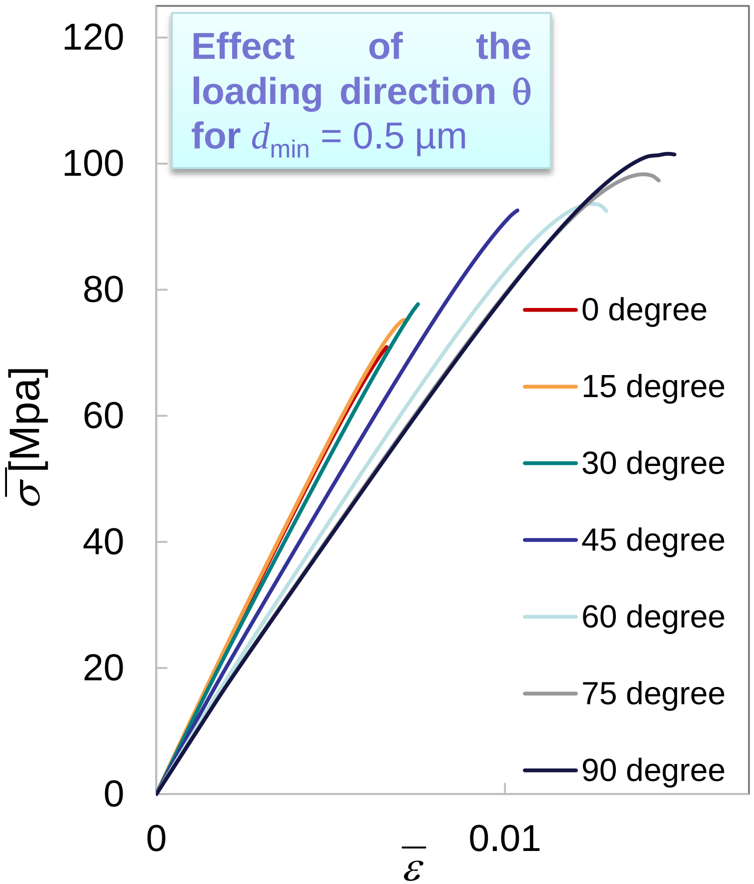
- Material uncertainties affect structural behaviors

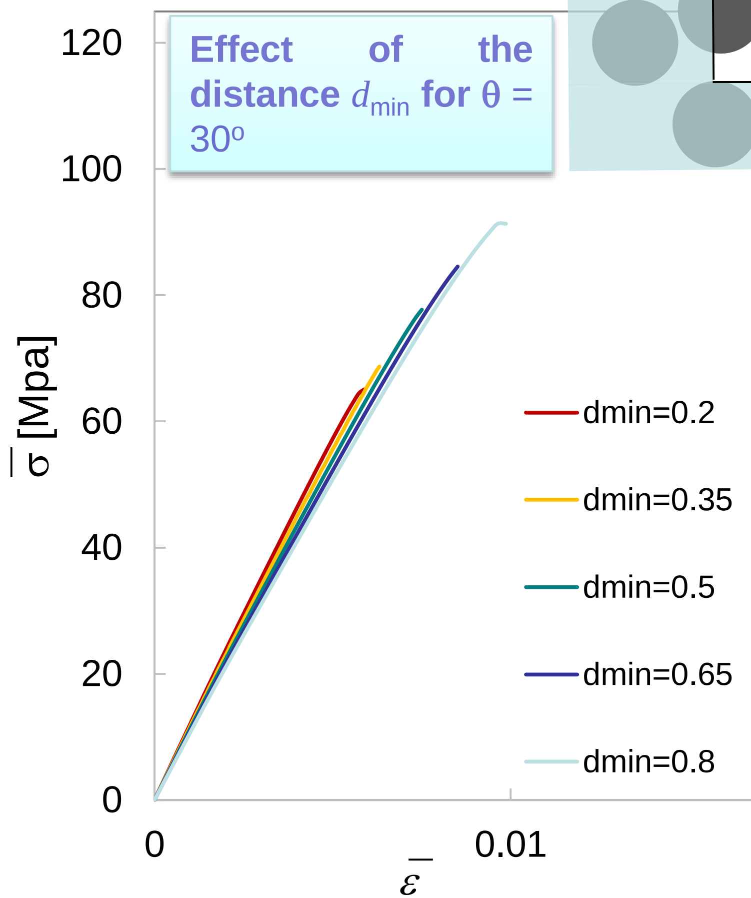


The problem

- Illustration assuming a regular stacking
  - 60%-UD fibers
  - Damage-enhanced matrix behavior





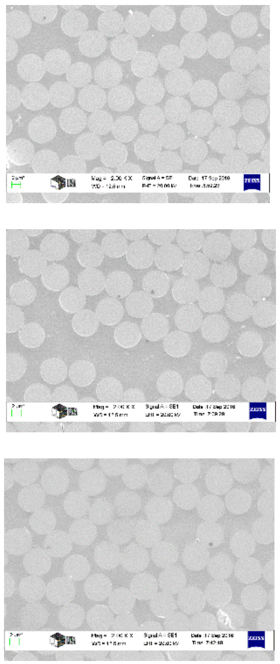


- Question: what does happen for a realistic fibre stacking?

The problem

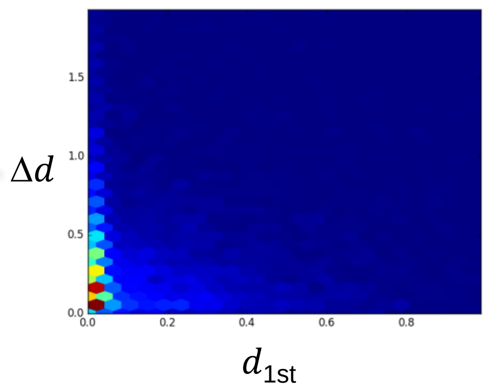
- Proposed methodology:

Experimental measurements

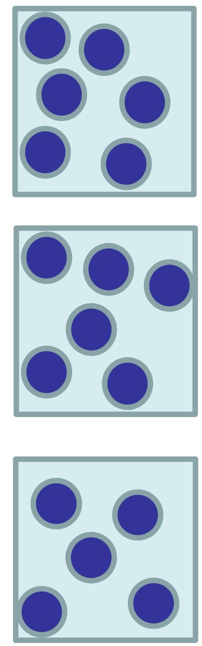


Micro-structure stochastic model

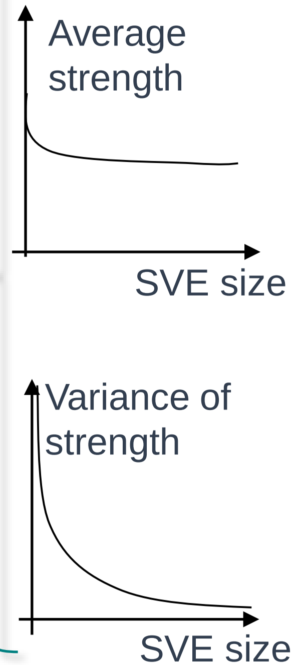
Copula (d_{1st}, Δd)



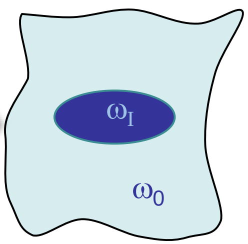
SVE realisations



Stochastic Homogenization



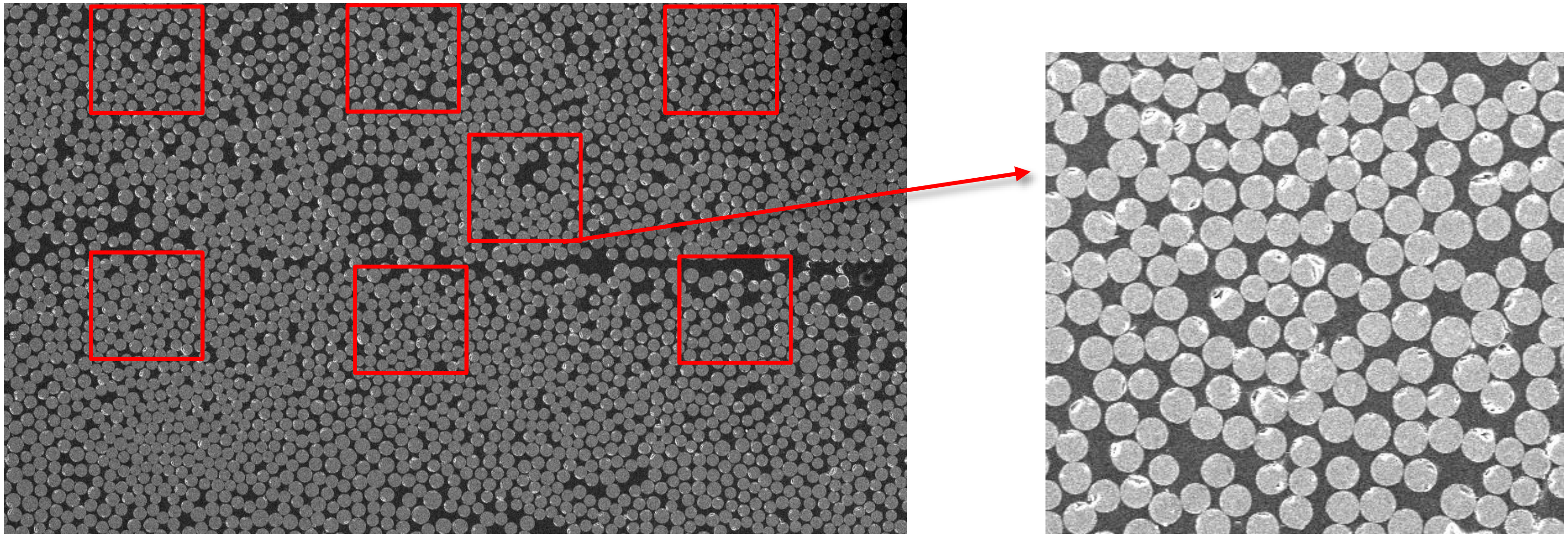
Stochastic reduced model



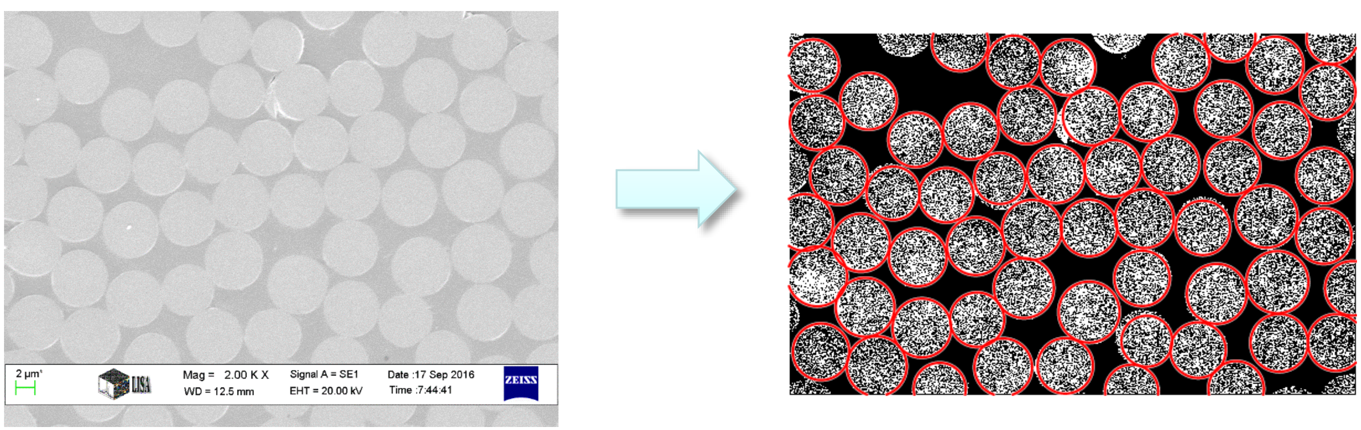
$\omega = \cup_i \omega_i$

Experimental measurements

- 2000x and 3000x SEM images



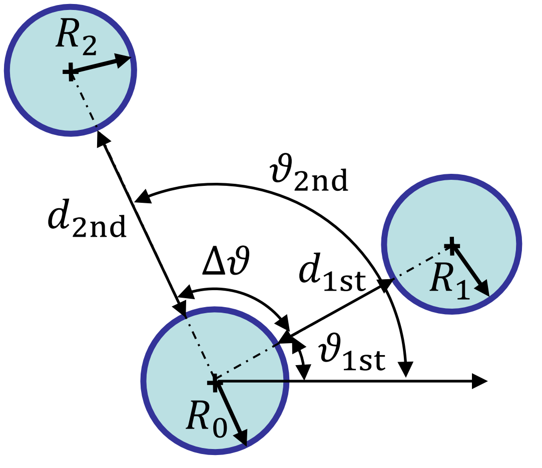
- Fibers detection



Micro-structure stochastic model

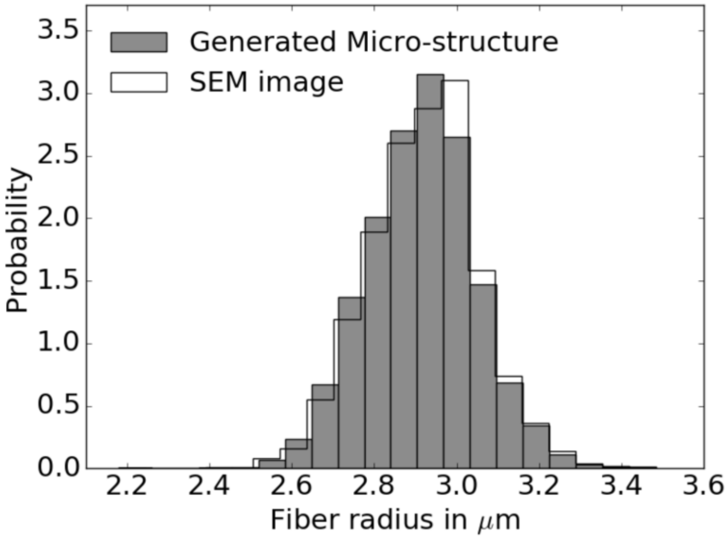
- Basic geometric information of fibers' cross sections
  - Fiber radius distribution $p_R(r)$

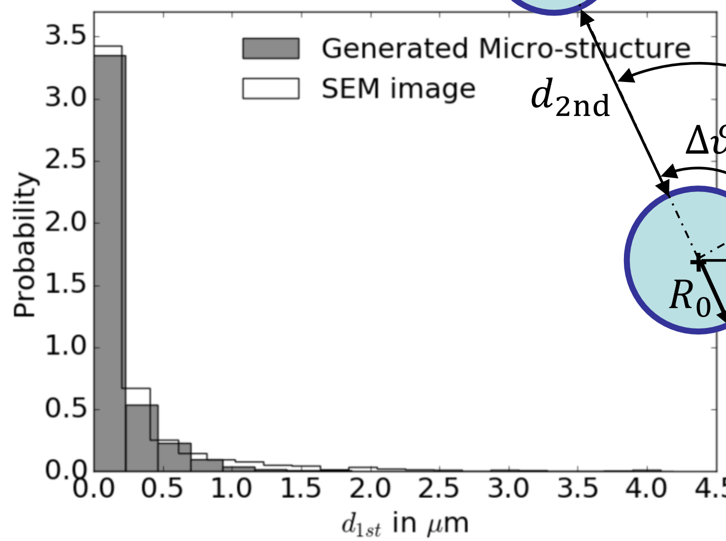
- Basic spatial information of fibers
  - The distribution of the nearest-neighbor net distance function $p_{d_{1st}}(d)$
  - The distribution of the orientation of the undirected line connecting the center points of a fiber to its nearest neighbor $p_{\vartheta_{1st}}(\theta)$
  - The distribution of the difference between the net distance to the second and the first nearest-neighbor $p_{\Delta d}(d)$ with $\Delta d = d_{2nd} - d_{1st}$
  - The distribution of the second nearest-neighbor's location referring to the first nearest-neighbor $p_{\Delta\vartheta}(\theta)$ with $\Delta\vartheta = \vartheta_{2nd} - \vartheta_{1st}$

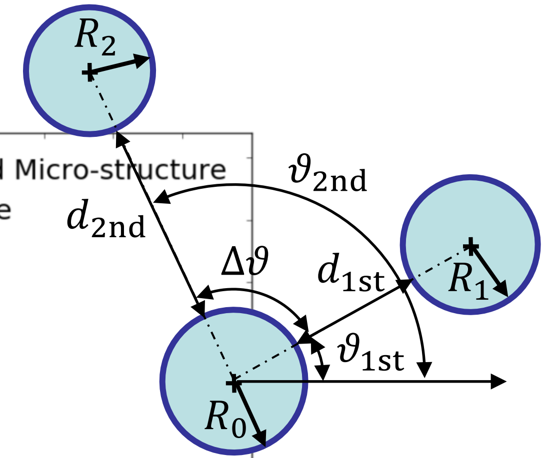


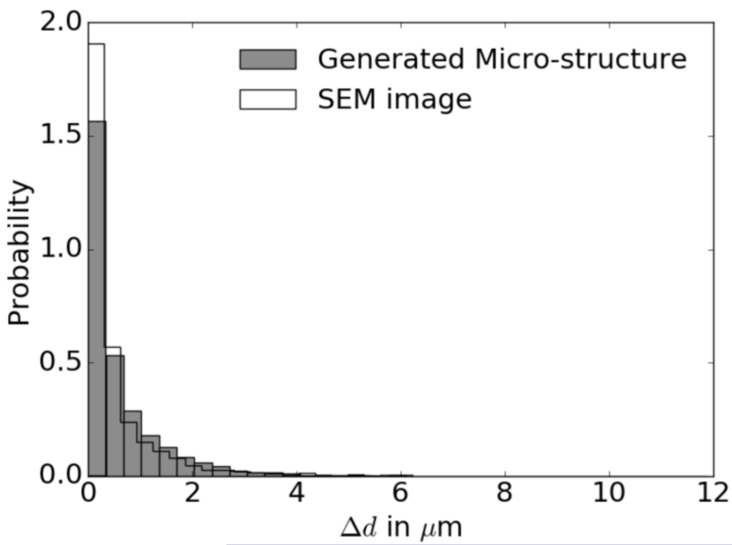
Micro-structure stochastic model

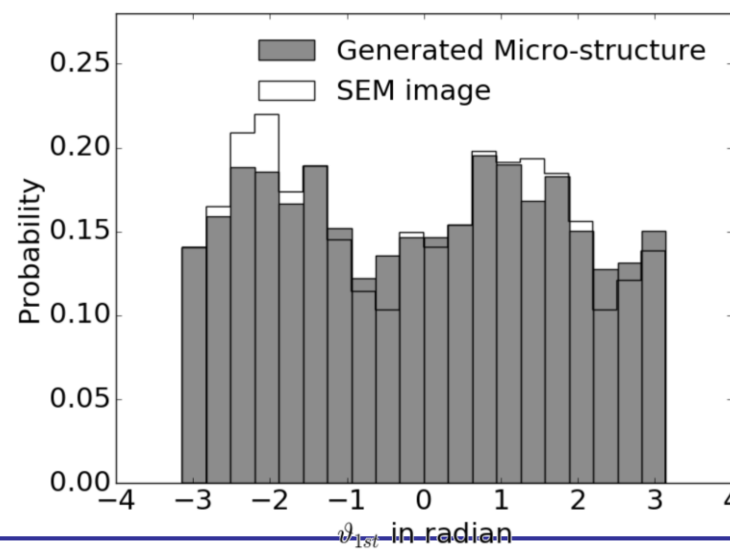
- Histograms of random micro-structures' descriptors





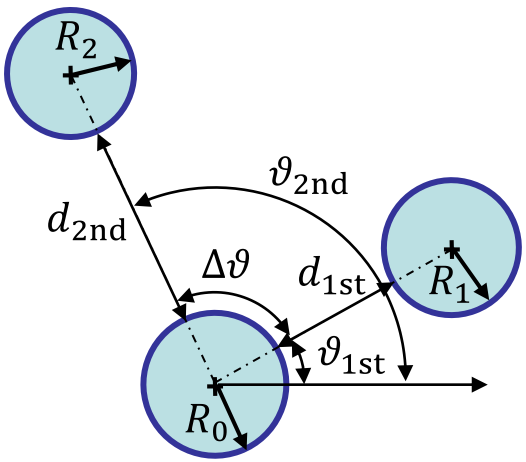






Micro-structure stochastic model

- Dependency of the four random variables d_{1st}, Δd, ϑ_{1st}, $\Delta\vartheta$
- Correlation matrix



	d_{1st}	Δd	ϑ_{1st}	$\Delta\vartheta$
d_{1st}	1.0	0.21	0.01	0.02
Δd		1.0	0.002	-0.005
ϑ_{1st}			1.0	0.02
$\Delta\vartheta$				1.0

- Distances correlation matrix

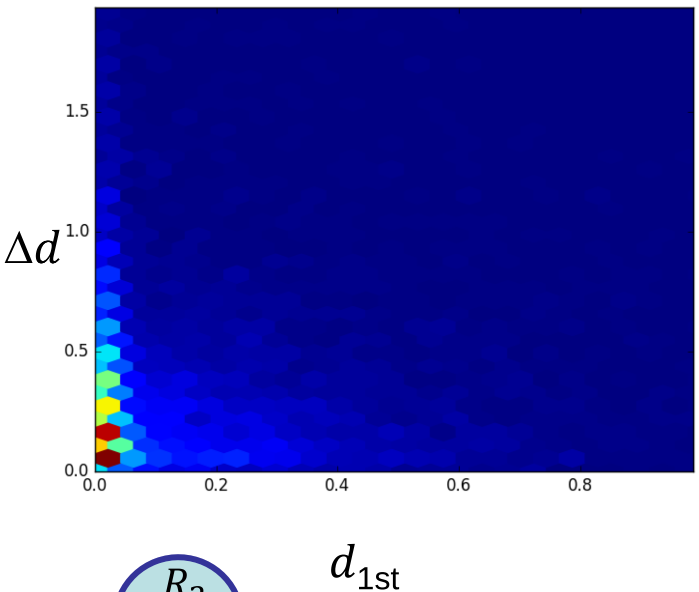
	d_{1st}	Δd	ϑ_{1st}	$\Delta\vartheta$
d_{1st}	1.0	0.27	0.04	0.08
Δd		1.0	0.05	0.06
ϑ_{1st}			1.0	0.05
$\Delta\vartheta$				1.0

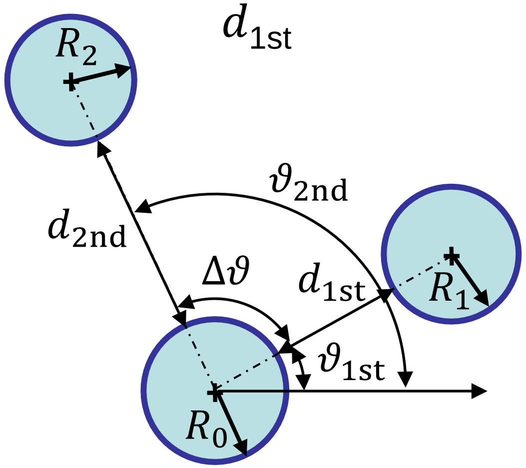
d_{1st} and Δd are dependent they will be generated from their empirical copula

Micro-structure stochastic model

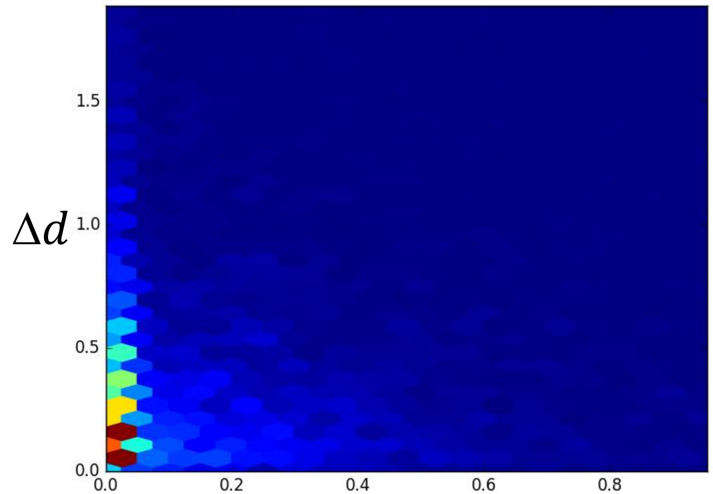
- d_{1st} and Δd should be generated using their empirical copula

SEM sample

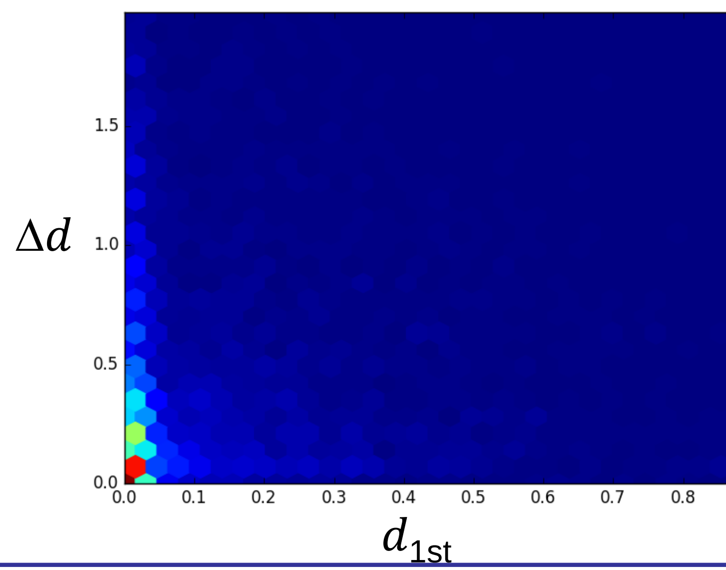




Generated sample



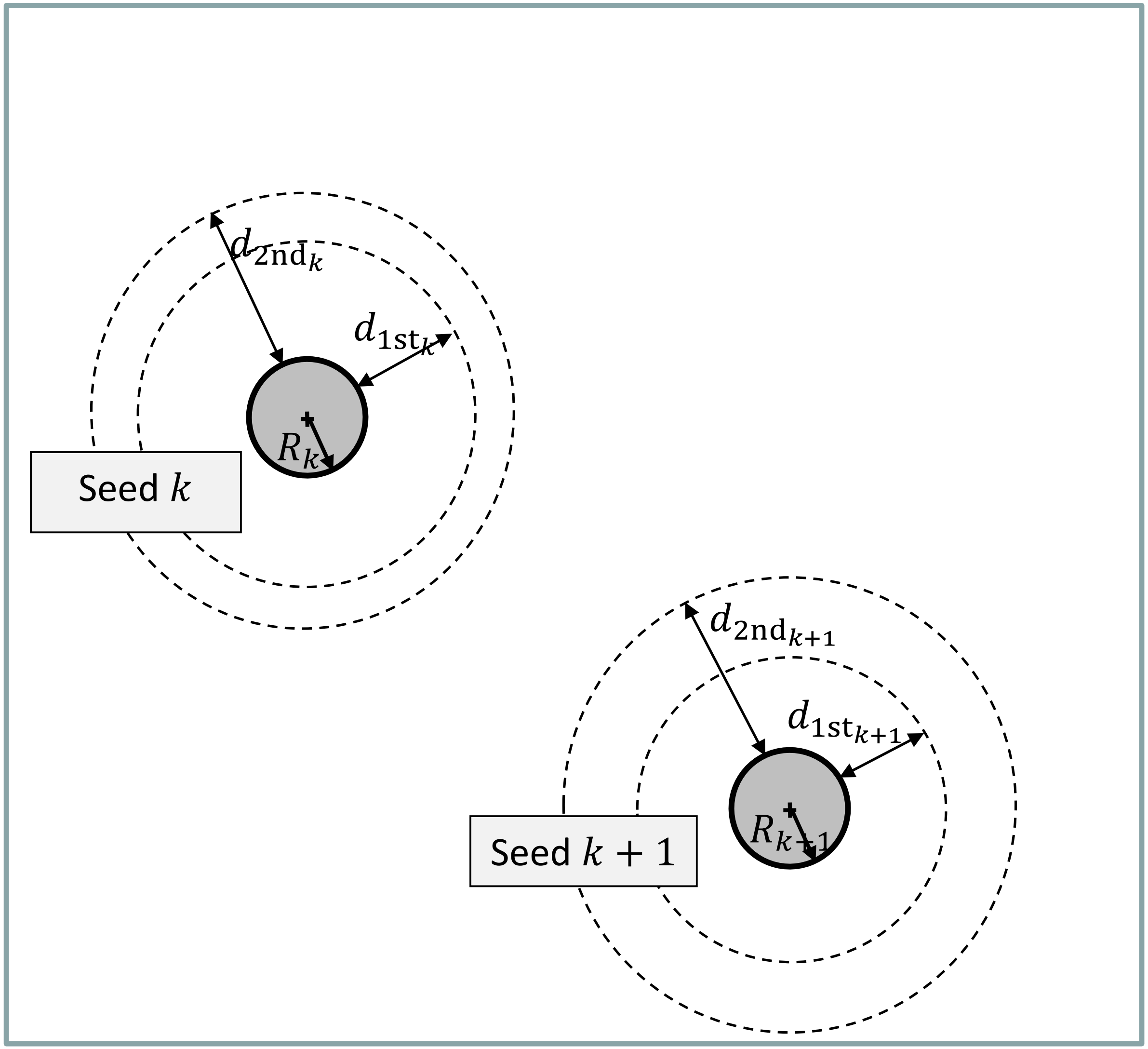
Directly from copula generator



Statistic result from generated SVE

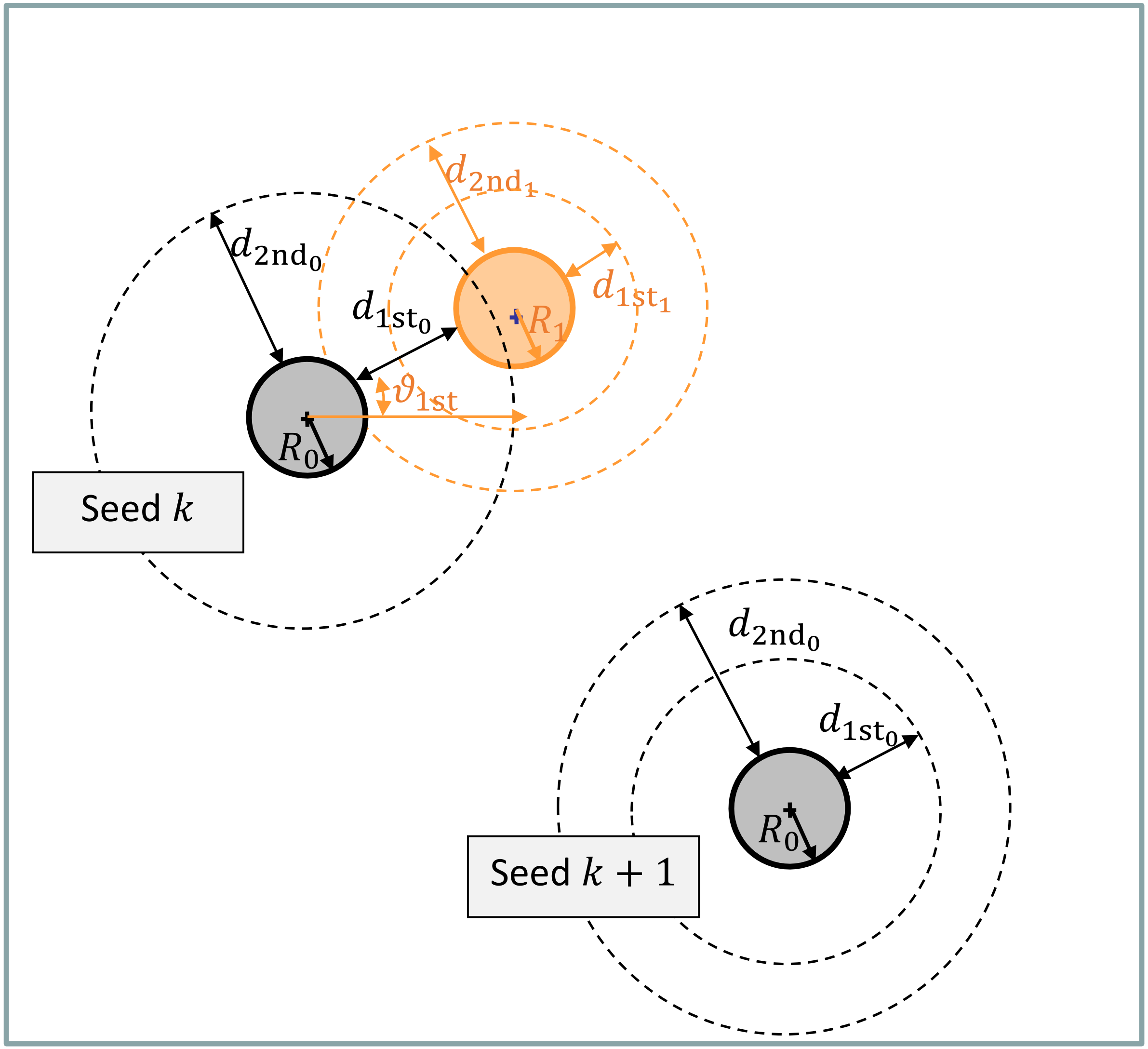
Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process
  1) Define N seeds with first and second neighbors distances



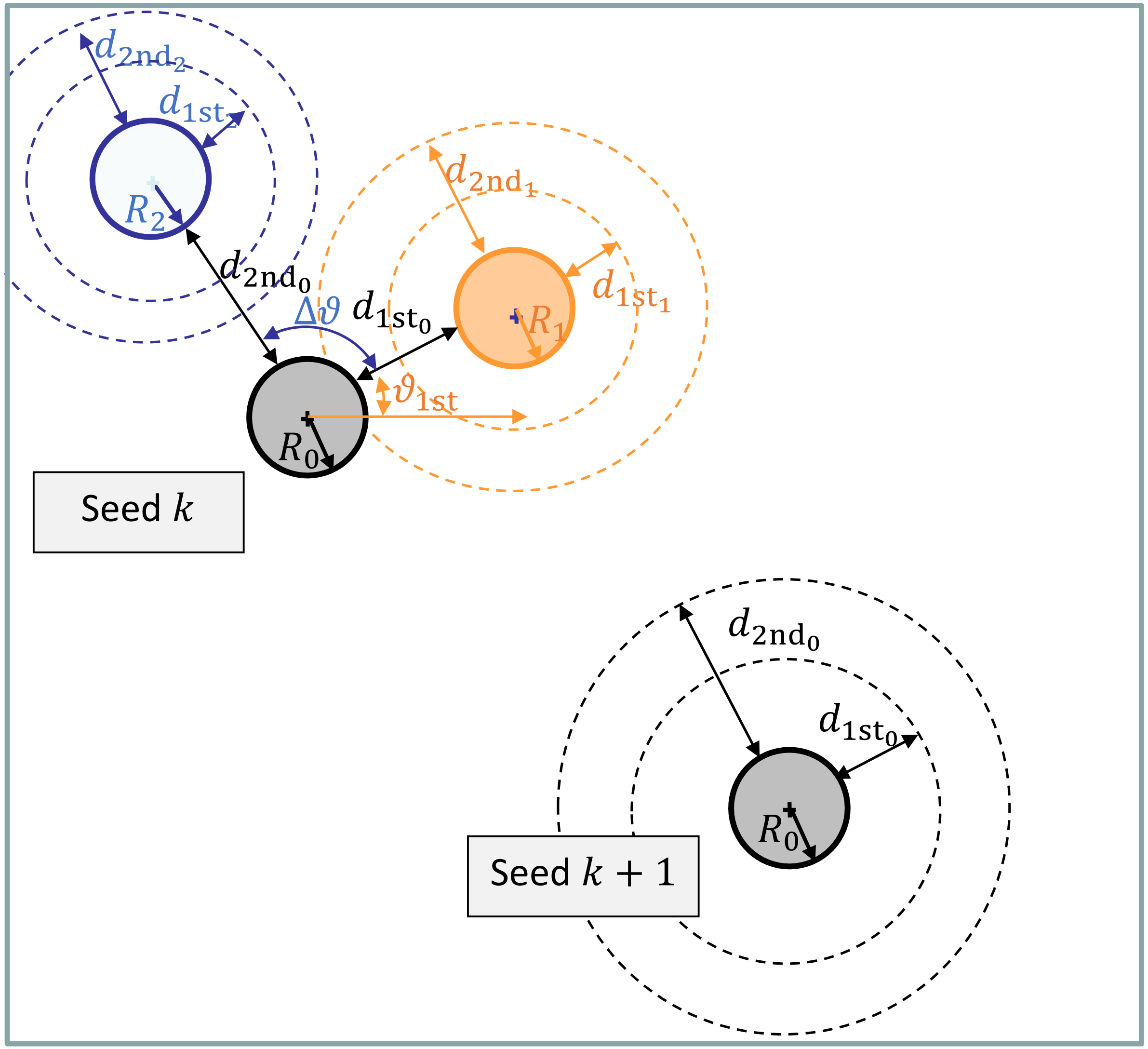
Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process
  1) Define N seeds with first and second neighbors distances
  2) Generate first neighbor with its own first and second neighbors distances

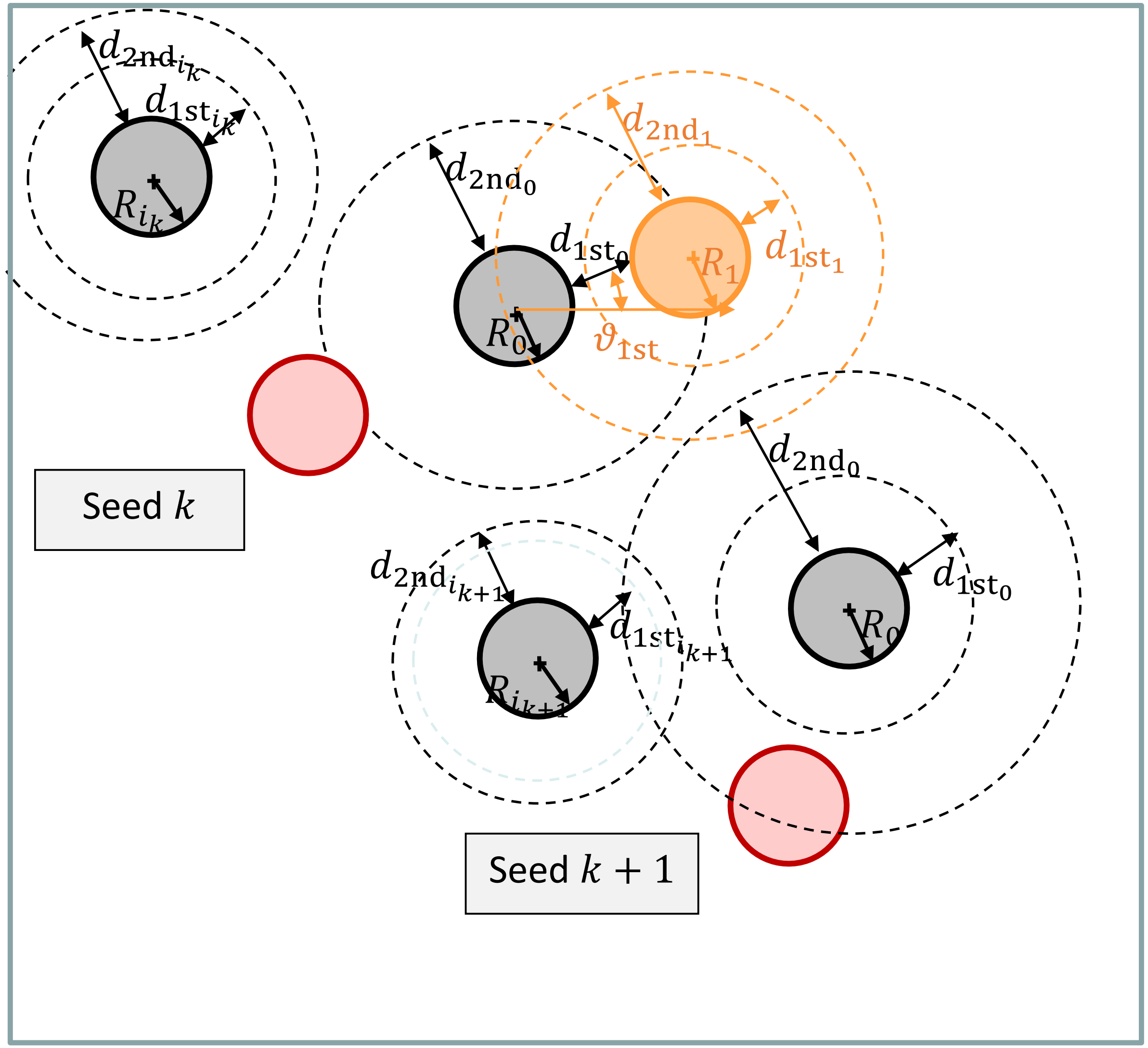


Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process
  1) Define N seeds with first and second neighbors distances
  2) Generate first neighbor with its own first and second neighbors distances
  3) Generate second neighbor with its own first and second neighbors distances



- The numerical micro-structure is generated by a fiber additive process
  1) Define N seeds with first and second neighbors distances
  2) Generate first neighbor with its own first and second neighbors distances
  3) Generate second neighbor with its own first and second neighbors distances
  4) Change seeds & then change central fiber of the seeds

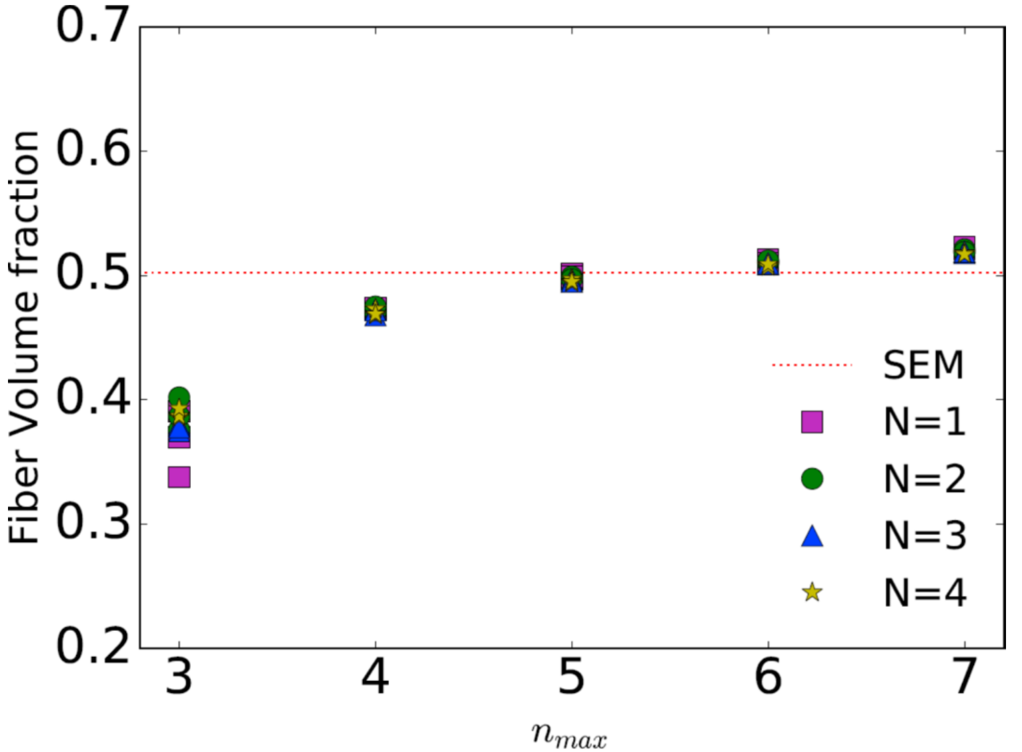


Micro-structure stochastic model

- The numerical micro-structure is generated by a fiber additive process
  - The effect of the initial number of seeds N and
  - The effect of the maximum regenerating times $n_{\max}$ after rejecting a fiber due to overlap

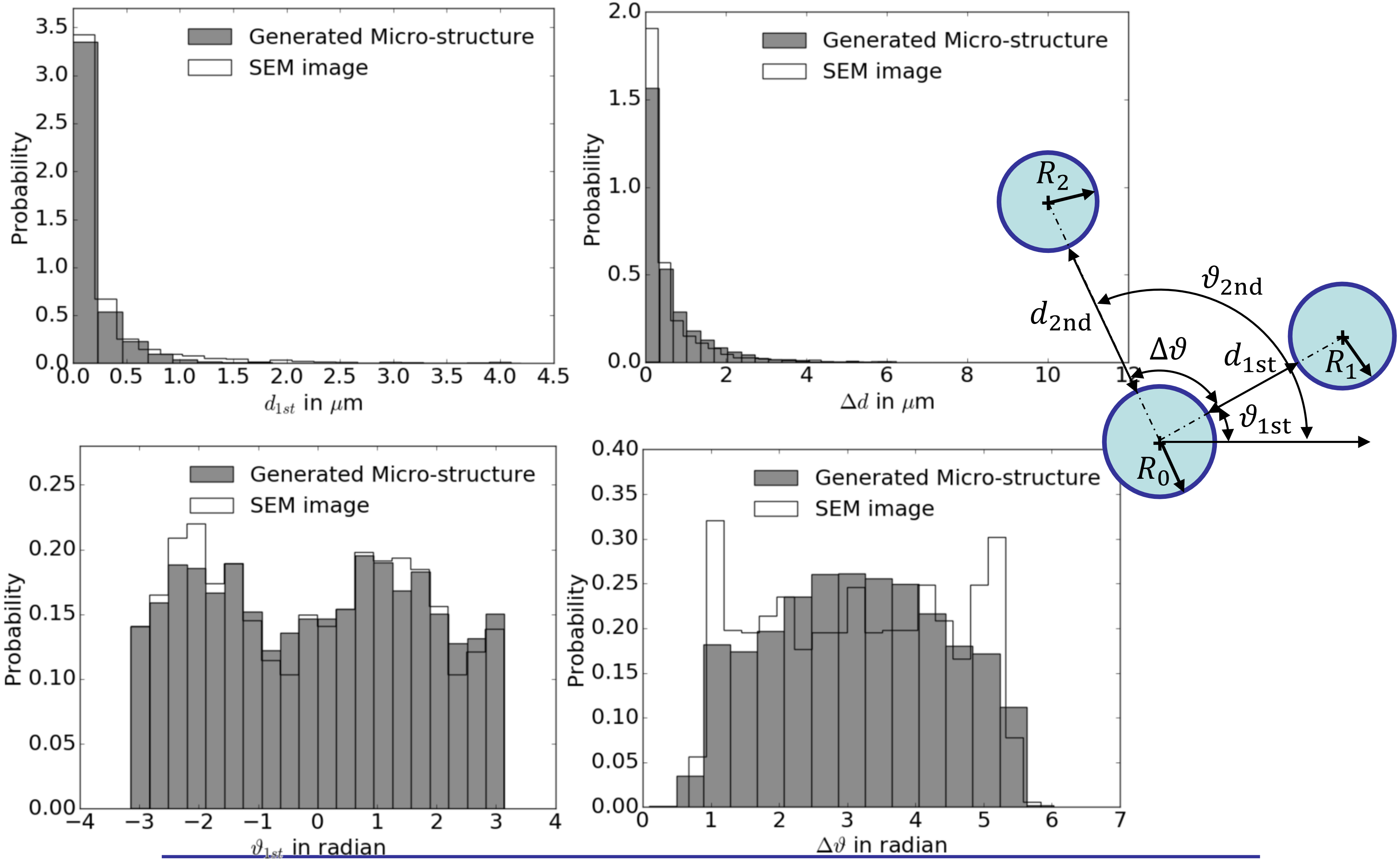
SEM: Average V_f of 103 windows;

Numerical micro-structures: Average V_f of 104 windows.



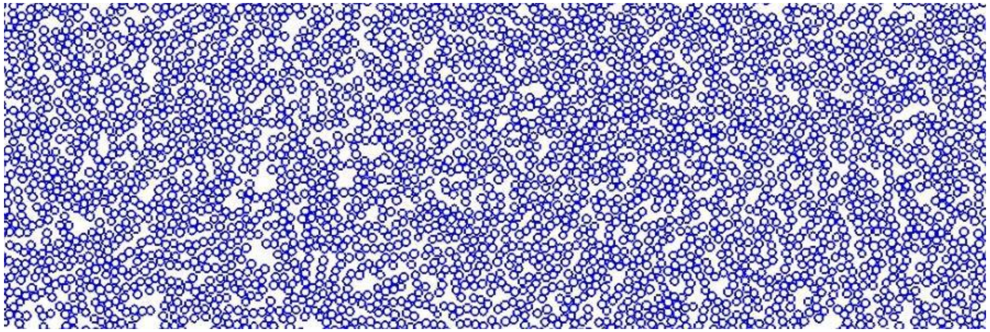
Micro-structure stochastic model

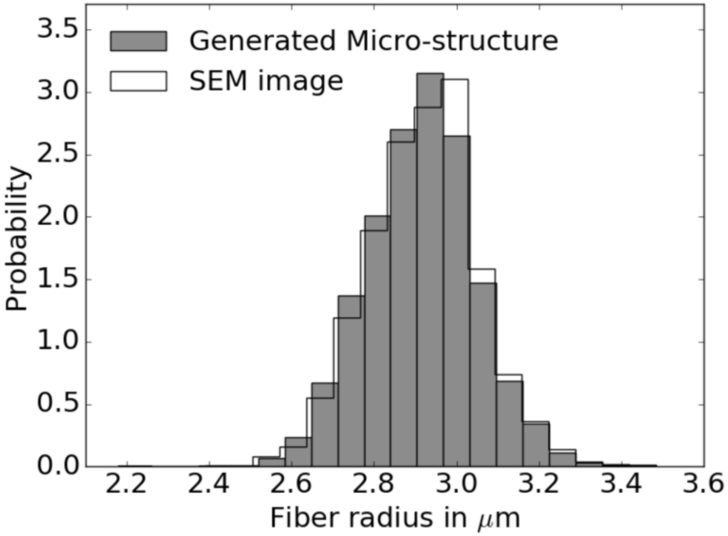
- Comparisons of fibers spatial information



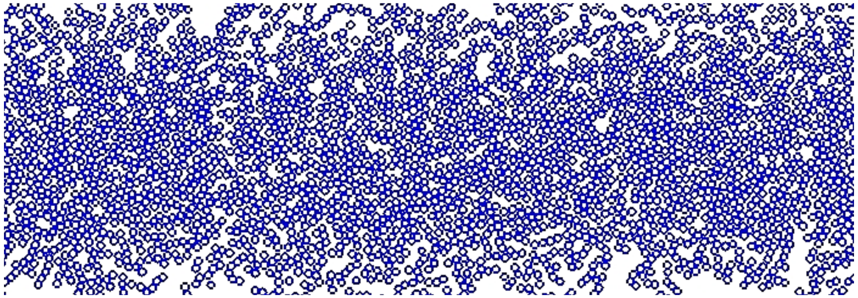
Micro-structure stochastic model

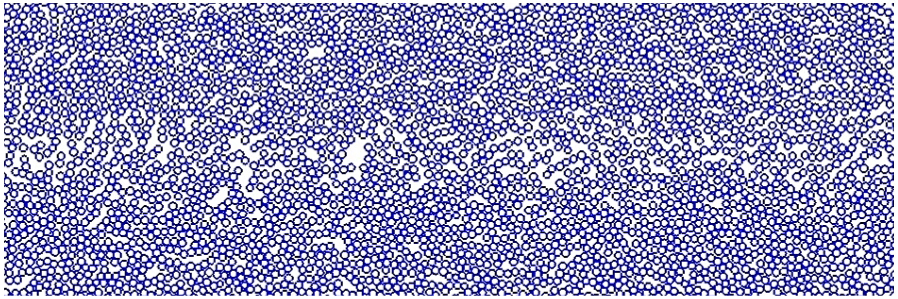
- Numerical micro-structures are generated by a fiber additive process
  - Arbitrary size
  - Arbitrary number





  - Possibility to generate non-homogenous distributions





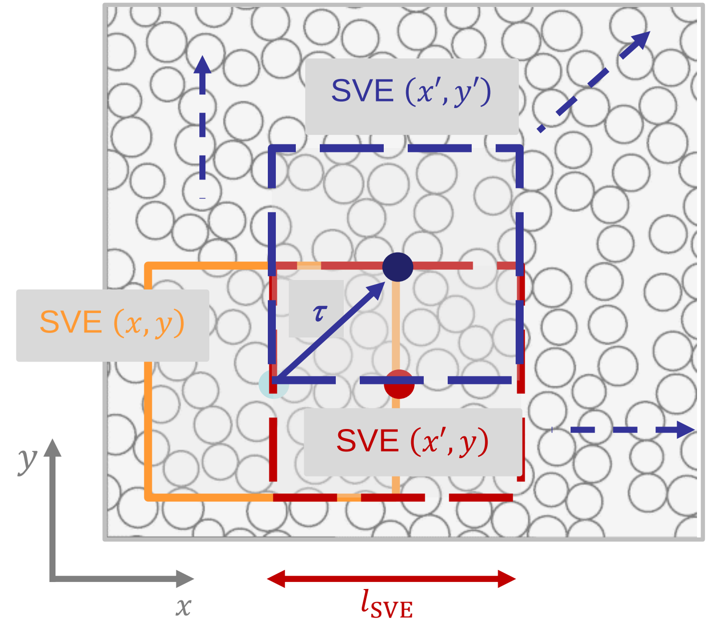
- Stochastic homogenization
  - Extraction of Stochastic Volume Elements
    - 2 sizes considered: $l_{\mathrm{SVE}} = 10\ \mu m$ & $l_{\mathrm{SVE}} = 25\ \mu m$
    - Window technique to capture correlation

$$R_{\mathbf{rs}}(\boldsymbol{\tau}) = \frac{\mathbb{E}\left[\left(r(\boldsymbol{x}) - \mathbb{E}(r)\right)\left(s(\boldsymbol{x}+\boldsymbol{\tau}) - \mathbb{E}(s)\right)\right]}{\sqrt{\mathbb{E}\left[\left(r - \mathbb{E}(r)\right)^2\right]}\sqrt{\mathbb{E}\left[\left(s - \mathbb{E}(s)\right)^2\right]}}$$

  - For each SVE
    - Extract apparent homogenized material tensor $\mathbb{C}_{\mathrm{M}}$

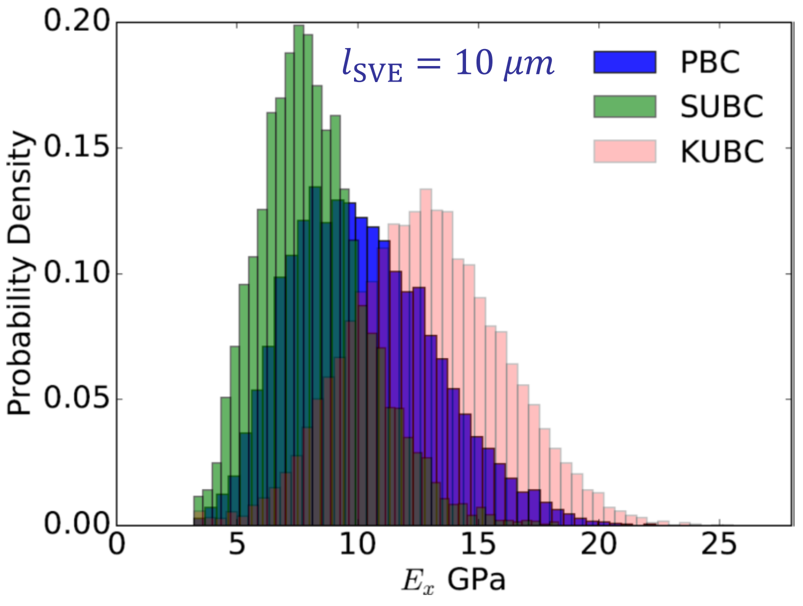
$$\begin{cases} \boldsymbol{\varepsilon}_{\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \boldsymbol{\varepsilon}_{\mathrm{m}} d\omega \\ \boldsymbol{\sigma}_{\mathrm{M}} = \dfrac{1}{V(\omega)}\displaystyle\int_{\omega} \boldsymbol{\sigma}_{\mathrm{m}} d\omega \\ \mathbb{C}_{\mathrm{M}} = \dfrac{\partial \boldsymbol{\sigma}_{\mathrm{M}}}{\partial \boldsymbol{u}_{\mathrm{M}} \otimes \boldsymbol{\nabla}_{\mathrm{M}}} \end{cases}$$

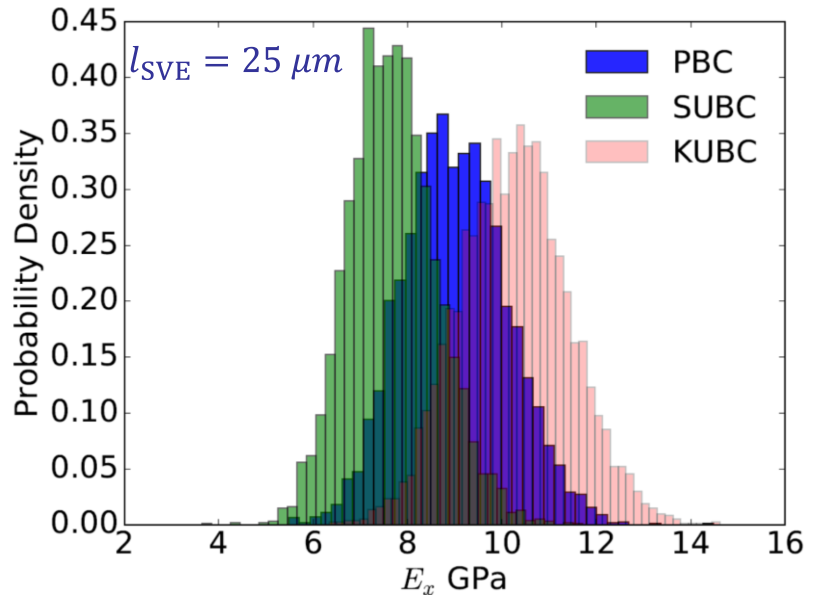
    - Consistent boundary conditions:
      - Periodic (PBC)
      - Minimum kinematics (SUBC)
      - Kinematic (KUBC)



Stochastic homogenization on the SVEs

- Apparent properties





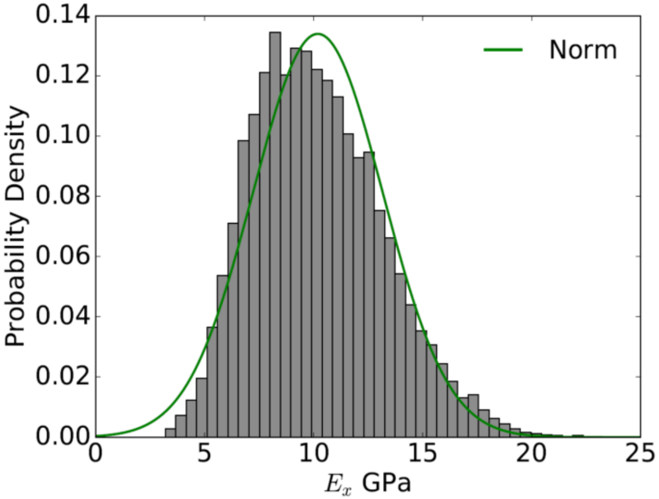
Increasing l_{SVE}

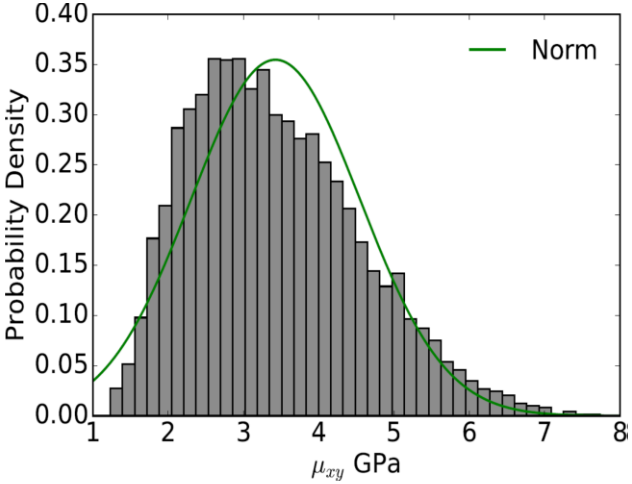
When l_{SVE} increases

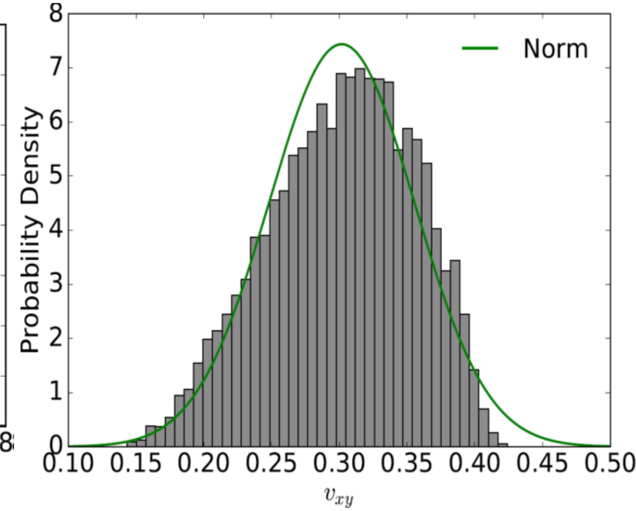
- Average values for different BCs get closer (to PBC one)
- Distributions narrow
- Distributions get closer to normal

Stochastic homogenization on the SVEs

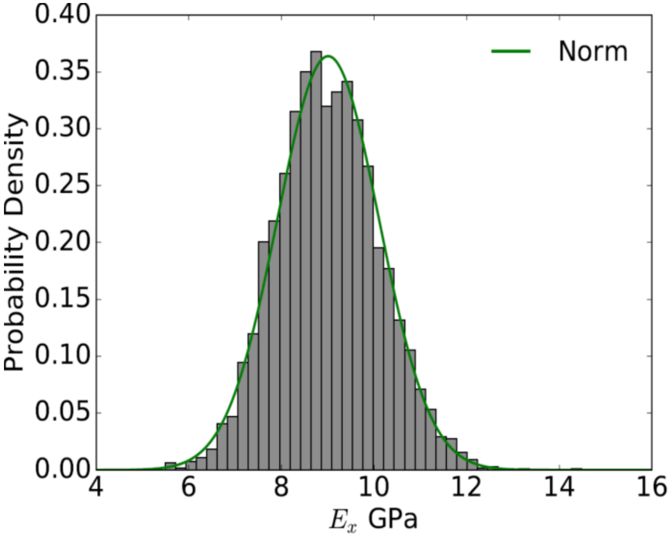
- When l_{SVE} increases: marginal distributions of random properties closer to normal
  - l_{SVE} = 10 μm

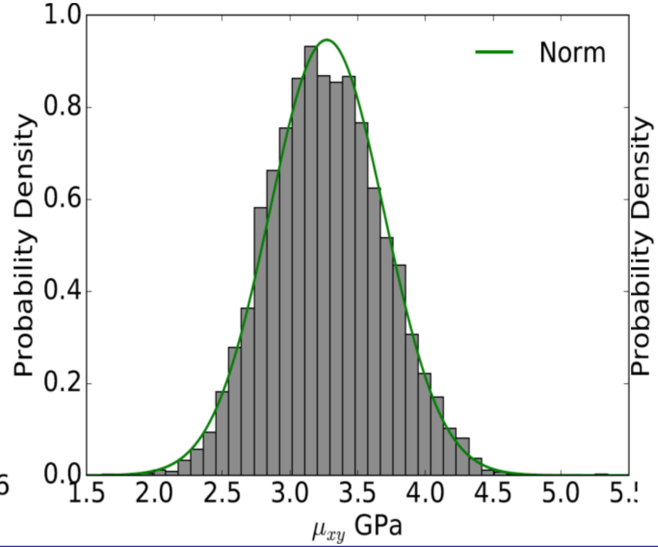


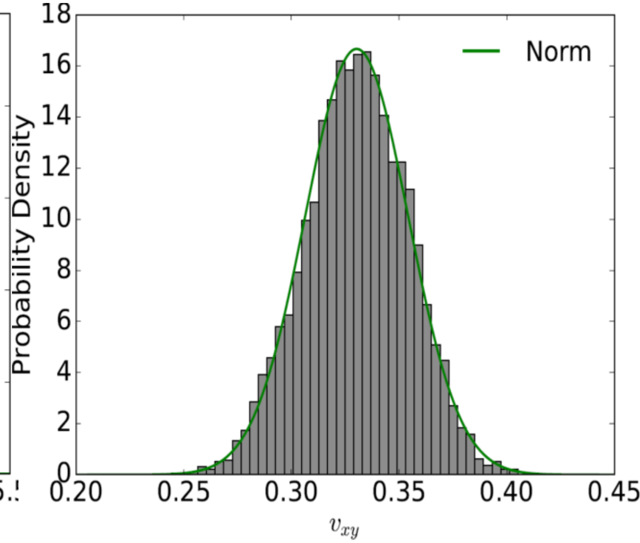




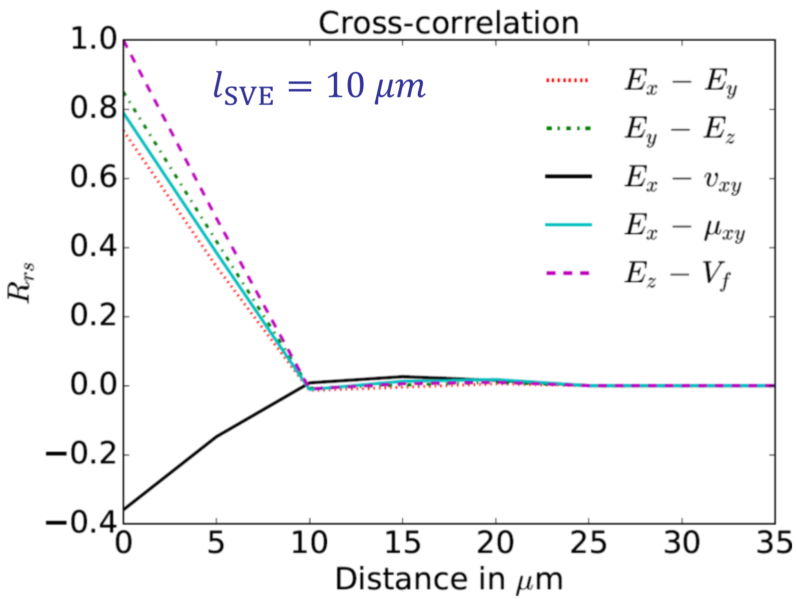
  - l_{SVE} = 25 μm

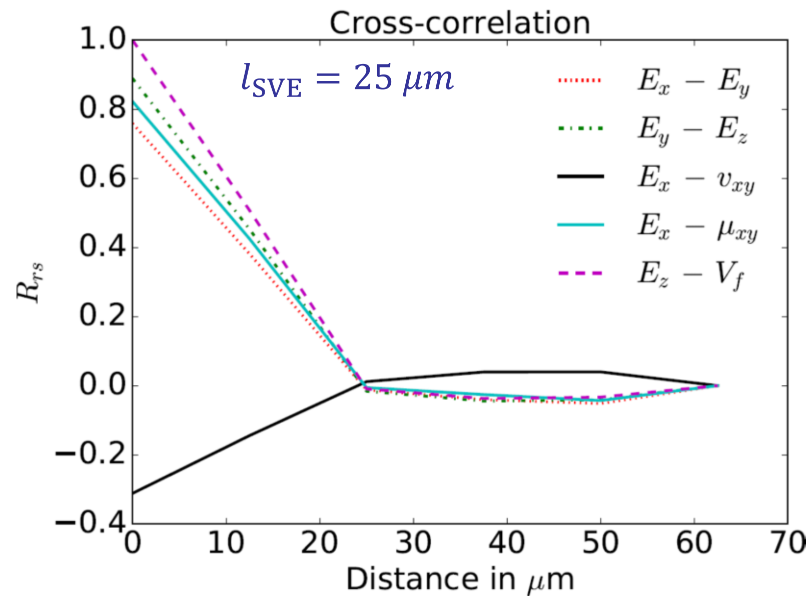






- Correlation





Increasing l_{SVE}

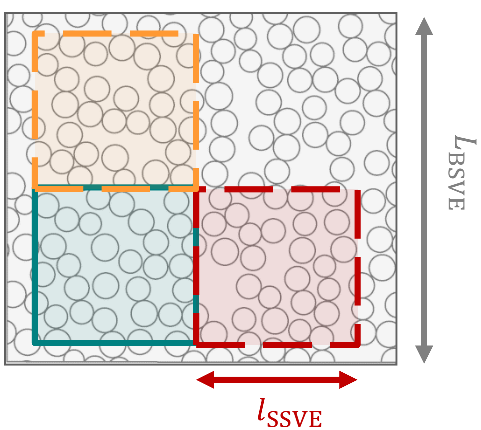
(1) Auto/cross correlation vanishes at $\tau = l_{\mathrm{SVE}}$
(2) When l_{SVE} increases, distributions get closer to normal

(1)+(2) Apparent properties are independent random variables
However the distribution depend on

- l_{SVE}
- The boundary conditions

Stochastic homogenization on the SVEs

- Quid larger SVEs?
  - Computational cost affordable in linear elasticity
  - Computational cost non affordable in failure analyzes
- How to deduce the stochastic content of larger SVEs?
  - Take advantages of the fact that the apparent tensors can be considered as random variables



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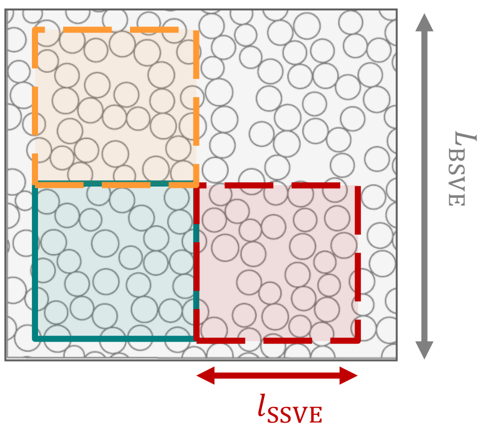
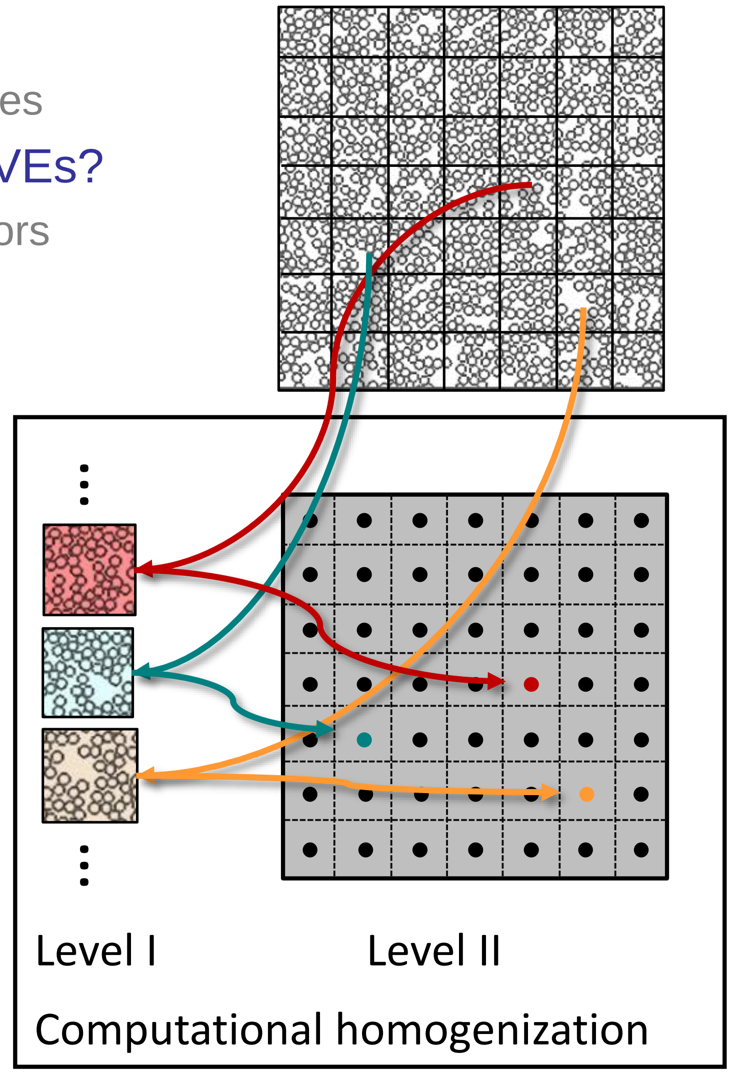
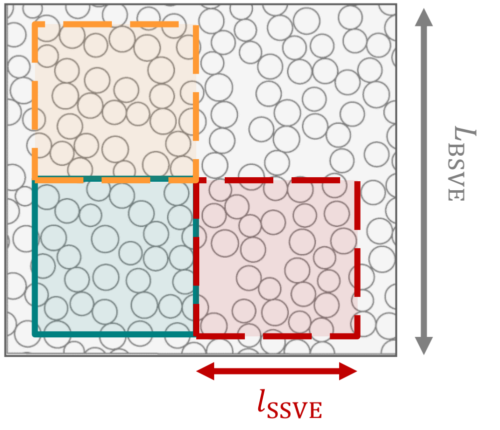
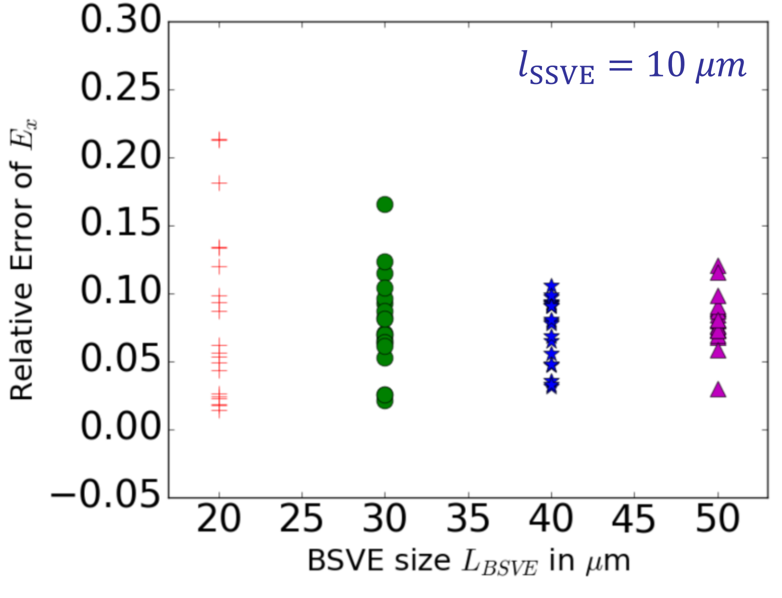


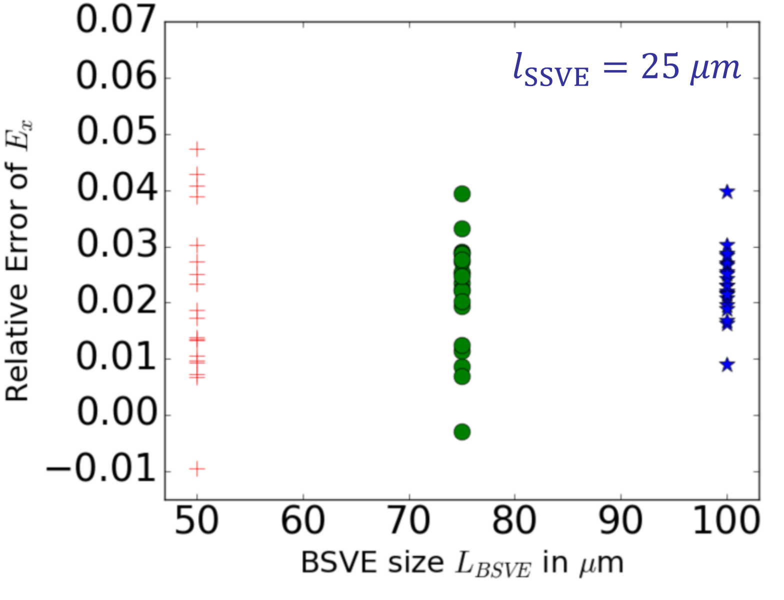


Stochastic homogenization on the SVEs

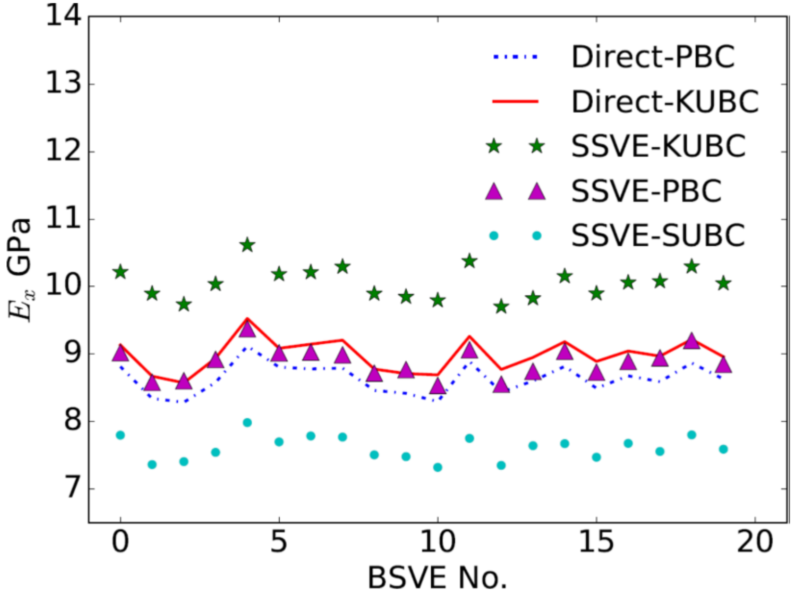
- Quid larger SVEs?
  - Computational cost affordable in linear elasticity
  - Computational cost non affordable in failure analyzes
- How to deduce the stochastic content of larger SVEs?
  - Take advantages of the fact that the apparent tensors can be considered as random variables
  - Accuracy depends on Small/Large SVE sizes

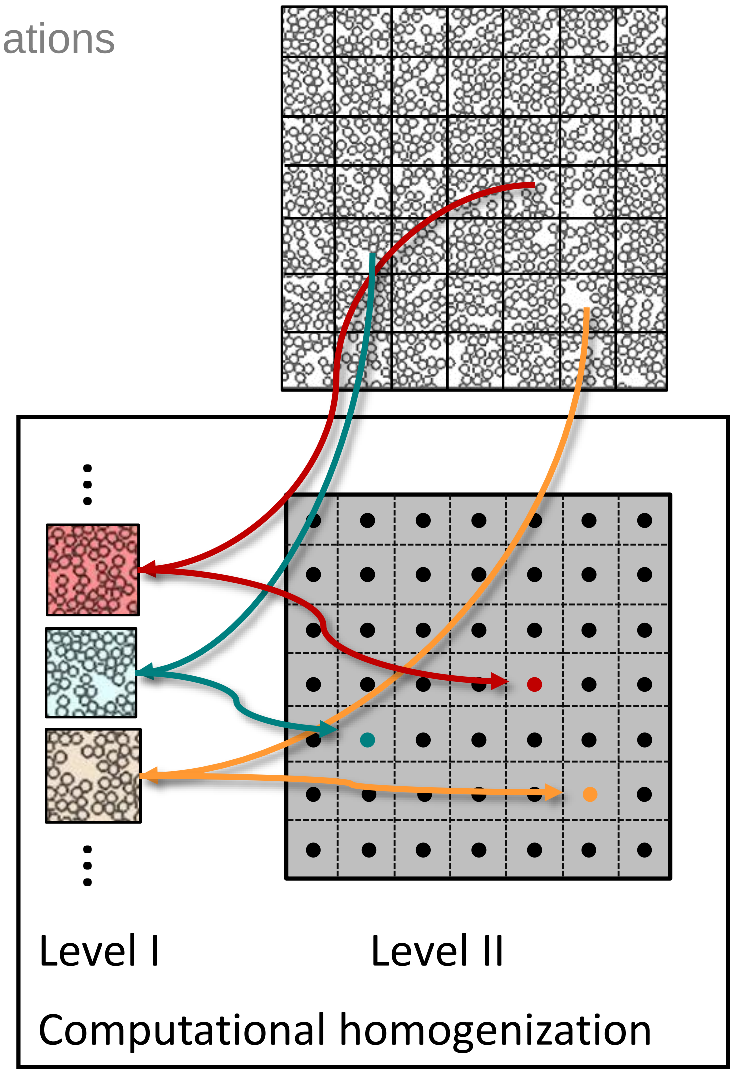






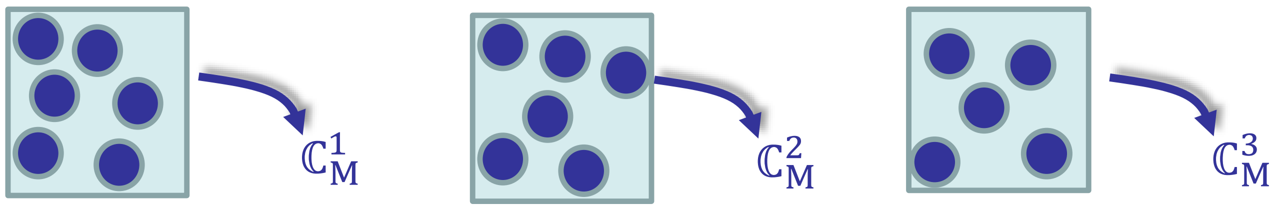
- Numerical verification of 2-step homogenization
  - Direct homogenization of larger SVE (BSVE) realizations
  - 2-step homogenization using BSVE subdivisions



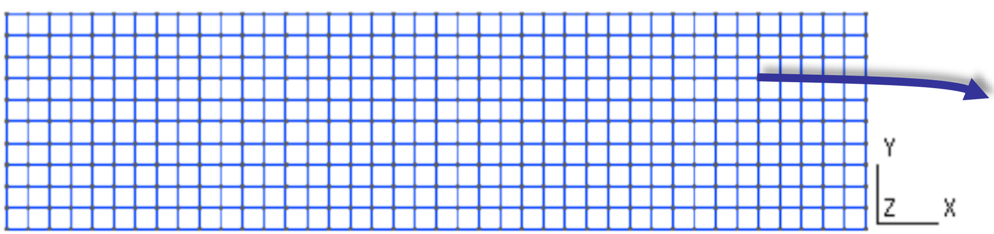


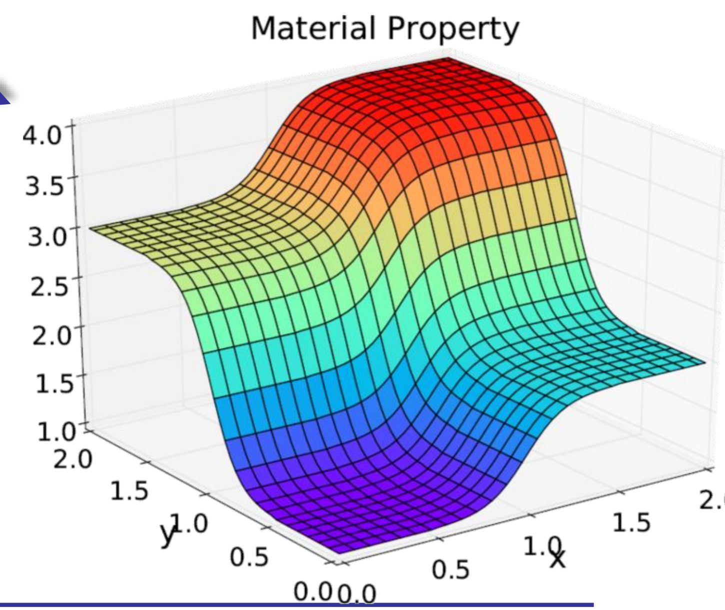
Stochastic reduced order model

- Stochastic model of the anisotropic elasticity tensor
  - Extract (uncorrelated) tensor realizations $\mathbb{C}_{\mathrm{M}}^{i}$

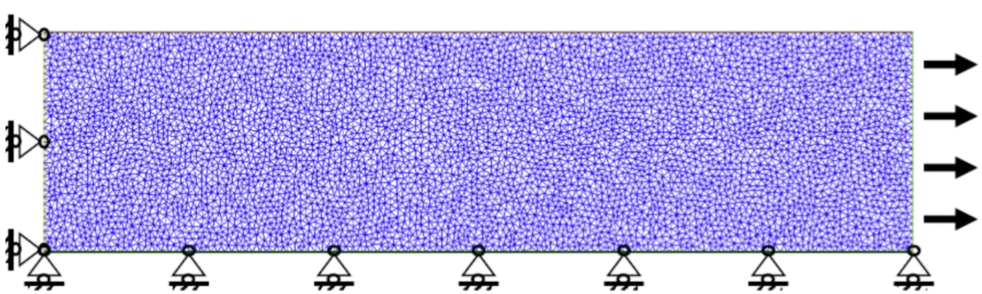


  - Represent each realization $\mathbb{C}_{\mathrm{M}}^{i}$ by a vector $\boldsymbol{\mathcal{V}}$ of 9 (dependant) $\boldsymbol{\mathcal{V}}^{(r)}$ variables
  - Generate random vectors $\boldsymbol{\mathcal{V}}$ using the Copula method
- Simulations require two discretizations
  - Random vector discretization



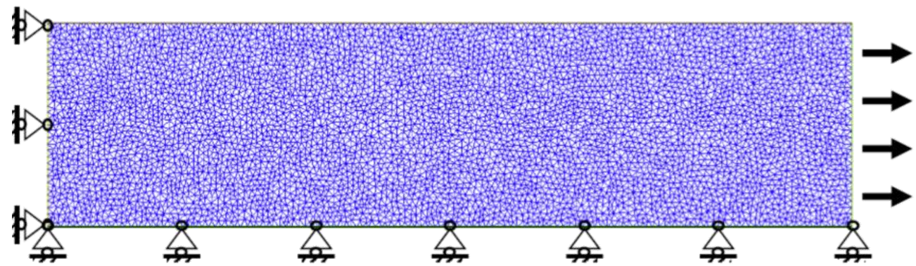


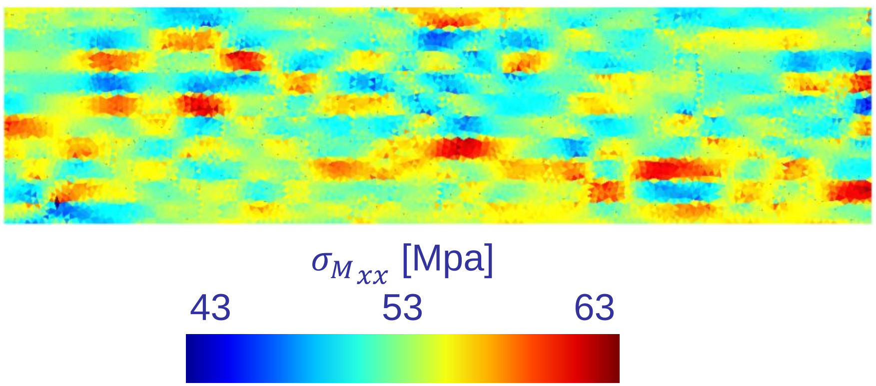
  - Finite element discretization

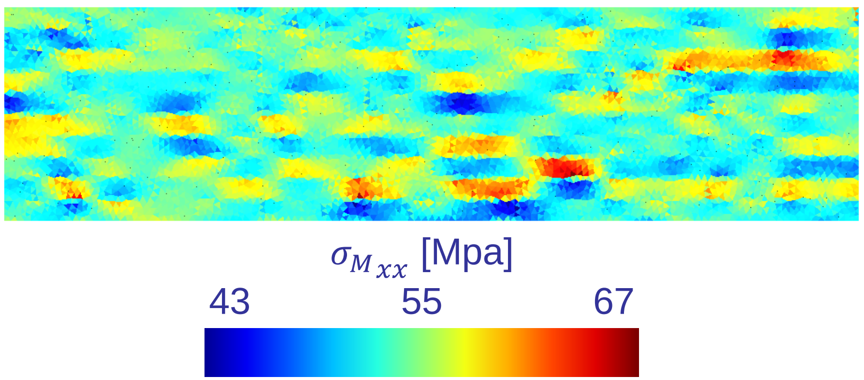


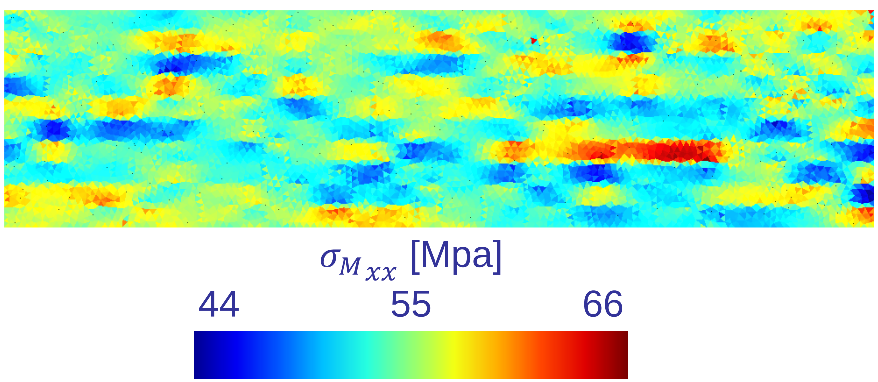
Stochastic reduced order model

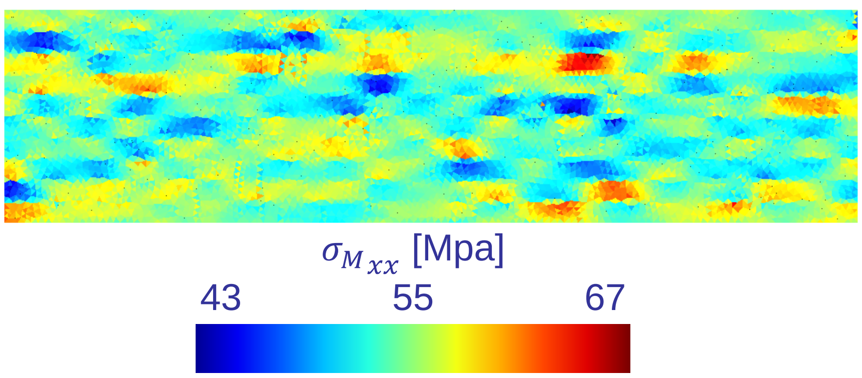
- Ply loading realizations
  - Non-uniform homogenized stress distributions
  - Different realizations yield different solutions









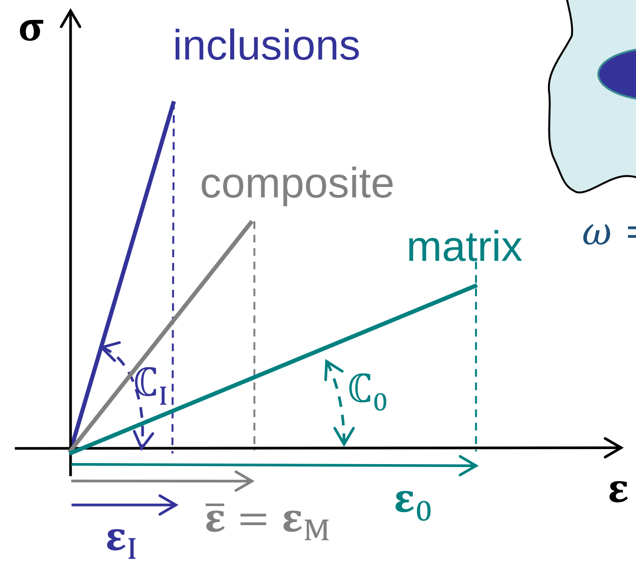


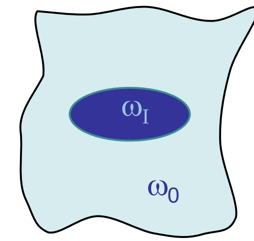
Stochastic Mean-Field Homogenization

- Mean-Field-homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$$\Rightarrow \quad \mathbb{C}_{\mathrm{M}} = \mathbb{C}_{\mathrm{M}}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}})$$





$\omega = \cup_i \, \omega_i$

  - We use Mori-Tanaka assumption for $\mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}})$
- Stochastic MFH
  - How to define random vectors $\boldsymbol{\mathcal{V}}_{\mathrm{MT}}$ of $\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}}$?

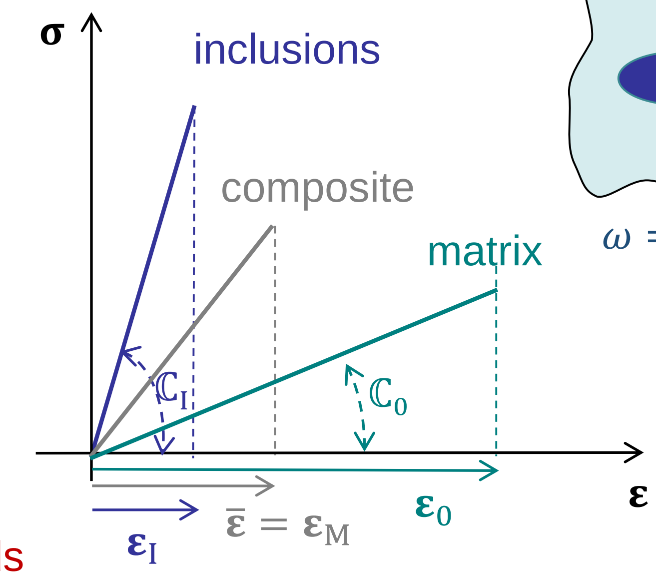
Stochastic Mean-Field Homogenization

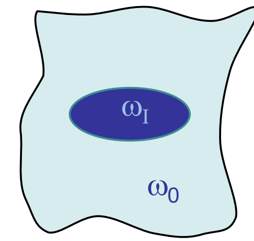
- Mean-Field-homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_\mathrm{M} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \boldsymbol{\varepsilon}_\mathrm{M} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \\ \boldsymbol{\varepsilon}_\mathrm{I} = \mathbf{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_\mathrm{I}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$$\Rightarrow \mathbb{C}_\mathrm{M} = \mathbb{C}_\mathrm{M}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_\mathrm{I}, v_\mathrm{I})$$

Defined as random fields

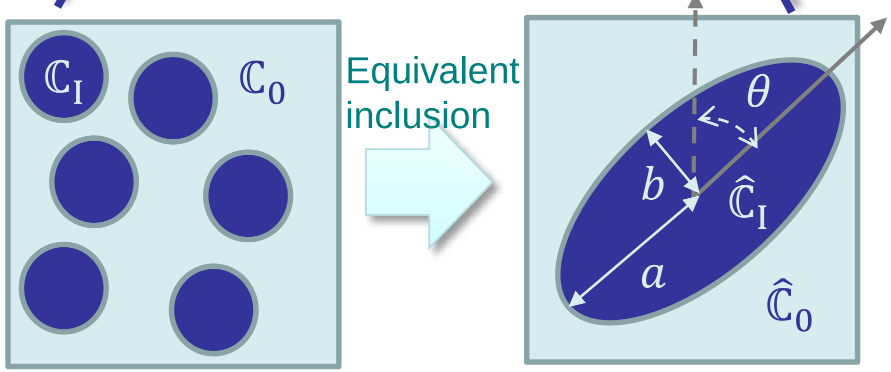




$\omega = \cup_i \, \omega_i$

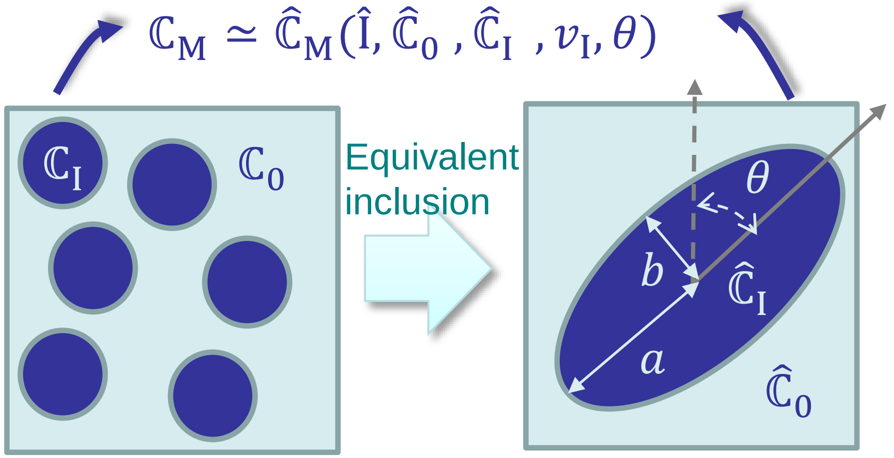
- Consider an equivalent system
  - For each SVE realization i:
    - $\Rightarrow$ $\mathbb{C}_\mathrm{M}$ and v_I known
  - Anisotropy from $\mathbb{C}_\mathrm{M}^i$
    - $\Rightarrow$ θ is evaluated
  - Fibre behaviour uniform
    - $\Rightarrow$ $\hat{\mathbb{C}}_\mathrm{I}$ for one SVE
  - Remaining optimization problem:

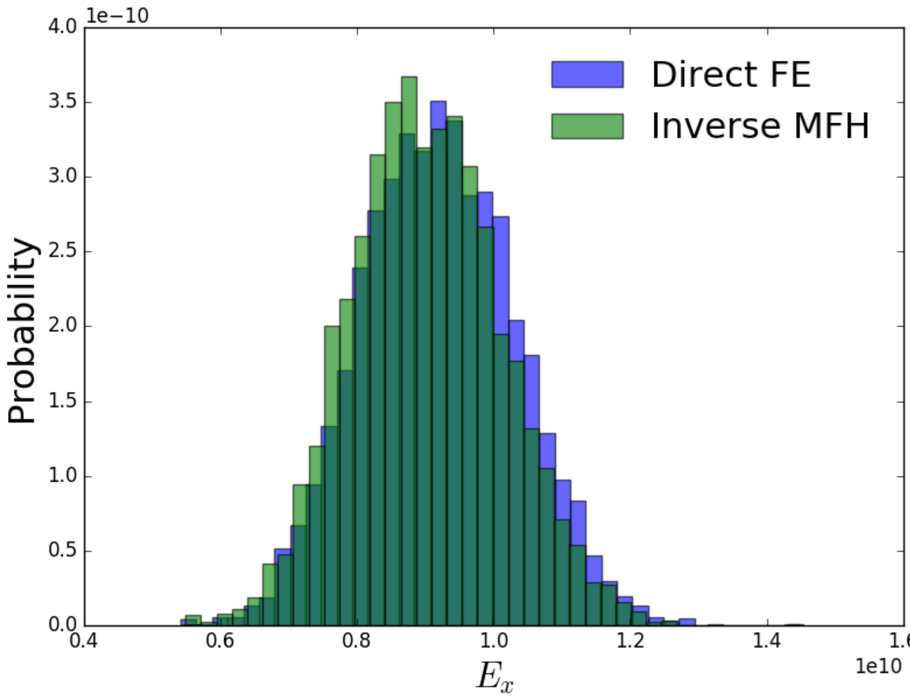
$$\mathbb{C}_\mathrm{M} \simeq \hat{\mathbb{C}}_\mathrm{M}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_0, \hat{\mathbb{C}}_\mathrm{I}, v_\mathrm{I}, \theta)$$

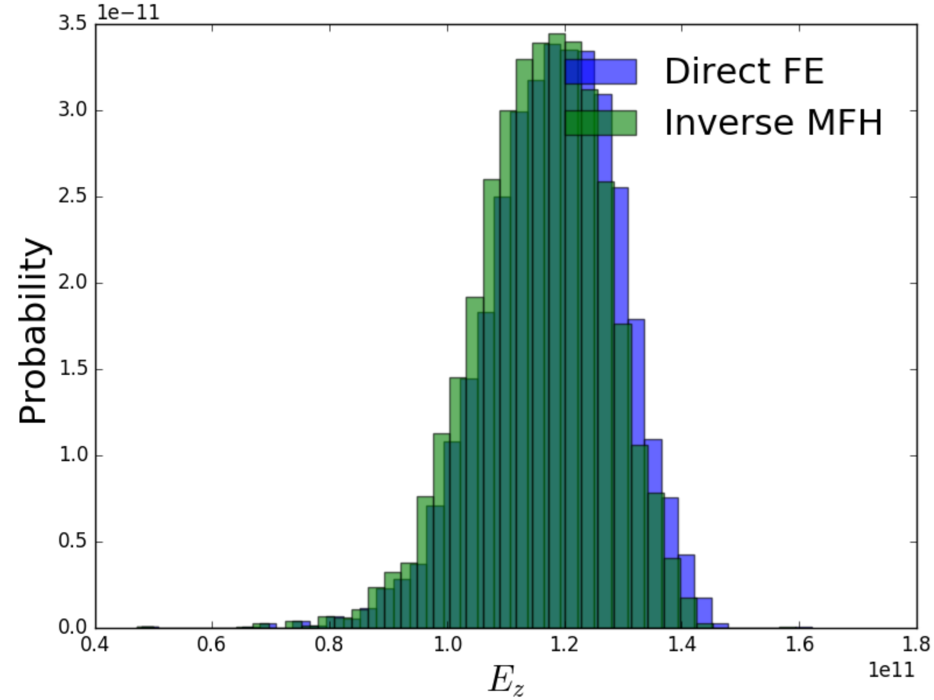


$$\min_{\frac{a}{b}, \hat{E}_0, \hat{\nu}_0} \left\| \mathbb{C}_\mathrm{M} - \hat{\mathbb{C}}_\mathrm{M}\left(\frac{a}{b}, \hat{E}_0, \hat{\nu}_0; v_\mathrm{I}, \theta, \hat{\mathbb{C}}_\mathrm{I}\right) \right\|$$

- Inverse stochastic identification
  - Comparison of homogenized properties from SVE realizations and stochastic MFH

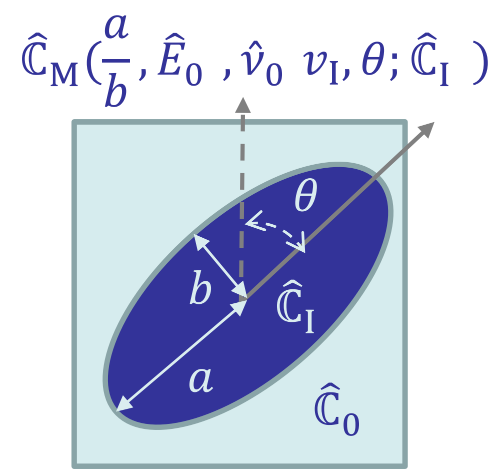






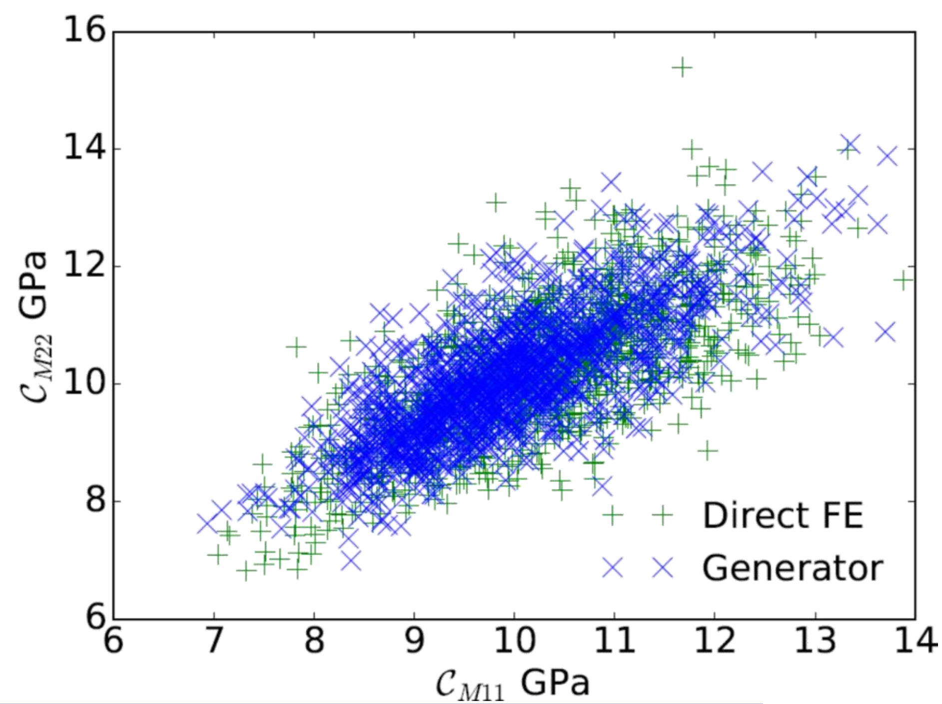
Stochastic Mean-Field Homogenization

- Stochastic MFH model:
  - Homogenized properties $\hat{\mathbb{C}}_{\mathrm{M}}(\frac{a}{b}, \hat{E}_0\ , \hat{\nu}_0\ v_{\mathrm{I}}, \theta; \hat{\mathbb{C}}_{\mathrm{I}}\ )$
  - Random vectors $\boldsymbol{\mathcal{V}}_{\mathrm{MT}}$
    - Realizations $\mathrm{v}_{\mathrm{MT}} = \left\{\frac{a}{b}, \hat{E}_0\ , \hat{\nu}_0\ v_{\mathrm{I}}, \theta\right\}$
    - Characterized by the distance correlation matrix
    - Generator using the copula method

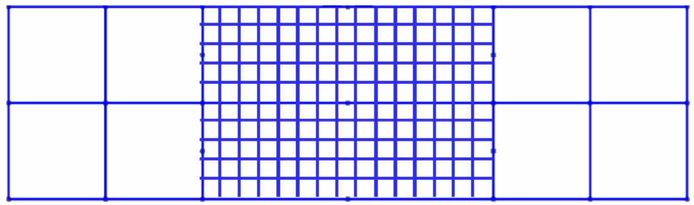


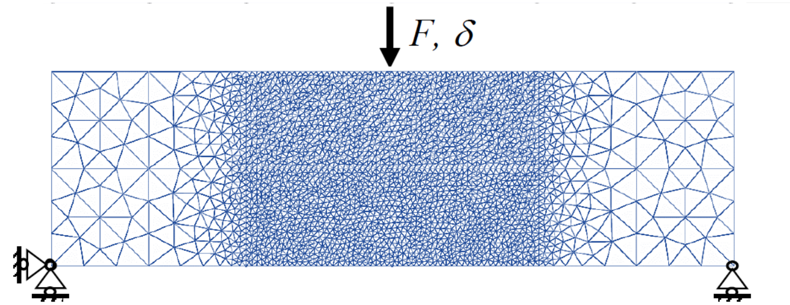
	v_{I}	θ	$\frac{a}{b}$	$\hat{E}_0$	$\hat{\nu}_0$
v_{I}	1.0	0.015	0.114	0.523	0.499
θ		1.0	0.092	0.016	0.014
$\frac{a}{b}$			1.0	0.080	0.076
$\hat{E}_0$				1.0	0.661
$\hat{\nu}_0$					

Distances correlation matrix

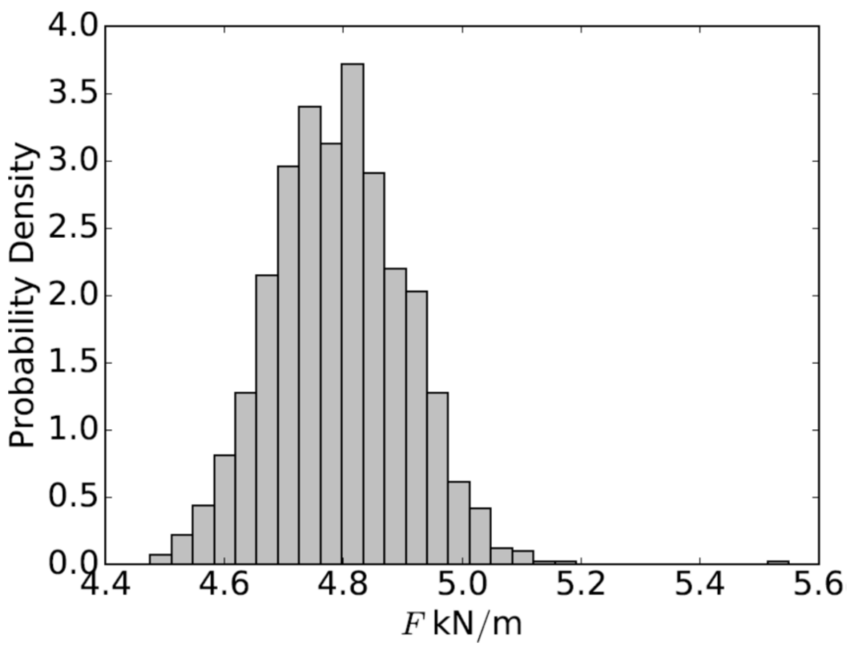


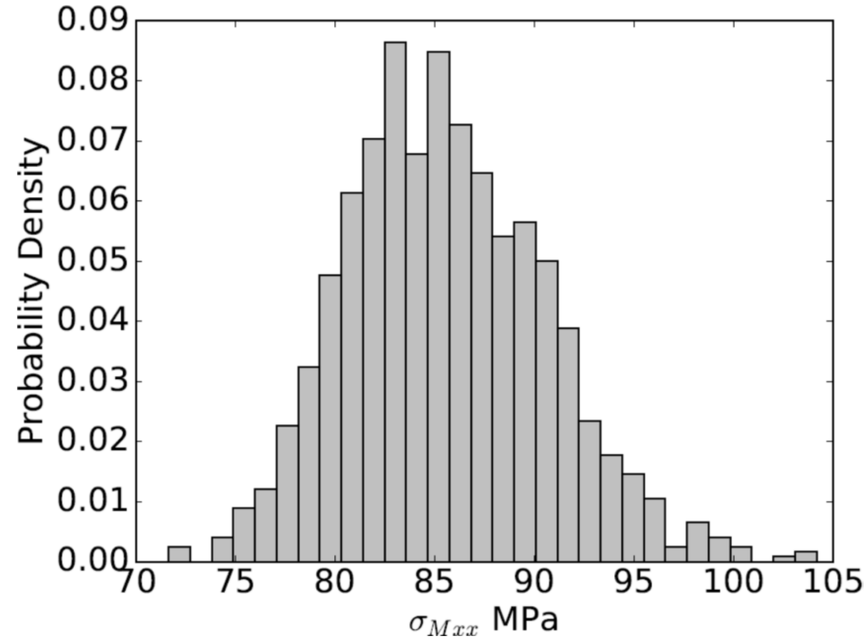
- Stochastic simulations
  - 2 discretization: Random field $\boldsymbol{\nu}_{\mathrm{MT}}$ & Stochastic finite-elements





  - Realizations to reach a given deflection $\boldsymbol{\delta}$





- Stochastic generator based on SEM measurements of unidirectional fibre reinforced composites
- Computational homogenization on SVEs
- Two-step computational homogenization for Big SVEs
- Definition of a Stochastic MFH method
- In progress: nonlinear and failure analyzes

Thank you for your attention !