NUMERICAL ERROR ANALYSIS OF CSL GEOREFERENCING PROCESSING IMPLEMENTATION

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ABSTRACT

We present the specific georeferencing and geoprojection process we have implemented into the CSL interferometric SAR processor.

The paper addresses the geometric aspects for georeferencing a point in a SAR image given in azimuth-slant range coordinates, and describes the implementation. Conversely, we present geometrical aspects of back projection, i.e. from a georeferenced position toward azimuth-slant range coordinate system, and the way it is implemented.

In order to evaluate numerical errors of both implementations, we performed back-and-forth projections of a given point to determine shifts between initial and final coordinates. Numerical errors are estimated with respect to azimuth - slant range position, Doppler centroïd and altitude of the point.

This procedure give information on the reliability and precision of the geoprojection process and implementation.

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1. INTRODUCTION

A cooperation program named ORFEO (Optic & Radar Federated Earth Observation) was set between Italy and France to develop an Earth observation dual system, optic and radar, with metric resolution. Italy is in charge of the radar component through Cosmo-SkyMed and France of the optic one (PLEIADES). Beside ORFEO, an accompanying program was set-up to prepare the use and exploitation of images that will be provided from this satellites constellation. This preparatory program is required because of the new capabilities and performances that will be offered by the ORFEO system, which imply new methodological developments in close cooperation with the final thematic user. In the frame of this ORFEO preparatory program, the Belgian Scientific Policy is financing several PhD researches among which the GEMITOR project (Multimodal Georeferencing of 3D VHR Optical and X-Band SAR Image). This project aims at using the 3D information issued from VHR SAR interferometry to geoproject and fuse both imagery modes on a common geographical system [1].

Since working at Very High Resolution, precision of georeferencing process is of prime importance to keep the metric resolution offered by the SAR sensor in geoprojected products.

This paper presents the implementation of the georeferencing process and the procedure followed to evaluate its numerical precision.

2. BACKGROUND

Our interferometric processor has been successfully tested on simulated CosmoSkymed data. We thus consider here we are starting from 3D information given in azimuth – slant range coordinate system.

2.1 Definitions

- Georeferencing: We call georeferencing or geocoding the operation consisting in referencing (i.e. locating) a point in longitude – latitude coordinate on a given datum.
- Geoprojection: Geoprojection, also called geocorrection, consist in interpolating and resampling the considered SAR product on a regular grid on the reference datum.
- Longitude and latitude: The geodetic latitude ϕ of a point above the ellipsoidal Earth is the angle between the normal to the ellipsoid passing through the point and the equatorial plane. The geodetic latitude λ of a point above the ellipsoid is the angle between the Greenwich meridian plane and the meridian plane passing through the point (fig. 1).

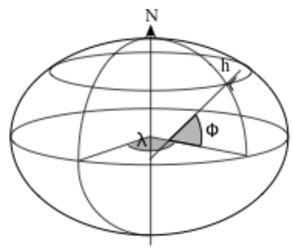

Figure 1. Geodetic coordinates

- Geodetic height: The geodetic height is the altitude of a point above the geoïd, measured perpendicularly (fig. 2). It is the distance between the considered point P above the Earth and its orthogonal projection Pg onto the ellipsoid.

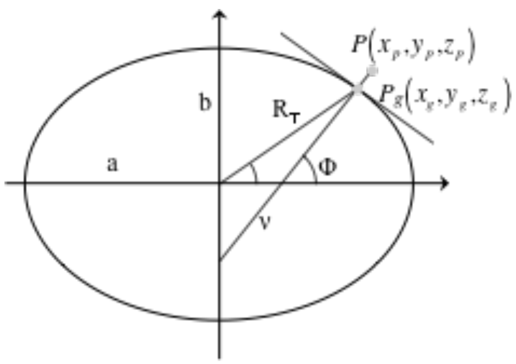

Figure 2. Geodetic height

Based on figures 1 and 2, we easily can deduce expressions relating the coordinates of a given point $P = (x_p, y_p, z_p)$ to its geographical coordinates and its geodetic height [2]:

$$x_p = (\nu + h)\cos\phi \ \cos\lambda$$
$$y_p = (\nu + h)\cos\phi \ \sin\lambda \qquad (1)$$
$$z_p = \left(\nu \frac{b^2}{a^2} + h\right)\sin\phi = \left(\nu\left(1 - e^2\right) + h\right)\sin\phi$$

where ν is the meridional curvature radius and e the excentricity:

$$\nu = \frac{a}{\sqrt{1 - e^2 \sin^2\phi}}$$
$$e^2 = \frac{a^2 - b^2}{a^2} \qquad (2)$$

2.2 Georeferencing

We describe here the procedure used to reference a point P given in slant range – azimuth coordinate and for which we know its altitude h. This latter information can come from interferometry or from an external DEM. We consider here that this altitude information is known for any considered point in the slant range – azimuth coordinate system, whatever the origin of the DEM.

2.2.1 Localisation in the geocentric cartesian reference system

The first step before geroreferencing the point in longitude – latitude coordinates consist in finding its Cartesian coordinates (x_p, y_p, z_p) in the geocentric coordinate system.

This first step can be explained on basis on the general viewing geometry of SAR system (fig. 3).

Let $\vec{S}$ be the position vector of the satellite, $\vec{R}$ the vector to the considered point P and $\vec{z_1}$ the range vector between the satellite position and the considered point P.

$\left\|\vec{z_1}\right\|$ is simply the range coordinate of the considered point. $\vec{z_1}$ lays in the observation Doppler plane.

$\vec{S}$ is obtained from the orbitography of the satellite. Interpolation of the state vectors allows calculating $\vec{S}$ for any azimuth line in the focused SAR image.

$\left\|\vec{R}\right\|$ is obtained either from interferometric processing or from the external DEM back projected to slant range – azimuth coordinate system. It is the sum of the local terrestrial radius and the local ellipsoidal height: $\left\|\vec{R}\right\| = R_T + h$

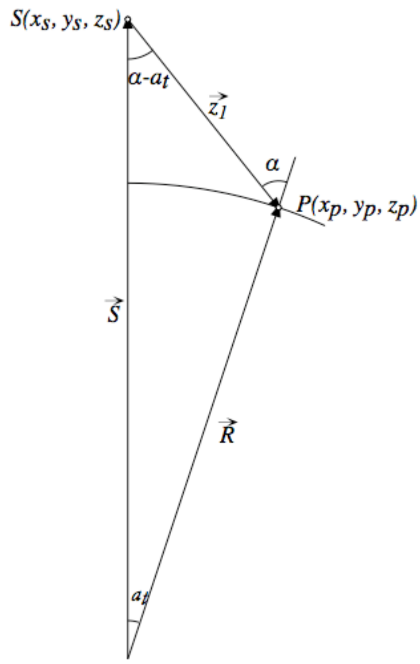

Figure 3. SAR viewing geometry

To calculate the Cartesian coordinates of the point P, we need three independent equations. These are chosen from the following geometrical properties:

The point P and the vector $\vec{z_1}$ belong to the Doppler plane Π.

$$\Pi \equiv a\,x_p + b\,y_p + c\,z_p = r \qquad (3)$$

where a, b, and c are the director cosines of the plane and r the distance between the plane and the origin. As we shall see after, this plane is given by the Doppler equation and its director cosines will be obtained from the relative speed of the sensor.

The point P belongs to a sphere of radius $R = \left\|\vec{R}\right\|$. R being the Earth local radius R_T added to the local geocentric height. This latter one is obtained through interferometric processing.

$$x_p^2 + y_p^2 + z_p^2 = R^2 = S^2 + z_1^2 - 2Sz_1 \cos(\alpha - a_t) \quad (4)$$

The point P belongs to a sphere centered on the satellite position and having a radius equal to the range position.

$$\vec{S} - \vec{R} = \vec{z}_1 \qquad (5)$$

Expanding eq.5 leads to:

$$x_p^2 + y_p^2 + z_p^2 - 2x_s x_p - 2y_s y_p - 2z_s z_p + x_s^2 + y_s^2 + z_s^2 = z_1^2 \qquad (6)$$

Eq.6 may be rewritten in a more convenient manner:

$$x_s x_p + y_s y_p + z_s z_p = \frac{S^2 + R^2 - z_1^2}{2} \qquad (7)$$

Our interferometer implementation calculates height above a spherical Earth. That is the reason why the second equation is a sphere and not the used datum.

2.2.2 The Doppler plane

The first equation of our system (eq. 3) is the equation of the Doppler plane. The squinting of this plane depends on the Doppler frequency around which the azimuth focusing is performed. The equation of the Doppler plane is given by the Doppler centroid equation [3]:

$$f_{DC} = -\frac{2}{\lambda z_1}\left(\overrightarrow{v_s} - \overrightarrow{v_p}\right) \bullet \left(\vec{S} - \vec{R}\right) \qquad (8)$$

where:

f_{DC} is the Doppler centroid frequency,

$\overrightarrow{v_s} = \begin{pmatrix} v_{sx} \\ v_{sy} \\ v_{sz} \end{pmatrix}$ is the speed vector of the satellite,

$\overrightarrow{v_p} = \begin{pmatrix} v_{px} \\ v_{py} \\ v_{pz} \end{pmatrix}$ is the speed vector of the observed point.

For fixed points, the speed of the observed point is the one induced by Earth rotation:

$$\overrightarrow{v_p} = \vec{\omega} \times \vec{R} = \begin{pmatrix} -\omega\, y_p \\ \omega\, x_p \\ 0 \end{pmatrix} \qquad (9)$$

$\vec{\omega}$ being the Earth angular speed vector.

Consequently, the Doppler equation becomes:

$$\frac{\lambda}{2} f_{DC} z_1 = \begin{pmatrix} -v_{sx} - \omega\, y_p \\ \omega\, x_p - v_{sy} \\ -v_{sz} \end{pmatrix} \bullet \begin{pmatrix} x_s - x_p \\ y_s - y_p \\ z_s - z_p \end{pmatrix}$$

$$\Rightarrow \left(v_{sx} + \omega\, y_s\right)x_p + \left(v_{sy} - \omega\, x_s\right)y_p + v_{sz} z_p = \frac{\lambda}{2} f_{DC} z_1 + v_{sx} x_s + v_{sy} y_s + v_{sz} z_s \qquad (10)$$

Equation 10 is only valid if focusing is well performed around the Doppler centroid frequency. If azimuth focusing is centered on another frequency, it is that focusing frequency that must be used to correctly define the plane to which belongs the focused point. In the case of ERS SAR images, focusing is performed at zero Doppler. In this case, equation 10 simplifies, canceling the first term of the right member.

2.2.3 Solving

We have to solve a three-equation system with three unknowns to determine the Cartesian coordinates of an observed point in the geocentric reference system:

$$\begin{cases} \left(v_{sx} + \omega\, y_s\right)x + \left(v_{sy} - \omega\, x_s\right)y + v_{sz} z \\ \quad = \frac{\lambda}{2} f_{DC} z_1 + \left(v_{sx} + \omega\, y_s\right)x_s + \left(v_{sy} - \omega\, x_s\right)y_s + v_{sz} z_s \\ x_s x + y_s y + z_s z = \frac{S^2 + R^2 - z_1^2}{2} \\ x^2 + y^2 + z^2 = R^2 = S^2 + z_1^2 - 2Sz_1 \cos(\alpha - a_t) \end{cases} \qquad (11)$$

Simplifying the notations, we obtain:

$$\begin{cases} Ax + By + Cz = K \\ x_s\, x + y_s\, y + z_s\, z = H \\ x^2 + y^2 + z^2 = R^2 \end{cases} \qquad (12)$$

We thus have to find the intersection between two planes and a sphere. This will lead to two solution points.

Let us first calculate the intersection between the two planes:

$$\begin{pmatrix} B & C \\ y_s & z_s \end{pmatrix}\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} K - Ax \\ H - x_s\, x \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} y \\ z \end{pmatrix} = \frac{1}{z_s B - y_s C}\begin{pmatrix} z_s & -C \\ -y_s & B \end{pmatrix}\begin{pmatrix} K - Ax \\ H - x_s\, x \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} y \\ z \end{pmatrix} = \frac{1}{z_s B - y_s C}\begin{pmatrix} \left(x_s C - z_s A\right)x + \left(z_s K - CH\right) \\ \left(y_s A - x_s B\right)x + \left(BH - y_s K\right) \end{pmatrix} \qquad (13)$$

Simplifying notations, we get:

$$\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} A_y x + B_y \\ A_z x + B_z \end{pmatrix} \qquad (14)$$

Replacing y and z in the equation of the sphere leads to the following quadratic equation:

$$\left(1 + A_y^2 + A_z^2\right)x^2 + 2\left(A_y B_y + A_z B_z\right)x + \left(B_y^2 + B_z^2 - R^2\right) = 0 \tag{15}$$

Solving eq. 15 and then eq. 14 leads us to two solutions for the Cartesian coordinates $\left(x_p, y_p, z_p\right)$ of the focused point P in the geocentric reference coordinate system, each on defining a left or right looking range.

To select the one of the solutions of eq. 15 that is meaningful, we must take into account the side toward which is looking the sensor with respect to its speed vector.

Solving our system leads us to two position vectors $\vec{R}$. These position vectors allow to define to slant range vectors $\vec{z_1} = \vec{R} - \vec{S}$, one pointing on the right, the other on the left of the orbit. To determine the side a slant range vector is pointing, we have to analyze the sign of the scalar product of the considered slant range vector $\vec{z_1}$ and the normal to the plane defined by $\vec{S}$ the position vector of the satellite and $\vec{v}$ its speed vector. This normal is defined by the vector product $\vec{v} \times \vec{S}$. Figure 4 hereafter shows the case of a right-side looking sensor, like ERS.

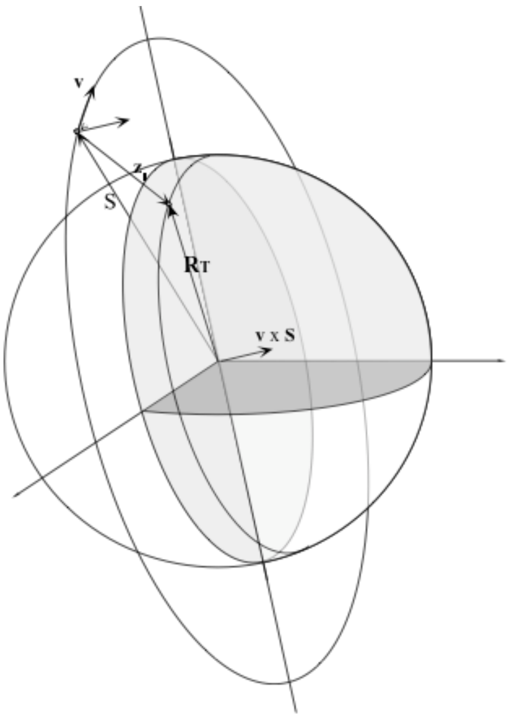

Figure 4. Right-side looking SAR sensor

In that case, the condition to be respected is:

$$\left(\vec{v} \times \vec{S}\right) \bullet \left(\vec{R} - \vec{S}\right) > 0 \tag{16}$$

An equivalent condition to condition (16) is the following:

$$\vec{v} \bullet \left(\vec{S} \times \vec{R}\right) > 0 \tag{17}$$

2.2.4 Projection on the reference ellipsoid

Once having the Cartesian coordinate of an observed point $P = \left(x_p, y_p, z_p\right)$ in the geocentric reference system, we can project it orthogonally on the ellipsoid to calculate its geodetic longitude-latitude coordinates. This projection point $P_g = \left(x_g, y_g, z_g\right)$ is the intersection between the ellipsoid and the normal to it passing through the observed point *P*. The index "g" stands for georeferenced.

The tangential plane to the ellipsoid to the point $P_g = \left(x_g, y_g, z_g\right)$ is given by [4]:

$$\Pi_{tg} \equiv \left(x - x_g\right)\left.\frac{\partial u}{\partial x}\right|_{x_g} + \left(y - y_g\right)\left.\frac{\partial u}{\partial y}\right|_{y_g} + \left(z - z_g\right)\left.\frac{\partial u}{\partial z}\right|_{z_g} = 0 \tag{18}$$

where $u\left(x, y, z\right)$ is the equation of the considered reference ellipsoid.

The normal to the ellipsoid to the point $P_g = \left(x_g, y_g, z_g\right)$ is thus the normal to this plane and is given by:

$$\frac{\left(x - x_g\right)}{\left.\frac{\partial u}{\partial x}\right|_{x_g}} = \frac{\left(y - y_g\right)}{\left.\frac{\partial u}{\partial y}\right|_{y_g}} = \frac{\left(z - z_g\right)}{\left.\frac{\partial u}{\partial z}\right|_{z_g}} \tag{19}$$

This normal must pass trough the point $P = \left(x_p, y_p, z_p\right)$ whose Cartesian coordinates where formerly found. This leads finally to a three-equation system with three unknowns $\left(x_g, y_g, z_g\right)$:

$$y_p - y_g = \frac{\left.\frac{\partial u}{\partial y}\right|_{y_g}}{\left.\frac{\partial u}{\partial x}\right|_{x_g}}\left(x_p - x_g\right)$$

$$z_p - z_g = \frac{\left.\frac{\partial u}{\partial z}\right|_{z_g}}{\left.\frac{\partial u}{\partial x}\right|_{x_g}}\left(x_p - x_g\right) \tag{20}$$

$$u(x_g, y_g, z_g) \equiv \frac{x_g^2 + y_g^2}{a^2} + \frac{z_g^2}{b^2} - 1 = 0$$

Therefore, the equation system we have to solve is:

$$\begin{cases} y_p - y = \dfrac{y}{x}\left(x_p - x\right) \\ z_p - z = \dfrac{a^2}{b^2}\dfrac{z}{x}\left(x_p - x\right) \\ \dfrac{x^2 + y^2}{a^2} + \dfrac{z^2}{b^2} - 1 = 0 \end{cases} \tag{21}$$

After a few manipulations, we get a fourth-order polynomial equation for x:

$$\begin{aligned} & e^4\left(x_p^2 + y_p^2\right) x^4 \\ & -2e^2 x_p\left(x_p^2 + y_p^2\right) x^3 \\ & +x_p^2\left(x_p^2 + y_p^2 + \frac{b^2}{a^2} z_p^2 - a^2 e^4\right) x^2 \\ & +2a^2 e^2 x_p^3\ x \\ & -a^2 x_p^4 = 0 \end{aligned} \tag{22}$$

This equation can only be solved numerically using, for example, a Newton-like method. An efficient seed point is the geocentric projection point leading to the geocentric longitude and latitude. This point is found as the intersection between the ellipsoid and the line connecting the center of the Earth and the point $P = \left(x_p, y_p, z_p\right)$. Consequently, this seed point is found solving the following equations system:

$$\begin{cases} y = \dfrac{y_p}{x_p}\ x \\ z = \dfrac{z_p}{x_p}\ x \\ x^2 + y^2 + \dfrac{a^2}{b^2}\ z^2 = a^2 \end{cases} \tag{23}$$

Which leads to:

$$x_0 = \pm \frac{a\ x_p}{\sqrt{x_p^2 + y_p^2 + \dfrac{a^2}{b^2} z_p^2}} \tag{24}$$

The sign of the seed point is to be taken identical to the one of x_p.

Starting from this seed point, the Newton method converges extremely rapidly. The second iteration leads already to a sub-metric correction of the root.

2.2.5 Conversion in longitude – latitude

Once having the Cartesian coordinates of the projected point $P_g = \left(x_g, y_g, z_g\right)$ on the ellipsoid, we still have to calculate the corresponding longitude ϕ and latitude λ. This coordinate conversion is made inverting the 3-equations system (eq. 1) supposing a null altitude h:

$$\begin{aligned} \lambda &= a\tan\left(\frac{y_g}{x_g}\right) \\ \phi &= a\sin\left(\frac{a^2}{b^2}\frac{z_g}{\nu}\right) \end{aligned} \tag{25}$$

where ν is the local curvature radius given by:

$$\nu = \sqrt{x_g^2 + y_g^2 + \frac{a^4}{b^4} z_g^2} \tag{26}$$

It must be noted that we do not use a direct inversion of eq. 1 system starting directly from $P = \left(x_p, y_p, z_p\right)$ [2]. In place we use the orthogonal projection scheme described here above before inverting the equation system. The reason of this choice is that InSAR generally leads to relative height measurements above a flat or spherical Earth. This relative measurement can in turn lead to an erroneous geolocalisation through eq. 1 inversion.

After orthonormal projection, the distance between P and Pg gives the geodetic height, which is different from the geocentric height given in input.

2.2.6 The local Earth radius

In the former equation systems and solving procedures, the local Earth radius is supposed known. In fact, this one must be determined either on a pixel per pixel basis or on a grid covering the whole scene for further interpolation.

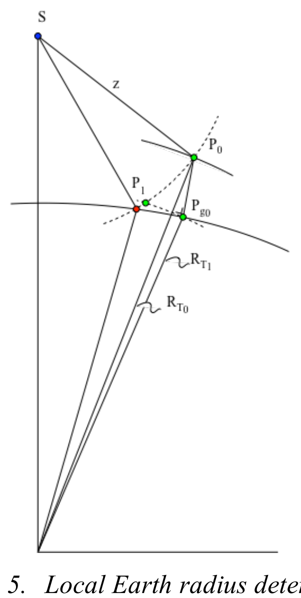

Figure 5. Local Earth radius determination

The local Earth radius at range z for a null local height is found iteratively using georeferencing procedure explained above. We start from an average Earth radius R_{T0}. With this Earth radius, we find an initial point P_0 in the geocentric coordinate system. Once projected onto the ellipsoid, we get a geolocation P_{g0} leading to a new local Earth radius R_{T1} (fig. 5). The process can then be reiterated to find a new point location P_1 in geocentric coordinate system for an observed point at range z and so on. In practice, after three iterations, the process converges with a centimeter precision.

2.3 Back-projection

Back-projection aims to project an external DEM into Slant range – Azimuth coordinates for a given reference scene. The problem is thus, having the latitude – longitude and height of a given observed point, to find it's coordinates in Slant range – Azimuth for a given orbit.

2.3.1 Geometrical approach

From a geometrical point of view, we can establish a very simple and efficient iterative method to find the solution of the inverse projection problem (fig. 6). The idea consists in finding a first rough approximation of the azimuth position S_0 corresponding to the observed point **P**. The Doppler plane passing through this azimuth first approximation is calculated (eq. 10). If we consider that the sensor speed vector varies slowly between the approximate azimuth position and the true position we are looking for, then the Doppler plane has simply to be translated to pass through the observed point.

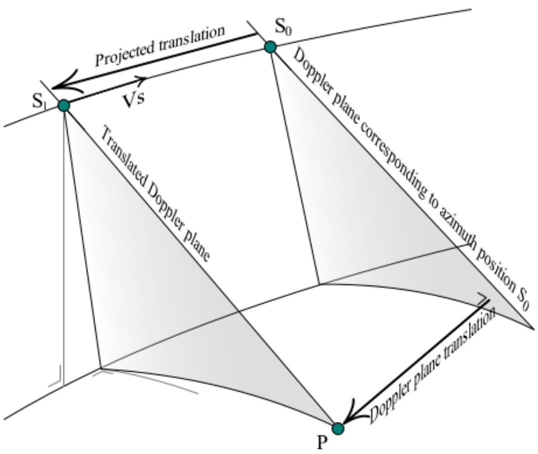

Figure 6. Geometrical iterative process for inverse projection

The intersection between this translated Doppler plane and the orbit gives us the new azimuth position. This new position is simply obtained by translating the approximate azimuth position along the orbit. Considering that the orbit can be approximate locally by a strait line, the translation to be applied is thus the projection along the speed vector of the translation vector applied to the Doppler plane. This process can be applied iteratively to refine the azimuth position. But in practice, it converges so rapidly that only one iteration appears to be sufficient.

To find the seed point, we georeference first the four corner of the reference scene and we establish a rough linear transform between both reference systems, i.e. longitude – latitude and azimuth – slant range.

3. PRECISION ESTIMATION OF BACK AND FORTH REFERENCING

Having implemented both referencing process, i.e. georeferencing and azimuth – slant range referencing allows testing the whole back and forth processing.

We took an ERS acquisition in order to have a reference orbit. Then, starting from a given azimuth – slant range point with a given geocentric height, we calculate its geolocation on WGS84. Then, starting from this geolocalisation and the found geodetic height, we referenced back our point in the azimuth – slant range coordinate system. The objective was to measure how close we are from our starting point. Each referencing process being independent, this procedure allows to estimate the cumulative error on both. Such estimation was performed using different azimuth and slant range position, but also for different heights and different Doppler centroïd, considering the focusing not necessarily done at zero Doppler. Whatever the variable, except for Doppler, return position is identical to the initial one within calculation noise error ($<10^{-6}$ pixel).

When limiting the orthogonal projection of the point on the ellipsoidal Earth to decimetric precision (eq. 25), the back and forth process lead to a return position that is correct within less than half a percent of a pixel in both range and azimuth.

For variable Doppler, there is a linear error in the back projection process (fig 7) when calculating the azimuth – slant range position using only one iteration of the process described here above (fig 6). The error can be up to 2 azimuth pixels for Doppler of 2500Hz. While the range error stays below a percent of a range pixel. When performing the calculation using two iterations, the azimuth position is correct within less than a thousandth of a pixel.

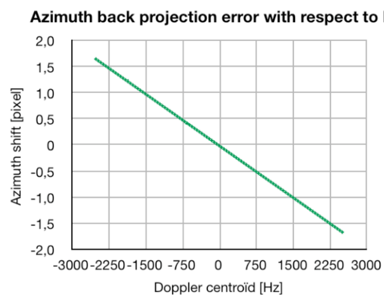

Figure 7. Azimuth back projection error with respect to DC using a single iteration.

4. CONCLUSIONS

We build a consistent geoprojection and back projection scheme revealing to be precise and robust.

Sub-metric precision can be reach provided a sufficient number of iteration are performed when required. Mainly, all iterative process described requires only one iteration to reach sub metric precision for both projection. Sole restriction concerns projection of images focused at non zero Doppler. In this case, back projection requires almost two iterations to reach sub metric precision.

It must be noted that the presented work concerns precision of the method and not necessarily accuracy. This latter one will mainly depends on orbital parameters, height measurement and Doppler estimate accuracy.

Finally, the developed georeferencing scheme is shown to be convenient for VHR SAR data geoprojection.

5. REFERENCES

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