The *np* Chart With Guaranteed In-control Average Run Lengths

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We evaluate the in-control performance of the *np*-control chart with estimated parameter conditional on the Phase I sample. We then apply the bootstrap method to adjust the control chart' limits to guarantee the desired in-control average run length (ARL_0) value in the monitoring stage. The adjusted limits ensure that the ARL_0 would take a value greater than the desired value (say, B) with a certain specified probability, that is $\Pr(ARL_0 > B) = 1 - \rho$. The results indicate that adjusting control limits is not always necessary. We present a method to design control charts such that in control and out of control run lengths are guaranteed with pre specified probabilities. This method is an improvement of the classical statistical design approach employing constraints on in control and out of control average run lengths (ARL) since, with this approach, there is a substantial probability that the actual run length in control may be too small. In addition, using the ARL approach may result in an actual out of control run length that is too large. Some numerical examples illustrate the efficacy of this design method.

***Keywords*:** *np*-charts, Bootstrap; Average run length (ARL); The average of ARL (AARL); Effect of estimation error; The standard deviation of ARL (SDARL)

1. Introduction

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Control charts are the important tools for monitoring a stable process to see if an assignable cause has impacted the process. This is labelled Phase II use in statistical process control. In Phase I, one wishes to try to estimate unknown process parameters through *m* available in-control samples each of size *n*. For a recent overview on Phase I issues and analyses, readers are referred to Jones-Farmer et al.[1]

ARL is the usual measure of control charts performance. When the process is in the in-control state, we would like no alarm to be triggered from a control chart and so we wish the in-control ARl to be large. The usual Shewhart control charts have an in control ARL=370. When an assignable cause of poor quality occurs, we wish to detect this rapidly. The speed at which control charts detect an assignable cause can be measured by the out-of-control ARL, or ARL_1

Considerable attention has been given to the effects of parameter estimation on the ARL_0 measure. Several authors have contributed to this topic, see for example, Chen[2&3], Braun[4], Jones et al.[5], Albers and Kallenberg[6], Chakraborti and Human[7&8], Bischak and Trietsch[9], Testik[10], Castagliola et al.[11&12], and Castagliola and Wu[13] among others. For thorough literature reviews on the impact of parameter estimation on the performance of different types of control charts, readers are referred to Jensen et al.[14] and Psarakis et al.[15].

Since different Phase I data sets give different process estimates, different practitioners or different sampling times will yield different conditional ARL values. Thus, there is a sampling distribution variation in the ARL values. Let AARL and SDARL be the average and standard deviation of the ARL distribution among different users, respectively. Most researchers advised practitioners to estimate parameters based on *m* Phase I samples each of size *n* such that $AARL_0$ is close enough to *B*. For example, Quesenberry[16] concluded $m=400/(n-1)$ Phase I samples guarantees the desired in-control performance, however, recent studies showed that $SDARL_0$ can be so large that the desired ARL_0 value cannot be guaranteed only with enormous number of Phase I samples (e.g., $m=3000$ or even higher). Jones and Steiner[17] are the first to recognize the importance of the $SDARL_0$ metric in determining the amount of Phase I data. Of course, small $SDARL_0$ values mean that the ARL_0 values will be close to the desired value *B*. Zhang et al.[18&19] , Lee et al.[20] and Faraz et al.[21] showed that only considering the $AARL_0$ measure can give misleading conclusions and it is common to have an $AARL_0$ value close to the desired ARL_0 value, but with very large $SDARL_0$ values.

The findings imply a need for alternative techniques for constructing control charts based on practical amounts of Phase I data.

The bootstrap method has become appealing because computing has become fast and inexpensive (Teyarachakul et al.[2222]). Recently, Jones and Steiner [1717] and Gandy and Kvalϕy [2323] applied the bootstrap method to adjust control limits' thresholds such that the desired in-control performance is guaranteed with a certain probability $(1-\rho)$ among practitioners. For example, the adjustments ensure that 90% of the constructed control charts would have ARL_0 values greater than or equal to B=370. Faraz et al.[2121] extended this approach to the S^2 chart, respectively. The proposed approach is effective, accurate and practical and therefore should be encouraged in practice.

In this paper, we adjust the control limits of *np*-charts based on the bootstrap method., a chart perhaps second in popularity only to the Xbar chart..

The paper is organized as follows: In Section 2, we review the *np*-control chart with estimated parameter. In Section 3, we study the sampling distribution variability when assessing the *np*-chart in terms of the $AARL_0$, $SDARL_0$, and some percentiles of the ARL_0 distribution. In Section 4, we adjust the control limits and compare the in-control performance of the adjusted charts with the classical *np*-chart. Finally,some concluding remarks and future research possibilities are given in the last section.

2. *np*-Control Charts with Estimated Parameters

Consider a stable production process with in-control nonconforming probability p_0, and that successive products are independent. Now suppose that a random sample of size n is selected and random variable X counts the number of nonconforming items. Then, X follows a binomial distribution with parameters n and p_0; i.e; $X \sim Bin(n, p_0)$. In practice, the parameter p_0 is usually unknown and therefore, it must be estimated from an available Phase I data set. Let $X_1, X_2, \ldots, X_m$ be the number of nonconforming items for the m initial samples each of size n. Then the MLE estimator of parameter p_0 is calculated as follows (See, Montgomery[2424]):

$$\hat{p}_0 = \bar{p} = \frac{\sum_{j=1}^{m} p_j}{m} = \frac{\sum_{j=1}^{m} X_j}{mn} \quad (1)$$

where $p_j = \frac{X_j}{n}$ is the fraction nonconforming for the sample j. Note that $T = \sum_{j=1}^{m} X_j \sim Bin(mn, p_0)$.

The estimated classical control limits for monitoring the number of nonconformities over time then are calculated as follows:

$$\widehat{UCL} = \left[n\bar{p} + \mathrm{K}\sqrt{n\bar{p}(1-\bar{p})}\right] \quad \text{and} \quad \widehat{LCL} = \max\left(0, \left[n\bar{p} - K\sqrt{n\bar{p}(1-\bar{p})}\right]\right) \qquad (2)$$

where $K = z_{1-\frac{\alpha}{2}}$ is the desired percentiles of the standard normal distribution. However, if $\widehat{LCL} \leq 0$, the $\widehat{UCL}$ should be adjusted to $\widehat{UCL} = \left[n\bar{p} + \mathrm{z}_{1-\alpha}\sqrt{n\bar{p}(1-\bar{p})}\right]$. At level $\alpha = 0.0027$, we have *K*=2.78 if $\widehat{LCL} = 0$, and *K*=3 otherwise.

In the following, we derive alternative control limits for the *np* charts by using the cumulative distribution function (CDF) of the binomial distribution with the desired false alarm rate. Here, we adjust the control limits so that the in-control ARL is at least the desired value and the false alarm probability is split as equally as possible between the two sides of the chart. At what follows, $\mathrm{F}^{-1}(\alpha, n, p_0)$ stands for the inverse CDF of a binomial distribution at point α with parameters n and p_0.

$$\widehat{LCL} = \mathrm{F}^{-1}\left(\frac{\alpha}{2}, n, \bar{p}\right)$$

$$\widehat{UCL} = \begin{cases} \mathrm{F}^{-1}\left(1-\frac{\alpha}{2}, n, \bar{p}\right) & if\ \widehat{LCL} \geq 1 \\ \mathrm{F}^{-1}(1-\alpha, n, \bar{p}) & if\ \widehat{LCL} = 0 \end{cases} \qquad (3)$$

In Phase II, since the number of nonconformities follow $Bin(n, p_0)$ and the chart limits are estimated, the ARL_0 of the *np*-chart becomes a random variable. However, the conditional ARL_0 given the estimate $\bar{p}$ can be calculated as follows:

$$ARL_0|\bar{p} = \frac{1}{\hat{\alpha}|\bar{p}}\ ;\ \hat{\alpha}|\bar{p} \neq 0 \qquad (4)$$

Where

$$\hat{\alpha}|\bar{p} = \begin{cases} 1 - \Pr\left(\widehat{LCL} \leq X_i \leq \widehat{UCL}\right) & if\ \widehat{LCL} \geq 1 \\ 1 - \Pr\left(X_i \leq \widehat{UCL}\right) & if\ \widehat{LCL} = 0 \end{cases}$$

$$\hat{\alpha}|\bar{p} = \begin{cases} 1 - \mathrm{F}\left(\widehat{UCL}, n, p_0\right) + F\left(\widehat{LCL} - 1, n, p_0\right) & if\ \widehat{LCL} \geq 1 \\ 1 - \mathrm{F}\left(\widehat{UCL}, n, p_0\right) & if\ \widehat{LCL} = 0 \end{cases} \qquad (5)$$

where $\mathrm{F}(x, n, p_0)$ stands for the cumulative distribution function of the binomial distribution at point x with parameters n and p_0. For example, suppose we have n=50, p_0=0.01 and $\alpha = 0.0027$. The classical control limits are calculated using Equation (2) as $\widehat{LCL} = 0$ and $\widehat{UCL} = 2$. Using equation (4), the false alarm probability is then 0.01382 and the ARL_0 is in fact 72.37.

Using the alternative control limits given in (3), we have $\widehat{LCL} = 0$ and $\widehat{UCL} = 3$ and it yields a false alarm rate of 0.0016 and an in-control ARL of 626.50. Hereafter, we shall use the alternative control limits due to their better in-control performance. In the next section, we study the estimation effect on the alternative control limits.

3. The AARL and SDARL measures

Equation (1) clearly shows that different practitioners might have different estimates and Equation (4) indicates that the ARL_0 measure is also a function of the estimated parameter $\bar{p}$ and hence it has a random behaviour with the mean $AARL_0$ and the standard deviation $SDARL_0$. Now, suppose that r different Phase I data sets consisting of m samples each of size n from a binomial distribution with parameters n and p_0 are available. Let $\bar{p}_i$ stand for the estimated in-control nonconforming probability for the ith Phase I sample, $i = 1,2, \dots, r$. Note that when the process parameters are known, the ARL_0 measure becomes a constant value. Tables 1-4 give the $AARL_0$, $SDARL_0$, the Median and the 10% and 25% percentiles ($Q_{0.10}$ and $Q_{0.25}$, respectively) of the ARL_0 distribution for different values of m, n, p and α for r=100,000 different simulated Phase I samples. The case $m = \infty$ corresponds to the case in which the process parameter p_0 is known. Results indicate that the alternative control limits have a good in-control performance. The degree at which the in-control performance is guaranteed depends on the sample size n, the number of Phase I samples m and the Type I error rate α.

[Insert Tables 1 and 2 here.]

Tables 1 – 4 can be used to see the effect of the amount of Phase I samples on the in-control performance of the *np*-charts with alternative control limits. For example, when p_0=0.02, n=100 and α=0.0027 we have $Q_{0.25}$=1073.03 for m=50. That is, with fifty Phase I samples the desired in-control ARL can be guaranteed with probability 75%. Moreover, $Q_{0.10}$ exceeds 370.4 for the first time when m=200 and so 200 samples are enough to guarantee the desired in-control ARL with probability 90%.

[Insert Tables 3 and 4 here.]

Tables 1 and 2 give the results for α=0.0027 and they show no specific pattern. However, the results are interesting when α=0.005. The rule of thumb is that the ARL_0=200 is guaranteed with probability 75% by alternative control limits, i.e, Pr(ARL_0>200)=75%. However, if one wants to design the *np*-charts such that Pr(ARL_0>200)=90% then at least m=75 samples are needed or the control limits should be adjusted. However, there are some exceptions where even m=25 samples are enough to have Pr(ARL_0>200)=90%. For example for the case m=25, p_0=0.01 and n=100, we get $AARL_0$=799.76, $SDARL_0$=1099.36 and 90% of practitioners would have an in-control ARL value greater than 291.53 which is bigger than the desired value 200.

In the next section, we use the bootstrap method to adjust the control chart limits to guarantee desired in-control performance with probability $(1-\rho)$ when the number of Phase I samples is limited.

4. The adjusted limits for the *np* charts

In this section, we apply the bootstrap method to obtain the control limit threshold of attribute control charts such that the conditional in-control ARL meets or exceeds the specified value with a certain probability. The bootstrap algorithm is summarized as follows:

1- Given a Phase I data set, use equation (1) to estimate the process parameter $\bar{p}$.

2- Generate i=1,2,…, D bootstrap estimates $p_i^* = t/mn$ where t comes from a $Bin(mn, \bar{p})$ and where D is a large number, e.g., D = 500.

3- For each bootstrap np-chart, construct the alternative limits at level of α as follows:

$$\widehat{LCL}_i = \mathrm{F}^{-1}\left(\frac{\alpha}{2}, n, p_i^*\right) \tag{6}$$

$$\widehat{UCL}_i = \begin{cases} \mathrm{F}^{-1}\left(1-\frac{\alpha}{2}, n, p_i^*\right) & if\ \widehat{LCL} \geq 1 \\ \mathrm{F}^{-1}\left(1-\alpha, n, p_i^*\right) & if\ \widehat{LCL} = 0 \end{cases} \tag{7}$$

4- Find the $(1-\rho)$ quantile of the bootstrap threshold $\widehat{UCL}_i$ (say, $\widehat{UCL}_\rho$) and the ρ quantile of the bootstrap threshold $\widehat{LCL}_i$(say, $\widehat{LCL}_\rho$) to guarantee the in-control performance with probability $(1-\rho)$100%.

Consider the case where p_0=0.1, m=25, n=50 and α=0.0027. Table 2 shows that no practical amount of Phase I samples can guarantee the in-control performance with probability 90%. Hence, the control limits should be adjusted. We then compare the in-control performance of the np-charts with and without adjustments for 500 different practitioners. First, for each given Phase I data set we estimate the process parameter using Equation (1). The alternative control limits and the conditional ARL_0 are then calculated using Equations (3) and (4), respectively. Second, for each given Phase I data set we adjust the control limits using the bootstrap algorithm given in section 4. The ARL_0 for the adjusted charts is then calculated. Figure 1 illustrates the boxplots of the ARL_0 distributions for the *np*-control charts with and without adjustment. These results indicate that approximately 90% of the adjusted control charts meet the desired in-control performance while almost 50% of the *np*-charts without adjustment have poor in-control performance. Now consider the same case but with $ARL_0 = 200$. Table 4 suggests that *m*=75 phase I samples are enough to guarantee the in-control performance with probability 90%. If this amount of Phase I samples is not available then the control limits should be adjusted otherwise only 75% of the *np*-charts without adjustment have good in-control performance.

[Insert Figure 1 here.]

It is worth mentioning that repeating the bootstrap method for a given Phase I sample gives different results which we call the within Phase I sample variation. Moreover, different results may be given for different Phase I samples, which we call the between Phase I sample variation. Faraz et al. (2015) proposed that one run the algorithm for a certain number of times, e.g., r = 1000 times, and then use the average as the final threshold. For example, Table 5 gives the suggested upper and lower thresholds for r = 10,000 different Phase I samples and for different values of m, n, p and α.

[Insert Table 5 here.]

The results indicate that the bootstrap method perfectly estimates the *np*-chart's thresholds when the parameter is unknown. These limits can be used by practitioners as a standard guide to adjust the alternative control limits such that the in-control performance is guaranteed with probability 90%.

5. Concluding Remarks and further research

In our paper, the in-control performance of the *np* - charts with alternative control limits and with estimated parameters are evaluated using the $AARL_0$ and $SDARL_0$ metrics. Additionally, we applied the bootstrap method to adjust the upper and lower thresholds such that the ARL_0 takes a value greater than the desired value with a certain probability. This approach can be extended to guarantee the in-control performance of the other type of attribute control charts such as the Cornish-Fisher corrected p-chart (Joekesa and Barbosab[25~~25~~]), the AFV-chart (Haridy et al[26~~26~~]) and the MON-chart (Wu and Jiao[27~~27~~]).

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Table 1. The distribution of in-control ARL for alternative limits with different values of n, m, p_0 when α=0.0027.

n		50					100				
p_0	m	*Lower Quartiles*		MARL$_0$	AARL$_0$	SDARL$_0$	*Lower Quartiles*		MARL$_0$	AARL$_0$	SDARL$_0$
		$Q_{0.10}$	$Q_{0.25}$				$Q_{0.10}$	$Q_{0.25}$			
0.01	25	626.50	626.50	626.50	2324.25	2799.57	291.35	291.35	1870.79	1639.65	2395.07
	50	626.50	626.50	626.50	2110.19	2660.26	291.35	291.35	1870.79	1357.19	1233.10
	75	626.50	626.50	626.50	1621.62	2285.13	291.35	291.35	1870.79	1398.82	812.71
	100	626.50	626.50	626.50	1517.09	2182.45	291.35	291.35	1870.79	1412.62	738.69
	125	626.50	626.50	626.50	1281.98	1912.84	291.35	291.35	1870.79	1427.01	713.46
	150	626.50	626.50	626.50	1205.32	1809.82	291.35	1870.79	1870.79	1481.64	680.59
	200	626.50	626.50	626.50	1013.72	1505.10	291.35	1870.79	1870.79	1507.52	664.68
	∞	**626.50**	**626.50**	**626.50**	**626.50**	**0.00**	**1870.79**	**1870.79**	**1870.79**	**1870.79**	**0.00**
0.02	25	311.55	311.55	2091.10	1557.67	2387.80	246.18	246.18	1073.03	1192.01	1344.27
	50	311.55	311.55	2091.10	1378.98	1211.41	246.18	1073.03	1073.03	1027.78	862.38
	75	311.55	311.55	2091.10	1397.18	926.16	246.18	1073.03	1073.03	968.91	598.12
	100	311.55	311.55	2091.10	1440.52	869.39	246.18	1073.03	1073.03	969.59	416.89
	125	311.55	311.55	2091.10	1470.66	848.04	246.18	1073.03	1073.03	983.78	317.82
	150	311.55	311.55	2091.10	1504.04	836.70	246.18	1073.03	1073.03	989.19	274.65
	200	311.55	311.55	2091.10	1563.21	812.86	1073.03	1073.03	1073.03	1013.43	220.68
	∞	**2091.10**	**2091.10**	**2091.10**	**2091.10**	**0.00**	**1073.03**	**1073.03**	**1073.03**	**1073.03**	**0.00**
0.05	25	313.64	313.64	1322.78	1145.24	1256.27	233.96	233.96	682.90	885.97	921.65
	50	313.64	313.64	1322.78	995.46	753.72	233.96	682.90	682.90	769.52	547.72
	75	313.64	313.64	1322.78	977.50	564.98	233.96	682.90	682.90	729.12	433.33
	100	313.64	313.64	1322.78	984.90	496.82	233.96	682.90	682.90	707.09	354.02
	125	313.64	313.64	1322.78	1000.13	478.19	682.90	682.90	682.90	695.77	291.93
	150	313.64	313.64	1322.78	1029.30	460.24	682.90	682.90	682.90	687.94	241.90
	200	313.64	313.64	1322.78	1051.64	447.66	682.90	682.90	682.90	682.27	171.27
	∞	**1322.78**	**1322.78**	**1322.78**	**1322.78**	**0.00**	**682.90**	**682.90**	**682.90**	**682.90**	**0.00**

Table 2. The distribution of in-control ARL for alternative limits with different values of *n*, *m*, p_0 when α=0.0027.

n		50					100				
p_0	***m***	***Lower Quartiles***		**MARL$_0$**	**AARL$_0$**	**SDARL$_0$**	***Lower Quartiles***		**MARL$_0$**	**AARL$_0$**	**SDARL$_0$**
		$Q_{0.10}$	$Q_{0.25}$				$Q_{0.10}$	$Q_{0.25}$			
0.10	25	310.57	310.57	995.40	915.26	853.20	434.74	434.74	443.10	619.28	235.96
	50	310.57	310.57	995.40	823.99	573.16	434.74	443.10	885.53	699.53	223.80
	75	310.57	310.57	995.40	785.38	420.93	434.74	443.10	885.53	745.58	208.68
	100	310.57	310.57	995.40	789.72	357.83	434.74	885.53	885.53	777.29	192.87
	125	310.57	310.57	995.40	803.37	327.48	434.74	885.53	885.53	800.47	176.53
	150	310.57	310.57	995.40	806.98	312.72	434.74	885.53	885.53	817.60	161.37
	200	310.57	995.40	995.40	833.98	291.50	443.10	885.53	885.53	839.57	136.69
	∞	**995.40**	**995.40**	**995.40**	**995.40**	**0.00**	**885.53**	**885.53**	**885.53**	**885.53**	**0.00**
0.15	25	337.26	445.37	1044.81	877.43	470.62	294.90	461.77	551.66	537.49	177.25
	50	445.37	445.37	1044.81	944.70	383.31	461.77	461.77	553.91	575.84	178.95
	75	445.37	1044.81	1044.81	964.52	325.48	461.77	461.77	553.91	591.19	179.94
	100	445.37	1044.81	1044.81	973.95	274.83	461.77	461.77	553.91	611.30	189.26
	125	445.37	1044.81	1044.81	984.23	239.02	461.77	461.77	553.91	619.05	191.35
	150	445.37	1044.81	1044.81	991.10	210.06	461.77	461.77	553.91	631.01	196.81
	200	1044.81	1044.81	1044.81	1001.55	171.15	461.77	553.91	553.91	649.18	203.43
	∞	**1044.81**	**1044.81**	**1044.81**	**1044.81**	**0.00**	**962.99**	**962.99**	**962.99**	**962.99**	**0.00**
0.20	25	369.84	450.89	622.63	602.56	202.49	293.54	374.58	547.22	498.72	138.90
	50	369.84	450.89	622.63	632.59	201.70	374.58	415.66	628.03	534.95	121.89
	75	450.89	450.89	622.63	646.46	203.27	415.66	415.66	628.03	549.39	112.48
	100	450.89	450.89	622.63	649.84	205.25	415.66	415.66	628.03	556.09	106.64
	125	450.89	450.89	622.63	642.56	206.32	415.66	415.66	628.03	561.01	103.10
	150	450.89	450.89	622.63	643.64	207.64	415.66	415.66	628.03	563.61	100.59
	200	450.89	450.89	450.89	638.04	209.11	415.66	415.66	628.03	568.50	96.75
	∞	**450.89**	**450.89**	**450.89**	**450.89**	**0.00**	**628.03**	**628.03**	**628.03**	**628.03**	**0.00**

Table 3. The distribution of in-control ARL for alternative limits with different values of *n*, *m*, p_0 when α=0.005.

n		50					100				
p_0	*m*	***Lower Quartiles***		**MARL₀**	**AARL₀**	**SDARL₀**	***Lower Quartiles***		**MARL₀**	**AARL₀**	**SDARL₀**
		$Q_{0.10}$	$Q_{0.25}$				$Q_{0.10}$	$Q_{0.25}$			
0.01	25	72.37	626.5	626.5	1078.12	1755.61	291.35	291.35	291.35	799.76	1099.36
	50	626.5	626.5	626.5	795.99	1120.26	291.35	291.35	291.35	697.65	703.14
	75	626.5	626.5	626.5	705.05	748.26	291.35	291.35	291.35	592.57	621.24
	100	626.5	626.5	626.5	659.51	495.36	291.35	291.35	291.35	557.18	591.09
	125	626.5	626.5	626.5	636.58	275.81	291.35	291.35	291.35	530.13	565.82
	150	626.5	626.5	626.5	629.51	163.41	291.35	291.35	291.35	479.56	511.73
	200	626.5	626.5	626.5	627.39	84.45	291.35	291.35	291.35	443.32	465.77
	∞	**626.5**	**626.5**	**626.5**	**626.5**	**0**	**291.35**	**291.35**	**291.35**	**291.35**	**0**
0.02	25	311.55	311.55	311.55	872.86	1196.09	246.18	246.18	246.18	637.17	725.48
	50	311.55	311.55	311.55	690.66	735.42	246.18	246.18	246.18	541.4	428.85
	75	311.55	311.55	311.55	598.91	655.58	246.18	246.18	246.18	509.69	390.05
	100	311.55	311.55	311.55	530.81	585.02	246.18	246.18	246.18	487.13	376.16
	125	311.55	311.55	311.55	509.52	559.6	246.18	246.18	246.18	467.67	366.22
	150	311.55	311.55	311.55	470.82	508	246.18	246.18	246.18	455.68	359.64
	200	311.55	311.55	311.55	411.74	410.19	246.18	246.18	246.18	425.06	340.46
	∞	**311.55**	**311.55**	**311.55**	**311.55**	**0**	**246.18**	**246.18**	**246.18**	**246.18**	**0**
0.05	25	84.84	313.64	313.64	575.5	645.39	87.17	233.96	233.96	393.49	277.13
	50	313.64	313.64	313.64	498.33	411.36	233.96	233.96	233.96	388.89	233.95
	75	313.64	313.64	313.64	441.48	343.41	233.96	233.96	233.96	387.16	221.04
	100	313.64	313.64	313.64	418.23	310.55	233.96	233.96	233.96	373.41	210.25
	125	313.64	313.64	313.64	397.15	279.44	233.96	233.96	233.96	369.88	206.88
	150	313.64	313.64	313.64	374.01	240	233.96	233.96	233.96	359.01	201.51
	200	313.64	313.64	313.64	353.41	196.5	233.96	233.96	233.96	346.28	194.48
	∞	**313.64**	**313.64**	**313.64**	**313.64**	**0**	**233.96**	**233.96**	**233.96**	**233.96**	**0**

Table 4. The distribution of in-control ARL for alternative limits with different values of n, m, p_0 when α=0.005.

n		50					100				
p_0	m	***Lower Quartiles***		**MARL$_0$**	**AARL$_0$**	**SDARL$_0$**	***Lower Quartiles***		**MARL$_0$**	**AARL$_0$**	**SDARL$_0$**
		$Q_{0.10}$	$Q_{0.25}$				$Q_{0.10}$	$Q_{0.25}$			
0.10	25	106.9	310.57	310.57	409.4	306.3	203.98	254.88	254.88	310.17	89.62
	50	183.86	310.57	310.57	406.82	271.3	203.98	254.88	254.88	312.14	77.94
	75	310.57	310.57	310.57	386.95	235.79	254.88	254.88	254.88	309.85	74.04
	100	310.57	310.57	310.57	370.89	206.51	254.88	254.88	254.88	305.61	71.26
	125	310.57	310.57	310.57	357.44	181.15	254.88	254.88	254.88	301.15	68.8
	150	310.57	310.57	310.57	346.12	156.89	254.88	254.88	254.88	296.8	66.14
	200	310.57	310.57	310.57	333.37	125.03	254.88	254.88	254.88	289.18	61.27
	∞	**310.57**	**310.57**	**310.57**	**310.57**	**0**	**254.88**	**254.88**	**254.88**	**254.88**	**0**
0.15	25	179.13	280.36	321.32	349.11	101.49	188.15	221.33	294.9	285.14	83.36
	50	280.36	280.36	445.37	372.49	93.22	221.33	221.33	341.01	303.36	72.17
	75	280.36	280.36	445.37	385.52	86.78	221.33	221.33	341.01	307.28	64.94
	100	280.36	280.36	445.37	392.64	83.28	221.33	221.33	341.01	310.03	60.43
	125	280.36	280.36	445.37	397.08	81.22	221.33	221.33	341.01	310.92	57.38
	150	280.36	445.37	445.37	400.76	79.36	221.33	294.9	341.01	312.25	55.06
	200	280.36	445.37	445.37	406.38	76.45	221.33	341.01	341.01	314.23	51.66
	∞	**445.37**	**445.37**	**445.37**	**445.37**	**0**	**341.01**	**341.01**	**341.01**	**341.01**	**0**
0.20	25	167.31	263.39	263.39	337.14	114.63	157.82	250.93	257.47	263.94	66.62
	50	263.39	263.39	369.84	354.78	100.8	250.93	250.93	257.47	280	60.6
	75	263.39	263.39	450.89	358.76	95.79	250.93	250.93	257.47	283.16	58.37
	100	263.39	263.39	450.89	359.18	94.38	250.93	257.47	257.47	286.12	60.23
	125	263.39	263.39	450.89	358.52	93.91	250.93	257.47	257.47	287.82	61.92
	150	263.39	263.39	450.89	357.93	93.8	250.93	257.47	257.47	288.39	62.57
	200	263.39	263.39	450.89	357.84	93.74	250.93	257.47	257.47	288.78	63.37
	∞	**450.89**	**450.89**	**450.89**	**450.89**	**0**	**257.47**	**257.47**	**257.47**	**257.47**	**0**

Table 5. The adjusted control limits for the *np*-charts to guarantee that

$Pr(ARL_0 > B) = 90\%$

B		***200***				***370.4***			
n		***50***		***100***		***50***		***100***	
p_0	***m***	***LCL_p***	***UCL_p***	***LCL_p***	***UCL_p***	***LCL_p***	***UCL_p***	***LCL_p***	***UCL_p***
0.01	25	0.00	3.42	0.00	4.84	0.00	3.70	0.00	5.23
	50	0.00	3.29	0.00	4.74	0.00	3.69	0.00	5.06
	75	0.00	3.18	0.00	4.64	0.00	3.60	0.00	4.99
	∞	**0.00**	**3.00**	**0.00**	**4.00**	**0.00**	**3.00**	**0.00**	**5.00**
0.02	25	0.00	4.83	0.00	7.03	0.00	5.12	0.00	7.46
	50	0.00	4.67	0.00	6.84	0.00	4.99	0.00	7.25
	75	0.00	4.60	0.00	6.77	0.00	4.95	0.00	7.15
	∞	**0.00**	**4.00**	**0.00**	**6.00**	**0.00**	**5.00**	**0.00**	**7.00**
0.05	25	0.00	7.84	0.00	12.30	0.00	8.28	0.00	12.79
	50	0.00	7.68	0.00	11.97	0.00	8.09	0.00	12.50
	75	0.00	7.53	0.00	11.87	0.00	7.99	0.00	12.43
	∞	**0.00**	**7.00**	**0.00**	**11.00**	**0.00**	**8.00**	**0.00**	**12.00**
0.10	25	0.00	12.20	2.17	20.23	0.00	12.54	1.92	21.02
	50	0.00	11.85	2.25	19.95	0.00	12.17	2.00	20.69
	75	0.00	11.70	2.31	19.82	0.00	12.10	2.00	20.56
	∞	**0.00**	**11.00**	**3.00**	**19.00**	**0.00**	**12.00**	**2.00**	**20.00**
0.15	25	1.05	16.19	5.28	26.90	0.92	16.72	4.78	27.65
	50	1.04	15.87	5.45	26.53	1.00	16.45	4.97	27.36
	75	1.03	15.78	5.54	26.36	1.00	16.34	5.02	27.17
	∞	**1.00**	**15.00**	**6.00**	**26.00**	**1.00**	**16.00**	**5.00**	**27.00**
0.20	25	2.46	19.41	8.80	33.03	2.07	19.99	8.19	33.93
	50	2.65	19.13	9.03	32.69	2.12	19.79	8.41	33.58
	75	2.80	19.01	9.12	32.53	2.12	19.65	8.51	33.42
	∞	**3.00**	**19.00**	**10.00**	**32.00**	**3.00**	**19.00**	**9.00**	**33.00**

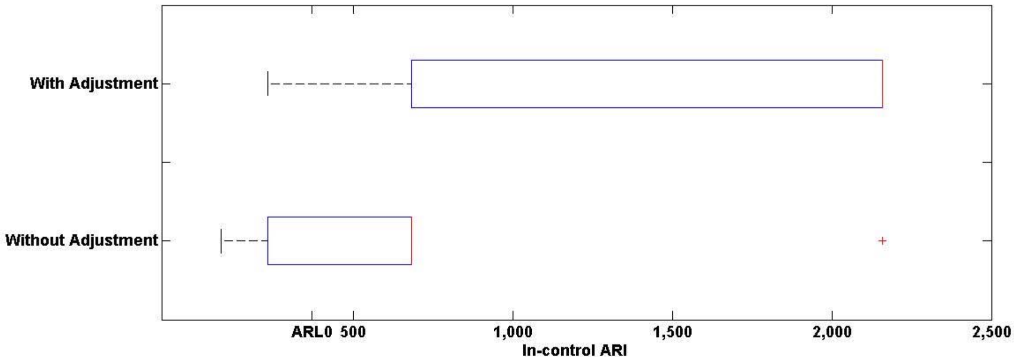

Figure 1. The box-plot of the in-control ARL with and without limits adjustment for m=25, n=100, p_0=0.05 and ARL_0=370. Median and Upper quartile are overlapped.