Post-contingency corrective control failure: a risk to neglect or a risk to control? Supplementary material

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January 2018

Nomenclature

The main mathematical symbols used in this document are defined as follows. Others may be defined as needed within the text.

Indices

c Index of contingencies.

d Index of demands.

g Index of dispatchable generating units.

k Index of piece-wise linear dispatchable generation cost curve segments.

ℓ Index of transmission elements (*i.e.* lines, cables and transformers).

n Index of nodes.

Sets

$\mathcal{C}_{N1}$ Set of contingencies.

$\mathcal{D}$ Set of demands.

$\mathcal{D}_n \subseteq \mathcal{D}$ Subset of demands connected at node n.

$\mathcal{G}$ Set of dispatchable generating units.

$\mathcal{G}_n \subseteq \mathcal{G}$ Subset of dispatchable generating units connected at node n.

$\mathcal{L}$ Set of transmission elements.

$\mathcal{N}$ Set of nodes.

Parameters

$a_{\ell,c}$ Binary parameter taking a zero value if transmission element $\{\ell \in \mathcal{L}\}$ is unavailable under contingency c.

c_g Marginal running cost of generating unit g.

P_g^{max} Capacity of generating unit g.

ΔP_g^- Ramp-down limit of generating unit g in corrective mode.

ΔP_g^+ Ramp-up limit of generating unit g in corrective mode.

P_d Active power demand of load d.

$voll_d$ Value of lost load of d.

f_ℓ^{max} Long-term thermal rating of transmission element ℓ.

r_ℓ Ratio of the short-term thermal rating to the long-term thermal rating of transmission element ℓ ($r_\ell \geq 1$).

X_ℓ Reactance of transmission element ℓ.

$\beta_{n,\ell}$ Element of the flow incidence matrix, taking a value of one if node n is the sending node of element ℓ, a value of minus one if node n is the receiving node of element ℓ, and a zero value otherwise.

π_c Probability of occurrence of contingency c.

pen A large penalty parameter.

Continuous Variables

$P_{g,0}$ Preventive dispatch of generating unit g.

$P_{g,c}^+$ Corrective ramp-up of generating unit g following contingency c.

$P_{g,c}^-$ Corrective ramp-down of generating unit g following contingency c.

$f_{\ell,0}$ Power flowing through transmission element ℓ under the pre-contingency state and scenario.

$f_{\ell,c}^{pc}$ Power flowing through transmission element ℓ following contingency c and prior to the application of corrective control.

$f_{\ell,c}$ Power flowing through transmission element ℓ following contingency c and the successful application of corrective control.

$\theta_{n,0}$ Voltage angle at node n under the pre-contingency state.

$\theta_{n,c}^{pc}$ Voltage angle at node n following contingency c and prior to the application of corrective control control.

$\theta_{n,c}$ Voltage angle at node n following contingency c and the successful application of corrective control.

ls_d Preventive involuntary shedding of load d.

$ls_{d,c}$ Corrective involuntary shedding of load d following contingency c.

Nb: All continuous variables are non-negative with the exception of the transmission element flow variables, and voltage angle variables.

Preventive N-1 SCOPF

$$\min\left(\sum_{g\in\mathcal{G}} c_g \cdot P_{g,0} + pen \cdot \sum_{d\in\mathcal{D}} ls_d\right) \tag{1}$$

subject to,

for all nodes $n \in \mathcal{N}$:

$$\sum_{g\in\mathcal{G}_n} P_{g,0} - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,0} = \sum_{d\in\mathcal{D}_n} \left(P_d - ls_d\right), \tag{2}$$

for all transmission elements $\ell \in \mathcal{L}$:

$$f_{\ell,0} - \frac{1}{X_\ell} \sum_{n\in\mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,0} = 0, \tag{3}$$

$$f_{\ell,0} \le f_\ell^{\max}, \tag{4}$$

$$- f_{\ell,0} \le f_\ell^{\max}, \tag{5}$$

for all generating units $g \in \mathcal{G}$

$$0 \le P_{g,0} \le P_g^{\max}, \tag{6}$$

for all loads $d \in \mathcal{D}$:

$$0 \le ls_d \le P_d, \tag{7}$$

for all nodes $n \in \mathcal{N}$ *& contingencies* $c \in \mathcal{C}_{N1}$:

$$\sum_{g\in\mathcal{G}_n} P_{g,0} \quad - \sum_{\ell\in\mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c}^{pc} = \sum_{d\in\mathcal{D}_n} \left(P_d - ls_d\right), \tag{8}$$

for all transmission elements $\ell \in \mathcal{L}$ *& contingencies* $c \in \mathcal{C}_{N1}$:

$$f_{\ell,c}^{pc} - a_{\ell,c} \cdot \frac{1}{X_\ell} \sum_{n\in\mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c}^{pc} = 0, \tag{9}$$

$$f_{\ell,c}^{pc} \le a_{\ell,c} \cdot f_\ell^{\max}, \tag{10}$$

$$- f_{\ell,c}^{pc} \le a_{\ell,c} \cdot f_\ell^{\max}, \tag{11}$$

Corrective N-1 SCOPF

$$\min\left[\sum_{g\in\mathcal{G}} c_g \cdot \left(P_{g,0} + \sum_{c\in\mathcal{C}_{N1}} \pi_c \cdot P_{g,c}^{+}\right) + pen \cdot \sum_{d\in\mathcal{D}} \left(|\mathcal{C}_{N1}| \cdot ls_d + \sum_{c\in\mathcal{C}_{N1}} ls_{d,c}\right)\right] \tag{12}$$

subject to,

for all nodes $n \in \mathcal{N}$:

$$\sum_{g \in \mathcal{G}_n} P_{g,0} - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,0} = \sum_{d \in \mathcal{D}_n} \left(P_d - ls_d\right), \tag{13}$$

for all transmission elements $\ell \in \mathcal{L}$:

$$f_{\ell,0} - \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,0} = 0, \tag{14}$$

$$f_{\ell,0} \leq f_\ell^{\max}, \tag{15}$$

$$- f_{\ell,0} \leq f_\ell^{\max}, \tag{16}$$

for all generating units $g \in \mathcal{G}$

$$0 \leq P_{g,0} \leq P_g^{\max}, \tag{17}$$

for all generating units $g \in \mathcal{G}$ *& contingencies* $c \in \mathcal{C}_{N1}$:

$$0 \leq P_{g,0} + \left(P_{g,c}^{+} - P_{g,c}^{-}\right) \leq P_g^{\max}, \tag{18}$$

$$0 \leq P_{g,c}^{+} \leq \Delta P_g^{+}, \tag{19}$$

$$0 \leq P_{g,c}^{-} \leq_c \Delta P_g^{-} \tag{20}$$

for all loads $d \in \mathcal{D}$ *& contingencies* $c \in \mathcal{C}_{N1}$:

$$0 \leq ls_d + ls_{d,c} \leq P_d, \tag{21}$$

for all nodes $n \in \mathcal{N}$ *& contingencies* $c \in \mathcal{C}_{N1}$:

$$\sum_{g \in \mathcal{G}_n} P_{g,0} \quad - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c}^{pc} = \sum_{d \in \mathcal{D}_n} \left(P_d - ls_d\right), \tag{22}$$

$$\sum_{g \in \mathcal{G}_n} \left(P_{g,0} + P_{g,c}^{+} - P_{g,c}^{-}\right) \quad - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,c} = \sum_{d \in \mathcal{D}_n} \left(P_d - ls_d - ls_{d,c}\right), \tag{23}$$

for all transmission elements $\ell \in \mathcal{L}$ *& contingencies* $c \in \mathcal{C}_{N1}$:

$$f_{\ell,c}^{pc} - a_{\ell,c} \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c}^{pc} = 0, \tag{24}$$

$$f_{\ell,c}^{pc} \leq a_{\ell,c} \cdot r_\ell \cdot f_\ell^{\max}, \tag{25}$$

$$- f_{\ell,c}^{pc} \leq a_{\ell,c} \cdot r_\ell \cdot f_\ell^{\max}, \tag{26}$$

$$f_{\ell,c} - a_{\ell,c} \cdot \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_{n,c} = 0, \tag{27}$$

$$f_{\ell,c} \leq a_{\ell,c} \cdot f_\ell^{\max}, \tag{28}$$

$$- f_{\ell,c} \leq a_{\ell,c} \cdot f_\ell^{\max}. \tag{29}$$

Emergency Stage OPF

$$\min \sum_{d \in \mathcal{D}} ls_d^e \tag{30}$$

subject to,

for all nodes $n \in \mathcal{N}$:

$$\sum_{g \in \mathcal{G}_n} \left(P_{g,0}^{\star} - e_g \cdot \sum_{d \in \mathcal{D}} ls_d^e \right) - \sum_{\ell \in \mathcal{L}} \beta_{n,\ell} \cdot f_{\ell,0} = \sum_{d \in \mathcal{D}_n} \left(P_d - ls_d^{\star} - ls_d^e \right), \tag{31}$$

for all loads $d \in \mathcal{D}$:

$$0 \leq ls_d^{*} + ls_d^e \leq P_d, \tag{32}$$

for all transmission elements $\ell \in \mathcal{L}$:

$$f_\ell^e - \gamma_\ell \frac{1}{X_\ell} \sum_{n \in \mathcal{N}_n} \beta_{n,\ell} \cdot \theta_n^e = 0, \tag{33}$$

$$f_\ell^e \leq f_\ell^{\max}, \tag{34}$$

$$- f_\ell^e \leq f_\ell^{\max}, \tag{35}$$

where superscript ($\star$) denotes the optimal values of the respective variables as per the corrective N-1 SCOPF, generation participation factors are set as $e_g = P_{g,0}^{\star} / \sum g \in \mathcal{G} P_{g,0}^{\star}$ and binary parameter γ_ℓ takes a value of zero to denote the tripping of the respective transmission element following any contingency and the failure of the respective corrective controls.

Socio-economic cost function

$$C_{SE}(u^{\star}(t), x(t)) = \Bigg\{ \sum_{g \in \mathcal{G}} c_g \cdot \left[P_{g,0}^{\star} + \sum_{c \in \mathcal{C}_{N1}} \pi_c \cdot \left(P_{g,c}^{+,\star} - Pg, c^{-,\star} \right) \right] + \sum_{d \in \mathcal{D}} voll_d \cdot \left(ls_d^{\star} + \sum_{c \in \mathcal{C}_{N1}} \pi_c \cdot ls_{d,c}^{\star} \right) \Bigg\}, \tag{36}$$

where superscript ($\star$) denotes the optimal values of the respective variables as per the solution of an OPF/SCOPF problem.

Severity function

$$S(u_E^{\star}(t), x(t)) = \sum_{d \in \mathcal{D}} voll_d \cdot ls_d^{e,\star}, \tag{37}$$

where superscript ($\star$) denotes the optimal values of the respective variables as per the solution of an OPF problem.

Corrective control failure probability function

$$\phi\left(p, u^{\star}_{N1c}(t), c\right) = \sum_{k\in[1,n_c]} (-1)^k \cdot p^k \cdot \frac{n_c!}{(n_c-k)!\ \ k!}, \tag{38}$$

where n_c denotes the number of elementary control operations following correctively secured contingency c.