Quasidegeneracy of Majorana Neutrinos and the Origin of Large Leptonic Mixing

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Abstract

We propose that the observed large leptonic mixing may just reflect a quasidegeneracy of three Majorana neutrinos. The limit of exact degeneracy of Majorana neutrinos is not trivial, as leptonic mixing and even CP violation may occur. We conjecture that the smallness of $|U_{13}|$, when compared to the other elements of U_{PMNS}, may be related to the fact that, in the limit of exact mass degeneracy, the leptonic mixing matrix necessarily has a vanishing element. We show that the lifting of the mass degeneracy can lead to the measured value of $|U_{13}|$ while at the same time accommodating the observed solar and atmospheric mixing angles. In the scenario we consider for the breaking of the mass degeneracy there is only one CP violating phase, already present in the limit of exact degeneracy, which upon the lifting of the degeneracy generates both Majorana and Dirac-type CP violation in the leptonic sector. We analyse some of the correlations among physical observables and point out that in most of the cases considered, the implied strength of leptonic Dirac-type CP violation is large enough to be detected in the next round of experiments.

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1 Introduction

The observed pattern of fermion masses and mixing continues being a major puzzle in particle physics and the discovery of large leptonic mixing rendered the question even more intriguing. A large number of models have been suggested in the literature for providing an understanding of neutrino masses and mixing. These models cover a large number of possibilities, going from models with discrete abelian or non-abelian symmetries [1] to the suggestion that in the neutrino sector, anarchy prevails [2].

In this paper we conjecture that the observed large mixing in the lepton sector may just reflect a Majorana character of neutrinos and quasidegeneracy of neutrino masses. It is well known that, for Dirac neutrinos, leptonic mixing can be rotated away in the limit of exact neutrino mass degeneracy. For Majorana neutrinos, it has been pointed out that leptonic mixing and even CP violation can occur in the limit of exact neutrino mass degeneracy and in this limit, leptonic mixing is characterized by two angles and one CP violating phase [3]. In this limit, the leptonic unitarity triangles are collapsed in a line, since one of the entries of the leptonic mixing vanishes, thus implying no Dirac-type CP violation. However, the Majorana triangles do not all collapse into the real or imaginary axis, thus implying [4] CP violation of Majorana type. We identify the zero entry of the leptonic mixing matrix with U_{13} and show that a small perturbation around the degenerate limit generates the observed neutrino mass differences as well as leptonic mixing in agreement with experiment, including the recent measurements of the smallest mixing angle, θ_{13}, at reactor [5], and accelerator [6] neutrino experiments. In this framework, one also finds a possible explanation for the smallness of $|U_{13}|$, compared to the other entries of the U_{PMNS}. This may just reflect the fact that, in the exact degenerate limit of Majorana neutrinos, one of the entries of U_{PMNS} necessarily vanishes.

As soon as it became clear that the experimental evidence favoured a nonvanishing U_{13}, many proposals [7] were put forward in the literature analysing how small perturbations around various textures obtained from symmetries, could accommodate a non-vanishing U_{13} while also correctly reproducing the data on the solar and atmospheric mixing angles. The distinctive feature of our proposal is the fact that we start from the non-trivial limit of exactly degenerate Majorana neutrinos.

This paper is organised as follows. In the next section, we study the limit of exact degeneracy of three Majorana neutrinos, pointing out that in this limit the Majorana mass matrix is proportional to a unitary matrix and describing the implications for leptonic mixing and CP violation. In section 3, we study the lifting of the mass degeneracy with the generation of neutrino mass differences and a non-vanishing U_{13}. We analyse in detail the case where the unperturbed leptonic mixing is given by some of the most popular Ansätze, allowing for Majorana-type CP violation, with special emphasis on the tribimaximal case [8]. We consider a scenario for the breaking of the degeneracy, where there is only one CP violating phase which, upon the lifting of the degeneracy, generates both Majorana and Dirac-type CP violation. Finally, in section 4 we present our conclusions.

2 The Limit of Exact Degeneracy

2.1 The Majorana Neutrino Mass Matrix

Without loss of generality, we choose to work in a weak basis (WB) where the charged lepton mass matrix is diagonal, real and positive. We assume three left-handed neutrinos and consider a Majorana mass term with the general form:

$$\mathcal{L}_{\text{mass}} = -(\nu_{L\alpha})^T C^{-1} (M_o)_{\alpha\beta} \quad \nu_{L\beta} + \text{h.c.} \tag{1}$$

where $\nu_{L\alpha}$ stand for the left-handed weak eigenstates and M_o is a 3×3 symmetric complex mass matrix. Since in general M_o is diagonalized by a unitary matrix U_o through $U_o^T \, M_o \, U_o = diag\ (m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, it follows that in the limit of exact neutrino mass degeneracy, M_o can be written:

$$M_o = \mu \, S_o \tag{2}$$

where μ is the common neutrino mass and $S_o = U_o^* U_o^\dagger$. In the limit of exact degeneracy, a novel feature arises, namely M_o is proportional to the symmetric unitary matrix S_o. Under a WB transformation corresponding to a rephasing of both ν_L and the charged lepton fields, the neutrino mass matrix transforms as:

$$M_o \to L \, M_o \, L \tag{3}$$

with $L \equiv diag(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3})$. As a result, the individual phases of M_o have no physical meaning, but one can construct polynomials in $(M_o)_{ij}$ which are rephasing invariant [9] such as $(M_o^*)_{11}(M_o^*)_{22}(M_o^*)_{12}^2$ or $(M_o)_{11}(M_o^*)_{33}(M_o)_{13}^2$. The fact that S_o is symmetric and unitary implies that in general S_o can be parametrized by two angles and one phase. In Ref. [3], the limit of exact degeneracy for Majorana neutrinos was analysed in some detail and it was shown that leptonic mixing and even CP violation can occur in that limit. Leptonic mixing can be rotated away if and only if there is CP invariance and all neutrinos have the same CP parity [10], [11]. Furthermore, it was also shown in Ref [3] that in the case of different CP parities, the most general matrix S_o can be parametrized in terms of two rotations with three-by-three orthogonal matrices having only a two-by-two non diagonal block each, corresponding to a single mixing angle, together with one diagonal matrix with one phase:

$$S_o = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & -c_\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \tag{4}$$

this equation is of the form:

$$S_o = O_{23}(\phi) \, O_{12}(\theta) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} O_{23}(\phi) \tag{5}$$

with each orthogonal matrix O_{ij} chosen to be symmetric. Using the fact that $S_o = U_o^* U_o^\dagger$ one concludes that , in this limit, the leptonic mixing matrix is given by:

$$U_o = O_{23}(\phi) \, O_{12}\left(\frac{\theta}{2}\right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & e^{-i\frac{\alpha}{2}} \end{pmatrix} \tag{6}$$

up to an orthogonal rotation of the three degenerate neutrinos.

Given the Majorana character of neutrino masses, it is clear that even in the limit of exact degeneracy with CP conservation, but with different CP-parities, one cannot rotate U_o away through a redefinition of the neutrino fields. It should be emphasized that the leptonic mixing matrix is only defined up to an orthogonal rotation of the three degenerate neutrinos. Indeed if U_o diagonalizes M_o so does U_oO, as it is evident from Eq. (2) and the fact that $S_o = U_o^* U_o^\dagger$. Without loss of generality one can eliminate the matrix O. It is important to notice that U_o always has one zero entry which in the above parametrization appears in the (13) position. This may be a hint that the limit of exact degeneracy is a good starting point to perform a small perturbation around it, leading to the lifting of the degeneracy and the generation of a non-zero U_{e3}. At this stage, it should be noted that although the limit of exact degeneracy necessarily implies a zero entry in U_o the location of the zero is not fixed. If we had interchanged the rôles of O_{23} and O_{12} in Eq. (4), the zero entry would appear in the (31) position. Our choice of Eq. (4) was dictated by the experimental fact that the leptonic mixing matrix has a small entry in the (13) position. It should be stressed that the identification of U_{PMNS} with U_oO can only be done after the lifting of the degeneracy, which will be done in the sequel. The matrix O will be fixed by the perturbation of M_o leading to the lifting of the degeneracy. In the exact degenerate limit the individual elements of the matrix U_oO have no physical meaning. But there are physical quantities which do have physical meaning even in the exact degenerate limit. These quantities are independent of the matrix O, depending only on combinations of the angles θ, ϕ and the phase α, entering in the parametrisation of U_o given in Eq. (4). An example of such a physical quantity, will be given in the next subsection, where we evaluate in the exact degenerate limit the strength of Majorana type CP violation, expressed in terms of the mixing angles θ, ϕ and the phase α. Of course, this quantity does not depend on the matrix O.

It is easy to understand why a symmetric unitary 3×3 matrix, such as S_o, can be parametrized by only two angles and one phase. On one hand, there is the freedom of choice of WB given by Eq. (3) on the other hand for a general 3×3 unitary matrix U, one can define an asymmetry parameter, given by [12]:

$$A_s \equiv |U_{12}|^2 - |U_{21}|^2 = |U_{31}|^2 - |U_{13}|^2 = |U_{23}|^2 - |U_{32}|^2 \tag{7}$$

In the case of a unitary symmetric matrix, one has $A_s = 0$, which leads to the loss of one parameter.

The parametrization of S_o in terms of two rotations and one phase is the most general one (apart from the unphysical complex phases which can be rotated away as in Eq. (3)). This can be seen by recalling that the parametrization of a general unitary matrix through Euler angles involves three orthogonal rotations, usually denoted by O_{12}, O_{13}, O_{23}. The fact that S_o is symmetric implies the loss of one parameter and, as a result, only two orthogonal matrices are needed. The rotation matrix O_{rs} that is left out, dictates the entry of U_o that is zero to be, (r,s) or (s,r) depending on the order chosen for the other two orthogonal matrices.

2.2 S_o unitarity triangles, leptonic mixing and CP violation

Since S_o is a unitary matrix, one can consider S_o unitarity triangles, which are analogous to the ones [4] encountered in the leptonic mixing matrix U_{PMNS}, but with a different physical meaning. A unitarity triangle corresponding to orthogonality of the two first columns of S_o is displayed in Figure 1. Note that the orientation of the S_o unitarity triangles rotates under the rephasing of Eq.(3) and therefore it has no physical

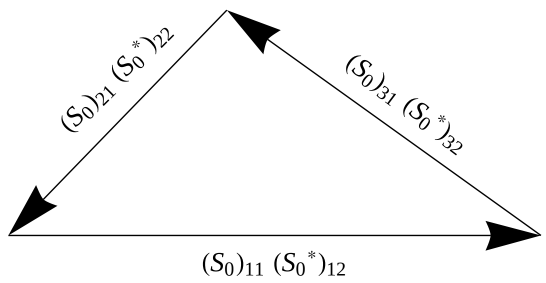

Figure 1: Unitarity triangle built from the first two columns of S_0, for a generic unitary matrix, assuming CP violation

meaning. However, the area of the S_o triangles has got physical meaning, giving a measure of the strength of Majorana-type CP violation in leptonic mixing in the case of exact degeneracy. All S_o unitarity triangles have the same area A, which equals twice the absolute value of any of the rephasing invariant quartets Q_s of S_o:

$$A = 2|\mathrm{Im} Q_s| = \frac{1}{2}\left|\cos(\theta)\sin^2(\theta)\sin^2(2\phi)\sin(\alpha)\right| \tag{8}$$

with $|Q_s| \equiv |(S_o)_{ij}(S_o)^*_{ik}(S_o)^*_{lj}(S_o)_{lk}|$ with $i \neq l,\ j \neq k$. In the limit of exact degeneracy, we have seen that the leptonic mixing matrix U_o has a zero entry, which implies that there is no Dirac-type CP violation and all the U_o unitarity triangles collapse to lines. However, there is CP violation of the Majorana-type, since the Majorana unitarity triangles for U_o are in general not collapsed along the real and imaginary axis [4]. Once the degeneracy is lifted the leptonic unitarity triangles open up and Dirac-type CP violation is generated. In Ref. [13], it was shown how to express, in this case, the full PMNS matrix, including the strength of Dirac-type CP violation in terms of arguments of the six independent rephasing invariant bilinears corresponding to the orientation of the sides of Majorana-type unitarity triangles, thus showing that Dirac-type CP violation in the leptonic sector with Majorana neutrinos, necessarily implies Majorana-type CP violation.

3 Lifting the Degeneracy

3.1 Rationale and Strategy

For definiteness and without loss of generality, we work in the weak basis where the charged lepton mass matrix is diagonal real. As emphasized in the previous section, in the case of exactly degenerate Majorana neutrinos, mixing is meaningful and it can be parametrized by two angles and one phase.

Several textures for the leptonic mixing matrix have been studied in the literature, often in the context of family symmetries [1]. In most of the proposed schemes, the pattern of leptonic mixing is predicted but the spectrum of masses is not constrained by the symmetries. It is therefore consistent to consider these schemes, together with the hypothesis of quasidegeneracy of Majorana neutrinos. A different approach connecting the leptonic mixing parameters with certain kinds of degeneracy of the neutrino mass spectrum was followed in [14].

Until recently one of the most favoured Ansätze, from the experimental point of view, seemed to be the tribimaximal mixing [8] which has a zero in the (13) entry. Other interesting textures which also have a zero entry in this location [15] include the democratic mixing [16], bimaximal mixing [17], golden ratio mixings [18], [19], hexagonal mixing [20] and bidodeca mixing [21], [22]. Recent measurements of θ_{13}, the smallest of the mixing angles of U_{PMNS} as given by the standard parametrization [23], have established a non-zero value for this angle [24]. In the standard parametrization, U_{PMNS} is given by:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P, \tag{9}$$

where $c_{ij} \equiv \cos\left(\theta_{ij}\right)$, $s_{ij} \equiv \sin\left(\theta_{ij}\right)$, with all θ_{ij} in the first quadrant, δ is a Dirac-type phase and $P = diag(1, e^{i\alpha}, e^{i\beta})$ with α and β denoting the phases associated with the Majorana character of neutrinos.

The clear experimental evidence for a non-zero θ_{13} has motivated a series of studies on how to generate a non-vanishing θ_{13} through a small perturbation of the tribimaximal and other schemes which predict $\theta_{13} = 0$ in lowest order. **The distinctive feature of our analysis**, is the fact that we start from a non-trivial limit of three exactly degenerate Majorana neutrinos. In the previous section, we presented the most general mixing matrix U_o in this limit and explained that it can be parametrized by two angles and one CP violating phase:

$$U_o = O(\theta, \phi) \cdot K \tag{10}$$

with K a diagonal matrix such as the one written in Eq. (6). This choice for the matrix K implies that in the CP conserving limit corresponding to $\alpha = 0$ or π, one neutrino has a CP parity different from the other two. Otherwise, in the limit of exact degeneracy, with CP conservation and all neutrinos having the same CP parity, the two angles ϕ and θ could be rotated away.

Lifting the degeneracy corresponds to adding a small perturbation to S_o

$$M = \mu\,(S_o + \varepsilon^2\, Q_o) \tag{11}$$

the matrix Q_o is fixed in such a way that the correct neutrino masses are obtained. It will be a function of the neutrino mass differences given in terms of Δm^2_{21} and Δm^2_{31} defined by:

$$\begin{aligned} \Delta m^2_{21} &= m^2_2 - m^2_1 \\ \Delta m^2_{31} &= |m^2_3 - m^2_1| \end{aligned} \tag{12}$$

as well as the overall mass scale μ. The parameter ε^2 is chosen as:

$$\varepsilon^2 \equiv \frac{\Delta m^2_{31}}{2\mu^2} \tag{13}$$

Quasidegeneracy forces the overall mass scale to be much larger than the neutrino mass differences and guarantees the smallness of the perturbation parameter ε^2. See Table 1 and subsequent comments.

Our strategy for confronting the data on neutrino masses and mixing is the following:

(i) We assume that the physics responsible for the lifting of the degeneracy, does not introduce new sources of CP violation beyond the phase α, already present in the limit of exact degeneracy. As a result, after the lifting of the degeneracy, the leptonic mixing matrix is given by:

$$U_{PMNS} = U_o \cdot O \tag{14}$$

where O is an orthogonal matrix, parametrized by small angles. The fact that O is orthogonal, rather than a general unitary matrix, implies that U_{PMNS} still diagonalizes S_o, thus establishing a strong connection between the degenerate and quasidegenerate case. This is particularly relevant since we shall take as starting point for U_o some of the most interesting examples considered in the literature based on symmetries and with a zero in the (13) entry of U_o.

(ii) After the lifting of the degeneracy, the single phase α will generate both Dirac and Majorana-type CP violations. This is a distinctive feature of our framework.

With the notation of Eq.(14), Q_o introduced in Eq. (11) is determined by:

$$\varepsilon^2\, Q_o = U_o^* \cdot O\, \left(\frac{1}{\mu}\, D_\nu - 1\!\mathrm{I}\right)\, O^T \cdot U_o^\dagger\,, \qquad D_\nu = diag(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) \tag{15}$$

In the limit of exact degeneracy the matrix O has no physical meaning, it only acquires meaning with the lifting of the degeneracy. A striking feature is the fact that new sources of CP violation are not introduced. However, once the matrix O is included, the CP violating phase present in K ceases to be a factorizable phase and in general gives rise to Dirac-type CP violation.

The matrix O will be parametrized by three mixing angles which we denote by:

$$O = O_{12}O_{13}O_{23} = \begin{pmatrix} c_{\phi_1} & s_{\phi_1} & 0 \\ -s_{\phi_1} & c_{\phi_1} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{\phi_3} & 0 & s_{\phi_3} \\ 0 & 1 & 0 \\ -s_{\phi_3} & 0 & c_{\phi_3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\phi_2} & s_{\phi_2} \\ 0 & -s_{\phi_2} & c_{\phi_2} \end{pmatrix} \tag{16}$$

Our choice of U_o's is based on the fact that θ_{13} is known to be a small angle. Furthermore, in each case, the resulting O matrices represent small perturbations around U_o matrices. Once the matrix O is fixed and the scale μ of neutrino masses is specified, Q_o can be computed from Eq. (15)

In our analysis, we use data from the global fit of neutrino oscillations provided in Ref. [24] requiring agreement within 1σ range. Table 1 summarizes the data obtained from Ref. [24]. From Table 1, assuming $\mu \sim 0.5$ eV, we obtain ε^2 of the order 5×10^{-3}.

In what follows we discuss separately several different cases of interest.

Table 1: Neutrino oscillation parameter summary. For Δm^2_{31}, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, and δ the upper (lower) row corresponds to normal (inverted) neutrino mass hierarchy.

Parameter	Best fit	1σ range
Δm^2_{21} $[10^{-5} eV^2]$	7.62	7.43 – 7.81
Δm^2_{31} $[10^{-3} eV^2]$	2.55	2.46 – 2.61
Δm^2_{31} $[10^{-3} eV^2]$	2.43	2.37 – 2.50
$\sin^2\theta_{12}$	0.320	0.303 – 0.336
$\sin^2\theta_{23}$	0.613 (0.427)	0.400 –0. 461 and 0.573 – 0.635
$\sin^2\theta_{23}$	0.600	0.569 – 0.626
$\sin^2\theta_{13}$	0.0246	0.0218 –0.0275
$\sin^2\theta_{13}$	0.0250	0.0223 – 0.0276
δ	0.80 π	0 –2 π
δ	-0.03 π	0 –2 π

3.2 Perturbing tribimaximal mixing

In this case our starting point is $U_o = U_{TBM} \cdot K$ with:

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad \text{and} \qquad K = diag(1, i, e^{-i\alpha/2}) \tag{17}$$

In the notation of Eq. (6), this ansatz corresponds to $\phi = 45^\circ$ and $\cos\left(\frac{\theta}{2}\right) = \frac{2}{\sqrt{6}}$ i.e., $\frac{\theta}{2} = 35.26^\circ$. We allow the angle α to vary, together with the three angles of the matrix O. In this example, agreement with the global fit for the experimental values requires lowering the values for the mixing angles θ_{12} and θ_{23} of U_o and at the same time generating a θ_{13} different from zero.

Denoting the entries of U_{PMNS} by U_{ij} we have:

$$\begin{aligned} |U_{11}| &= |\tfrac{2}{\sqrt{6}} O_{11} + \tfrac{i}{\sqrt{3}} O_{21}| = c_{12} c_{13} \\ |U_{12}| &= |\tfrac{2}{\sqrt{6}} O_{12} + \tfrac{i}{\sqrt{3}} O_{22}| = s_{12} c_{13} \\ |U_{13}| &= |\tfrac{2}{\sqrt{6}} O_{13} + \tfrac{i}{\sqrt{3}} O_{23}| = s_{13} \\ |U_{23}| &= |\tfrac{1}{\sqrt{6}} O_{13} - \tfrac{i}{\sqrt{3}} O_{23} + \tfrac{1}{\sqrt{2}}\, e^{-i\alpha/2}\, O_{33}| = s_{23} c_{13} \end{aligned} \tag{18}$$

The first three equations allow to determine ϕ_1, ϕ_3 and ϕ_2, the fourth one puts bounds on the phase α thus constraining the strength of leptonic CP violation [25]. At this stage it is worth emphasizing that there is strong experimental evidence that in the quark sector the V_{CKM} matrix is complex even if one assumes the possible presence of physics beyond the Standard Model [26]. As a result, it is natural to assume that the leptonic sector also violates CP.

This scenario allows for a particularly simple solution since, one can reach agreement with the experimental data by choosing a matrix O with only one parameter different from zero, namely the angle ϕ_2. In this case the relevant O_{ij} simplify significantly and one can express $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $\sin^2(\theta_{13})$ simply in terms of ϕ_2, and the phase α, or else. equivalently, in terms of $|U_{13}|$ and the phase α:

$$\sin^2(\theta_{13}) \equiv |U_{13}|^2 = \frac{\sin^2(\phi_2)}{3} \tag{19}$$

$$\sin^2(\theta_{12}) \equiv \sin^2(\theta_{solar}) = \frac{1-\sin^2\phi_2}{3-\sin^2\phi_2} = \frac{\frac{1}{3}-|U_{13}|^2}{1-|U_{13}|^2} \tag{20}$$

$$\sin^2(\theta_{23}) \equiv \sin^2(\theta_{atm}) = \frac{1}{2} - \frac{\sqrt{6}\sin(\frac{\alpha}{2})\sin\phi_2\cos\phi_2}{3-\sin^2\phi_2} = \frac{1}{2} - \frac{\sqrt{2}\sin(\frac{\alpha}{2})|U_{13}|\sqrt{1-3|U_{13}|^2}}{1-|U_{13}|^2} \tag{21}$$

Clearly, $|U_{13}|$ fixes the allowed range for the angle ϕ_2 and in this limit only $\sin^2(\theta_{atm})$ depends on the phase α. From Eq. (19) and taking the best fit value from Table 1 we obtain $\sin(\phi_2) = 0.27$. It is instructive to determine Q_o for this value of $\sin(\phi_2)$. Making use of Eq. (15), and keeping only the dominant terms, by making the following approximations:

$$\mu\sqrt{1+\frac{\Delta m^2_{21}}{\mu^2}} \simeq \mu \qquad ; \qquad \mu\sqrt{1+\frac{\Delta m^2_{31}}{\mu^2}} \simeq \mu\left(1+\frac{\Delta m^2_{31}}{2\mu^2}\right) = \mu(1+\varepsilon^2) \tag{22}$$

the explicit expression for Q_o simplifies to:

$$Q_o = \begin{pmatrix} -0.0243 & 0.0243-0.1061e^{i\frac{\alpha}{2}} & 0.0243+0.1061e^{i\frac{\alpha}{2}} \\ 0.0243-0.1061e^{i\frac{\alpha}{2}} & (0.1559i+0.6808e^{i\frac{\alpha}{2}})^2 & -0.0243-0.4634e^{i\alpha} \\ 0.0243+0.1061e^{i\frac{\alpha}{2}} & -0.0243-0.4634e^{i\alpha} & (0.1559i-0.6808e^{i\frac{\alpha}{2}})^2 \end{pmatrix} \tag{23}$$

It should be noticed that, even after factoring out ε^2, most entries of the matrix Q_o have modulus much smaller than one, thus confirming that we are doing a very small perturbation around the degeneracy limit.

We find that the angle ϕ_2 cannot deviate significantly from the value of the Cabibbo angle. The constraints on the phase α obtained from Eq. (21), translate into bounds for the Dirac CP violating phase δ. The strength of Dirac-type CP violation is often given in terms of the modulus of the parameter I_{CP} defined as the imaginary part of a quartet of the mixing matrix U_{PMNS}, i.e., $I_{CP} \equiv \mathrm{Im}|U_{ij}U^*_{ik}U^*_{lj}U_{lk}|$ with $i \neq l,\ j \neq k$. Due to the unitariry of U_{PMNS} all quartets have the same modulus. For the standard parametrization, given in Eq. (9), we have:

$$I_{CP} \equiv \frac{1}{8}|\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\cos(\theta_{13})\sin(\delta)| \tag{24}$$

In our framework, with only ϕ_2 and α different from zero, I_{CP} is given by:

$$I_{CP} = \left|\frac{\cos(\alpha/2)\sin(\phi_2)\cos(\phi_2)}{3\sqrt{6}}\right| \tag{25}$$

and is predicted to be of order 10^{-2}, meaning that it could be within reach of future neutrino experiments. This is a special prediction for this framework since from the values of Table 1 we can conclude that the experimental bounds at 1σ level allow for the leptonic strength of Dirac-type CP violation to range from

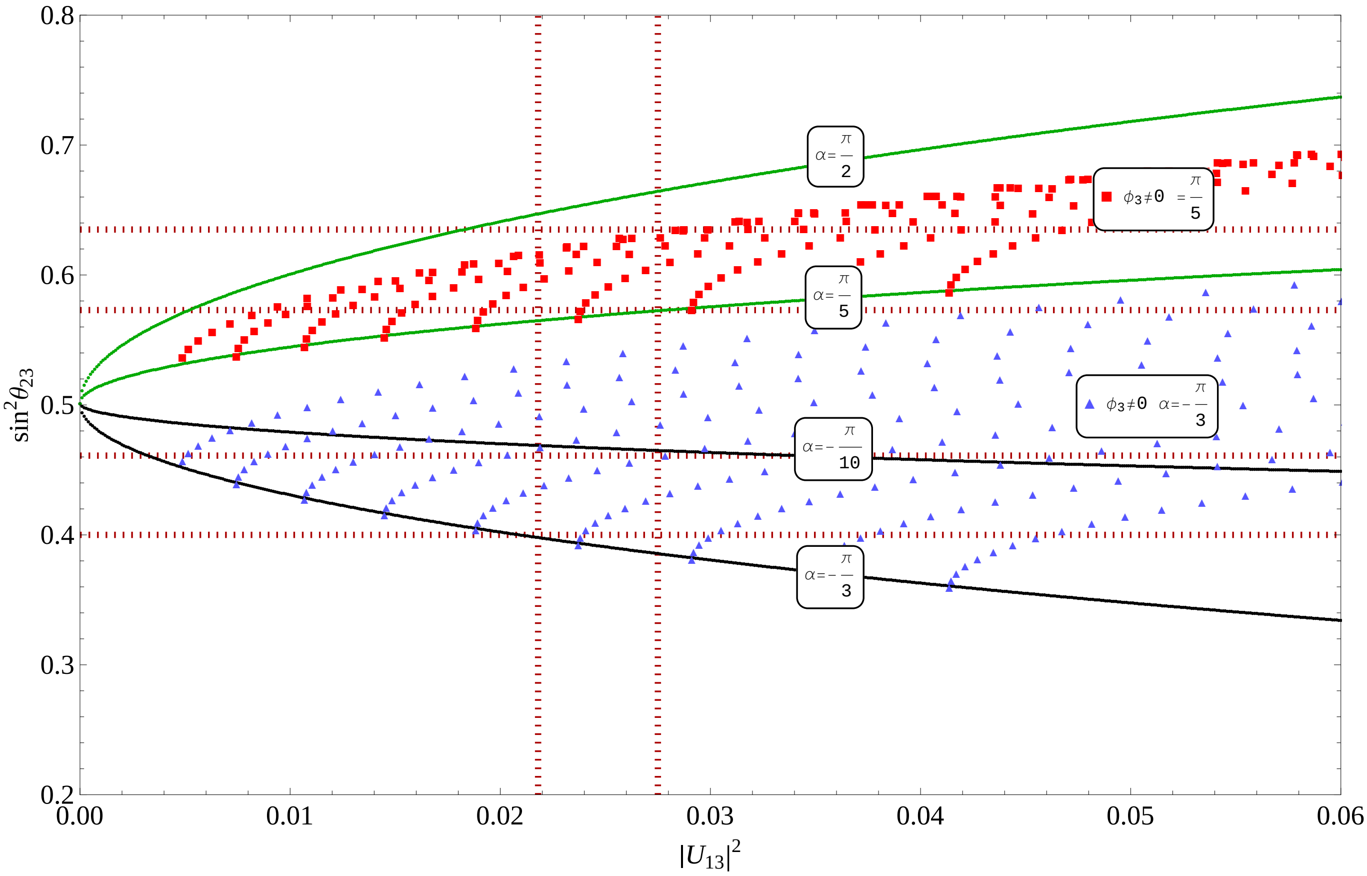

Figure 2: $\sin^2\theta_{23}$ versus $|U_{13}|^2$ obtained by perturbing tribimaximal mixing with $\phi_3 = 0$. Each curve corresponds to a fixed α and to $\phi_1 = 0$, therefore ϕ_2 is the only variable. The points drifting away from each curve were obtained by varying also ϕ_3.

0 to about 4×10^{-2}. In Figure 2 we present $\sin^2(\theta_{atm})$ versus $|U_{13}|^2$. The dotted vertical lines delimit the allowed experimental values for $|U_{13}|^2$. The dotted horizontal lines delimit the two allowed experimental regions for $\sin^2(\theta_{atm})$ according to Table 1. The authors of Ref. [24] consider the region of lower $\sin^2(\theta_{atm})$ to be experimentally favoured, therefore in our analysis we require that this region can be reached even though we also indicate the above region. The different solid lines correspond to our framework with only one parameter different from zero, the angle ϕ_2, and for different values of the phase α as indicated in the figure. The values for this phase are chosen in such a way as to give an indication of the intervals that are compatible with the experimental data. Points represented by squares and triangles where obtained with one additional mixing angle, ϕ_3, different from zero. Squares and triangles correspond to different values of the phase α respectively, as indicated in the figure. In Figure 3 we plot I_{CP} versus $|U_{13}|^2$. Again the dotted vertical lines delimit the allowed experimental values for $|U_{13}|^2$ and the different solid lines correspond to our framework with only one mixing angle different from zero and for different values of the phase α as indicated in the figure. The values chosen for this phase are based on the information contained in Figure 2. Points represented by squares and triangles where obtained with one additional mixing angle different from zero, which in this case was chosen to be ϕ_1. Squares and triangles correspond to different values of

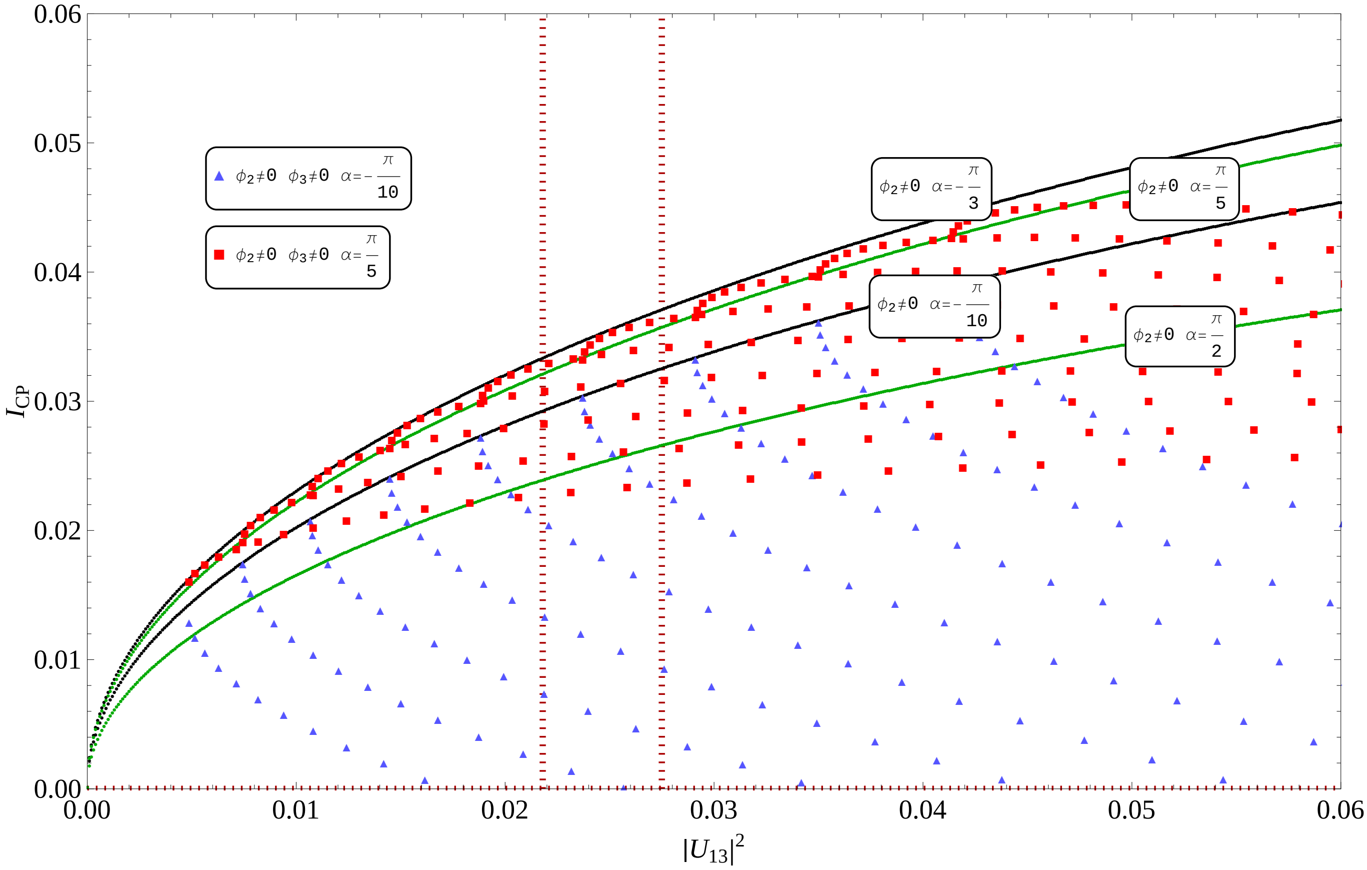

Figure 3: I_{CP} versus $|U_{13}|^2$ obtained by perturbing tribimaximal mixing with $\phi_3 = 0$. Each curve corresponds to a fixed α and to $\phi_1 = 0$, therefore ϕ_2 is the only variable. The points drifting away from each curve were obtained by varying also ϕ_3.

the phase α respectively, as indicated in the figure.

Concerning the neutrinoless double beta decay, this process depends on the effective Majorana mass, m_{ee}, defined by;

$$m_{ee} = \left| \sum_{k=1}^{3} U_{1k}^2 m_k \right| \tag{26}$$

In the above framework, the dominant terms, ignoring in particular corrections of order $\Delta m^2_{21}/\mu^2$ are:

$$\begin{aligned} m_{ee} &= \frac{\mu}{3} \left(1 - \frac{\Delta m^2_{31}}{2\mu^2} \sin^2(\phi_2)\right) \\ &= \frac{\mu}{3} \left(1 - 3\frac{\Delta m^2_{31}}{2\mu^2} |U_{13}|^2\right) \end{aligned} \tag{27}$$

Agreement with the present experimental bounds, [27] taking into account nuclear physics uncertainties [29] requires $|m_{ee}|$ to be smaller than 0.4 eV. The Heidelberg-Moscow experiment [28] claimed to have obtained a non-zero result close to 0.38 eV which would imply all three neutrino masses close to 1 eV. These masses are somewhat above the bound favoured by cosmology, however the cosmological bound depends on model assumptions and on the data set that is taken into consideration [30].

In this framework, the angle ϕ_2 cannot deviate significantly from the value of the Cabibbo angle even when we extend it to include other non-zero mixing angles. In fact, the range of the allowed experimental parameters given in Table 1 can accommodate non zero values for the two other angles in the matrix O requiring them to be smaller than the Cabibbo angle. In this case the simple expressions given above must be replaced by somewhat more cumbersome and less transparent ones. The solar angle obtained in the unperturbed tribimaximal mixing case is larger than the allowed experimental values. The angle ϕ_2 is the only one in O capable of lowering its value. The effect of the other two mixing angles is the opposite.

3.3 Perturbing other interesting schemes

As stated before, we analysed perturbations around some of the well known mixing textures considered in the literature with a zero in the (13) entry. Examples of such textures include the democratic mixing, U_{DM} [16], bimaximal mixing U_{BM} [17], golden ratio mixings U_{GRM1} [18], U_{GRM2} [19], hexagonal mixing U_{HM} [20] and bidodeca mixing U_{BDM} [21], [22]. The democratic mixing and the bimaximal mixing are of the form:

$$U_{DM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}; \qquad U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

These two cases are very constrained in our framework, since they correspond to $\sin^2(\theta_{sol}) = 0.5$ which lies significantly above the favoured experimental range given in Table 1. Although it is still possible to bring it down to acceptable values making use of ϕ_2, agreement with the experimental values given in Table 1 is hardly possible at 1σ level. Therefore, we do not further analyse these two cases.

The other textures mentioned above are U_{GRM1}, U_{GRM2} and the hexagonal mixing U_{HM} which coincides with the bidodeca mixing U_{BDM}:

$$U_{GRM1} = \begin{pmatrix} \sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)} & \sqrt{\frac{2}{5+\sqrt{5}}} & 0 \\ -\frac{1}{\sqrt{5+\sqrt{5}}} & \frac{1}{2}\sqrt{1+\frac{1}{\sqrt{5}}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{5+\sqrt{5}}} & -\frac{1}{2}\sqrt{1+\frac{1}{\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix}; \quad U_{GRM2} = \begin{pmatrix} \frac{1}{4}(1+\sqrt{5}) & \frac{1}{2}\sqrt{\frac{1}{2}(5-\sqrt{5})} & 0 \\ -\frac{1}{4}\sqrt{5-\sqrt{5}} & \frac{1+\sqrt{5}}{4\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{4}\sqrt{5-\sqrt{5}} & -\frac{1+\sqrt{5}}{4\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_{HM} = U_{BDM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

The case of the golden ratio mixing 2 is less favourable than the golden ratio mixing 1, due to the fact that the corresponding solar angle is larger. We analysed in more detail only the cases starting with U_{GRM1} and U_{HM} . We have scanned the allowed region of parameter space for the angles ϕ_1, ϕ_3, ϕ_2 of our perturbation, and for the phase α. Both examples have very similar features. The exact analytic expressions are obtained

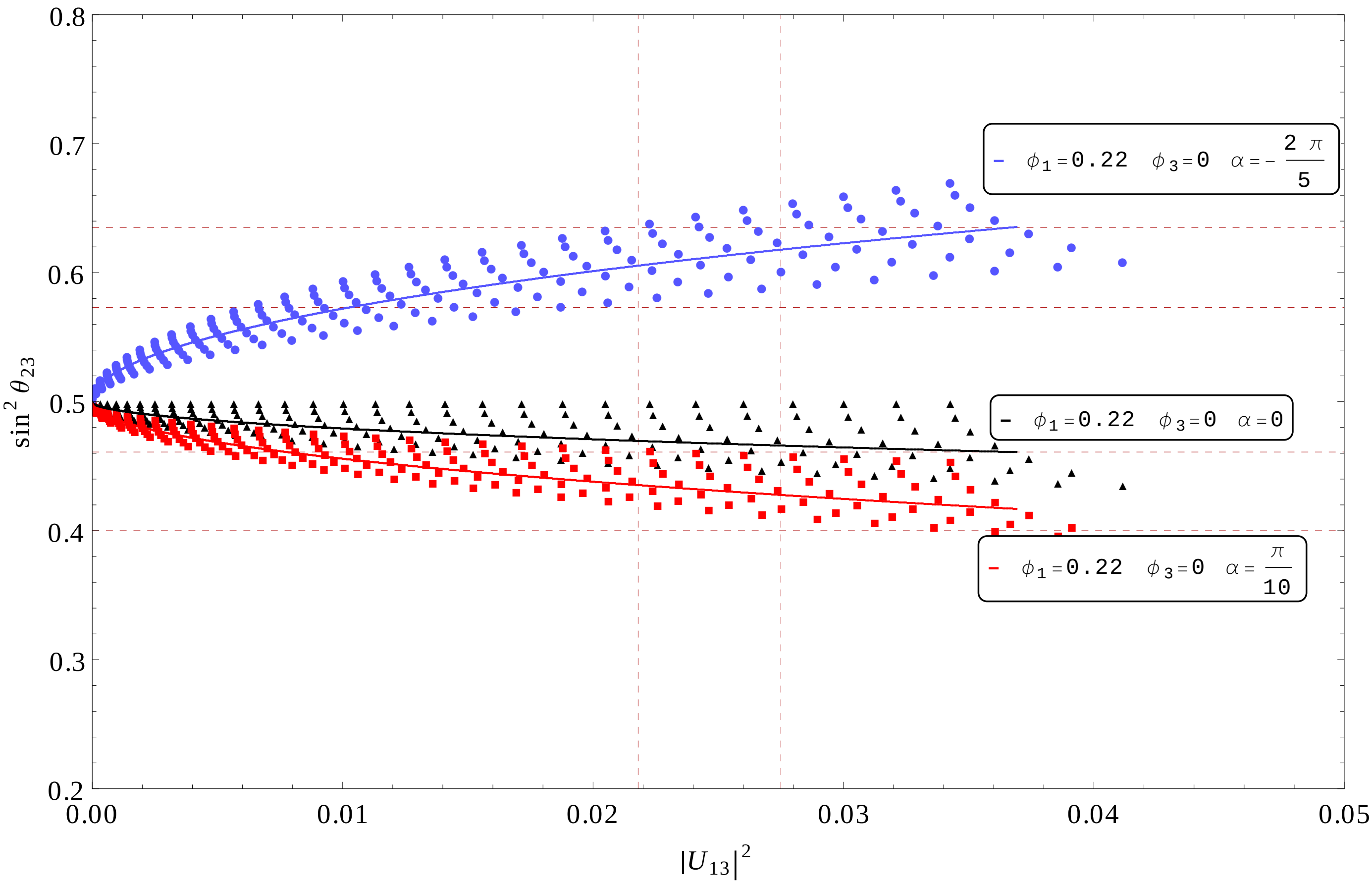

Figure 4: $\sin^2\theta_{23}$ versus $|U_{13}|^2$ obtained by perturbing golden ratio 1 with $\phi_3 = 0$. Each curve corresponds to a fixed α and fixed value of ϕ_1. The points drifting away from each curve were obtained by varying ϕ_1.

from Eqs. (14) and (16). A novel feature of these examples is the fact that agreement with experiment cannot be obtained with the matrix O parametrized by one mixing angle only. Furthermore, unlike the tribimaximal mixing case, it is ϕ_1 that is required to differ from zero and on the other hand either ϕ_2 or ϕ_3 can be zero, although not simultaneously. These new features are related to the fact that in both cases the corresponding solar angle lies below the experimental range unlike in the tribimaximal case. As pointed out, in the tribimaximal case the angle ϕ_2 played a fundamental rôle in lowering this angle. In the case of ϕ_3 equal to zero, U_{13} is then given by:

$$U_{13} = (U_o)_{11}\sin(\phi_1)\sin(\phi_2) + (U_o)_{12}\cos(\phi_1)\sin(\phi_2) \tag{28}$$

it is the second term that gives the dominant contribution. The fact that there are two independent parameters in the matrix O does not allow to express $\sin^2(\theta_{solar})$, $\sin^2(\theta_{atm})$ and I_{CP} in terms of $|U_{13}|$ only. However it is still instructive to plot these quantities as a function of $|U_{13}|$ for certain choices of the parameters of the matrix O. For illustration, we present in Figure 4 $\sin^2(\theta_{23})$ versus $|U_{13}|^2$ with $\sin(\phi_3) = 0$. This plot is done for golden ratio 1. The hexagonal mixing case presents similar features. Each curve in the figure corresponds to a fixed value of the parameter ϕ_1 and of the phase α and is therefore obtained by varying

ϕ_2. The points drifting away from each curve were obtained by varying in turn ϕ_1 still keeping α fixed and $\phi_3 = 0$. Circles, triangles and squares are associated to different choices of the phase α, respectively, as indicated in the figure. The figure shows that it is possible to accommodate $\alpha = 0$, corresponding to the CP conserving case, however agreement with experiment in this case is only possible for a small range of the parameter space. On the other hand, fixing $\phi_3 = 0$ allows both ϕ_1 and ϕ_2 to be close to the Cabibbo angle.

4 Conclusions

In this paper, we present a novel proposal for the understanding of the observed pattern of leptonic mixing, which relies on the assumption that neutrinos are Majorana particles. It is argued that the observed large leptonic mixing may arise from a quasidegeneracy of three Majorana neutrinos. The essential point is the fact that the limit of exact mass degeneracy of three Majorana neutrinos is non-trivial as lepton mixing and even CP violation can arise. This limit is particularly interesting since in this case leptonic mixing can be parametrized by only two mixing angles and one phase, implying that without loss of generality the leptonic mixing matrix can be written with one zero entry. We have then conjectured that the smallness of $|U_{13}|$ when compared to the other elements of U_{PMNS} may result from this fact. We show that the observed pattern of mixing and neutrino mass differences can be generated through a small perturbation of the exact degenerate case, without the introduction of additional CP violating sources. A key point in our work is the assumption that the physics responsible for the lifting of the degeneracy does not introduce new sources of CP violation. Our perturbation requires the multiplication on the right by an orthogonal matrix. The resulting unitary matrix U_oO which can be identified as the U_{PMNS} matrix, also diagonalizes the neutrino mass matrix in the fully degenerate case. This allows to establish a strong connection between the degenerate and quasidegenerate cases and at the same time reducing the number of free parameters. Upon the lifting of degeneracy, this single phase generates both Majorana and Dirac-type CP violation in the leptonic sector. For definiteness, we have used as the starting point for the perturbation around the limit of exact degeneracy, some of the most interesting Ansätze considered in the literature, which were proposed in the past assuming $\theta_{13} = 0$. We analyse correlations among physical observables, and point out that in most of the cases considered, the implied strength of leptonic Dirac-type CP violation is large enough to be detectable in the next round of experiments.

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References

[1] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada and M. Tanimoto, Prog. Theor. Phys. Suppl. **183** (2010) 1 [arXiv:1003.3552 [hep-th]]; S. F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, New J. Phys. **16** (2014) 045018 [arXiv:1402.4271 [hep-ph]]. R. M. Fonseca and W. Grimus, arXiv:1405.3678 [hep-ph]; For a recent review and further references see: G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82** (2010) 2701 [arXiv:1002.0211 [hep-ph]].

[2] L. J. Hall, H. Murayama, and N. Weiner, Phys. Rev. Lett. 84 (2000) 2572, [hep-ph/9911341]; N. Haba and H. Murayama, Phys. Rev. D63 (2001) 053010, [hep-ph/0009174]; A. de Gouvea and H. Murayama, Phys. Lett. B573 (2003) 94, [hep-ph/0301050]; Guido Altarelli, Ferruccio Feruglio, Isabella Masina, Luca Merlo, JHEP 1211 (2012) 139; Andre de Gouvea and Hitoshi Murayama, arXiv:1204.1249.

[3] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Rev. Lett. **82** (1999) 683 [hep-ph/9810328].

[4] J. A. Aguilar-Saavedra and G. C. Branco, Phys. Rev. D **62** (2000) 096009 [hep-ph/0007025].

[5] F. P. An *et al.* [DAYA-BAY Collaboration], Phys. Rev. Lett. **108** (2012) 171803 [arXiv:1203.1669 [hep-ex]]; J. K. Ahn *et al.* [RENO Collaboration], Phys. Rev. Lett. **108** (2012) 191802 [arXiv:1204.0626 [hep-ex]]; Y. Abe *et al.* [Double Chooz Collaboration], Phys. Rev. D **86** (2012) 052008 [arXiv:1207.6632 [hep-ex]]; F. P. An *et al.* [Daya Bay Collaboration], Chin. Phys. C **37** (2013) 011001 [arXiv:1210.6327 [hep-ex]].

[6] K. Abe *et al.* [T2K Collaboration], Phys. Rev. D **88** (2013) 032002 [arXiv:1304.0841 [hep-ex]].

[7] K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552** (2003) 207 [hep-ph/0206292]. A. Datta, L. Everett and P. Ramond, Phys. Lett. B **620** (2005) 42 [hep-ph/0503222]; G. C. Branco, R. Gonzalez Felipe, M. N. Rebelo and H. Serodio, Phys. Rev. D **79** (2009) 093008 [arXiv:0904.3076 [hep-ph]]; E. Ma and D. Wegman, Phys. Rev. Lett. 107 (2011) 061803 [arXiv:1106.4269]; X. -G. He and A. Zee, Phys. Rev. D **84** (2011) 053004 [arXiv:1106.4359 [hep-ph]]; S. Zhou, Phys. Lett. B **704** (2011) 291 [arXiv:1106.4808 [hep-ph]]; P. S. Bhupal Dev, R. N. Mohapatra and M. Severson, Phys. Rev. D **84** (2011) 053005 [arXiv:1107.2378 [hep-ph]]; R. d. A. Toorop, F. Feruglio and C. Hagedorn, Phys. Lett. B **703** (2011) 447 [arXiv:1107.3486 [hep-ph]]; S. -F. Ge, D. A. Dicus and W. W. Repko, Phys. Rev. Lett. **108** (2012) 041801 [arXiv:1108.0964 [hep-ph]]; D. A. Eby and P. H. Frampton, Phys. Rev. D **86** (2012) 117304 [arXiv:1112.2675 [hep-ph]]; P. S. Bhupal Dev, B. Dutta, R. N. Mohapatra and M. Severson, Phys. Rev. D **86** (2012) 035002 [arXiv:1202.4012 [hep-ph]]; G. C. Branco, R. G. Felipe, F. R. Joaquim and H. Serodio, Phys. Rev. D **86** (2012) 076008 [arXiv:1203.2646 [hep-ph]]; I. de Medeiros Varzielas and G. G. Ross, JHEP **1212** (2012) 041 [arXiv:1203.6636 [hep-ph]]; D. Hernandez and A. Y. Smirnov, Phys. Rev. D **86** (2012) 053014 [arXiv:1204.0445 [hep-ph]]; P. M. Ferreira, W. Grimus, L. Lavoura and P. O. Ludl, JHEP **1209** (2012) 128 [arXiv:1206.7072 [hep-ph]]; S. M. Boucenna, S. Morisi, M. Tortola and J. W. F. Valle, Phys. Rev. D **86** (2012) 051301 [arXiv:1206.2555 [hep-ph]]; G. Altarelli, F. Feruglio, I. Masina and L. Merlo, JHEP **1211** (2012) 139 [arXiv:1207.0587 [hep-ph]]; S. Bhattacharya, E. Ma, A. Natale and D. Wegman, Phys. Rev. D **87** (2013) 013006 [arXiv:1210.6936 [hep-ph]]; F. Feruglio, C. Hagedorn and R. Ziegler, JHEP **1307** (2013) 027 [arXiv:1211.5560 [hep-ph]]; D. Hernandez and A. Y. Smirnov, Phys. Rev. D **87** (2013)

053005 [arXiv:1212.2149 [hep-ph]]; B. Wang, J. Tang and X. -Q. Li, Phys. Rev. D **88** (2013) 073003 [arXiv:1303.1592 [hep-ph]]; D. V. Forero, S. Morisi, J. C. Romao and J. W. F. Valle, Phys. Rev. D **88** (2013) 016003 [arXiv:1305.6774 [hep-ph]]. Y. Shimizu and M. Tanimoto, arXiv:1405.1521 [hep-ph]; S. F. King and C. Luhn, Rept. Prog. Phys. **76** (2013) 056201 [arXiv:1301.1340 [hep-ph]], and references therein.

[8] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530** (2002) 167 [hep-ph/0202074].

[9] Y. Farzan and A. Y. .Smirnov, JHEP **0701** (2007) 059 [hep-ph/0610337].

[10] L. Wolfenstein, Phys. Lett. B **107** (1981) 77.

[11] G. C. Branco, L. Lavoura and J. P. Silva, Int. Ser. Monogr. Phys. **103** (1999) 1.

[12] G. C. Branco and P. A. Parada, Phys. Rev. D **44** (1991) 923.

[13] G. C. Branco and M. N. Rebelo, Phys. Rev. D **79** (2009) 013001 [arXiv:0809.2799 [hep-ph]].

[14] D. Hernandez and A. Y. Smirnov, Phys. Rev. D **88** (2013) 093007 [arXiv:1304.7738 [hep-ph]].

[15] B. Wang, J. Tang and X. -Q. Li, Phys. Rev. D **88** (2013) 073003 [arXiv:1303.1592 [hep-ph]].

[16] H. Fritzsch and Z. -Z. Xing, Phys. Lett. B **372** (1996) 265 [hep-ph/9509389].

[17] V. D. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Lett. B **437** (1998) 107 [hep-ph/9806387].

[18] Y. Kajiyama, M. Raidal and A. Strumia, Phys. Rev. D **76** (2007) 117301 [arXiv:0705.4559 [hep-ph]].

[19] W. Rodejohann, Phys. Lett. B **671** (2009) 267 [arXiv:0810.5239 [hep-ph]].

[20] C. H. Albright, A. Dueck and W. Rodejohann, Eur. Phys. J. C **70** (2010) 1099 [arXiv:1004.2798 [hep-ph]].

[21] J. E. Kim and M. -S. Seo, JHEP **1102** (2011) 097 [arXiv:1005.4684 [hep-ph]].

[22] J. E. Kim and M. -S. Seo, Int. J. Mod. Phys. A **27** (2012) 1250017 [arXiv:1106.6117 [hep-ph]].

[23] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86** (2012) 010001.

[24] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D **86** (2012) 073012 [arXiv:1205.4018 [hep-ph]].

[25] For a recent review see: G. C. Branco, R. G. Felipe and F. R. Joaquim, Rev. Mod. Phys. **84** (2012) 515 [arXiv:1111.5332 [hep-ph]].

[26] F. J. Botella, G. C. Branco, M. Nebot and M. N. Rebelo, Nucl. Phys. B **725** (2005) 155 [hep-ph/0502133].

[27] H. V. Klapdor-Kleingrothaus, A. Dietz, L. Baudis, G. Heusser, I. V. Krivosheina, S. Kolb, B. Majorovits and H. Pas *et al.*, Eur. Phys. J. A **12** (2001) 147 [hep-ph/0103062]; C. E. Aalseth *et al.* [IGEX Collaboration], Phys. Rev. D **65** (2002) 092007 [hep-ex/0202026]; M. Auger *et al.* [EXO Collaboration], Phys. Rev. Lett. **109** (2012) 032505 [arXiv:1205.5608 [hep-ex]]; A. Gando *et al.* [KamLAND-Zen Collaboration], Phys. Rev. Lett. **110** (2013) 062502 [arXiv:1211.3863 [hep-ex]]; M. Agostini *et al.* [GERDA Collaboration], Phys. Rev. Lett. **111** (2013) 122503 [arXiv:1307.4720 [nucl-ex]].

[28] See H. V. Klapdor-Kleingrothaus, et. al. in [27]

[29] J. D. Vergados, H. Ejiri and F. Simkovic, Rept. Prog. Phys. **75** (2012) 106301 [arXiv:1205.0649 [hep-ph]].

[30] Y. Y. Y. Wong, Ann. Rev. Nucl. Part. Sci. **61** (2011) 69 [arXiv:1111.1436 [astro-ph.CO]].