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# Nonzero $\theta_{13}$ for neutrino mixing in the context of $A_4$ symmetry

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## Abstract

In the original 2004 paper which first derived tribimaximal mixing in the context of  $A_4$ , i.e. the non-Abelian finite symmetry group of the tetrahedron, as its simplest application, it was also pointed out how  $\theta_{13} \neq 0$  may be accommodated. On the strength of the new T2K result that  $0.03(0.04) \leq \sin^2 2\theta_{13} \leq 0.28(0.34)$  for  $\delta_{CP} = 0$  and normal (inverted) neutrino mass hierarchy, we perform a more detailed analysis of how this original idea may be realized in the context of  $A_4$ .

Neutrino oscillations require nonzero neutrino masses as well as nonzero neutrino mixing angles. The current combined world data imply [1]

$$7.05 \times 10^{-5} \text{ eV}^2 \leq \Delta m_{21}^2 \leq 8.34 \times 10^{-5} \text{ eV}^2, \quad (1)$$

$$2.07 \times 10^{-3} \text{ eV}^2 \leq \Delta m_{32}^2 \leq 2.75 \times 10^{-3} \text{ eV}^2, \quad (2)$$

$$0.36 \leq \sin^2 \theta_{23} \leq 0.67, \quad 0.25 \leq \sin^2 \theta_{12} \leq 0.37, \quad (3)$$

$$\sin^2 \theta_{13} \leq 0.035 \text{ (90\% CL).} \quad (4)$$

However, the T2K Collaboration recently announced that a new measurement [2] has yielded a nonzero  $\theta_{13}$  at 90% confidence level, i.e.

$$0.03(0.04) \leq \sin^2 2\theta_{13} \leq 0.28(0.34) \quad (5)$$

for  $\delta_{CP} = 0$  and normal (inverted) neutrino mass hierarchy.

For several years now, the mixing matrix  $U_{l\nu}$  linking the charged leptons ( $e, \mu, \tau$ ) to the neutrino mass eigenstates ( $\nu_1, \nu_2, \nu_3$ ) has often been assumed to be of tribimaximal form [3], i.e.

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad (6)$$

which predicts  $\theta_{13} = 0$ . This is particularly appealing because it was derived in 2004 [4] from the simple application of the symmetry group  $A_4$ , first used for understanding maximal  $\nu_\mu - \nu_\tau$  mixing in 2001 [5]. However, even in that original 2004 paper [4], the possibility of  $\theta_{13} \neq 0$  was already anticipated. Although the new T2K result [2] is only  $2.5\sigma$  away from zero, it is the most solid experimental indication to date of this possibility. Here we offer a more detailed analysis of how  $\theta_{13} \neq 0$  may be realized in the context of  $A_4$ .

As is well-known,  $A_4$  is the group of the even permutation of 4 objects. It is also the symmetry of the perfect three-dimensional tetrahedron [6]. It has 12 elements and 4 irreducible

representations:  $\underline{1}$ ,  $\underline{1}'$ ,  $\underline{1}''$ ,  $\underline{3}$ , with the multiplication rule

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \quad (7)$$

The first step in understanding neutrino mixing is to show that  $A_4$  allows the charged-lepton mass matrix to be diagonalized by the Cabibbo-Wolfenstein matrix [7, 8]

$$U_{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \quad (8)$$

where  $\omega = e^{2\pi i/3} = -1/2 + i\sqrt{3}/2$ , with three independent eigenvalues, i.e.  $m_e, m_\mu, m_\tau$ . This has been achieved in two ways. One is the original proposal of 2001 [5]. The other was discovered later in 2006 [9]. In the former, the lepton assignments are  $L_i = (\nu_i, l_i) \sim \underline{3}$ ,  $l_1^c \sim \underline{1}$ ,  $l_2^c \sim \underline{1}'$ ,  $l_3^c \sim \underline{1}''$ , with 3 Higgs doublets  $\Phi_i = (\phi_i^0, \phi_i^-) \sim \underline{3}$ . In the latter, they are  $L_i = (\nu_i, l_i) \sim \underline{3}$ ,  $l_i^c \sim \underline{3}$ , with 4 Higgs doublets  $\Phi_i = (\phi_i^0, \phi_i^-) \sim \underline{3}$ ,  $\Phi_0 \sim \underline{1}$ . Assuming  $v_1 = v_2 = v_3$  for the vacuum expectation values of  $\Phi_i$ , which correspond to a  $Z_3$  residual symmetry (lepton triality) [10, 11, 12, 13], the seemingly impossible result of a diagonal charged-lepton matrix is always obtained from  $U_{CW}$  of Eq.(8), independent of the values of  $m_e, m_\mu, m_\tau$ . This is a highly nontrivial result, which motivates how the otherwise arbitrary  $3 \times 3$  neutrino mass should be organized. It argues strongly for an underlying non-Abelian symmetry with a three-dimensional irreducible representation, the smallest of which is  $A_4$ .

We now consider the neutrino mass matrix in the original  $A_4$  basis. Let there be 6 heavy Higgs triplets [14]:

$$\xi_1 \sim \underline{1}, \quad \xi_2 \sim \underline{1}', \quad \xi_3 \sim \underline{1}'', \quad \xi_i \sim \underline{3} \quad (i = 4, 5, 6), \quad (9)$$

where  $\xi_i = (\xi_i^{++}, \xi_i^+, \xi_i^0)$ . Then

$$\mathcal{M}_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}, \quad (10)$$

where  $a$  comes from  $\langle \xi_1^0 \rangle$ ,  $b$  from  $\langle \xi_2^0 \rangle$ ,  $c$  from  $\langle \xi_3^0 \rangle$ ,  $d$  from  $\langle \xi_4^0 \rangle$ ,  $e$  from  $\langle \xi_5^0 \rangle$ ,  $f$  from  $\langle \xi_6^0 \rangle$ . As it stands, there is of course no prediction at all. For a pattern to emerge, the way  $A_4$  breaks into its subgroups must be considered. For  $b = c$  and  $e = f = 0$ , which breaks  $A_4$  to  $Z_2$ , the neutrino mass matrix, written in the basis where the charged-lepton mass matrix is diagonal, is given by

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_{CW}^\dagger \mathcal{M}_\nu U_{CW}^* = \begin{pmatrix} a + (2d/3) & b - (d/3) & b - (d/3) \\ b - (d/3) & b + (2d/3) & a - (d/3) \\ b - (d/3) & a - (d/3) & b + (2d/3) \end{pmatrix}, \quad (11)$$

which is indeed diagonalized by  $U_{TB}$  of Eq.(6), with eigenvalues  $m_1 = a - b + d$ ,  $m_2 = a + 2b$ , and  $m_3 = -a + b + d$ . It has been shown [15] how this pattern is obtained from  $A_4$  alone with the help of lepton number.

Deviations from tribimaximal mixing may be obtained for  $b \neq c$ . This will allow  $\nu_1$  to mix with  $\nu_3$  and  $\theta_{13}$  becomes nonzero. However, this same mixing will move  $\theta_{12}$  to a larger value [4] so that  $\tan^2 \theta_{12} > 0.5$  which is not favored by current data. To allow  $\tan^2 \theta_{12} < 0.5$ , it was proposed [4] that  $e = -f \neq 0$  in Eq.(10). This is maintained by an assumed residual symmetry of the  $\xi\Phi\Phi$  soft terms of the Higgs potential under which  $\xi_5 \leftrightarrow -\xi_6$  and  $\Phi_2 \leftrightarrow \Phi_3$ . As a result, the neutrino mass matrix under  $U_{TB}$  is no longer diagonal, but is given by [4]

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} m_1 & 0 & m_4 \\ 0 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix}, \quad (12)$$

where  $m_1 = a - (b + c)/2 + d$ ,  $m_2 = a + b + c$ ,  $m_3 = -a + (b + c)/2 + d$ ,  $m_4 = \sqrt{3}/2(c - b)$  and  $m_5 = -i\sqrt{2}e$ . If  $m_4 = 0$ , then  $\nu_2$  mixes with  $\nu_3$  and it can be shown that the allowed range of  $\theta_{23}$  from Eq.(3) implies  $\sin^2 2\theta_{13} \leq 0.04$  which lies on the outer edge of the allowed region of Eq.(5). In the following, we consider both  $m_4$  and  $m_5$  to be nonzero and study various numerical solutions to the T2K data.

The atmospheric neutrino mixing is assumed to be maximal, i.e.  $\sin^2 2\theta_{23} = 1$ , which is also the assumption of T2K in obtaining their new result. The solar neutrino mixing is

taken to be  $\sin^2 2\theta_{12} = 0.87 \pm 0.3$  [1]. We also use  $\Delta m_{32}^2 = 2.40 \times 10^{-3}$  eV<sup>2</sup> which is the value used by T2K, and  $\Delta m_{21}^2 = 7.65 \times 10^{-5}$  eV<sup>2</sup>. For the central value of  $\theta_{12} = 34.43^\circ$ , we have  $\tan^2 \theta_{12} = 0.47$  which is rather close to the tribimaximal prediction of 0.5. Using this and assuming the central value of  $\sin \theta_{13} = 0.168$  ( $\sin^2 \theta_{13} = 0.11$ ), the zero entry of the neutrino mass matrix of Eq.(12) implies the condition

$$0.007655m'_1 - 0.020990m'_2 + 0.013342m'_3 = 0, \quad (13)$$

where  $m'_{1,2,3}$  are the mass eigenvalues of Eq.(12). Hence they are related to the measured  $\Delta m_{32}^2$  and  $\Delta m_{21}^2$  by

$$m'_2 = \pm \sqrt{m'_1{}^2 + \Delta m_{21}^2}, \quad (14)$$

$$m'_3 = \pm \sqrt{m'_1{}^2 + \Delta m_{21}^2/2 \pm \Delta m_{32}^2}. \quad (15)$$

There is only one solution to Eq.(13), i.e.

$$m'_1 = 0.0246 \text{ eV}, \quad m'_2 = -0.0261 \text{ eV}, \quad m'_3 = -0.0552 \text{ eV}, \quad (16)$$

which exhibits normal mass hierarchy. From this solution, we then obtain  $m_{1,2,3,4,5}$  and the original  $A_4$  parameters  $a, b, c, d, e$ . The  $\nu_e$  mass observed in nuclear beta decay is given by  $\sum_i |U_{ei}|^2 |m'_i| = 0.026$  eV. The effective mass  $m_{ee}$  for neutrinoless double beta decay is

$$m_{ee} = |a + (2/3)d| = |(2/3)m_1 + (1/3)m_2|. \quad (17)$$

We plot in Figs. 1 to 3 the solutions for  $|m'_{1,2,3}|$  and  $m_{ee}$  as a function of  $\sin^2 2\theta_{13}$  in the range 0.03 to 0.135 [corresponding to the upper bound given in Eq.(4)] for  $\sin^2 \theta_{23} = 1$  and the values  $\sin^2 2\theta_{12} = 0.84, 0.87, 0.90$ . Thus  $m_{ee}$  is predicted to be at most 0.04 eV. As for the  $\nu_e$  mass in nuclear beta decay, it can be read off as approximately given by  $(2|m'_1| + |m'_2|)/3$ . We also plot in Fig. 4 the values of  $m_{1,2,3,4,5}$  for  $\sin^2 2\theta_{12} = 0.87$ . This shows that  $m_4$  and  $m_5$ , i.e. the parameters of  $A_4$  which deviate from tribimaximal mixing, are indeed small. In

terms of  $A_4$  symmetry, the following breaking patterns are in effect: in the charged-lepton sector,  $A_4$  breaks to  $Z_3$  (which may be verified experimentally from Higgs-boson decay [13]); in the neutrino sector,  $A_4$  breaks first to  $Z_2$  (the tribimaximal limit), and then  $Z_2$  is also broken with the pattern  $b \neq c$  and  $f = -e$ , which may be maintained by a suitably chosen set of soft terms.

In conclusion, on the strength of the recent observation [2] of a nonzero  $\theta_{13}$  for neutrino mixing, the original  $A_4$  proposal of 2004 [4] is updated. We find that solutions are indeed possible with the most recent data but only in a normal hierarchy of neutrino masses, i.e.  $|m'_1| < |m'_2| < |m'_3|$ . We confirm that the parameters of  $A_4$  which deviate from tribimaximal mixing, i.e.  $m_4$  and  $m_5$ , are indeed small. We also make predictions on the effective  $m_{ee}$  in neutrinoless double beta decay.

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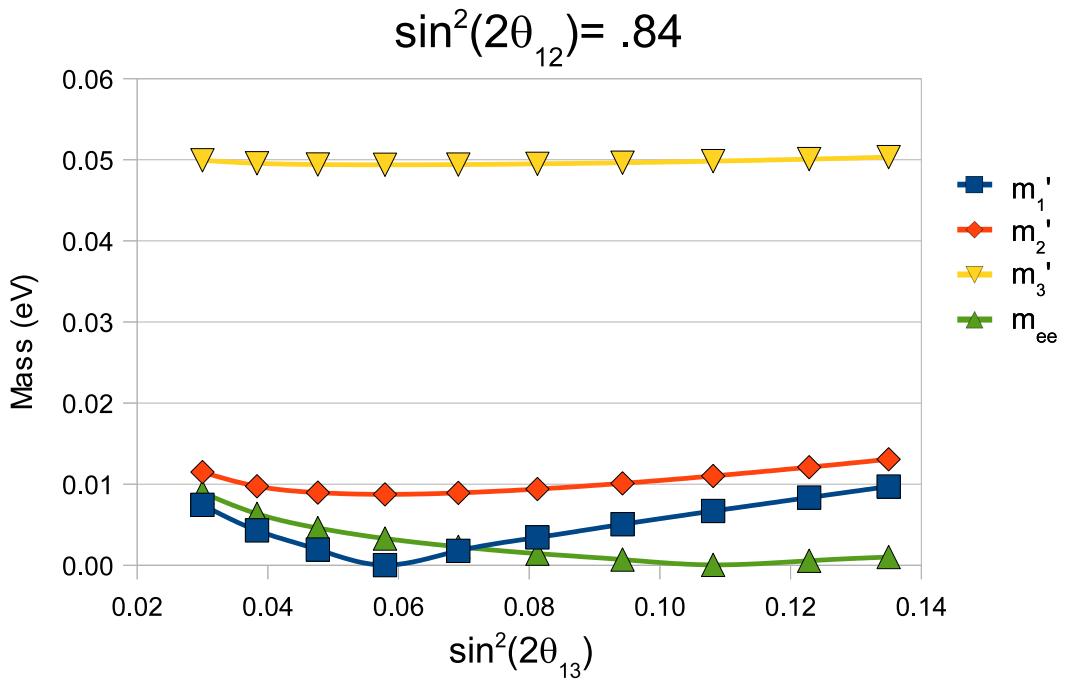


Figure 1: Physical neutrino masses  $|m'_{1,2,3}|$  and the effective  $m_{ee}$  for neutrinoless double beta decay of this model in the range  $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$  for  $\sin^2 2\theta_{23} = 1$  and  $\sin^2 2\theta_{12} = 0.84$ .

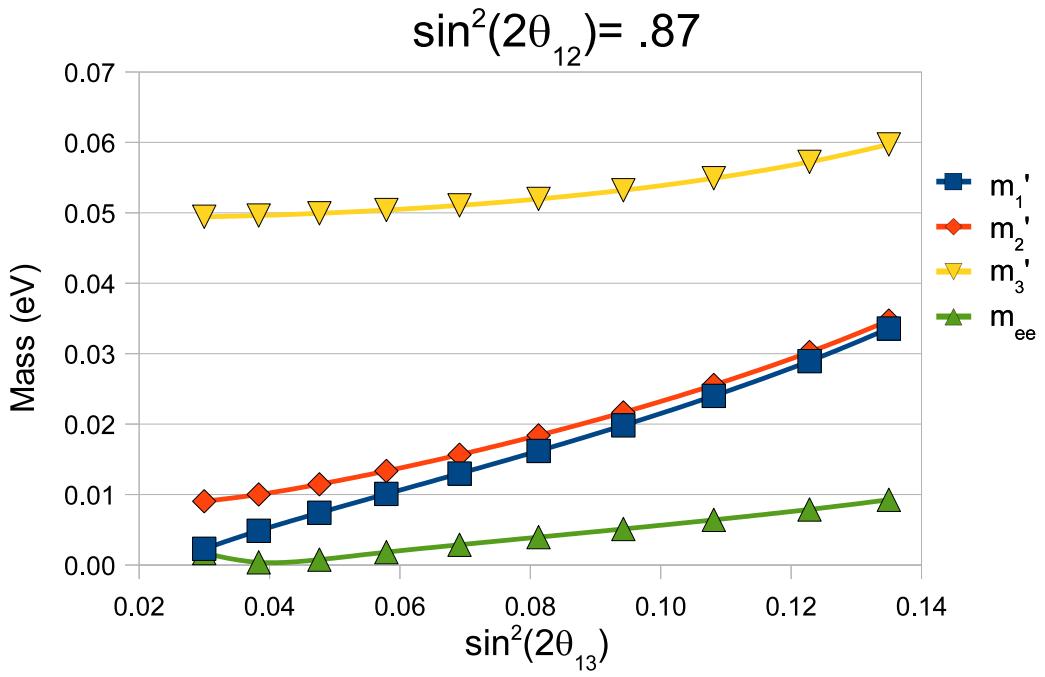


Figure 2: Physical neutrino masses  $|m'_{1,2,3}|$  and the effective  $m_{ee}$  for neutrinoless double beta decay of this model in the range  $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$  for  $\sin^2 2\theta_{23} = 1$  and  $\sin^2 2\theta_{12} = 0.87$ .

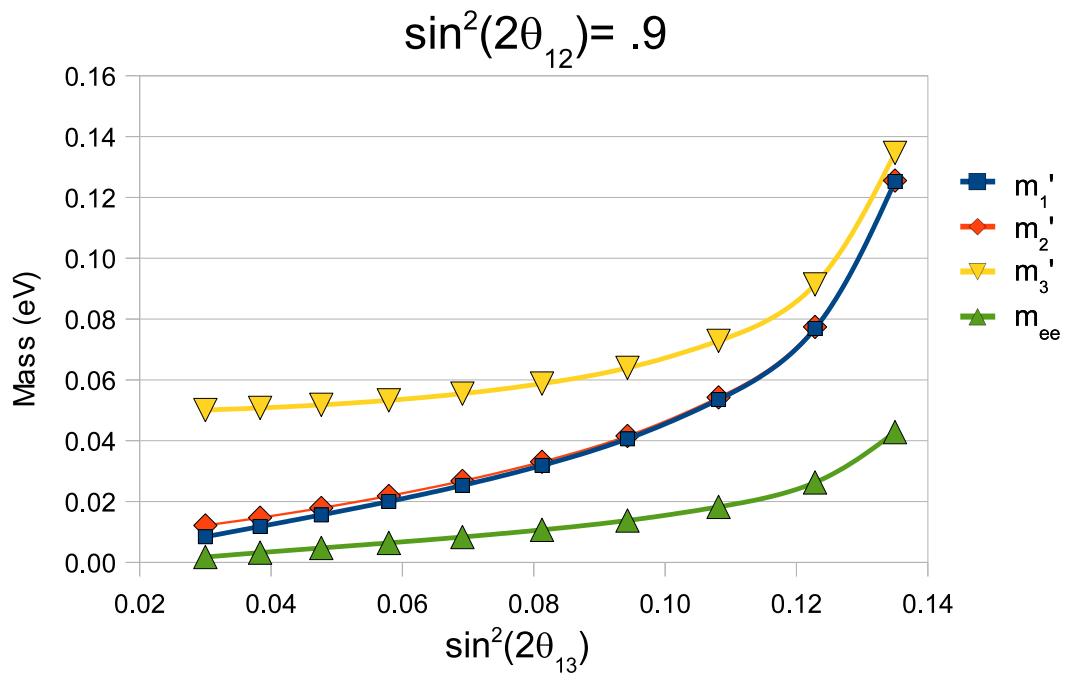


Figure 3: Physical neutrino masses  $|m'_{1,2,3}|$  and the effective  $m_{ee}$  for neutrinoless double beta decay of this model in the range  $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$  for  $\sin^2 2\theta_{23} = 1$  and  $\sin^2 2\theta_{12} = 0.90$ .

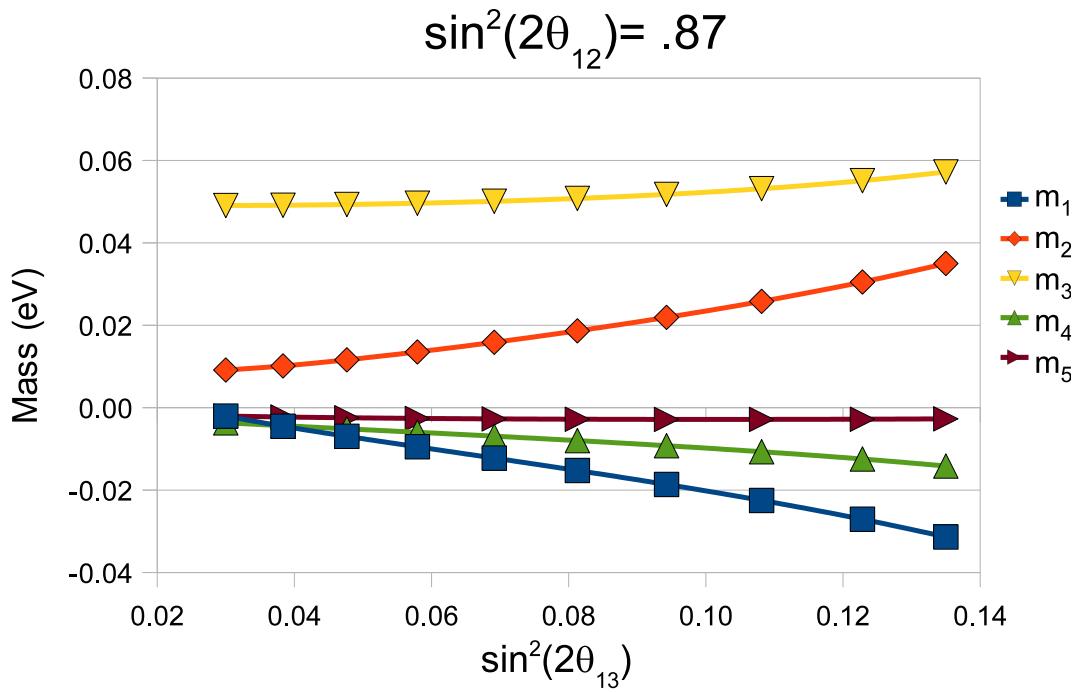


Figure 4: The  $A_4$  parameters  $m_{1,2,3,4,5}$  of this model in the range  $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$  for  $\sin^2 2\theta_{23} = 1$  and  $\sin^2 2\theta_{12} = 0.87$ .