Polynomial nonlinear state-space modelling of vortex-induced vibrations: black-box vs grey-box approach

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Introduction

The kinematics of oscillating bluff bodies in a fluid flow have been a hot research topic for decades. Most important are the potentially harmful vortex-induced vibrations (VIVs) where the structure is excited by alternating vortex shedding in its wake. Classical examples are the vibration of chimney stacks exposed to wind, pipelines on the sea-bed excited by the ocean currents or water tubes in heat exchangers [1].

For these kind of fluid-structure interactions, typically an analytical solution cannot be found. Yet accurate predictions of the kinematics and the resulting dynamics are vital during for instance a design process, in monitoring applications or for control. Obtaining predictions with high fidelity have so far been restricted to solving the Navier-Stokes equations via computation fluid dynamic (CFD) simulations or to performing experiments. Both approaches are cumbersome and, in the case of CFD, requires a lot of computing power. These drawbacks make the current methods disadvantageous towards many intended applications where only limited time and resources are available to assess a certain risk.

What is needed is an efficient and powerful model, flexible enough to span a wide domain in parameter space with a single set of coefficients. A task for which, we believe, system identification can be a very powerful tool.

Work description

The challenges in modelling the system at hand are, however, substantial. First off all, fluid-structure interactions are inherently nonlinear [2]. Moreover is VIV a self-excited yet self-limited oscillation, resulting in a stable limit cycle [3]. In addition, the vortex shedding behaviour is known to be hysteretic of nature [4].

In this work, a novel approach to modelling flow problems by applying state-of-the-art nonlinear identification techniques is proposed. The powerful framework of nonlinear state-space models is explored and put to the test.

Two approaches are proposed and compared. On the one hand, the nonlinear functions of the state-space equations are formed out of multi variate polynomials of the state variables and the input (f and g in Eq.1).

$$\begin{cases} x(t+1) = \mathbf{A}x(t) + \mathbf{B}y(t) + f(x(t), y(t)) & \text{(1a)} \\ C_y\ (t) = \mathbf{C}x(t) + \mathbf{D}y(t) + g(x(t), y(t)) & \text{(1b)} \end{cases}$$

Where $y(t)$, the displacement of the structure, is the input to the system, and C_y(t), the force coefficient of the resultant force in the y-direction, is the output. Since no prior knowledge is used in construction these general nonlinear functions we will refer to this approach as the black-box modelling approach. Identification of such a model consists of 3 steps:

- First a nonparametric estimate of the best linear approximation is constructed.
- Using linear identification techniques, estimates of the coefficients in the linear part (**A**,**B**,**C**,**D**) are derived.
- Finally, nonlinear optimisation is used to optimise all coefficients, including the nonlinear terms.

In the second approach, the nonlinear functions will be build up out of a deliberate choice of basis functions. In this case nonlinear functions of the output will be used.

$$\begin{cases} x(t+1) = \mathbf{A}x(t) + \mathbf{B}y(t) + h(y(t)) & \text{(2a)} \\ C_y\ (t) = \mathbf{C}x(t) + \mathbf{D}y(t) & \text{(2b)} \end{cases}$$

As inspiration for the nonlinear basis functions in terms of the output (h in Eq. 2), we will rely on the literature of heuristic modelling attempts of VIV, which typically were in the form of a Van der Pol equation [5] [6] [7]. The second approach will be referred to as the grey-box approach. Identifying such a structure is done using a 2-step approach:

- Use nonlinear subspace identification to directly obtain a nonlinear initial model.
- Use nonlinear optimisation to remove systematic errors.

The advantage of constructing these tailored nonlinear basis functions lies in a enormous reduction in the number of model parameters [8].

Both approaches will be demonstrated on data acquired from the Van der Pol model equation. In a second step the methods are applied to data obtained from CFD simulations of vortex shedding behaviour.

Preliminary results

Both approaches are demonstrated on data obtained from the nonlinear Van der Pol equation (Eq. 3).

$$\ddot{C_y} + \mu\Omega_{\text{aut}}(\gamma C_y^2 - 1)\dot{C_y} + \Omega_{\text{aut}}^2 C_y = b\dot{y} \tag{3}$$

Where C_y represents the non-dimensional force coefficient (output), y is the displacement (input) and Ω_{aut} is the autonomous angular frequency (with $f_{\text{aut}} = 3$ Hz in this case). Validation result for both the black and the grey-box modelling approach are plotted in Fig. 1.

Both approaches are able to accurately capture the dynamics of the Van der Pol equation. The grey-box approach is, however far less computationally expensive and results in a system with far less parameters.

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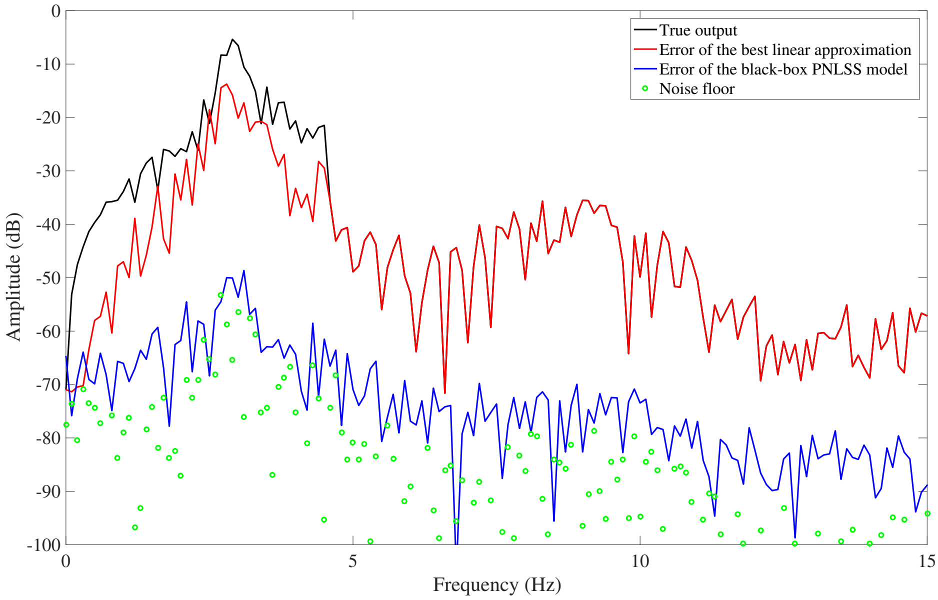
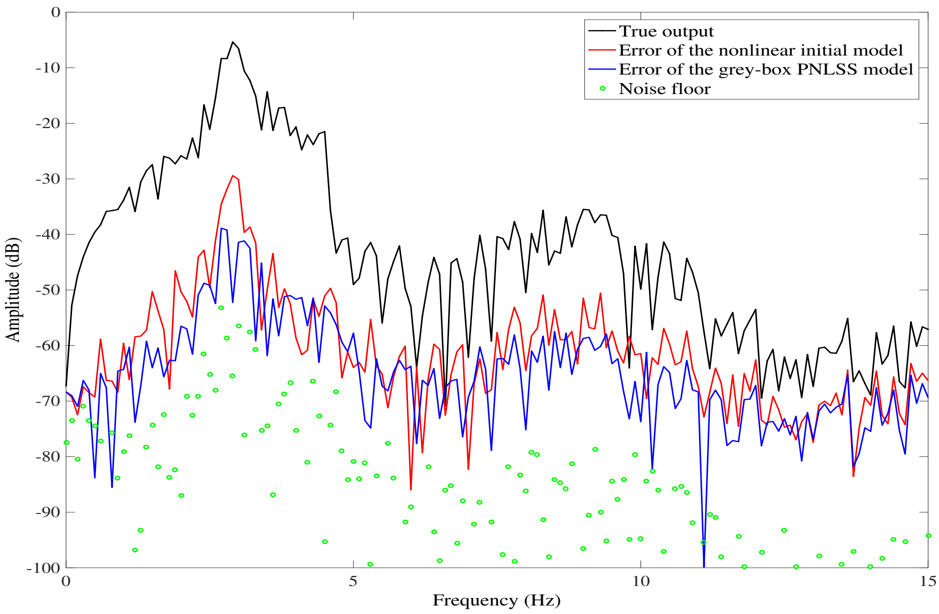


(a)



(b)

Figure 1: Validation results of the black-box (a) and grey-box (b) modelling approach of the Van der Pol equation.