Locating Nonlinearity in Mechanical Systems - A Dynamic Network Perspective

J.P. Noël[1]**, M. Schoukens**[2]**, P.M.J. Van den Hof**[2]

[1] Aerospace and Mechanical Engineering Department
University of Liège, Liège, Belgium
jp.noel@uliege.be

[2] Control Systems Group
Technical University of Eindhoven, Eindhoven, The Netherlands
m.schoukens@tue.nl, p.m.j.vandenhof@tue.nl

ABSTRACT

Though it is a crucial step for most identification methods in nonlinear structural dynamics, nonlinearity location is a sparsely addressed topic in the literature. In fact, locating nonlinearities in mechanical systems turns out to be a challenging problem when treated nonparametrically, that is, without fitting a model. The present contribution takes a new look at this problem by exploiting some recent developments in the identification of dynamic networks, originating from the systems and control community.

Keywords: nonlinear structural dynamics; nonlinear system identification; nonlinearity location; best linear approximation; dynamic networks.

1 INTRODUCTION

The identification of linear dynamic networks has attracted increasing attention over the past few years. An example of linear network is displayed in Fig. 1. It consists of three nodes, where the signal $w_i(t)$ measured at node i is the sum of the applied reference signal $r_i(t)$, the noise signal $v_i(t)$ and the outputs $y_{i,k}(t)$ of the linear blocks $G_{i,k}(j\omega)$ that point towards it. The majority of the contributions in the field has so far dealt with obtaining consistent estimates of linear block dynamics under different noise assumptions [1−3]. In a recent study [4], some research attention has eventually been turned to the detection and location of nonlinearities in networks using linear approximation techniques.

Approximating nonlinear systems for topology detection is not a new idea and various approaches exist [5, 6], but a common thread is the definition of an optimality criterion for the approximation that holds within a given class of systems and under a certain class of excitation signals. In the present work, the Best Linear Approximation (BLA) of Volterra nonlinear systems is exploited, assuming Gaussian excitation signals [7]. Applying the BLA to mechanical systems has already proved successful for nonlinearity detection [8, 9]. In this contribution, by interpreting mechanical systems as dynamic networks and following the methodology introduced in Ref. [4], and described in Section 2, we extend the use of the BLA in structural dynamics to nonlinearity *location*. A demonstration on a three-degree-of-freedom system is proposed in Section 3.

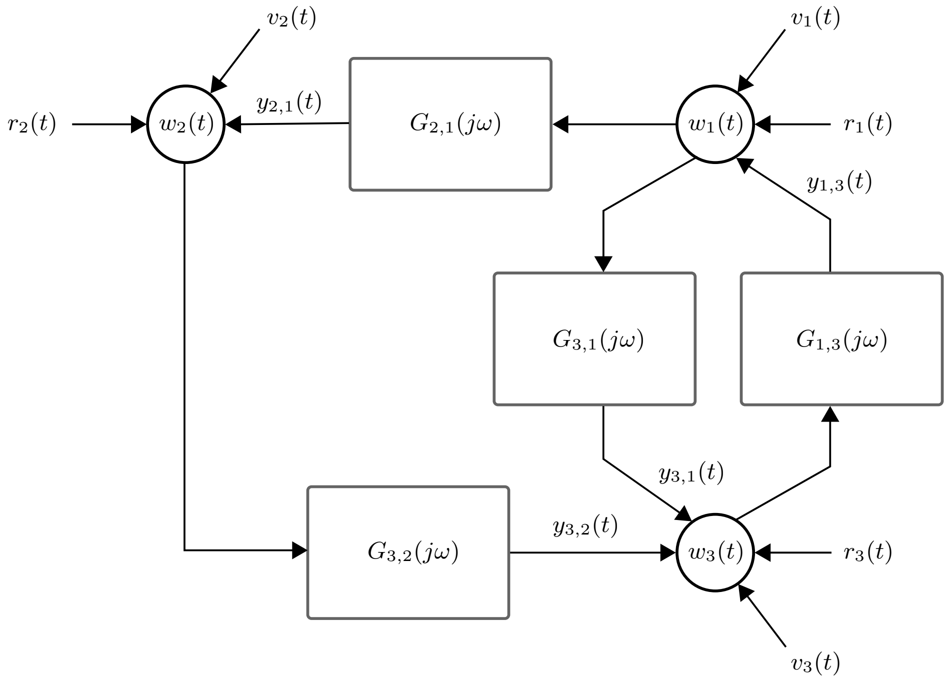


Figure 1: An example of a linear dynamic network with three nodes and four linear dynamic blocks.

2 METHODOLOGY

The key assumption to a proper application of the BLA framework is that noise must only corrupt output measurements [7]. This prevents one from directly calculating the BLA from node to node in a dynamic network setting. To circumvent this issue, a three-step methodology is followed, leading a rigorous nonlinearity detection and location procedure.

2.1 Step 1: BLA analysis from references to nodes

The first step consists in calculating a multi-input, multi-output (MIMO) BLA from all references to all nodes. It is defined from reference k to node i as the transfer function $S^{i,k}_{BLA}(j\omega)$ providing the best linear approximation of the nonlinear network dynamics in least-squares sense, *i.e.*

$$S^{i,k}_{BLA}(j\omega) = \text{arg}_{S^{i,k}} \min E_{r,v} \left\{ \sum_{i=1}^{n_w} \left| w_i(j\omega) - \sum_{k=1}^{n_r} S_{i,k}(j\omega)\, r_k(j\omega) \right|^2 \right\}. \tag{1}$$

The expectation $E_{r,v}\{\bullet\}$ is taken with respect all possible realisations of the reference signal r_k and noise signal v_i, within the considered signal class. In Eq. (1), reference and node signals are assumed to be zero-mean.

Eq. (1) allows one to calculate a noise-free linear approximation $\bar{w}_i(j\omega)$ of the node signal $w_i(j\omega)$ that writes

$$\bar{w}_i(j\omega) = \sum_{k=1}^{n_w} S^{i,k}_{BLA}(j\omega) r_k(j\omega). \tag{2}$$

2.2 Step 2: BLA analysis from nodes to nodes

A second MIMO BLA analysis is carried out in between nodes, considering as inputs the noise-free node signals from Eq. (2), and taking into account the direct contributions of the references. This results in BLA estimates $G_{BLA}^{i,k}(j\omega)$ from node i to node k of the form

$$G_{BLA}^{i,k}(j\omega) = \text{arg}_{G^{i,k}} \text{ min } E_{r,v} \left\{ \sum_{i=1}^{n_w} \left| w_i(j\omega) - \sum_{k=1}^{n_r} S_{i,k}(j\omega)\, r_k(j\omega) \right|^2 \right\}. \tag{3}$$

Node signals can now be simulated using the node-to-node linear approximate description $G_{BLA}^{i,k}(j\omega)$ of the network dynamics, leading to the relation

$$\bar{\bar{w}}_i(j\omega) = r_i(j\omega) + \sum_{k=1,\neq i}^{n_w} G_{BLA}^{i,k}(j\omega) w_k(j\omega). \tag{4}$$

2.3 Step 3: Residual analysis and nonlinearity location

By comparing the simulated node signals $\bar{\bar{w}}_i(j\omega)$ and the corresponding measurements $w_i(j\omega)$ over multiple periods and realisations, a residual analysis can be conducted. Let P and M be the number of measured periods and realisations of the network signals, and $\bar{\bar{w}}_i^{[m,p]}(j\omega)$ and $w_i^{[m,p]}(j\omega)$ be the simulated and measured node signals over period p of realisation m, respectively. The variances of the total distortions σ_t^2, noise distortions σ_n^2 and nonlinear distortions σ_{nl}^2 in the residual are then separated following the expressions [10]

$$\sigma_t^2(j\omega) = \frac{1}{P}\frac{1}{M-1} \sum_{p=1}^{P}\sum_{m=1}^{M} \left(e_i^{[m,p]}(j\omega) - \frac{1}{P}\sum_{m=1}^{M} e_i^{[m,p]}(j\omega) \right)^2 ; \tag{5}$$

$$\sigma_n^2(j\omega) = \frac{1}{M}\frac{1}{P-1} \sum_{m=1}^{M}\sum_{p=1}^{P} \left(e_i^{[m,p]}(j\omega) - \frac{1}{P}\sum_{p=1}^{P} e_i^{[m,p]}(j\omega) \right)^2 ; \tag{6}$$

$$\sigma_{nl}^2(j\omega) = \sigma_t^2(j\omega) - \sigma_n^2(j\omega), \tag{7}$$

where

$$e_i^{[m,p]}(j\omega) = \bar{\bar{w}}_i^{[m,p]}(j\omega) - w_i^{[m,p]}(j\omega). \tag{8}$$

The comparison along the frequency axis of the three variance levels $\sigma_t^2(j\omega)$, $\sigma_n^2(j\omega)$ and $\sigma_{nl}^2(j\omega)$ provides a quantitative way of detecting and locating nonlinearities in any network arrangement of linear dynamic subsystems.

3 APPLICATION TO A THREE-DEGREE-OF-FREEDOM MECHANICAL SYSTEM

The present section illustrates the methodology described in Section 2 considering the three-mass system of Fig 2. The physical parameters and modal properties of the system are listed in Tables 1 and 2, respectively. A cubic spring is introduced between mass 3 and the ground. Multisine excitations are applied to the three masses, and displacements are processed as output data. Multisines are periodic signals with a user-defined amplitude spectrum and randomly-chosen phases [11]. In this application, they excite the system in the 5-100 Hz band with a root-mean-square amplitude of 50 N. Response signals are generated using a nonlinear Newmark integration algorithm with a sampling frequency of 5000 Hz, and numerical experiments are repeated over 10 periods and 30 realisations with 8192 measured samples per period. The resulting displacement time signals are corrupted with white Gaussian noise with a signal-to-noise ratio of 40 dB.

The result of the residual analysis performed through Eqs. (5-7) is shown in Fig. 3. The spectra of the signals $\bar{\bar{w}}_i(t)$ and the noise levels are displayed using grey and black lines, respectively. The total and nonlinear distortions are plotted using orange crosses and blue squares, respectively. It is easily concluded from these three graphs that mass 3 in Fig. 3 (c) is the only node in the system connected to a nonlinear element. For masses 1 and 2 in Fig. 3 (a-b), the observed total distortion level is fully explained by the noise disturbances, implying that they are not directly connected to a nonlinearity.

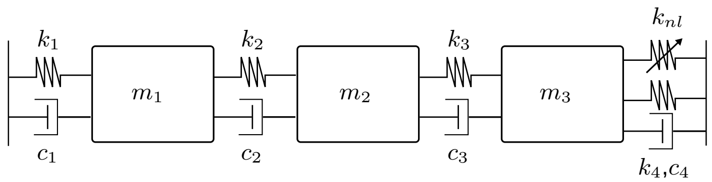


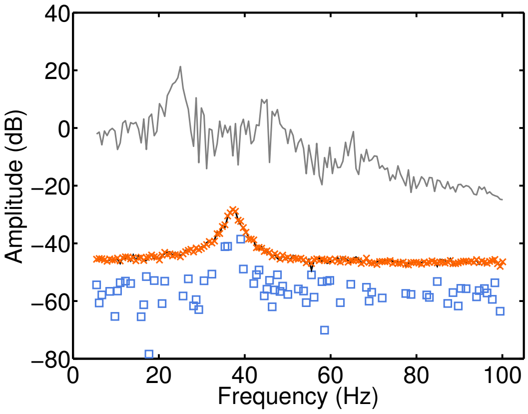
Figure 2: A three-mass system with one cubic spring connecting mass 3 to the ground.

Parameter	Value
m_1	$1\ kg$
m_2	$0.8\ kg$
m_3	$1.2\ kg$
k_1	$20\ 10^3\ N/m$
k_2	$35\ 10^3\ N/m$
k_3	$50\ 10^3\ N/m$
k_4	$80\ 10^3\ N/m$
k_{nl}	$1\ 10^8\ N/m^3$
c_{1-4}	8 Ns/m

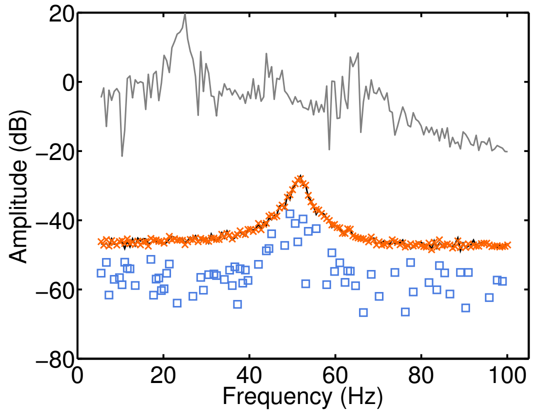
TABLE 1: Physical parameters of the three-mass system.

Mode	Frequency (Hz)	Damping ratio ($\%$)
1	25.0	2.0
2	44.8	2.7
3	64.8	3.4

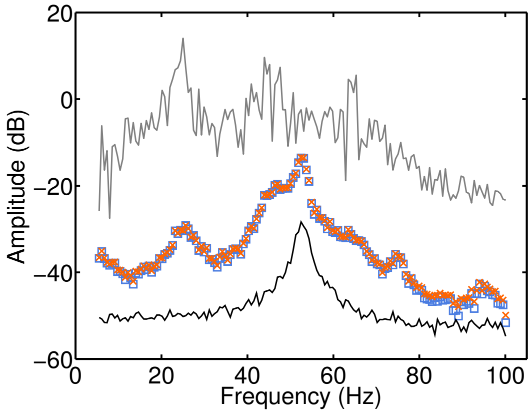
TABLE 2: Linear modal properties of the three-mass system.



(a)



(b)



(c)

Figure 3: Locating nonlinearity in the three-mass system. (a-c) Mass 1-3. The spectra of the signals $\bar{\bar{w}}_i(t)$ and the noise levels are displayed using grey and black lines, respectively. The total and nonlinear distortions are plotted using orange crosses and blue squares, respectively.

4 CONCLUSIONS AND PERSPECTIVES

This contribution introduced an original methodology to locate nonlinearities in mechanical systems using random excitation signals. The proposed technique is fully nonparametric, *i.e.* it requires no estimation of parameters. It relies on the use of the Best Linear Approximation (BLA) of the nonlinear dynamics taking place in between the masses of the considered system, interpreted as a network arrangement of nodes. Its graphical outcome is a frequency-domain plot per measurement location comparing the amplitude of the nonlinear and the noise distortions, and hence enabling the user to assess quantitatively and systematically the impact of nonlinearity across the tested structure. Further developments in this research will focus on relaxing the current need for multi-input, multi-output experiments, through the concatenation of data measured sequentially in single-input conditions.

ACKNOWLEDGEMENTS

The author J.P. Noël is a Postdoctoral Researcher of the *Fonds de la Recherche Scientifique – FNRS* which is gratefully acknowledged.

REFERENCES

[1] **Van den Hof, P., Dankers, A., Heuberger, P.** and **Bombois, X.**, *Identification of dynamic models in complex networks with prediction error methods - Basic methods for consistent module estimates*, Automatica, Vol. 49, pp. 2994–3006, 2013.

[2] **Dankers, A., Van den Hof, P., Bombois, X.** and **Heuberger, P.**, *Errors-in-variables identification in dynamic networks - Consistency results for an instrumental variable approach*, Automatica, Vol. 62, pp. 39–50, 2015.

[3] **Weerts, H. H., Van den Hof, P.** and **Dankers, A. G.**, *Identification of dynamic networks with rank-reduced noise*, IFAC PapersOnLine, Vol. 50, No. 1, pp. 10562–10567, 2017.

[4] **Schoukens, M.** and **Van den Hof, P.**, *Detecting and quantifying nonlinearity in dynamic networks*, Automatica (in review), 2017.

[5] **Schoukens, J., Pintelon, R., Rolain, Y., Schoukens, M., Tiels, K., Vanbeylen, L., Van Mulders, A.** and **Vandersteen, G.**, *Structure discrimination in block-oriented models using linear approximations: A theoretic framework*, Automatica, Vol. 53, pp. 225–234, 2015.

[6] **Schoukens, M.** and **Tiels, K.**, *Identification of block-oriented nonlinear systems starting from linear approximations: A survey*, Automatica, Vol. 85, pp. 272–292, 2017.

[7] **Schoukens, J., Vaes, M.** and **Pintelon, R.**, *Linear system identification in a nonlinear setting*, IEEE Control Systems, Vol. 36, No. 3, pp. 38–69, 2016.

[8] **Wernholt, E.** and **Gunnarsson, S.**, *Estimation of nonlinear effects in frequency domain identification of industrial robots*, IEEE Transactions on Instrumentation and Measurement, Vol. 57, No. 4, pp. 856–863, 2008.

[9] **Vaes, M., Schoukens, J., Peeters, B., Debille, J., Dossogne, T., Noël, J., Grappasonni, C.** and **Kerschen, G.**, *Nonlinear ground vibration identification of an F-16 aicraft - Part 1: Fast nonparametric analysis of distortions in FRF measurements*, *Proceedings of the International Forum on Aeroelasticity and Structural Dynamics (IFASD)*, Saint Petersburg, Russia, 2015.

[10] **Schoukens, J., Swevers, J., Pintelon, R.** and **Van der Auweraer, H.**, *Excitation design for FRF measurements in the presence of non-linear distortions*, Mechanical Systems and Signal Processing, Vol. 18, pp. 727–738, 2004.

[11] **Pintelon, R.** and **Schoukens, J.**, System Identification: A Frequency Domain Approach, IEEE Press, Piscataway, NJ, 2001.