NEW SIMPLIFIED ANALYTICAL METHOD FOR THE PREDICTION OF GLOBAL STABILITY OF COMPOSITE SWAY FRAMES

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INTRODUCTION

In the last years, the construction of taller composite buildings and larger composite industrial halls without wind bracing systems is susceptible to make global instability a relevant failure mode. This one is not yet covered by EC4 [2]. This paper reflects investigations carried out of Liege University on this topic. In particular, an innovative simplified analytical method to predict the ultimate loading factor and the associated collapse mode of composite frames is derived.

Several analytical methods are proposed in the codes for the design of steel sway frames such as the “Amplified sway moment”, “Sway-mode buckling length”, “Merchant-Rankine”, “Simplified second-order plastic” or “Wind moment” methods. The latter allow analysing structures without high capacity software taking into account the sway effects and the non-linearities. However, the accuracy of these methods is not always satisfactory; also conditions of applicability have to be respected which are generally linked to the ratio $V_{Ed}/V_{cr}$.

One important factor that makes the analyses between steel and composite frames different is the concrete cracking. Indeed, concrete cracking tends to increase the lateral deflection of the frame, amplifies consequently the second-order effects, and then reduces the ultimate resistance of the frames. In other word, for a same number of plastic hinges formed at a given loading level, larger sway displacements are observed for the composite frames than the steel frames. That explains the difficulty to apply directly the design methods mentioned above to composite sway frames.

In the last decade, European researches have been dedicated to this topic, [3] and [6]. In particular, Liege University, as part of these projects, has investigated the applicability of two simplified analytical methods, initially developed for steel sway frames, to composite sway frames [4]:

- the “Amplified sway moment” method shows a good accuracy for the prediction of the elastic loading factor $\lambda_e$ of composite sway frames;
- and the “Merchant-Rankine” approach may be used to determine the ultimate loading factor $\lambda_u$ of a given composite frame, but with a moderate accuracy.

In fact, it has been shown that the plastic mechanism associated to the first-order rigid-plastic loading factor introduced in the Merchant-Rankine formula does not always correspond to the one occurring at the failure of the frame. In other words, different types of plastic mechanisms are not affected the same way by the second-order effects, or it deals with the fact that the first-order and second-order rigid-plastic loading factors may associate to different plastic mechanisms.

The solution proposed here is to develop three formulas, one for each type of plastic mechanisms that could occur in the studied frame:

- $\text{formula}_1(\lambda_{p,\text{beam}}, \lambda_{cr}) \rightarrow \lambda_{u,\text{beam}}$;
- $\text{formula}_2(\lambda_{p,\text{panel}}, \lambda_{cr}) \rightarrow \lambda_{u,\text{panel}}$;
- and $\text{formula}_3(\lambda_{p,\text{combined}}, \lambda_{cr}) \rightarrow \lambda_{u,\text{combined}}$.

Three predicted ultimate loading factors are computed and the smallest one is then considered as the ultimate loading factor of the given frame. The developed formulas are founded on the Ayrton-Perry formulations. The latter is already used in the EC3 [1] to treat member stabilities (table 1). A great advantage is that permits explicitly to respect the following limit conditions:
when the frame is very stiff ($\lambda_{cr} >> \lambda_p$), no instability appears and the failure is reached by a plastic mechanism ($\lambda_u \rightarrow \lambda_p$);
and when the frame is very flexible ($\lambda_p >> \lambda_{cr}$), no yielding appears and the collapse is reached by an instability.

Table 1: From the Ayrton-Perry formulation to the new simplified analytical design method

<table>
<thead>
<tr>
<th>Ayrton-Perry formulations</th>
<th>New method for sway frames</th>
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<tbody>
<tr>
<td>$N_{pl,k} = \frac{\chi \alpha_{pl,k}}{\gamma_{pl}}$</td>
<td>$\lambda_u = \chi \lambda_p$</td>
</tr>
<tr>
<td>$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$</td>
<td>$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$</td>
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<tr>
<td>$\lambda = \frac{\sqrt{\alpha_{pl,k}}}{\alpha_{cr}}$</td>
<td>$\lambda = \frac{\lambda_p}{\lambda_{cr}}$</td>
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<tr>
<td>$\phi = 0.5 \left[ 1 + \alpha (\lambda - \lambda_u) + \lambda^2 \right]$</td>
<td>$\phi = 0.5 \left[ 1 + \mu (\lambda - \lambda_u) + \lambda^2 \right]$</td>
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Satisfactory results are obtained when the method is applied to steel sway frames [5]. This paper illustrates how this method has been extended to the composite ones. Parametrical investigations on about 200 composite frames have been performed to validate the new method as it will be illustrated herein.

1 METHODOLOGY

Within the developed method, $\lambda_u$ (table 1), representing the length of the plateau, is taken as equal to zero as no strain hardening and cladding effects are considered (the same assumption as the one adopted for the Merchant-Rankine approach). As for the parameter $\mu$, it has to be calibrated through a parametrical investigation; in total, three values of $\mu$ are defined: $\mu_{beam}$, associated to the beam plastic mechanism; $\mu_{combined}$, to the combined plastic mechanism and $\mu_{panel}$, to the panel plastic mechanism.

1.1 Calibration of $\mu$

For each composite frame considered in the parametrical investigations, the value of $\lambda_{cr}$ is first calculated through a critical elastic analysis, and then the values of $\lambda_p$ ($\lambda_{p,beam}$, $\lambda_{p,combined}$ or $\lambda_{p,panel}$), through a first-order rigid-plastic analysis and then the value of $\lambda_u$, through a full non-linear analysis. Accordingly, the values of $\lambda$ ($\sqrt{\lambda_p / \lambda_{cr}}$) and $\chi$ ($\lambda_u / \lambda_p$) can be also computed; $\lambda_p$ ($\lambda_{p,beam}$, $\lambda_{p,combined}$ or $\lambda_{p,panel}$) must be the one having the same plastic mechanism as $\lambda_u$. Reporting all these couples ($\chi, \lambda$) in the diagram of the fig. 1, three groups can be identified, corresponding to each collapse mechanism: beam mechanism (BM), combined mechanism (CM) and panel mechanism (PM). By fitting the analytical curves (BM, CM and PM) derived from the proposed method to these points, three calibrated coefficients $\mu$ ($\mu_{beam}$, $\mu_{combined}$ and $\mu_{panel}$) are found out.

1.2 Critical elastic analyses

It deals with the determination of the critical loading factor of frames ($\lambda_{cr}$). There are several ways to find out the latter, by either numerical simulations or analytical models. In the presented development, FINELG (homemade FEM software developed at Liege University) is used to minimize as well as possible any sources of error to be able to validate the proposed method.
1.3 Full non-linear analyses

The ultimate loading factor $\lambda_u$ of a given sway composite frame is computed through a full non-linear numerical simulation. Initial deformations are introduced according to the EC4 recommendations. The formula used to estimate the value of this initial out-of-plumb $\phi_{ini}$ is the same as the one proposed in EC3:

$$\Phi_{ini} = \alpha_c \Phi_0$$  \hspace{1cm} (1)

where

$$\Phi_0 = 1/200$$  \hspace{1cm} (2)

$$\alpha_g = 2/\sqrt{h_{struc}}$$, but $2/3 \leq \alpha_g \leq 1$  \hspace{1cm} (3)

$$\alpha_e = \sqrt{0.5(1+1/m)}$$  \hspace{1cm} (4)

with $h_{struc}$, the total height of the structure in meters and $m$, the number of columns in a row including only ones which carry a vertical load $N_{Ed}$ not less than 50% of the average value of the columns in the vertical plane considered. The shape of the initial deformations is in fact proportional to the first global instability mode obtained through the critical elastic analysis. The use of the first mode of instability as the initial deformations of the structure permits to introduce at the same time the global initial deformations (to initiate P-Δ effects) and the local initial deformations for the members (to initiate P-δ effects).

1.4 First-order rigid-plastic analyses

It deals with the determination of the first-order rigid-plastic loading factor ($\lambda_p$). FINELG may be used, but gives only the minimum among the three $\lambda_{p,beam}$, $\lambda_{p,combined}$ and $\lambda_{p,panel}$. So, a software based on an analytical approach is developed to be able to calculate at the same time the three values of $\lambda_p$. This tool has been validated by comparing its results to the numerical ones.

2 STUDIED COMPOSITE FRAMES

Different configurations of “academic” composite frames have been defined to cover all the plastic mechanisms through the first-order rigid-plastic or full non-linear analyses. The parameters that varied within these frames are properties of the joints, type of structural elements; height of the columns and applied loading (fig. 2).

2.1 Materials

The steel is modelled as an elastic-perfectly plastic material characterized by the elastic modulus $E$ ($0.21 \times 10^6$ N/mm²) and the yield stress $f_y$ (fig. 3.A). S355 ($f_y = 355$ N/mm²) is used for all steel profiles and S500 ($f_y = 500$ N/mm²) for steel rebars.
For the concrete, material class C25/30 according to EC2, a bilinear law ($\sigma_{cc} = f_{cc}$) with tension stiffening is adopted for the modelling of concrete material (fig. 3.B). Shrinkage and creep phenomena are not introduced in the computations. Through a preliminary study, an important observation has been highlighted. Even if the cross-sections of a composite sway frame can be considered as class 1 according to the EC4 recommendations, the ductility of the concrete in compression ($\epsilon_{cu} = 0.35\%$ as defined in EC2) is not sufficient to allow the development of a full plastic mechanism. As it deals with the problem of cross-section classification, which is not under investigations, a sufficient ductility for the concrete has been assumed and the value $\epsilon_{ccu}$ has been unlocked. In future investigations, criteria on the maximum compressive deformation to be ensured by the concrete should be added in the evaluation of the cross-section class.

![Fig. 2: Types of studied frames](image)

2.2 Joints

The beam-to-column composite joints may be rigid or semi-rigid, full-strength or partial-strength in different configurations of frames. They are assumed to have a sufficient ductility to develop plastic hinges and to allow plastic analyses. The column bases are assumed rigid and fully resistant.

2.3 Beam and column members

Two types of composite beam configurations bent around their major axis are considered: hot-rolled profiles with the upper flange fully connected to a concrete slab (fig. 4.A) and hot-rolled profiles with the upper flange fully connected to a composite slab (fig. 4.B). Different steel IPE profiles and composite slabs (the thickness ranging from 65 to 150 mm and the width, from 600 to 730 mm) are also adopted for the investigated frames.

As for the column members, there are also two types of columns bent around their major axis: steel hot-rolled profile columns (fig. 4.C) and partially encased composite columns (fig. 4.D). Different steel HEA profiles are introduced to vary the response of the studied frames.
A particular attention is paid to the cross-section resistances of column members due to the presence of M-N interaction. For steel profiles, many analytical formulations, proposed for double T cross-section, can be used to calculate the plastic loading. For composite profiles, EC4 proposes a simplified procedure to evaluate the M-N interaction through a polygonal curve as illustrated in the fig. 5.

Different collapse modes and a wide range of \( \lambda_p/\lambda_{cr} \) (ranging between 0.05 and 0.36) are covered. As mentioned previously, three values of \( \mu \) (\( \mu_{\text{beam}} \), \( \mu_{\text{combined}} \) and \( \mu_{\text{panel}} \)) have been calibrated: \( \mu_{\text{beam}} = 0.02 \), \( \mu_{\text{combined}} = 0.42 \) and \( \mu_{\text{panel}} = 0.7 \). Comparisons between the predicted values of \( \lambda_u \) obtained through the analytical methods (the new one and the Merchant-Rankine approach) and the numerical simulations are given in the fig. 6 and fig. 7. From the latter, it can be observed that:

- the new method produces more accurate results than the Merchant-Rankine approach. In addition, more points in the “unsafe side” are present for the latter, 40.7 % of the investigated frames, than for the new method, 7.54 % of the investigated frames (fig. 6).
- for differences on the values of \( \lambda_u \) between 0 % and 10 %, there are 83.92 % of the studied frames with the new method and 25.63 % with the Merchant-Rankine approach (fig. 7).

Moreover, it is noted that there are 19.10 % of the investigated frames for which the collapse modes associated to \( \lambda_p \) do not correspond to the one associated to \( \lambda_u \); but the new method is able to predict the collapse mode associated to \( \lambda_u \) for 99.5 % of the investigated frames, what is not possible with the Merchant-Rankine approach.

Through about 200 numerical simulations, it is demonstrated that the conclusions drawn for the steel sway frames [5] are still valid for the composite sway frames. The new method gives safe values of the ultimate loading factor with a satisfactory accuracy. Furthermore, it can predict the collapse mode of the structure at the ultimate failure.
Fig. 6: Comparison between the analytical and the numerical results for the prediction of $\lambda_{u}$

![Graph showing comparison between analytical and numerical results for $\lambda_{u}$]

Fig. 7: Evaluation of the accuracy of the analytical methods

Differences between analytical and numerical ultimate loading factors [%]

- UNSAFE SIDE
- SAFE SIDE

% of studied frames

New method
Merchant Rankine

REFERENCES


