A MODAL ANALYSIS METHOD TO IDENTIFY TENSION IN STAY-CABLES EQUIPPED WITH ADDITIONAL UNKNOWN MASSES IN THE SPAN

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Introduction

This paper presents a low-order model to identify the tension in a cable with additional masses in the span. The method is applicable in a general sense but is illustrated with low Irvine parameters (slightly extensible) cables. The number of masses is arbitrary; however the method performs best with a limited number of masses, say less than 12 or 15; otherwise a Galerkin scheme would perform equally well. The paper illustrates the method with numerically simulated data. Validation with field measurement is provided in the oral presentation.

CONTEXT AND MOTIVATION

The tension in a stay-cable can be identified from the relation existing between the natural frequency of the cable and the tension in the cable [1, 2]. Based on this formula of the theory of free vibrating strings, the measurement of the natural frequencies of a cable allows for the determination of the tension in the cable. This identification procedure is known to be robust because the natural frequencies of a long stay cable with uniform mass and negligible bending stiffness are known to be regularly spaced, and integer multiples of the fundamental natural frequency. A least-square fitting approach provides therefore the robustness to the identification algorithm [3].

Several improvement of the basic identification procedure have been proposed, in particular, when the bending stiffness of the stay-cable is not negligible [4]. It that case, the successive natural frequencies are not uniformly distributed anymore. The slight departure from uniformity, which is known to have monotonically increasing importance for higher modes since wavelengths are shorter, might be used to identify not only the tension in the cable, but also the bending stiffness [3]. The same procedure could also apply to cables with a small bending stiffness and built-in end conditions. It this case, it is recognized that boundary layers exist in the neighborhood of singularities, such as boundary conditions [5] and might affect the dynamics of the cable [6].

In this paper, we treat a similar problem related to the identification of the tension in a stay-cable, with negligible bending stiffness but equipped with some additional masses in the span. The addition of concentrated masses in the span, which might be relatively very light (stockbridge dampers on transmission lines) or up to very heavy and massive (isolators on stay cables of telecommunication antenna's), also completely disturbs the regular arrangement of natural frequencies. A batch-data identification procedure is proposed for that class of problem. It is based on a semi-analytical model of the cable equipped with the masses. An experimental investigation of this problem with a single mass in the span has been undertaken by [7]. This work consists in the analytical counterpart of the study and extends the analysis to an arbitrary number of masses in the span.

A direct low-order model is first derived for the dynamics of a cable with concentrated masses in the span. Then, by keeping the masses and their locations as unknown parameters, as well as the tension in the cable, the natural frequencies of this low order analytical model are determined. The choice to recourse to a semi-analytical model rather than a numerical model is motivated by (i) the wish to bypass the well-known drop in accuracy in estimating higher natural frequencies with discrete models, (ii) the possibility

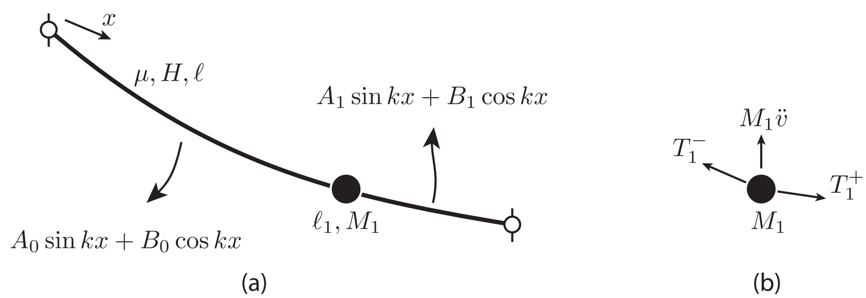

Figure 1: Free body diagram of a cable with a localized mass in span

to accurately derive the Jacobian of the cost function of the model updating problem with respect to the problem parameters. Based on this model, an efficient optimization algorithm is developed in order to match the computed and measured natural frequencies, by adjusting the unknown masses and their position, as well as the tension in the stay-cable.

DIRECT LOW-ORDER MODEL

Cable with uniform mass an without any additional concentrated mas

Under the shallow cable assumptions [REF], the dynamics of a cable with uniform mass per unit lenght μ and (assumed) constant horizontal tension H is governed by the wave equation

$$H\partial_x^2 v = \mu\partial_t^2 v \tag{1}$$

where $v(x,t)$ is the transverse displacement of the cable, measured transversely to the chord. This governing equation is usually supplemented with the boundary conditions $v(0) = 0$, $v_0(\ell) = 0$ where ℓ is the cable length. Eigen vibration modes for this cable are obtained by substituting $v = v_0(x)e^{i\omega t}$ (synchronous motion) in the governing equation, which yields

$$Hv_0'' + \mu\omega^2 v_0 = 0 \tag{2}$$

whose solution is $v_0 = A\sin kx + B\cos kx$ with $k^2 = \mu\omega^2/H$. Considering the boundary conditions, the non-trivial solutions read $\omega_i = \frac{i\pi}{\ell}\sqrt{\frac{H}{\mu}}$ which might be reversed to obtain the tension in a cable from a known mass per unit length and measured length and fundamental natural frequency ω_1. The horizontal tension H in the cable might also be determined with a least square approach, and by adjustment with several natural frequencies.

Cable with uniform mass and with one additional localized mass

Under the same assumptions as before, the equation of motion governing the vibrations of a cable with localized masses reads

$$H\partial_x^2 v = \mu\partial_t^2 v + \sum_{k=1}^{n} M_k \delta(x-\ell_k)\,\partial_t^2 v(\ell_k,t). \tag{3}$$

The Dirac delta function $\delta(x-\ell_k)$ being equal to 0 for $x \neq \ell_r$, the above solution for the homogenous system is still valid over each of the sub-domains in-between the concentrated masses. Considering for now that there is a single concentrated mass M_1 located at abscissa ℓ_1 ($n = 1$), the two solutions $v_0(x,t)$ and $v_1(x,t)$ are

$$v_r(x,t) = A_r \sin kx + B_r \cos kx \qquad x \in]\ell_r, \ell_{r+1}[\tag{4}$$

for $r = 0, 1$ and where we have used the notations $\ell_0 = 0$ and $\ell_2 = \ell$ (to anticipate the multiple mass configuration). The classical boundary conditions,

$$v_0(0,t) = 0; \quad v_1(\ell,t) = 0; \tag{5}$$

together with the continuity condition and the local equilibrium equation of the mass

$$v_0(\ell_1,t) = v_1(\ell_1,t) \quad ; \quad \lfloor \partial_x v \rfloor_{\ell_1^-}^{\ell_1^+} = \partial_x v_1(\ell_1,t) - \partial_x v_0(\ell_1,t) = \frac{M_1}{H} \ddot{v}_0(\ell_1,t) \tag{6}$$

form the set of four additional equations necessary to determine the four constants A_0, A_1, B_0 and B_1. These four equations read

$$\begin{aligned} B_0 &= 0 \\ A_1 \sin k\ell + B_1 \cos k\ell &= 0 \\ A_0 \sin k\ell_1 + B_0 \cos k\ell &= A_1 \sin k\ell_1 + B_1 \cos k\ell_1 \\ kH\left(A_1 \cos k\ell_1 - B_1 \sin k\ell_1 - A_0 \cos k\ell_1 + B_0 \sin k\ell_1\right) &= -M\omega^2 \left(A_0 \sin k\ell_1 + B_0 \cos k\ell_1\right) \end{aligned} \tag{7}$$

and might be written in the dimensionless matrix format

$$\mathbf{A}\boldsymbol{\varphi} = \mathbf{0} \tag{8}$$

where $\boldsymbol{\varphi} = (A_0, B_0, A_1, B_1)^T$ and

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sin\kappa & \cos\kappa \\ \sin\kappa\lambda_1 & \cos\kappa\lambda_1 & -\sin\kappa\lambda_1 & -\cos\kappa\lambda_1 \\ \cos\kappa\lambda_1 - \mathcal{M}_1\kappa\sin\kappa\lambda_1 & -\sin\kappa\lambda_1 - \mathcal{M}_1\kappa\cos\kappa\lambda_1 & -\cos\kappa\lambda_1 & \sin\kappa\lambda_1 \end{pmatrix} \tag{9}$$

and where $\mathcal{M}_1 = \frac{M_1}{\mu\ell}$, $\lambda_1 = \frac{\ell_1}{\ell}$ and $\kappa = k\ell$ are the ratio of the concentrated mass to the mass of the cable, the relative position of the concentrated mass and the dimensionless wave number. The non-trivial eigen solutions of this problem are such that matrix $\mathbf{A}$ is singular. The form a countably infinite set $\{\kappa_i\}$ and are obtained by stating that $\det(\mathbf{A}) = 0$, which yields after some simplifications

$$\kappa_i \mathcal{M}_1 \cos\left[\kappa_i (1 - 2\lambda_1)\right] - \kappa_i \mathcal{M}_1 \cos\kappa_i - 2\sin\kappa_i = 0. \tag{10}$$

As particular cases, it is seen that if $\lambda_1 = 0$ or $\lambda_1 = 1$ (the concentrated mass is located at either end of the cable), the characteristic equation boils down to $2\sin\kappa_i = 0$, which further yields $\kappa_i = i\pi$, that is, the solutions obtained in the previous case without concentrated mass.

When $\mathcal{M}_1 \ll 1$, when the additional concentrated mass is much smaller than the mass of the cable, the eigen values are the discrete solutions of a perturbed equation of the form $2\sin\kappa_i = \varepsilon\kappa_i f(\kappa_i)$ where $\varepsilon \ll 1$ and $f(\cdot)$ is of order 1 at most. This indicates that the natural frequencies up to $\kappa_i = O\left(\mathcal{M}_1^{-1}\right)$ are unchanged at first order; the presence of the small mass however affects higher modes. On the contrary when $\mathcal{M}_1 \gg 1$, the characteristic polynomial reads $\kappa_i \sin\left[\kappa_i (1 - \lambda_1)\right] \sin(\kappa_i\lambda_1) + \mathcal{M}_1^{-1} \sin\kappa_i = 0$, so that at first order, the root

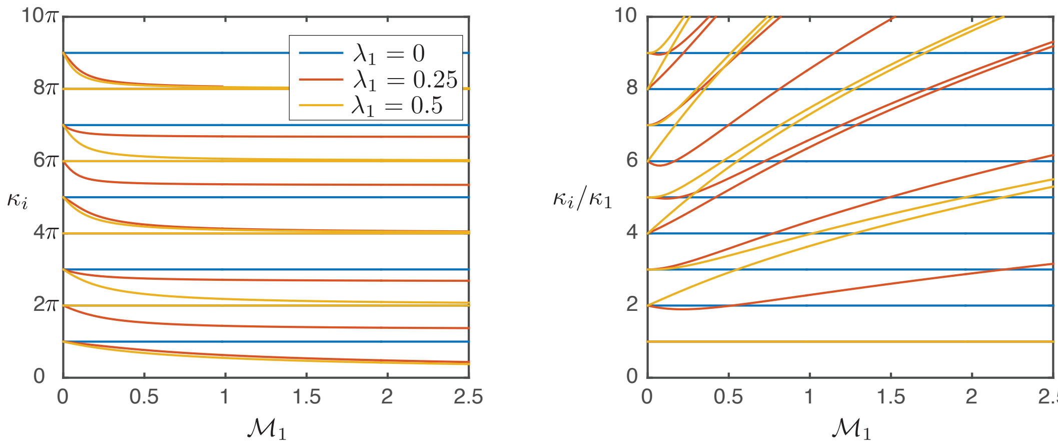

Figure 2: Natural circular frequencies $\omega_i = \frac{\kappa_i}{\ell}\sqrt{\frac{H}{\mu}}$ of a cable with a mass $M_1 = \mathcal{M}_1 \mu \ell$ located at abscissa $\ell_1 = \lambda_1 \ell$.

is given by either $\sin[\kappa_i(1-\lambda_1)] = 0$, either $\sin(\kappa_i \lambda_1) = 0$, which are the natural frequencies corresponding to independent vibrations of the two sub-spans of the cable, the large concentrated mass acting therefore as an intermediate support.
Figure 2 shows the dimensionless frequency κ_i as a function of the added mass, for several relative positions of the mass. These graphs confirm the statements formulated in the above analysis. The right plot also shows that the natural frequencies (even of the lower modes!) are no longer organized with a uniform pattern as soon as there is a significant mass located at a significant distance from the cable end.

Cable with uniform mass with several additional localized masses

The case with multiple masses is readily obtained following the same procedure. Let n be the number of masses in the span. There are located at abscissa ℓ_k with $k = 1, \cdots, n$. Introducing $\ell_0 = 0$ and $\ell_{n+1} = \ell$, and organizing the numbering of masses by increasing distance from a cable end, we have $0 = \ell_0 \leq \ell_1 \leq \cdots \leq \ell_n \leq \ell_{n+1} = \ell$. Along each interval $]\ell_r, \ell_{r+1}[$ with $r = 0, \cdots, n$, the governing equation is given by (1) whose solution is (4) where now $r = 0, \cdots, n$. The $2n+2$ unknowns $\{A_r, B_r\}$ are determined thanks to the two boundary conditions (5), n continuity conditions and n local equilibrium equations of the kind (6). The resulting set of equation is written in the form

$$\mathbf{A}(k; \boldsymbol{\ell}, \mathbf{M}, \mu, H)\, \boldsymbol{\varphi} = \mathbf{0} \tag{11}$$

where $\boldsymbol{\varphi} = (A_0, B_0, \cdots, A_n, B_n)^T$ and $\mathbf{A}(k; \boldsymbol{\ell}, \mathbf{M}, \mu, \ell, H)$ is a matrix of coefficients expressed a a function of k and the other parameters of the problem, i.e. μ, ℓ, H, $\boldsymbol{\ell} = \{\ell_1, \cdots, \ell_n\}$ and $\mathbf{M} = \{M_1, \cdots, M_n\}$.
The scaling is also readily extended to the multiple mass case by introducing

$$\mathcal{M}_k = \frac{M_k}{\mu \ell} \qquad ; \qquad \lambda_k = \frac{\ell_k}{\ell} \qquad (k = 1, \cdots, n) \tag{12}$$

and $\kappa = k\ell = \omega \ell \sqrt{\mu / H}$. With these dimensionless parameters, the eigen value problem (11) reads

$$\mathcal{A}(\kappa; \boldsymbol{\lambda}, \boldsymbol{\mathcal{M}})\, \boldsymbol{\varphi} = \mathbf{0} \tag{13}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\mathcal{M}}$ summarize the information related to the additional masses and their positions. Figure 3 shows an example of normal modes for a cable with 4 additional masses in the span. They correspond

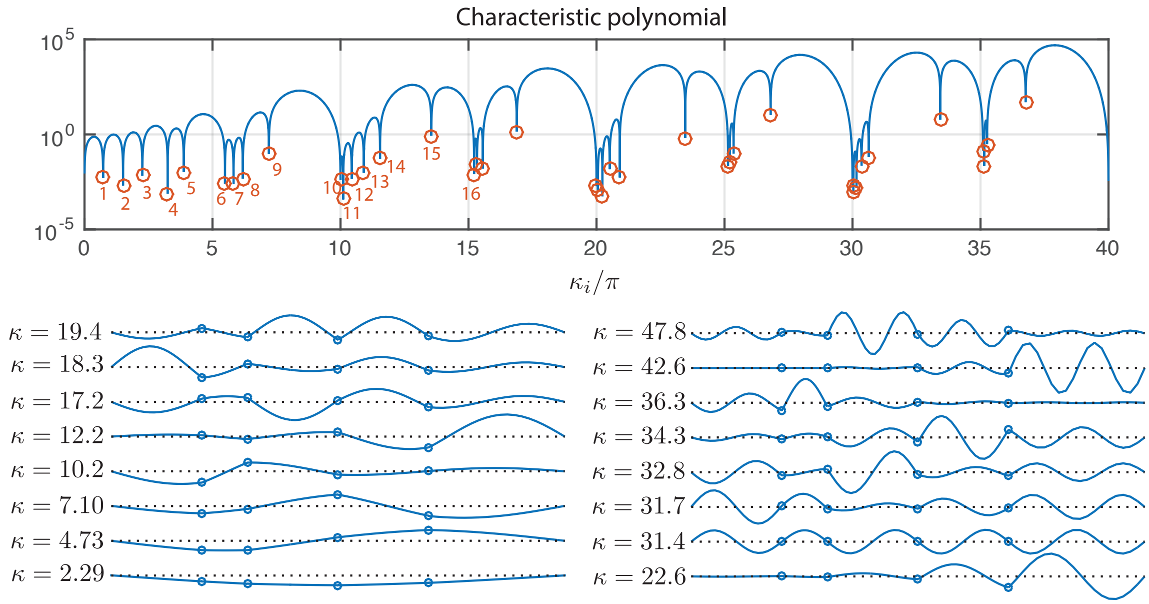

Figure 3: First 16 natural frequencies and modes shapes of a cable with 4 additional masses in the span ($\boldsymbol{\lambda} = \{0.2, 0.3, 0.5, 0.7\}$ and $\boldsymbol{\mathcal{M}} = \{0.1, 0.2, 0.2, 0.1\}$). The upper plot shows the norm of the characteristic polynomial

to $\boldsymbol{\lambda} = \{0.2, 0.3, 0.5, 0.7\}$ and $\boldsymbol{\mathcal{M}} = \{0.1, 0.2, 0.2, 0.1\}$. The upper part of this figure shows the characteristic polynomial, as a function of κ. Although the total mass of the additional masses only represent 60% of the own mass of the cable, $\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4 = 0.6$, the presence of the additional masses is sufficient to create a non-uniform spacing of the natural frequencies. The total additional mass (60%) being relatively small, the natural frequency of the first few eigen modes is only slightly affected, but it is observed that higher modes are clustered, see for instance modes {6,7,8) or {10,...,14}. This observation is specific to the example, but this clustering of natural frequencies tends to be a rule as soon as higher modes of a cable with a few additional masses is considered. The advantage of this low-order model, over a finite element model, is that it does not require to worry about the space discretization since it is perfectly and accurately captured by the explicit solution. The proposed analytical approach is also very fast and as accurate as desired.

SENSITIVITY OF NATURAL FREQUENCIES TO CHANGES OF MASS OR MASS POSITION

Now we have a direct model for the computation of the natural frequencies of a given cable with given information about additional masses, the inverse problem might be tackled. To do so, let us imagine now that the eigen frequencies of the cable are known (for instance by peak picking from an experimental campaign) and that we would like to determine the values of the additional masses and their positions. Starting from best guesses for these unknowns, a model updating procedure needs to be developed. In order to avoid search algorithm running in all directions (because of the high-gradient sensitivity of the natural frequencies to small changes of masses and mass positions), and even if the dimensions of the search space are limited, it is interesting to establish explicitly the sensitivity of the natural frequencies to small changes in the masses or mass positions.

First the sensitivity of the natural frequencies and mode shapes to changes in the mass positions is expressed by means of the derivatives of the natural frequencies $\partial_\lambda \kappa$ (let us consider just one mode, so κ is a scalar here) and associated mode shapes $\partial_\lambda \boldsymbol{\varphi}$. These $n + 1$ unknowns are determined by considering the first derivative of

$$\mathcal{A}(\kappa;\boldsymbol{\lambda},\boldsymbol{\mathcal{M}})\,\boldsymbol{\varphi}=\mathbf{0} \tag{14}$$

with respect to $\boldsymbol{\lambda}$ and notice that $\kappa=\kappa(\boldsymbol{\lambda},\boldsymbol{\mathcal{M}})$ and $\boldsymbol{\varphi}=\boldsymbol{\varphi}(\boldsymbol{\lambda},\boldsymbol{\mathcal{M}})$ are also both functions of $\boldsymbol{\lambda}$. This yields

$$(\partial_{\lambda}\mathcal{A}+\partial_{\kappa}\mathcal{A}\partial_{\lambda}\kappa)\boldsymbol{\varphi}+\mathcal{A}\partial_{\lambda}\boldsymbol{\varphi}=\mathbf{0} \tag{15}$$

where $\partial_{\lambda}\mathcal{A}$ is the Jacobian of $\mathcal{A}$ associated with changes of the mass positions. This forms a set of n equations which is complemented by stating that the vector norm of $\boldsymbol{\varphi}$ is normalized to 1, i.e. $\boldsymbol{\varphi}^T\boldsymbol{\varphi}=1$. Again, a side-by-side derivation yields $\left(\partial_{\lambda}\boldsymbol{\varphi}^{\mathbf{T}}\right)\boldsymbol{\varphi}+\boldsymbol{\varphi}^T\partial_{\lambda}\boldsymbol{\varphi}=0$, i.e.

$$\boldsymbol{\varphi}^T\partial_{\lambda}\boldsymbol{\varphi}=0, \tag{16}$$

so that the derivatives of the eigen value and eigenvector with respect to changes in the mass position might be expressed as the solution of equations (15) and (16). In a matrix form, they read

$$\begin{pmatrix}\partial_{\lambda}\boldsymbol{\varphi}\\ \partial_{\lambda}\kappa\end{pmatrix}=-\begin{pmatrix}\boldsymbol{\varphi}^T & 0\\ \mathcal{A} & (\partial_{\kappa}\mathcal{A})\boldsymbol{\varphi}\end{pmatrix}^{-1}\begin{pmatrix}0\\ (\partial_{\lambda}\mathcal{A})\boldsymbol{\varphi}\end{pmatrix}. \tag{17}$$

The same developments hold for changes of masses

$$\begin{pmatrix}\partial_{\mathcal{M}}\boldsymbol{\varphi}\\ \partial_{\mathcal{M}}\kappa\end{pmatrix}=-\begin{pmatrix}\boldsymbol{\varphi}^T & 0\\ \mathcal{A} & (\partial_{\kappa}\mathcal{A})\boldsymbol{\varphi}\end{pmatrix}^{-1}\begin{pmatrix}0\\ (\partial_{\mathcal{M}}\mathcal{A})\boldsymbol{\varphi}\end{pmatrix}. \tag{18}$$

In equations (17) and (18) the derivatives of matrix $\mathcal{A}$ with respect to κ, $\boldsymbol{\lambda}$ and $\boldsymbol{\mathcal{M}}$ required. They can be obtained explicitly as well since the band matrix $\mathcal{A}$ is nothing but a assembly of smaller matrices, of the type of (9). These details are omitted from this paper.

Iterative nonlinear least-square for the low-order model updating

Let us assume for a moment that the cable tension H, the mass per unit length μ and the cable length ℓ are known. A set of measured circular natural frequencies $\{\hat{\omega}_i\}$ might therefore be transformed into a set of dimensionless wave numbers

$$\hat{\kappa}_i=\hat{\omega}_i\ell\sqrt{\frac{\mu}{H}}. \tag{19}$$

For a given set of additional masses $\boldsymbol{\mathcal{M}}$ and positions $\boldsymbol{\lambda}$, the low-order model described before provides a set of circular natural frequencies $\kappa_i(\boldsymbol{\lambda},\boldsymbol{\mathcal{M}})$. Provided they can be paired up appropriately, the measured and estimated dimensionless wave numbers might be compared. For example, the cost function

$$\mathcal{R}\left(\hat{\boldsymbol{\kappa}},\boldsymbol{\lambda},\boldsymbol{\mathcal{M}}\right)=\sum_{i=1}^{n}\left(\kappa_i(\boldsymbol{\lambda},\boldsymbol{\mathcal{M}})-\hat{\kappa}_i\right)^2$$

is a very good candidate for the model updating procedure. Starting from initial guesses $\boldsymbol{\mathcal{M}}^{(0)}$ and $\boldsymbol{\lambda}^{(0)}$, the nonlinear least-square minimization of this residue yields so-called normal equations in ordinary least square fitting. Indeed, in a small perturbation approach, the small increments $\Delta\boldsymbol{\lambda}=\boldsymbol{\lambda}^{(p+1)}-\boldsymbol{\lambda}^{(p)}$ and $\Delta\boldsymbol{\mathcal{M}}=\boldsymbol{\mathcal{M}}^{(p+1)}-\boldsymbol{\mathcal{M}}^{(p)}$ are expressed by equating to the measured frequencies, the linearized version of the natural frequencies estimated with the model, that is

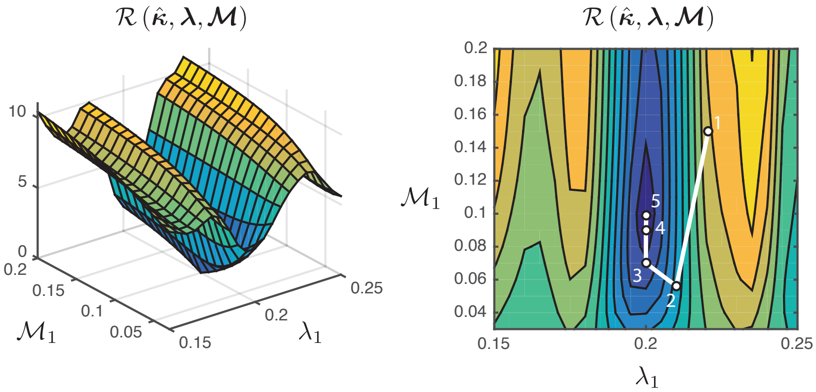

Figure 4: Residue of the model updating process. Only the first mass is initially badly evaluated (mass and position) Five iterations are sufficient for the algorithm to converge.

$$\hat{\boldsymbol{\kappa}} = \boldsymbol{\kappa}(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\mathcal{M}}^{(0)}) + \partial_{\boldsymbol{\mathcal{M}}}\kappa\Delta\boldsymbol{\mathcal{M}} + \partial_{\lambda}\kappa\Delta\boldsymbol{\lambda} \qquad \rightarrow \qquad \begin{pmatrix} \partial_{\boldsymbol{\mathcal{M}}}\kappa & \partial_{\lambda}\kappa \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\mathcal{M}} \\ \Delta\boldsymbol{\lambda} \end{pmatrix} = \hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\mathcal{M}}^{(0)})$$

The normal equations are obtained by projection on the subspace $\begin{pmatrix} \partial_{\boldsymbol{\mathcal{M}}}\kappa & \partial_{\lambda}\kappa \end{pmatrix}$; they read

$$\begin{pmatrix} (\partial_{\boldsymbol{\mathcal{M}}}\kappa)^T \partial_{\boldsymbol{\mathcal{M}}}\kappa & (\partial_{\boldsymbol{\mathcal{M}}}\kappa)^T \partial_{\lambda}\kappa \\ (\partial_{\lambda}\kappa)^T \partial_{\boldsymbol{\mathcal{M}}}\kappa & (\partial_{\lambda}\kappa)^T \partial_{\lambda}\kappa \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\mathcal{M}} \\ \Delta\boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} (\partial_{\boldsymbol{\mathcal{M}}}\kappa)^T \\ (\partial_{\lambda}\kappa)^T \end{pmatrix} \left(\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\mathcal{M}}^{(0)})\right)$$

and are easily solved for $\Delta\boldsymbol{\lambda}$ and $\Delta\boldsymbol{\mathcal{M}}$, with the help of the derivatives established in (17) and (18).This closes the marching procedure that generates the sequence of $\boldsymbol{\mathcal{M}}^{(p)}$ and $\boldsymbol{\lambda}^{(p)}$ minimizing the residue in an iterative nonlinear least-square sense. Iteration are stopped after desired accuracy on the residue. This kind of algorithm is known to be particularly efficient.

EXAMPLE OF APPLICATION

As an illustration, we consider again the cable whose normal modes of vibration are given in Figure 3, but we pretend this time that the additional masses $\boldsymbol{\mathcal{M}} = \{0.1, 0.2, 0.2, 0.1\}$ and their positions $\boldsymbol{\lambda} = \{0.2, 0.3, 0.5, 0.7\}$ are unknown. Figure 4 shows the residue $\mathcal{R}$ as a function of $\mathcal{M}_1$ and λ_1 . This function exhibits significant gradients in the parameter space, especially in the λ_1-direction, which heralds difficulties in convergence. The white trajectory on the right side shows the successive iterates, starting from $\boldsymbol{\mathcal{M}}^{(0)} = \{0.15, 0.2, 0.2, 0.1\}$) and $\boldsymbol{\lambda}^{(0)} = \{0.22, 0.3, 0.5, 0.7\}$, i.e. only the first mass was initially badly evaluated and localized. Five iterations are sufficient to converge towards the global optimum of the problem.

In a second example, the identification process is initiated with $\boldsymbol{\lambda}^{(0)} = \{0.22, 0.32, 0.52, 0.72\}$ and $\boldsymbol{\mathcal{M}} = \{0.15, 0.15, 0.15, 0.15\}$), i.e. all positions of the four masses are overestimated (3% to 10%) and the mass, although being well evaluated in total, starts with some 50% off in the worst case. Figure 5 shows the successive ratios of the current estimates $\mathcal{M}_k^{(p)}$ and $\lambda_k^{(p)}$ and the exact values. This example shows that the positions of the masses converge earlier than the values of the additional masses. The residual of the problem (see rightmost plot) monotonically drops down to the minimum of the problem. After 10 iterations, the model parameters are accurately estimated: ratios $\mathcal{M}_k^{(p)}/\hat{\mathcal{M}}_k$ and $\lambda_k^{(p)}/\hat{\lambda}_k$ tend to 1, $\forall k = 1, \cdots, 4$.

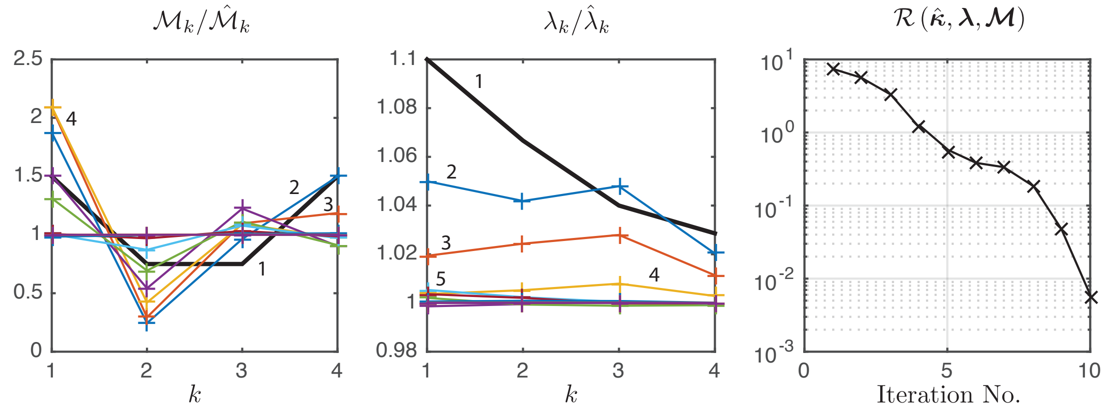

Figure 5: Illustration of the convergence of the algorithm, starting from $\boldsymbol{\lambda}^{(0)} = \{0.2, 0.3, 0.5, 0.7\}$ and $\boldsymbol{\mathcal{M}}^{(0)} = \{0.1, 0.2, 0.2, 0.1\}$).

CONCLUSION

The paper has presented a low-order model for the modal analysis of a cable with equipped masses in the span. This model offers a rapid evaluation of the natural frequencies and mode shapes. As soon as the devices installed in the span of a cable have similar masses than the mass of the cable, there is a certain influence on the natural frequencies, whose arrangement is then far from the well set of regularly spaces values. The derivatives of the natural frequencies with respect to the masses of the additional devices and their position as derived in order to develop a more robust optimization tool for the model parameter identification, that is, determine the masses and position, and finally the tension in a cable based on measured natural frequencies only. Although this has not been studied here, it is expected that the simultaneous consideration of mode shapes would improve the identification procedure. This is however more difficult to implement since several sensors are required to identify the mode shape with sufficient accuracy.

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