Predicting the transverse volume distribution under an agricultural spray boom

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ABSTRACT. The volume distribution of spray below individual agricultural flat-fan spray nozzles was fitted to a truncated normal distribution. This expresses the parameters as a function of the spray liquid pressure, the boom height and the nozzle orifice size. This model was used to predict the transverse distribution below an agricultural spray boom.

KEYWORDS: Flat-fan nozzle; spray volume distribution; truncated normal distribution

Introduction

The variability of the spray deposited on the ground by an agricultural spraying boom has two components, longitudinal and transverse. The longitudinal component is affected by the regularity of the relationship between flow and forward speed of the sprayer. The transverse component is linked to the characteristics of the nozzles, to their spacing and to the distribution of spray from each nozzle across the boom. This depends on the individual characteristics of the nozzle, on its height and the pressure of the liquid while spraying.

The objective of this work was to establish a mathematical model of the transverse spray pattern from a fan nozzle based on experimental data, and to use the results of this model to predict the spray pattern below a horizontal boom with several nozzles. This model should enable optimization of the spray pattern beneath the boom, with different nozzle spacing, by varying parameters such as height of the boom and spraying pressure.

In order to study the effects of spray pattern from individual nozzles and boom roll angle on the uniformity of application, Mawer (1988) and Mawer and Miller (1989) have modelled two idealized spray volume distributions, namely rectangular and triangular, and showed that both normal and beta distributions could also be used to describe the spray volume distribution beneath a flat-fan nozzle.

Model of the spray pattern of a fan nozzle

Theoretical model

If the falling of each unit of volume (the drop) is considered as a random variable, one can consider the volumes collected in the tubes of the patternator as observed frequencies of this random variable, grouped in classes. The unit of measure of these class frequencies will be cm^3, which corresponds to the smallest volume distinguishable in the tubes. One can then interpret the pattern of the volume sprayed as an observed probability distribution, and represent it by a theoretical probability distribution with similar characteristics. This problem is twofold: first, the choice of the model or probability density function and second, the estimation of its parameters from those of the measured spray pattern, considered as a sample. The estimated parameters of the theoretical distribution will then be expressed as a function of the factors which condition spraying: pressure, height and orifice size of the nozzle.

Three common makes of nozzles were used with 80 degree and 110 degree spray angles. Among these nozzles and spray angles, three typical examples were chosen and each nozzle tested using a standard patternator (ISO 5682 l^{-1}) at a range of pressures (between 1 and 4 bar) and at three different heights (40, 60 and 80 cm). Henceforth, only one make is referred to as the results for the other two are similar.

Variance analysis of the results obtained by repeating the patternator test after re-mounting the experimental device showed that the influence of slight mounting errors was not significant at the 0·05 level. The same method has been applied to study the influence of the differences between nozzles of the same manufacture batch, and gave the same conclusions, namely the non-significance of this factor at the 0·05 level.

In order to assess the influence of other random factors, computing the number of replicates needed to determine the mean volume collected in each tube with a maximum relative error of 5% showed that

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one simple replicate was sufficient, and even more so when the volume was greater. However, this was not the case for the extremes of the pattern, where the volume collected is relatively low.

Description of spray volume distributions

The observed spray volume distributions are approximately bell-shaped symmetrical curves with finite margins, all of which are properties of a truncated normal distribution.

A random variable X has a double truncated normal distribution when its probability density funtion is (Johnson and Kotz, 1970)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2\sigma^2)(x-\xi)^2} \times \left[\frac{1}{\sigma\sqrt{2\pi}}\int_A^B e^{-(1/2\sigma^2)(t-\xi)^2}\,dt\right]^{-1}$$
$$= \sigma^{-1} Z\left(\frac{x-\xi}{\sigma}\right)\left[\phi\left(\frac{B-\xi}{\sigma}\right) - \phi\left(\frac{A-\xi}{\sigma}\right)\right]^{-1},$$
$$\text{for } A \le X \le B,$$

where σ is the standard deviation of the normal distribution, $Z(X)$ is the probability density function of the unit normal distribution at the value X, and $\phi(x)$ is the value of the cumulative distribution function of the same distribution for the value X. The upper and lower points of truncature are respectively A and B. If

$$\frac{A-\xi}{\sigma} = -\left(\frac{B-\xi}{\sigma}\right) = -\delta$$

then

$$E(X) = \xi$$

and

$$\mathrm{var}(X) = \left[1 - \frac{2\delta Z(\delta)}{2\phi(\delta) - 1}\right]\sigma^2.$$

The double truncated normal distribution is entirely defined by four parameters: ξ, σ and the two points of truncature A and B.

If one supposes that the transverse pattern of the flow sprayed by a nozzle placed vertically from the central point of the central class (tube) or class O, is symmetrical, then one effectively has $A = -B$ and $\xi = 0$. Thus, the spray width of a nozzle is symmetrical to the nozzle orifice and is contained between the two points of truncature. Moreover, population variance is estimated by that of the sample.

Critique of the fitting

In most of the observed patterns two irregularities occur: (1) two lateral shoulderings more or less symmetrical to the central class; (2) a flattening of the top of the spray pattern. The percentage of the total volume represented by these irregularities was low (0–5%) and varied with the nozzle and spraying conditions.

In general, the shape of the pattern was correctly described by the model; however, this degree of resemblance should be quantified more rigorously by a test of 'goodness of fit' (Dagnelie, 1975).

The χ^2 test consists in computing

$$\chi^2_{\text{obs}} = \sum_{i=1}^{p} \frac{(n_i - nP_i)^2}{nP_i}$$

where p is the number of tubes, n_i is the volume collected in the tube i (in cm^3), n is the total volume (in cm^3) and P_i is the theoretical probability of tube i. The quantity χ^2_{obs} has to be compared with the value of a χ^2 variable with $p-1$ degrees of freedom, corresponding to a probability of $1-\alpha$, if one admits a first-order error α. Thus the fraction $\chi^2_{\text{obs}}/\chi^2_{1-\alpha}$ has to be <1 to accept the 'goodness of fit'. Henceforth, $\chi^2_{\text{obs}}/\chi^2_{1-\alpha}$ is termed the χ^2 ratio and is used to measure the fitting quality.

The characteristics of a series of 110 degree fan

TABLE 1. Fitting parameters of F110 nozzles

		Nozzle 11002				Nozzle 11004				Nozzle 11006			
Height (cm)	Pressure (bar)	Width (cm)	SD (cm)	Flow (l min^{-1})	χ^2 ratio	Width (cm)	SD (cm)	Flow (l min^{-1})	χ^2 ratio	Width (cm)	SD (cm)	Flow (l min^{-1})	χ^2 ratio
40	1	105	21·5	0·42	3·53	115	22·6	0·87	4·35	115	22·1	1·35	3·90
	2	145	27·9	0·63	1·75	145	27·9	1·27	2·15	145	27·3	1·91	3·63
	3	155	29·0	0·78	1·70	155	29·4	1·55	1·63	155	28·8	2·36	3·67
	4	155	30·1	0·90	0·83	155	30·3	1·79	1·24	165	29·6	2·72	4·54
60	1	155	30·6	0·42	4·20	165	33·2	0·87	4·50	165	32·6	1·35	3·98
	2	195	36·2	0·63	1·53	195	39·0	1·27	2·13	205	37·9	1·91	3·57
	3	205	38·5	0·78	1·52	205	40·8	1·55	1·70	215	39·9	2·36	3·68
	4	215	39·6	0·90	1·63	215	41·7	1·79	1·35	225	40·8	2·72	3·51
80	1	195	37·7	0·42	3·40	205	41·7	0·87	3·82	215	41·7	1·35	3·63
	2	235	43·5	0·63	1·53	255	47·8	1·27	2·17	255	47·7	1·91	2·74
	3	245	45·8	0·78	2·23	255	49·8	1·55	2·15	255	49·5	2·36	3·05
	4	245	46·9	0·90	3·79	255	50·6	1·79	2·62	255	49·7	2·72	2·98

SD, standard deviation; the χ^2 ratio is calculated for α equal to 0.05

nozzles and the values of parameters that determine the model are given in *Table 1*.

Out of the 153 spray patterns measured with the three makes of nozzles, only six gave a χ^2 ratio <1, and $46<2$. Fitting to the normal (untruncated) distribution always yielded less good results.

Two of the curves for F11002 nozzles derived from the model are shown in *Figure 1* (χ^2 ratio <1) and *Figure 2* (χ^2 ratio >3).

Relationships between the parameters of the model and those of spraying

Regression of the standard deviation as a function of the height, pressure and flow

The standard deviation of the observed spray pattern increased with Napierian logarithms of pressure, height of the nozzle and standard flow.

The multiple regression equation is

$$\sigma = -78{\cdot}0 + 27{\cdot}1\ln(H) + 6{\cdot}15\ln(P) + 1{\cdot}72\ln(Q2),\ \text{and}\ R^2 = 0{\cdot}980,$$

where σ is the standard deviation (cm), H the height of the nozzle (cm), P the spraying pressure in bars, and $Q2$ the standard flow at the pressure of two bars (l min^{-1}); R^2 is the determination coefficient. All the coefficients are very highly significant. The residual standard deviation is equal to 1·26.

An analysis of variance showed that most of the variation of the standard deviation is explained by the spraying height (80% of this variation) while the rest is due to pressure, and, to a lesser extent, liquid flow.

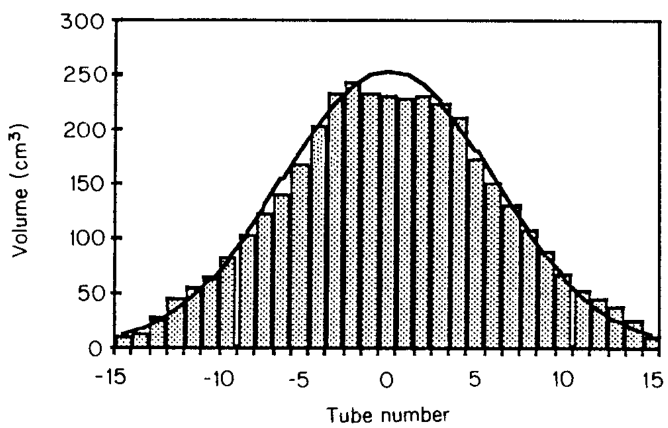

FIGURE 1. Curve derived from model fits the experimental data. Nozzle XR11002 (40 cm, 4 bar) χ^2 ratio 0·83. —, Volume expected; ▒, volume collected

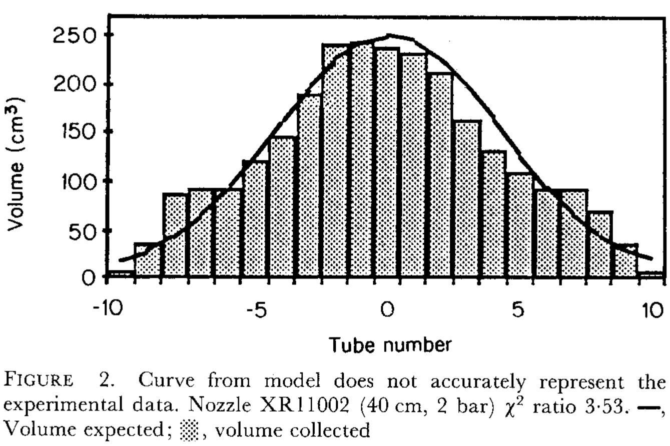

FIGURE 2. Curve from model does not accurately represent the experimental data. Nozzle XR11002 (40 cm, 2 bar) χ^2 ratio 3·53. —, Volume expected; ▒, volume collected

Regression of the spray width

The spray width is, in our model, the number of tubes containing a significant volume, to which 1 may be added in order to have an identical number on either side of the central tube.

The spray width was considered in relation to the Napierian logarithms of height, pressure and standard flow. For example, the regression equation for the same 110 degree fan nozzles was

$$W = -401^{***} + 1{\cdot}39^{***}\ln(H) + 36{\cdot}5^{***}\ln(P) + 8{\cdot}94^{***}\ln(Q2),$$

where W is the spray width (cm). The coefficients are highly (**) or very highly (***) significant and $R^2 = 0{\cdot}976$. The residual standard deviation is equal to 7·29.

Model of the spray pattern under the boom

Introduction

The individual patterns of adjacent nozzles along a boom overlap to improve the overall homogeneity of the spray. In most cases, each individual pattern is different because of the wear of the nozzles and their different individual characteristics. Conversely, the simulation of the spray pattern under the boom using only one individual spray pattern presupposes that all the nozzles have an identical pattern. This supposition is compatible with the conclusions of the study of non-controlled factors. However, this is only applicable to correctly mounted new nozzles of a same manufacture batch.

Henceforth, the spray pattern under a boom obtained by simulation using observed individual spray patterns will be called the 'observed spray pattern', and the pattern obtained using individual adjusted distributions will be termed the 'theoretical spray pattern'.

Simulation of the spray pattern under a boom

The simulated spray pattern (*Figure 3*) obtained from a spray pattern experiment correctly fitted to the truncated normal distribution shows that the overlapping of identical individual spray patterns spaced at a constant distance can produce regular peaks and troughs on both observed and adjusted spray patterns. However, the variation in the ob-

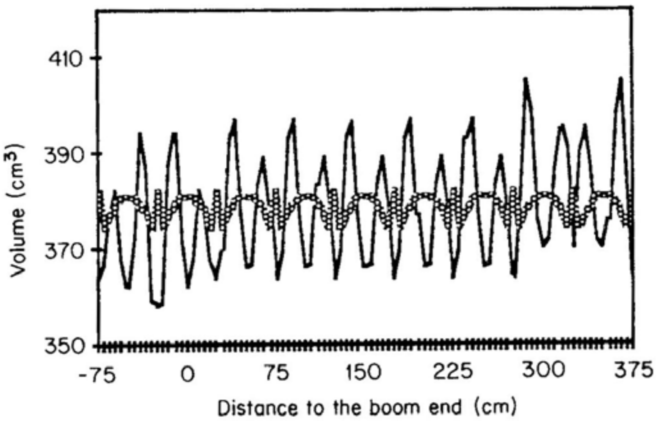

FIGURE 3. Simulation of the spray pattern under a boom. Nozzle XR11002 (40 cm, 4 bar) χ^2 ratio 0.83.ooooo,volume expected; —, volume collected

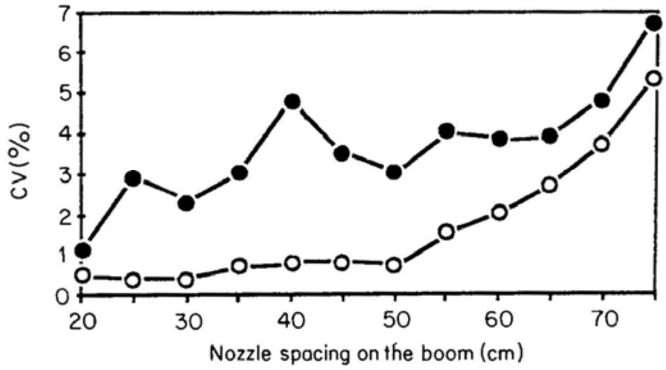

FIGURE 4. CV versus nozzle spacing. Nozzle XR11002 (40 cm, 4 bar) χ^2 ratio 0.83. ●, Observed CV; ○, theoretical CV

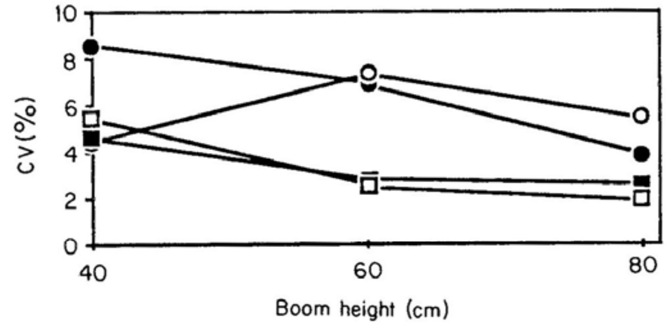

FIGURE 5. CV relating to height and pressure (●, 1 bar; ○, 2 bar; ■, 3 bar; □, 4 bar). Nozzle XR11006; spacing, 50 cm

served spray pattern is much greater, and its peaks (or troughs) do not correspond to those of the theoretical spray pattern. Also, it can be noted that the quality of the fitting does not influence these differences qualitatively. The process of summing individual patterns amplifies their irregularities compared with the model distribution. The resulting theoretical spray pattern is generally more even than the observed one.

Discussion of the results

The coefficients of variation relative to the theoretical and observed spray patterns, designated respectively CVT and CVO, have been calculated for each nozzle, for the whole working conditions and for different nozzle spacings on the boom. *Figures 4* and *5* show for two F110 nozzles these coefficients as a function of nozzle spacing and spraying conditions.

Examination of the spray patterns under a boom have demonstrated differences between simulated observed and theoretical spray patterns, with no defined relationship with spacing (*Figure 4*) despite the close fit of the model in this example (χ^2 ratio < 1). Consequently, the theoretical model cannot be used to describe and optimize the observed spray pattern under a boom, and therefore only the latter is discussed.

In general, the coefficient of variation decreased as nozzle spacing was reduced. However, the relationship between CV and spacing is unpredictable and cases may occur where the CV increased when spacing was reduced. The CV of the spray pattern decreased when the spraying height was raised from 40 to 60 cm, with little further change from 60 to 80 cm. Nevertheless, there is an interaction with pressure: thus at a height of 40 cm, increasing pressure improves homogeneity slightly. At 60 cm, the effect of pressure on the CV is slight and does not have a systematic character. At 80 cm, this effect becomes negligible. This means that at certain heights the CV is hardly affected by variation in pressure. For nozzles spaced at 50 cm, these heights range varied from 60 to 80 cm.

Conclusions

In practice, the truncated normal distribution fits experimental data, even when it is not acceptable in a statistical sense. Its parameters can easily be computed from liquid pressure, height and size of the orifice of the nozzle. However, when one uses this model to produce the transverse volume distribution under a boom, the overlapping of individual patterns increases the differences between model and data so that the model cannot be used to simulate and optimize the real pattern under a boom.

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