Magnetic and electrical characterization of superconductors

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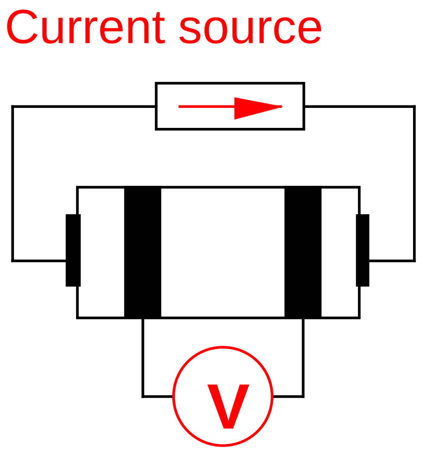
What kind of **measurements** can we make to characterize superconductors ?

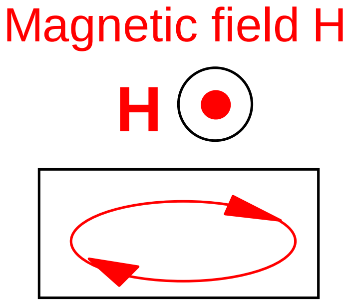
What kind of **information** can we extract from measurements ?

Purpose of this lecture

To better understand how we can characterize the electrical and magnetic propeties of materials through

TRANSPORT measurements and MAGNETIC measurements





Transport current
(applied externally)

Induced current
(by the applied magnetic field)

Outline

- Transport measurements - R(T)
- Transport measurements - E(J)

- Magnetic measurements (general)
- Magnetic measurements - M(H)

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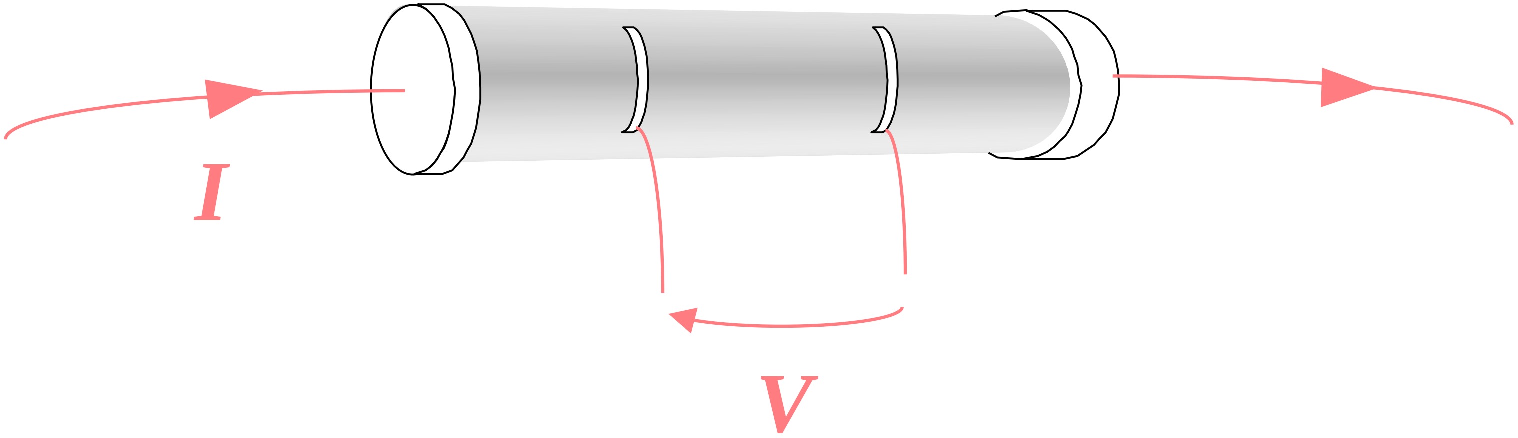
- Magnetic measurements (general)
- Magnetic measurements - M(H)

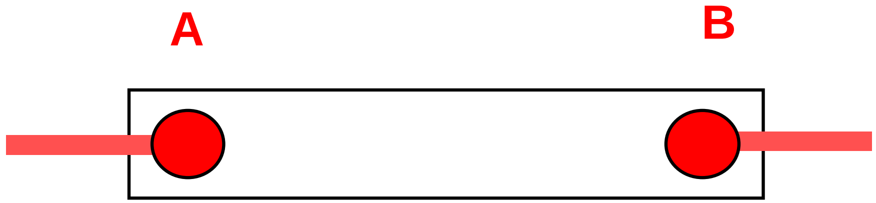
The main difficulty for transport measurements on superconductors = ?

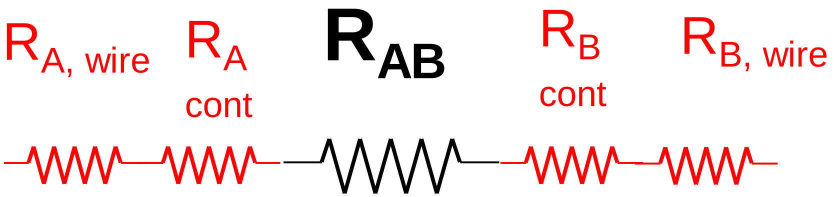


The finite resistance of electrical contacts

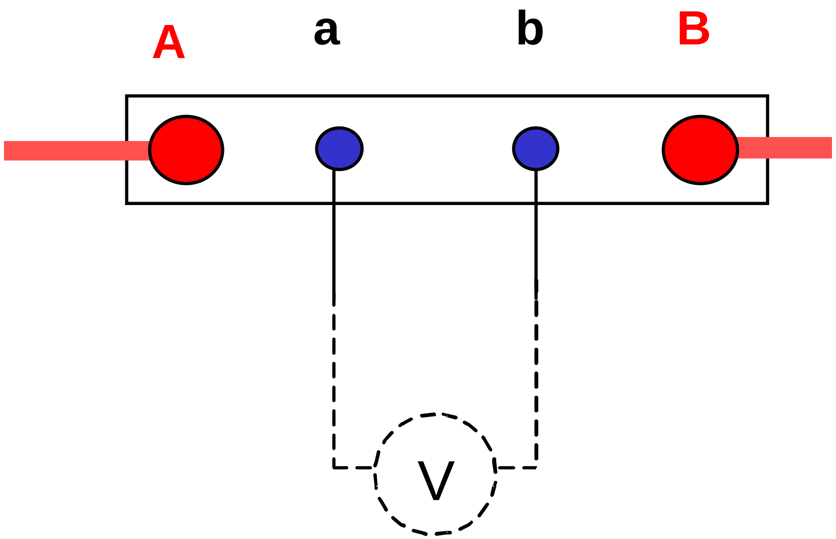
Influence of contact resistance & wire resistance

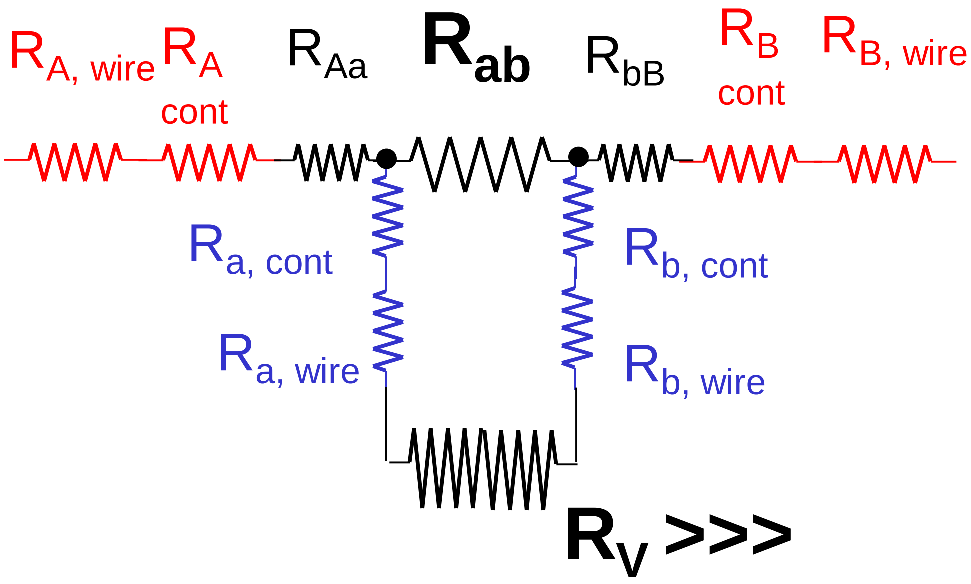
2-wire connexions





4-wire connexions





4-contact measurement (Kelvin connections)

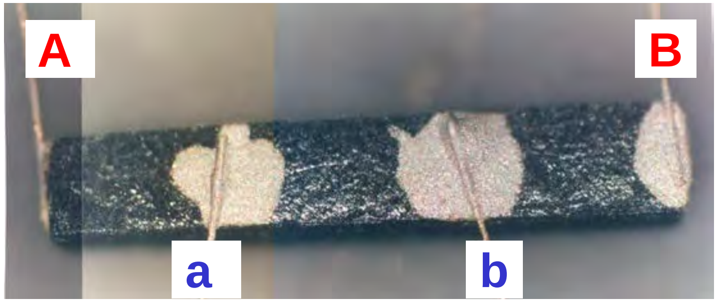
4-wire connexions are used to eliminate contact resistances and wire resistances

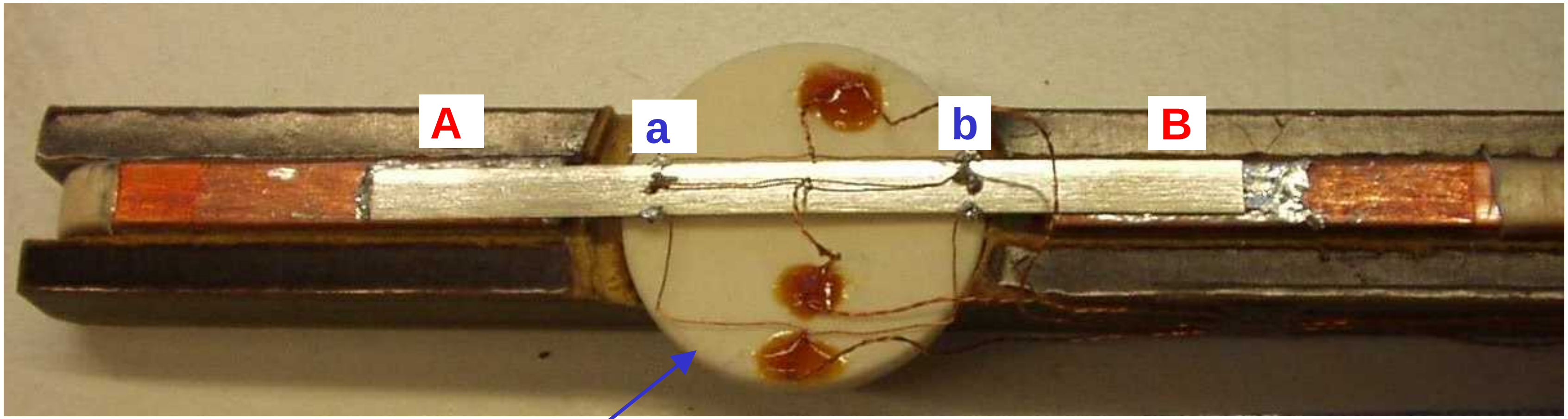
(i) The **current** contact resistances and wire resistances are outside the measurement circuit

(ii) The **voltage** contact resistances and wire resistances can be neglected with respect to the resistance of the voltmeter

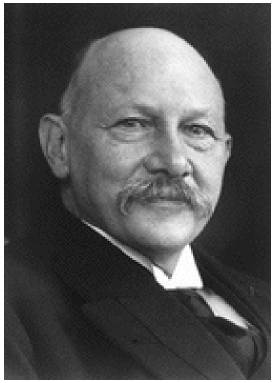
Examples :

A, B = current contacts
a, b = voltage contacts



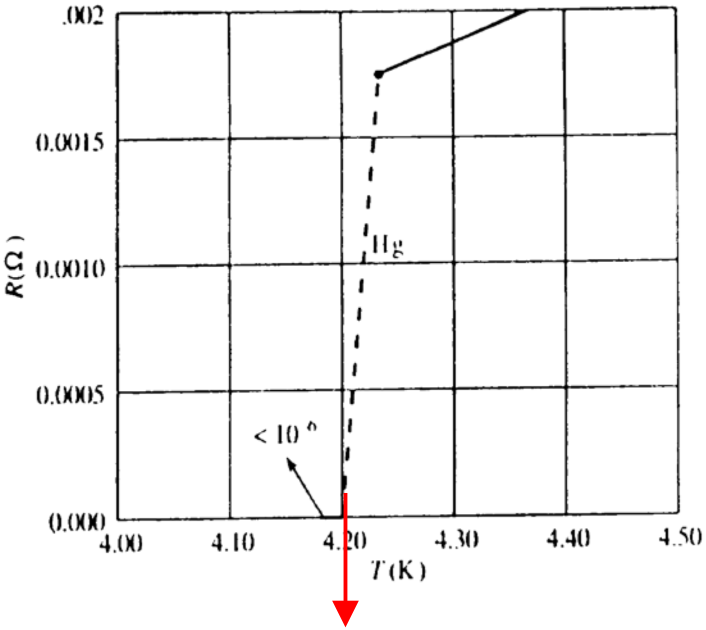


NB : for AC measurements : **twisted wires** are required to avoid inductive pick-up loops !!!



H. Kamerlingh-Onnes

Example for type-I superconductor (Hg)



T_c critical temperature

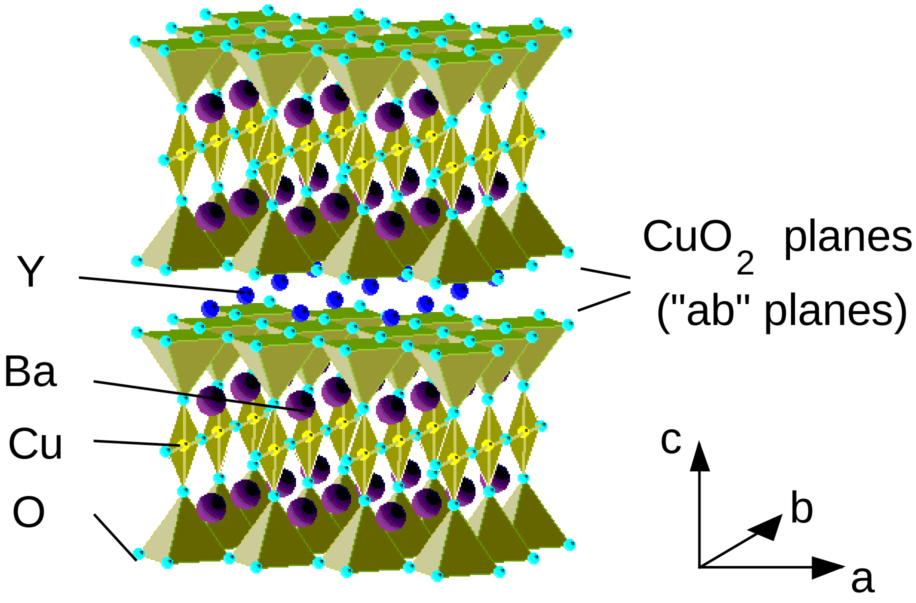
In addition to giving the critical temperature of the superconductor, a R(T) measurement in the presence of a magnetic field can be helpful in characterizing

(i) **anisotropy** effects
(ii) **granularity** and connectivity between grains
(iii) the phase diagram **(irreversibility line of the material)**

These characteristics of HTS materials are briefly recalled hereafter

(i) Anisotropy

Ex : Y - 123 single crystal



The **flow of current density J** is easier in the ab planes than along the c-axis :

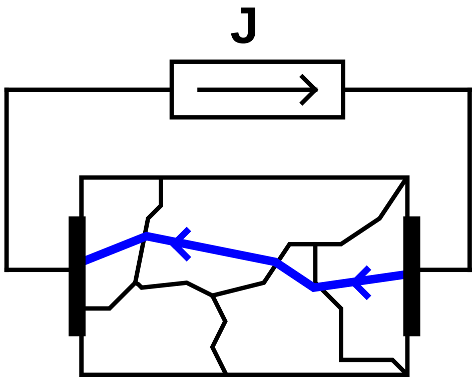
$$\mathbf{J_c (\| ab) > J_c (\| c)}$$

The pinning of **flux lines B** is larger for **B || ab** than for **B || c**

(J || B) = "force-free" configuration

(ii) Granularity

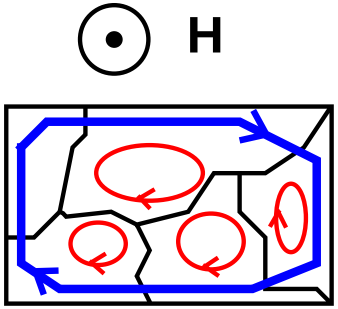
Transport current



Intergranular current J_{CJ}

Shielding currents

Applied magnetic field



Intergranular current J_{CJ}

Intragranular current J_{CG}

$$\boxed{J_{CJ} < J_{CG}}$$

Grain alignment - or **texturation** - is a key ingredient
to improve the **intergranular** critical current density

VOLUME 61, NUMBER 2 PHYSICAL REVIEW LETTERS 11 JULY 1988

Orientation Dependence of Grain-Boundary Critical Currents in $YBa_2Cu_3O_{7-\delta}$ Bicrystals

D. Dimos, P. Chaudhari, J. Mannhart, and F. K. LeGoues
Thomas J. Watson Research Center, IBM Research Division,
Yorktown Heights, New York, 10598
(Received 4 May 1988)

The critical current densities across grain boundaries have been measured as a function of misorientation angle in the basal plane of bicrystals of $YBa_2Cu_3O_{7-\delta}$. For small misorientation angles, the ratio of the grain-boundary critical current density to the bulk critical current density is roughly proportional to the inverse of the misorientation angle; for large angles, this ratio saturates to a value of about $\frac{1}{50}$. These results imply that achieving a high degree of texture both normal to and within the basal plane is important for the obtaining of very high critical currents in pure polycrystalline samples.

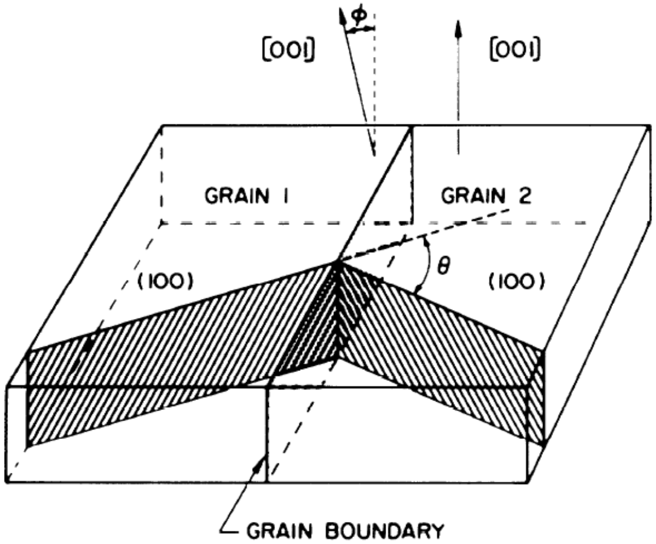
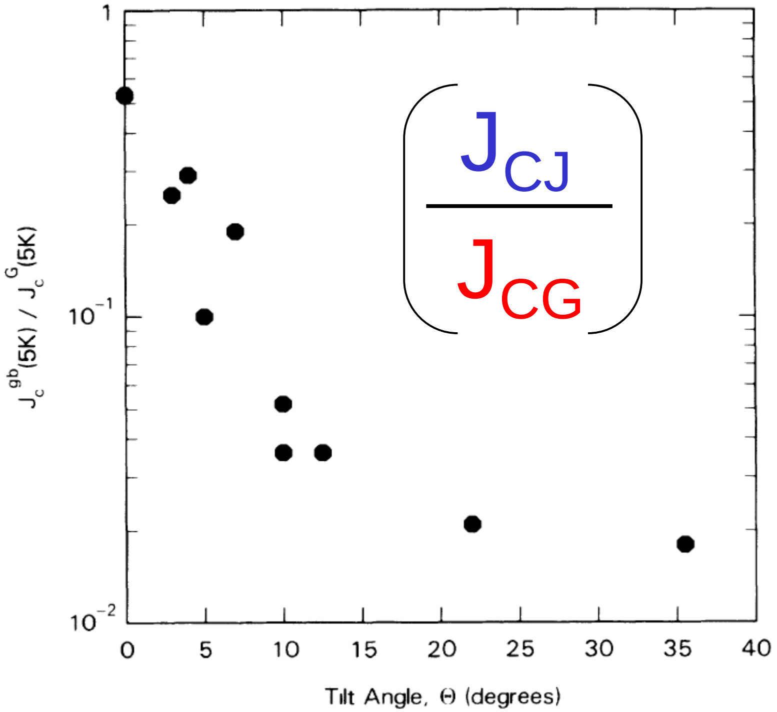


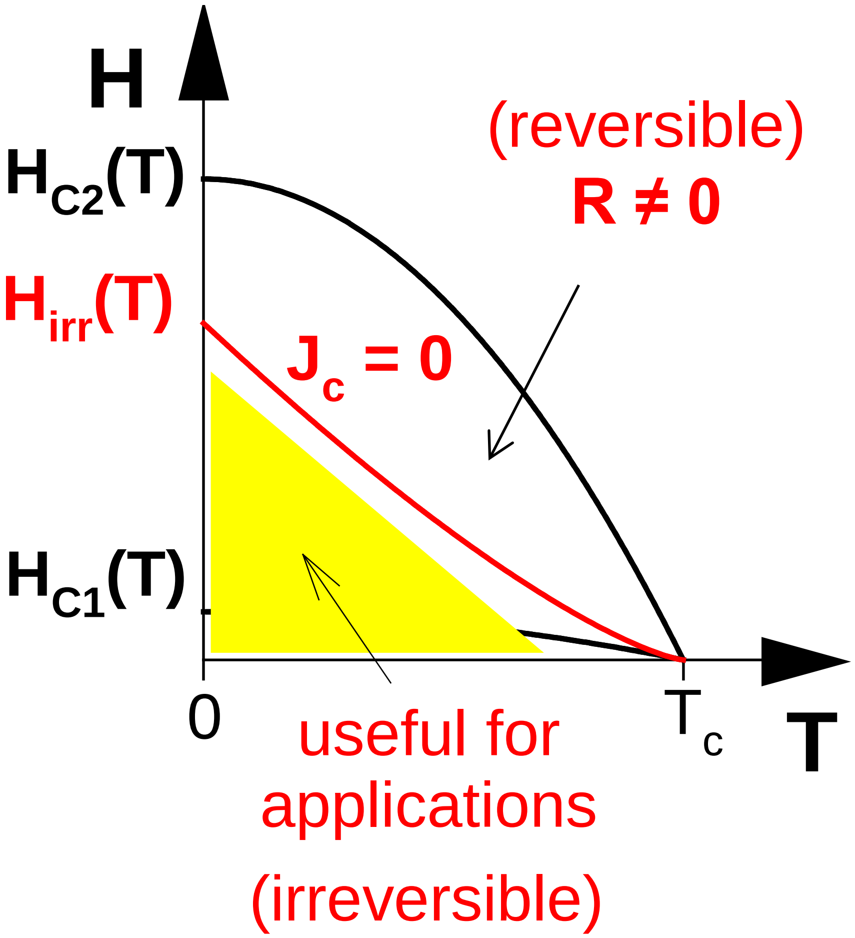
FIG. 1. Schematic diagram showing the important crystallography of the SrTiO₃ bicrystals which were used as substrates for the thin-film deposition.



$$\left(\frac{J_{CJ}}{J_{CG}}\right)$$

(iii) Irreversibility line

(relevant for high-temperature superconductors)

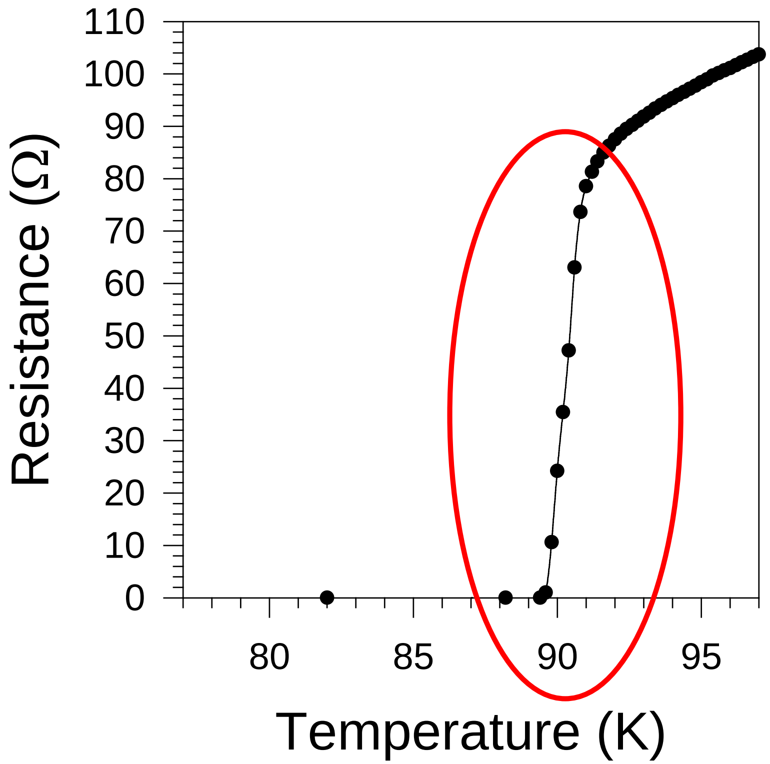


Irreversibility fields of some HTS materials at T = 77 K

Bi-2212 :	< 0.1 T
Bi-2223 :	0.3 T
Y-123 :	7-10 T

Typical R(T) curve

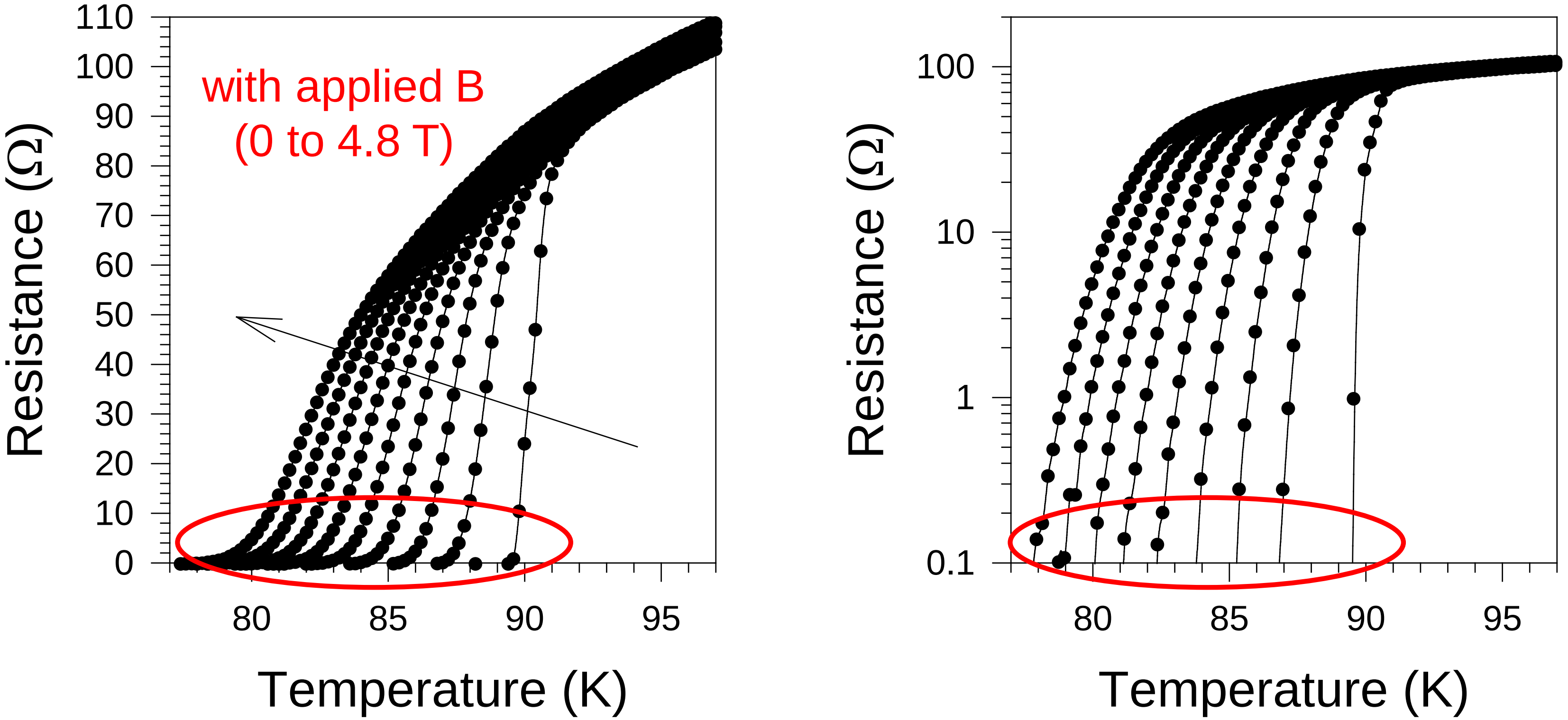
Ex: $YBa_2Cu_3O_7$



The width of the transition requires a **given criterion** to define Tc
(usual criterion : inflexion point [change of curvature] but others are possible)

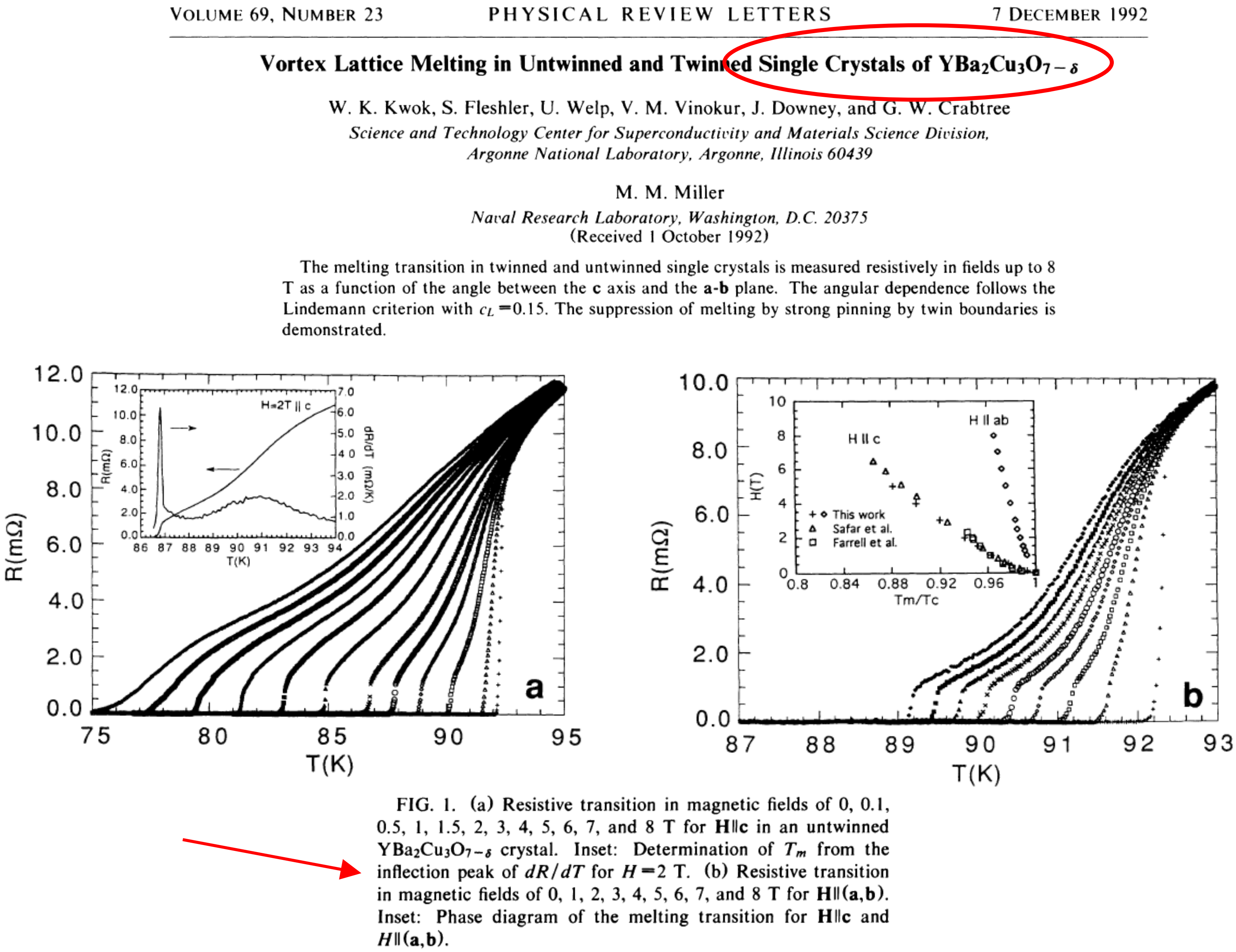
Typical R(T) curve

Ex: $YBa_2Cu_3O_7$



The use of a log scale can be very useful the temperature above which electrical resistance merges from the noise level (= irreversibility line ?)

Some (slightly more complicated) examples…



VOLUME 69, NUMBER 23 PHYSICAL REVIEW LETTERS 7 DECEMBER 1992

Vortex Lattice Melting in Untwinned and Twinned Single Crystals of $YBa_2Cu_3O_{7-\delta}$

W. K. Kwok, S. Fleshler, U. Welp, V. M. Vinokur, J. Downey, and G. W. Crabtree
Science and Technology Center for Superconductivity and Materials Science Division,
Argonne National Laboratory, Argonne, Illinois 60439

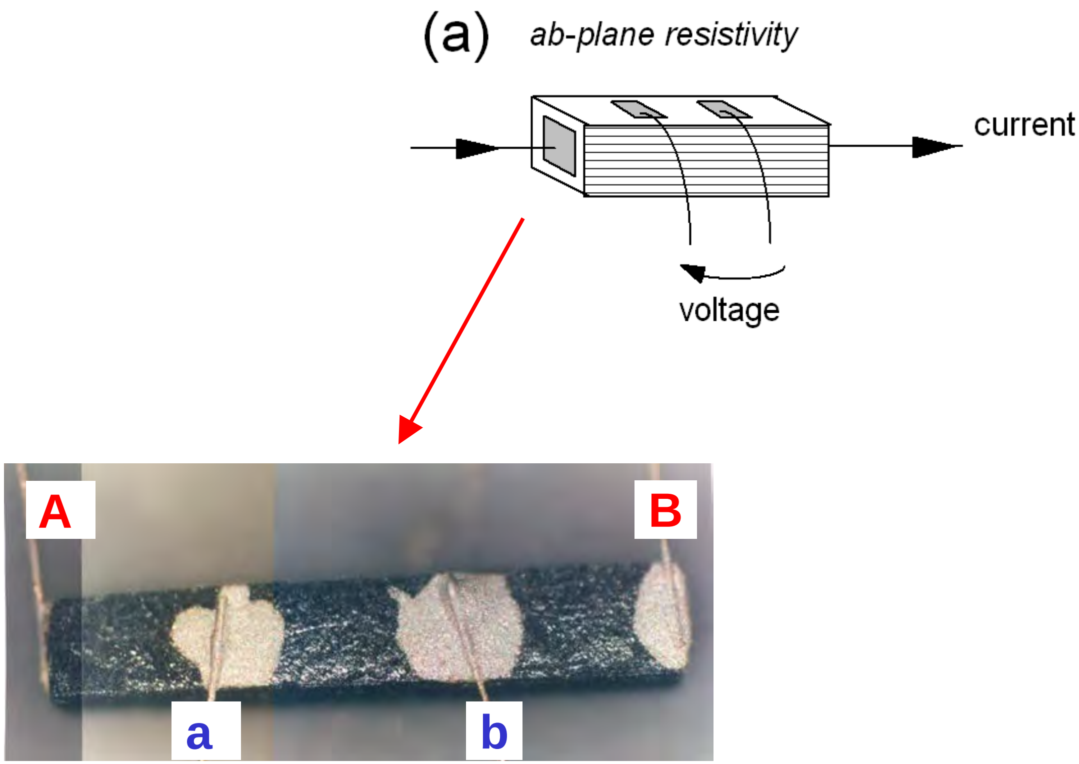
M. M. Miller
Naval Research Laboratory, Washington, D.C. 20375

(Received 1 October 1992)

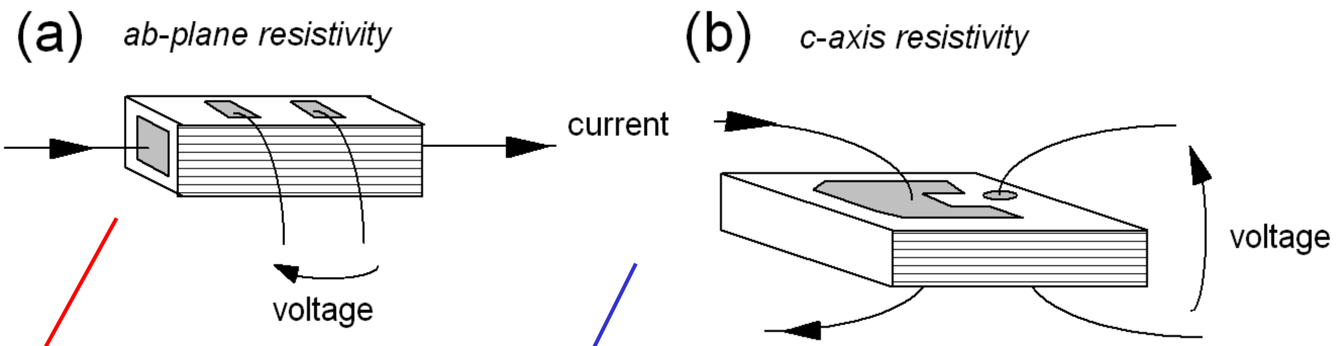
The melting transition in twinned and untwinned single crystals is measured resistively in fields up to 8 T as a function of the angle between the **c** axis and the **a-b** plane. The angular dependence follows the Lindemann criterion with $c_L = 0.15$. The suppression of melting by strong pinning by twin boundaries is demonstrated.

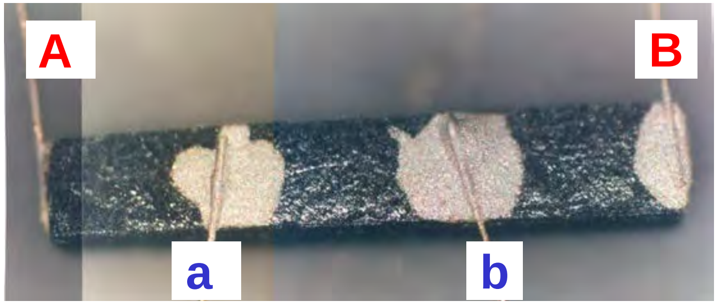
FIG. 1. (a) Resistive transition in magnetic fields of 0, 0.1, 0.5, 1, 1.5, 2, 3, 4, 5, 6, 7, and 8 T for **H**∥**c** in an untwinned $YBa_2Cu_3O_{7-\delta}$ crystal. Inset: Determination of T_m from the inflection peak of dR/dT for $H = 2$ T. (b) Resistive transition in magnetic fields of 0, 1, 2, 3, 4, 5, 6, 7, and 8 T for **H**∥(**a**,**b**). Inset: Phase diagram of the melting transition for **H**∥**c** and H∥(**a**,**b**).

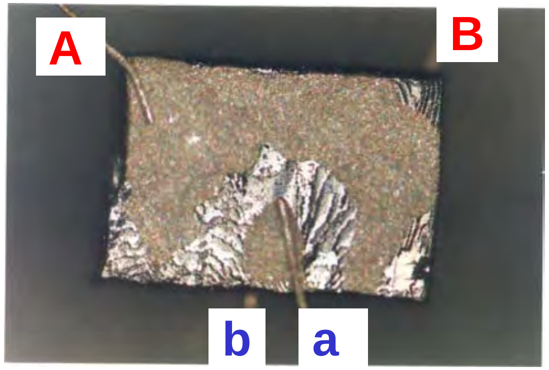
Anisotropy

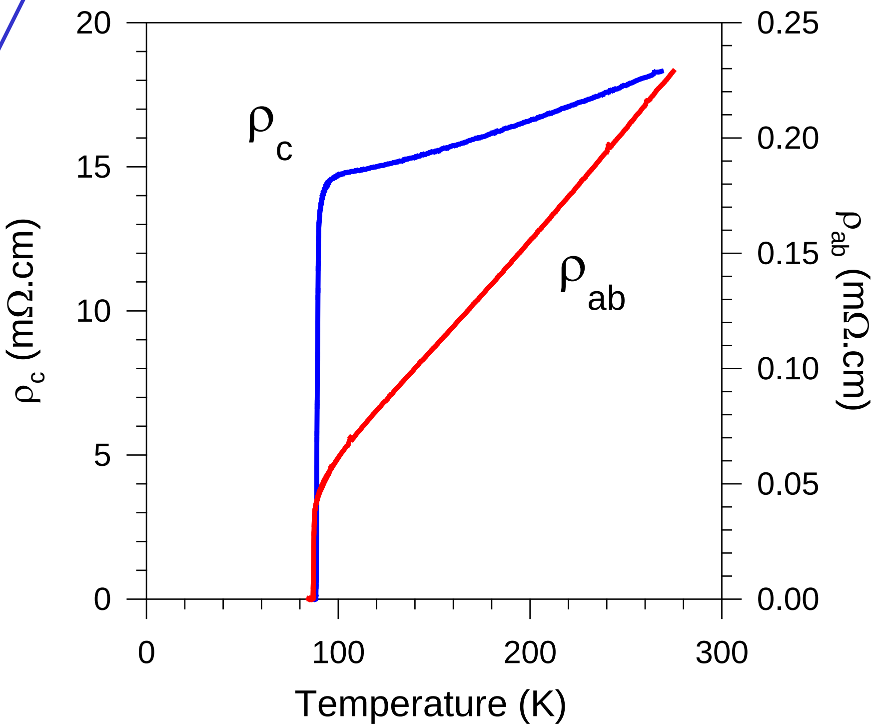


Anisotropy

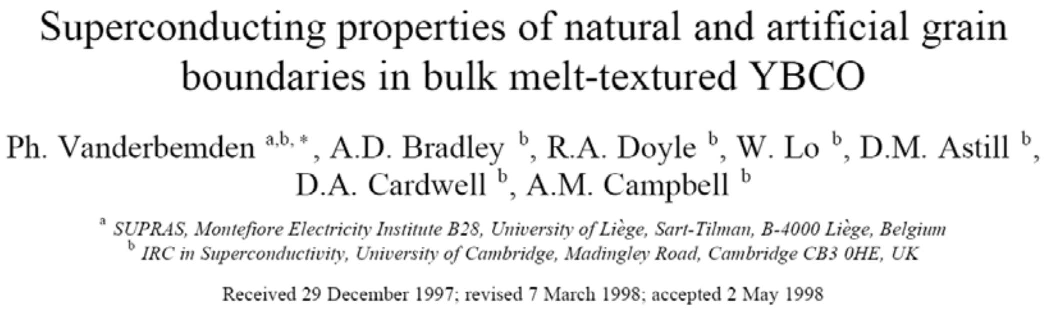

(a) ab-plane resistivity
voltage
current
(b) c-axis resistivity
voltage


A
B
a
b


A
B
b
a


ρc
ρab
ρc (mΩ.cm)
ρab (mΩ.cm)
Temperature (K)
0
5
10
15
20
0.00
0.05
0.10
0.15
0.20
0.25
0
100
200
300

Granularity



Superconducting properties of natural and artificial grain boundaries in bulk melt-textured YBCO

Ph. Vanderbemden [a,b,*], A.D. Bradley [b], R.A. Doyle [b], W. Lo [b], D.M. Astill [b],
D.A. Cardwell [b], A.M. Campbell [b]

[a] *SUPRAS, Montefiore Electricity Institute B28, University of Liège, Sart-Tilman, B-4000 Liège, Belgium*
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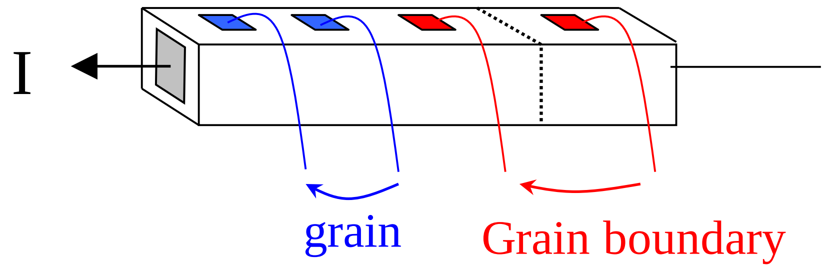
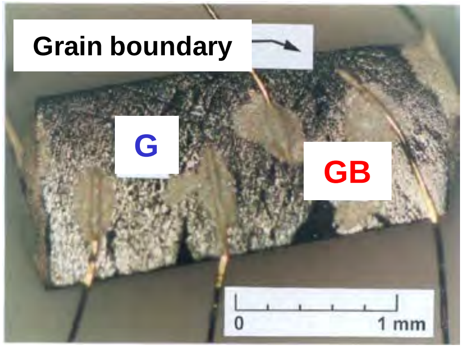
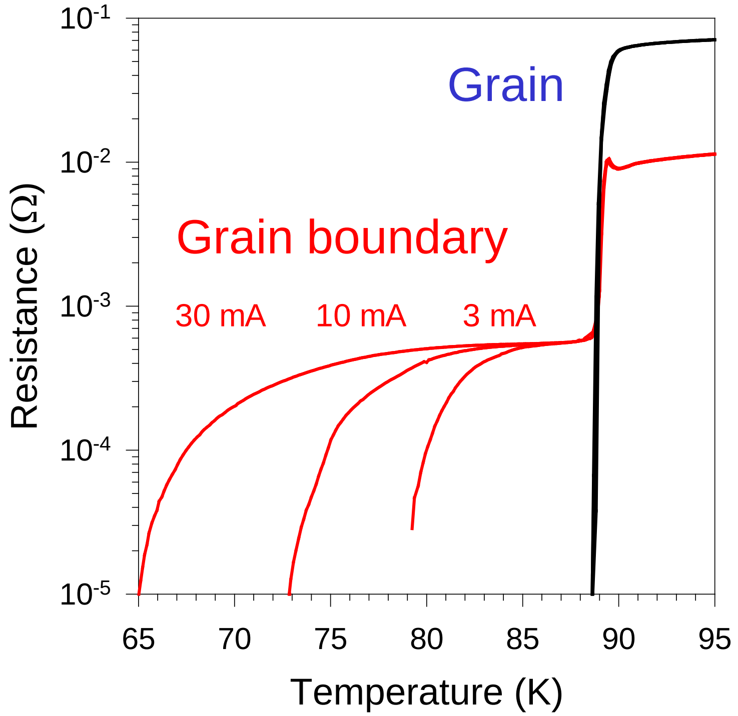
Received 29 December 1997; revised 7 March 1998; accepted 2 May 1998

PHYSICA C

Physica C 302 (1998) 257–270

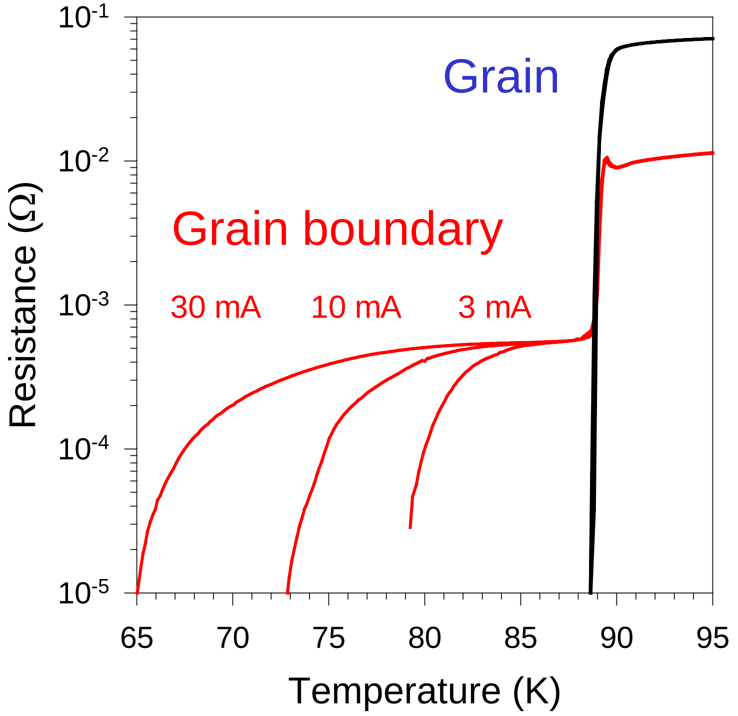




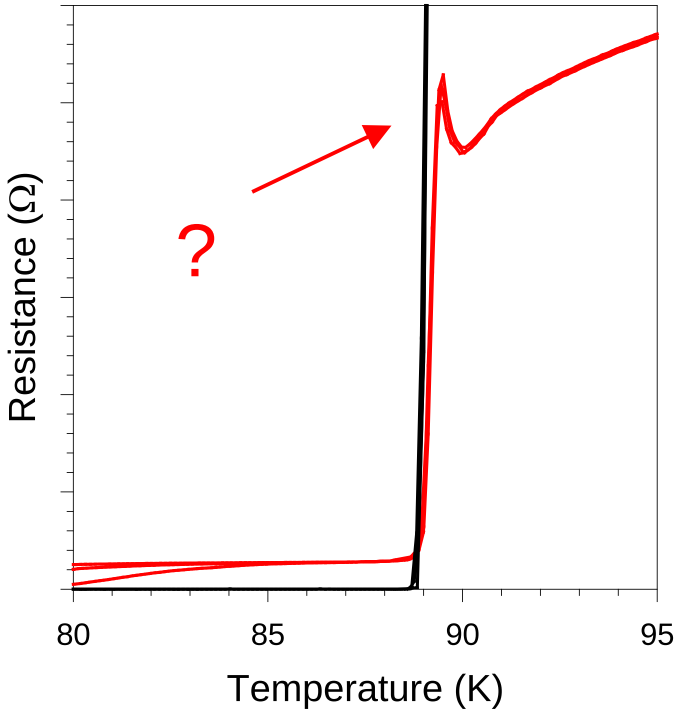


A shoulder in R(T) – possibly using a log scale for R
is a clear signature of the presence of one or more grain boundaries

Some artefacts or difficulties …



Back to
LINEAR
SCALE



The peak in R(T) just above the superconducting transition is a (relatively) common feature usually attributed to inhomogeneities and current redistribution

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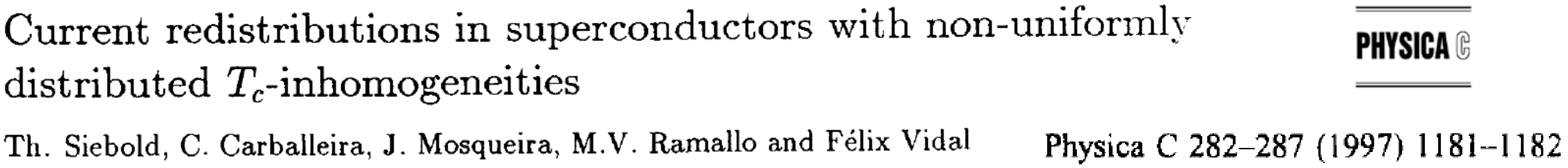


Current redistributions in superconductors with non-uniformly distributed T_c-inhomogeneities

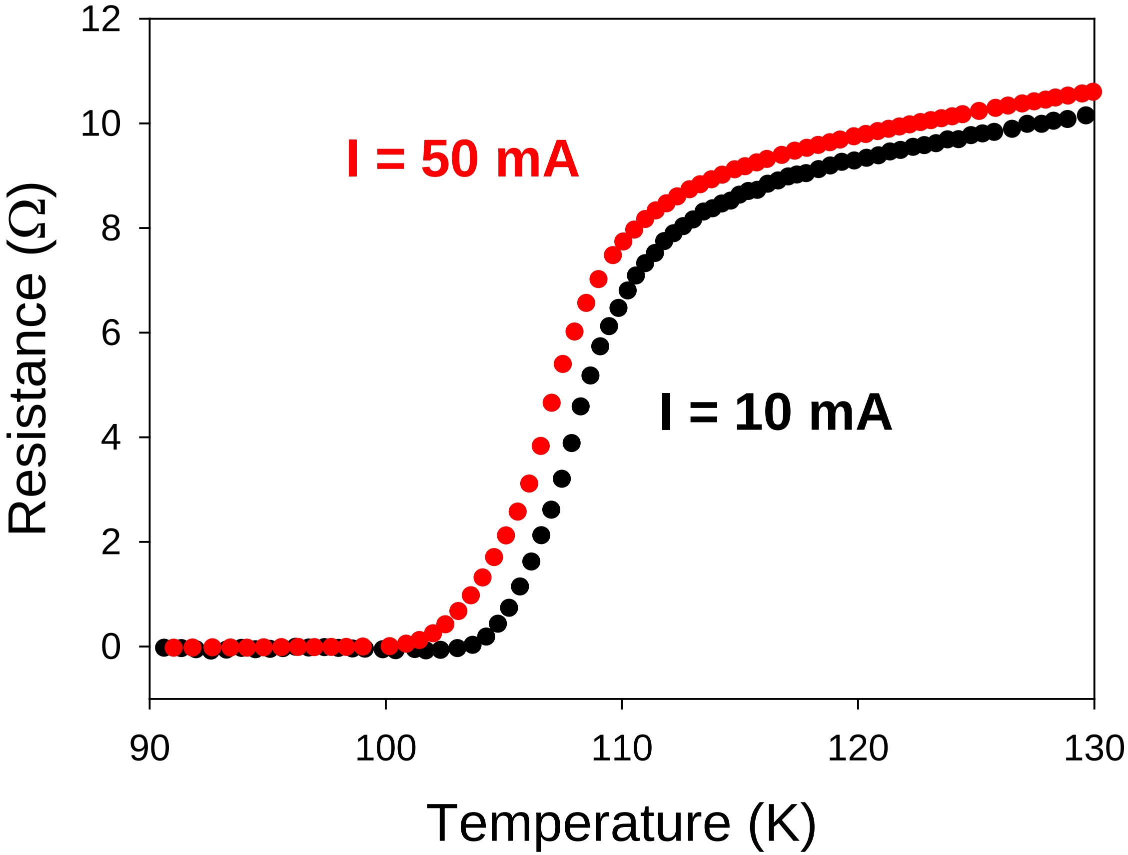
PHYSICA C

Th. Siebold, C. Carballeira, J. Mosqueira, M.V. Ramallo and Félix Vidal

Physica C 282–287 (1997) 1181–1182

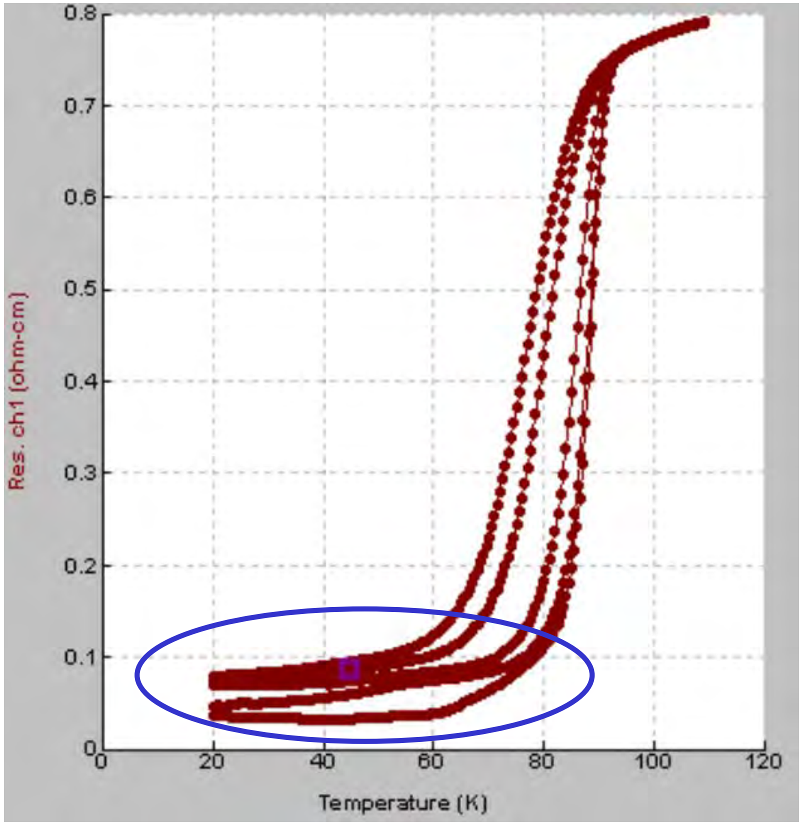
Some artefacts or difficulties …

Ex: Bi-2223 ceramic



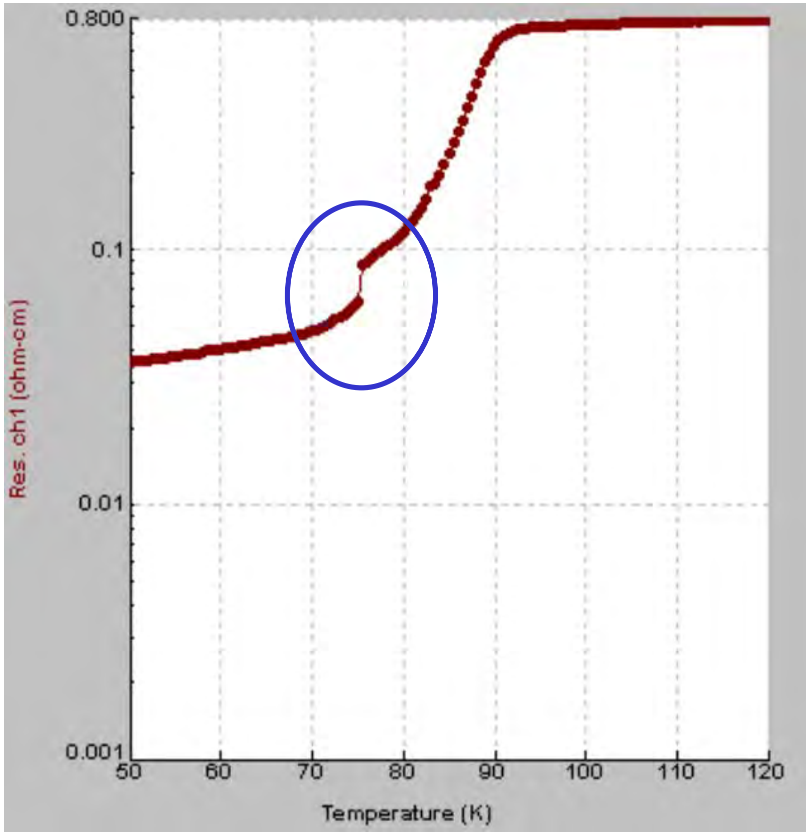
A larger current means also a much larger power dissipated in current contacts ($P = R I^2$!) and, possibly, sample heating and error in the temperature measurement

Other errors …



Bad sample or bad contact resistance

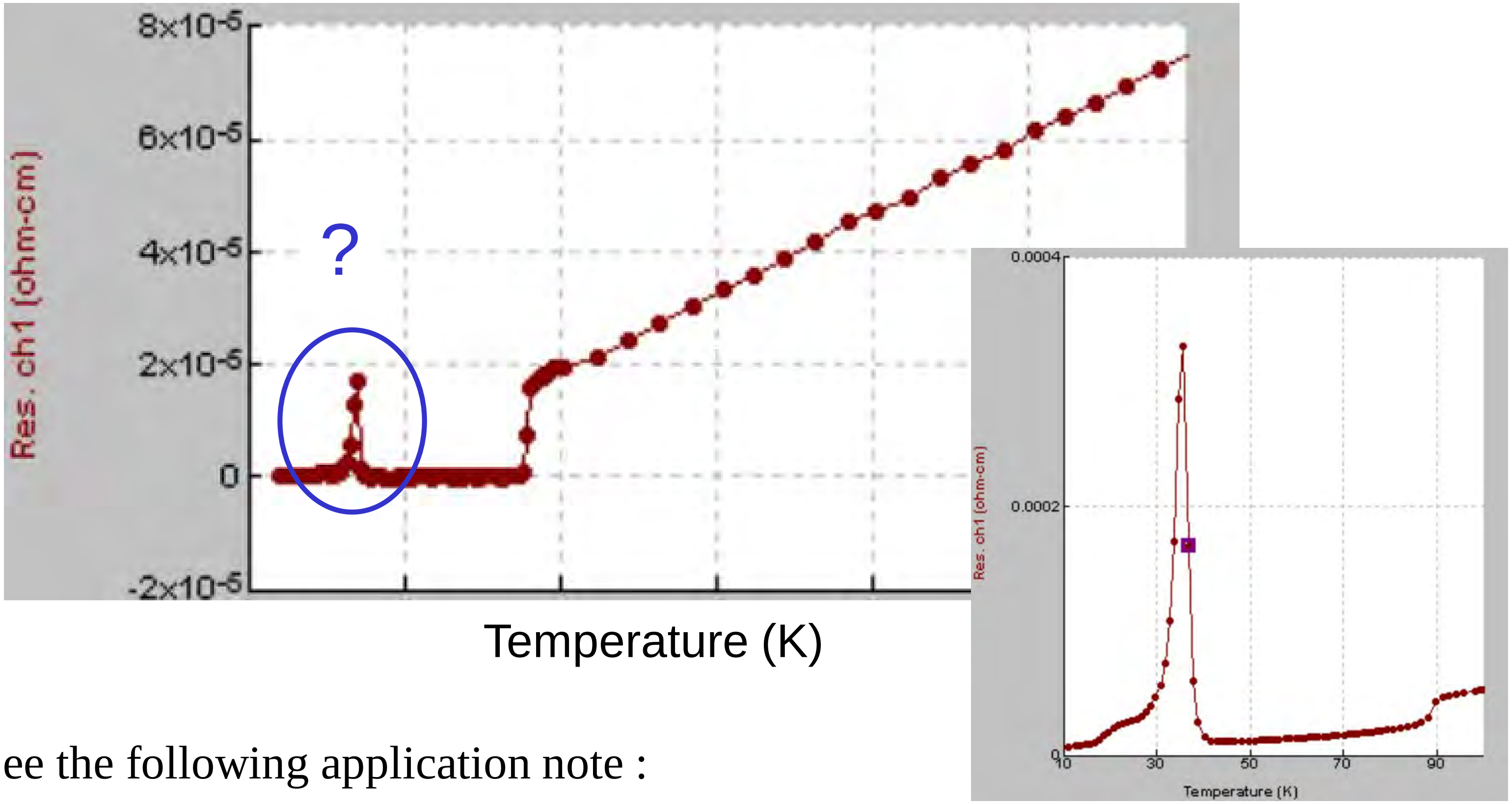
Try again with new contacts !



« Jumping » contact

Try again with new contacts !

A well-known error from the QD Physical Property Measurement System (PPMS)



See the following application note :

Quantum Design

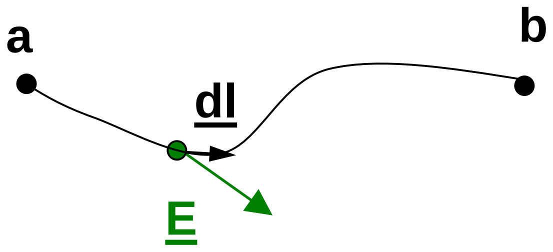
Distorted low-level signal readback of AC signals in the PPMS in the temperature range 25-35 K due to Inconel mitigation of inductive cross talk

Outline

- Transport measurements - R(T)
- Transport measurements - E(J)

- Magnetic measurements (general)
- Magnetic measurements - M(H)

Electric field E (V/m)



(OK when no time-dependent magnetic flux density)

$$\underline{E} \quad \Rightarrow \quad V_a - V_b = \int_a^b \underline{E}.\underline{dl}$$

Electric field

[V/m]

Local quantity

voltage difference
voltage drop
[volts], [V]

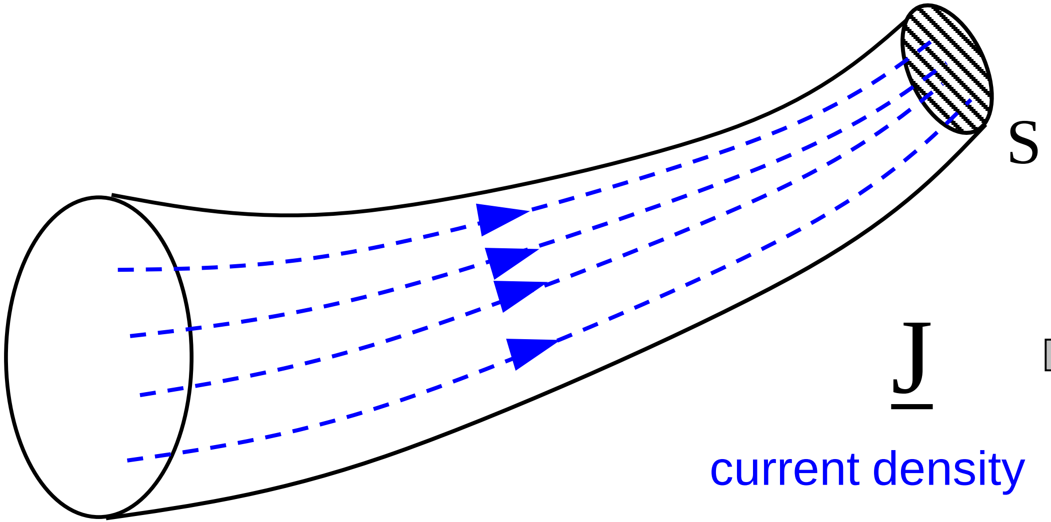
Global quantity

Particular case :

E uniform and parallel to the segment between a and b

$$\boxed{E = \frac{V_a - V_b}{\ell}}$$

Current density J (A/m²)



$$\underline{J} \Longrightarrow I = \iint_S \underline{J}.\underline{n}\, dS$$

current density

[A/m²]

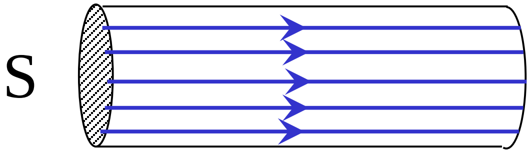
current

[A]

<u>Local</u> quantity

<u>Global</u> quantity

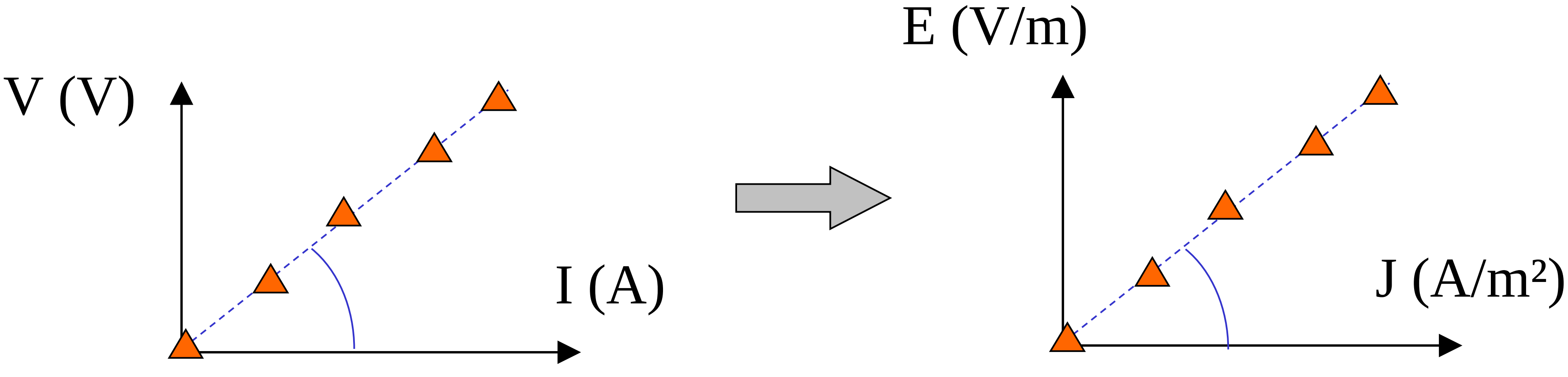
Particular case :



<u>J</u> uniform and ⊥ S

$$\boxed{J = \frac{I}{S}}$$

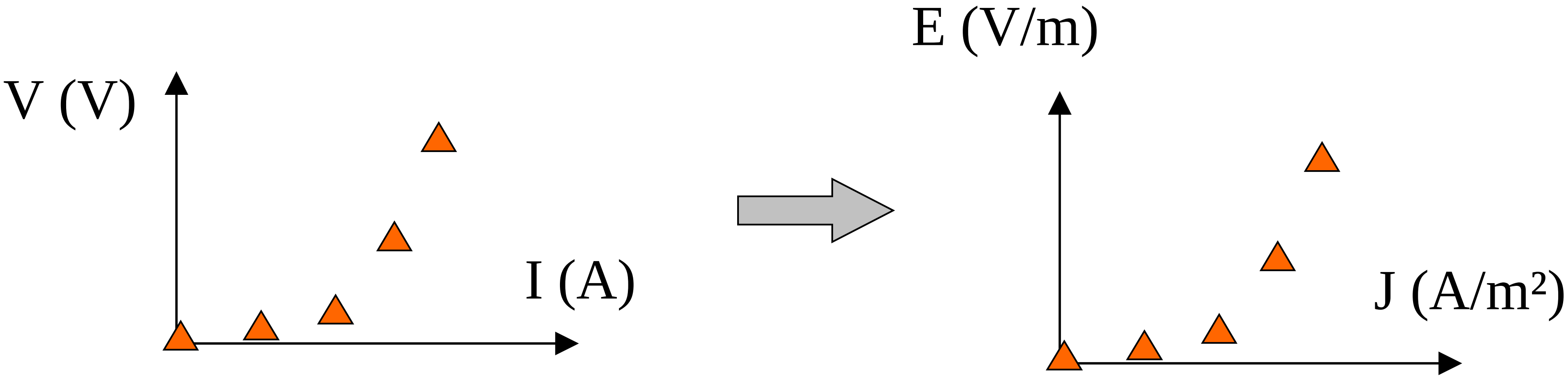
Linear conductor



(slope) R = resistance (Ω)

(slope) ρ = resistivity (Ω.m)

Non-linear conductor



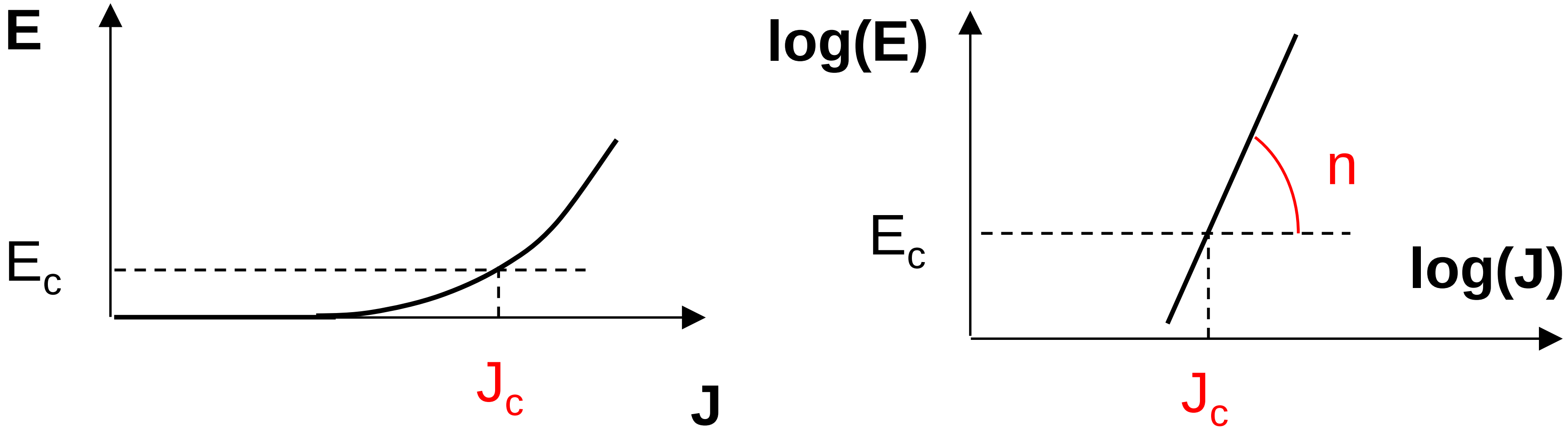
(slope) = ???

(slope) = ???

In practice …

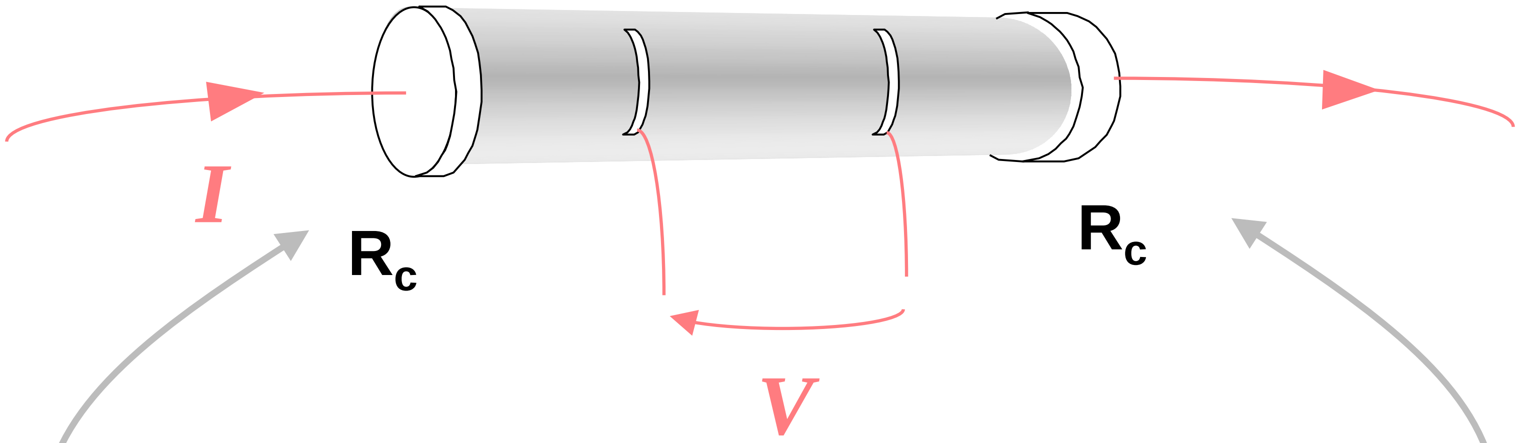
Most high-Tc superconductors have a non-linear characteristic which can be described by a **power law**

$$\boxed{E(J) = E_c \left(\frac{J}{J_c}\right)^n}$$



The definition of J_c requires a electric field threshold often (by convention) referred as **E_c = 1 μV/cm.**

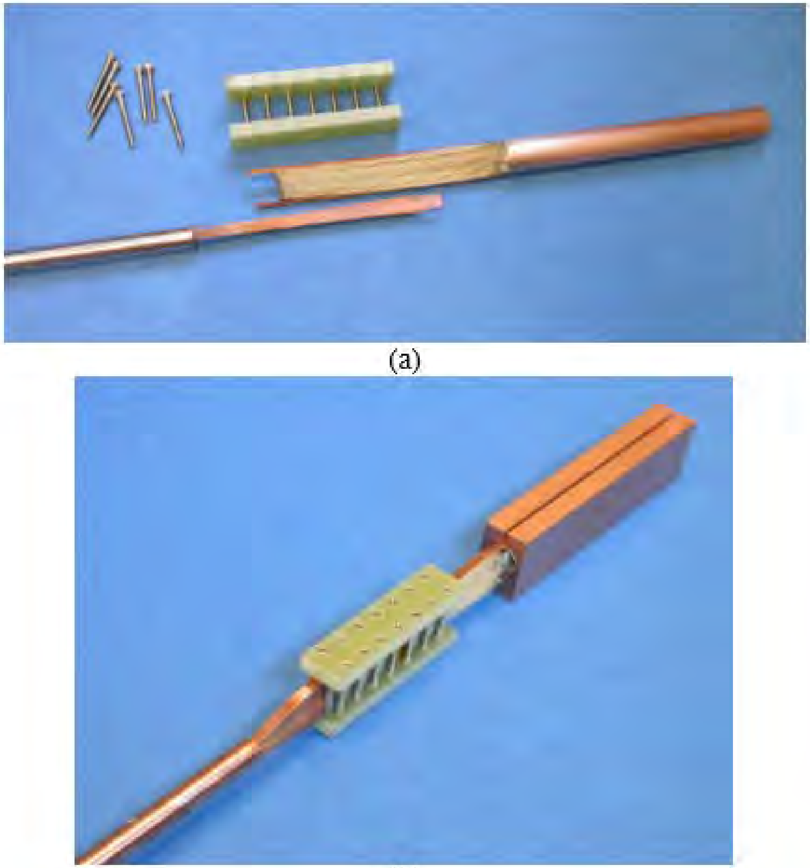
The main difficulty for transport measurements on superconductors = ?



The finite resistance of electrical contacts

HEATING P = Rc I² (>>)

Achieving a small contact resistance is essential



(a)

(b)

Fig. 10 Termination of a 40 YBCO tape cable tested at KIT, Germany in August 2012. It was operated at 10 kA. (a) Assembling parts of a BSCCO terminator with a stacked tape cable. (b) Assembled termination. The joint section of YBCO and BSCCO tapes was clamped with 70 mm length G10 plate without soldering.

PSFC/JA-16-41

Termination Methods for REBCO Tape High-Current Cable Conductors

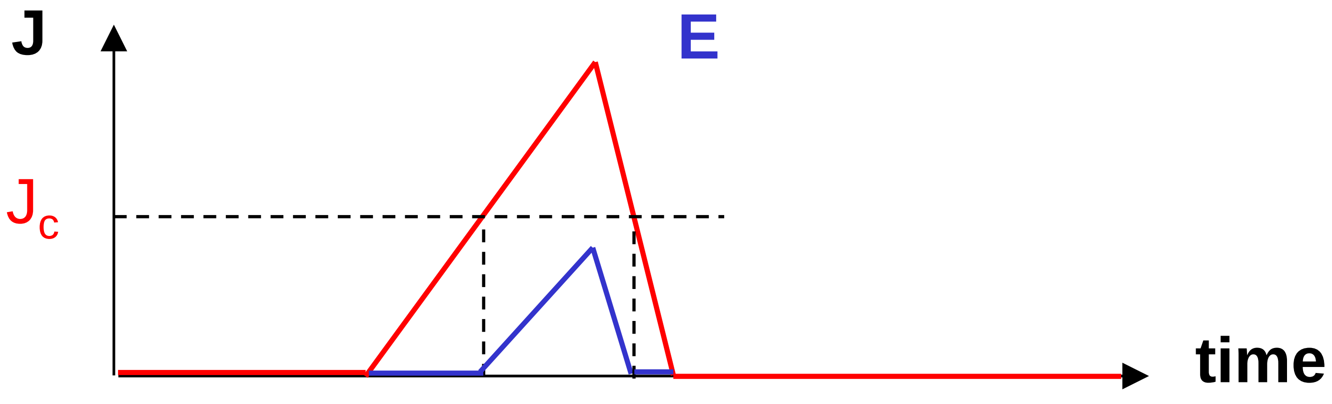
M. Takayasu, L. Chiesa[*], and J.V. Minervini

[*]Tufts University, Mechanical Engineering, Medford, MA 02155, USA

June 19, 2016

Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139

Use of pulsed currents



Study of the superconducting transition at high pulsed current of bulk Bi-2223 sintered and textured by hot forging

J.G. Noudem [a,b,*], L. Porcar [a,b], O. Belmont [b,c], D. Bourgault [a], J.M. Barbut [b], J. Beille [c], P. Tixador [d], M. Barrault [b], R. Tournier [a]

PHYSICA C

Physica C 281 (1997) 339–344

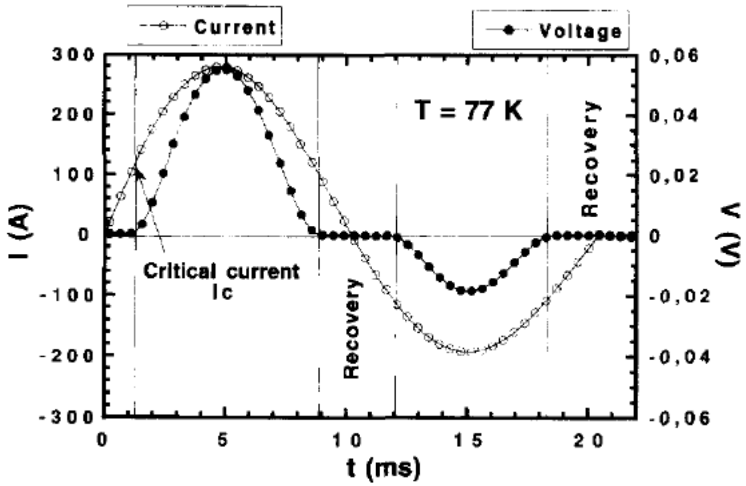
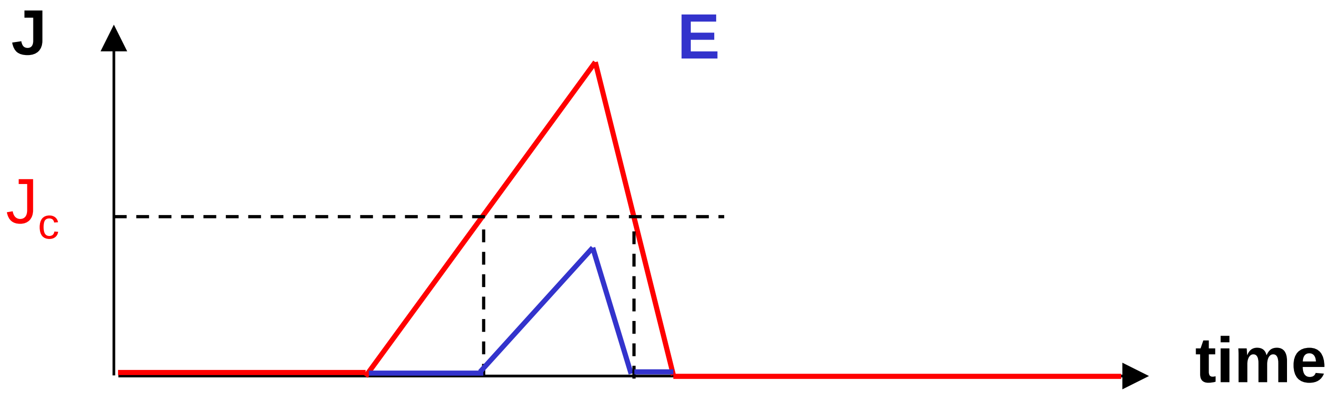
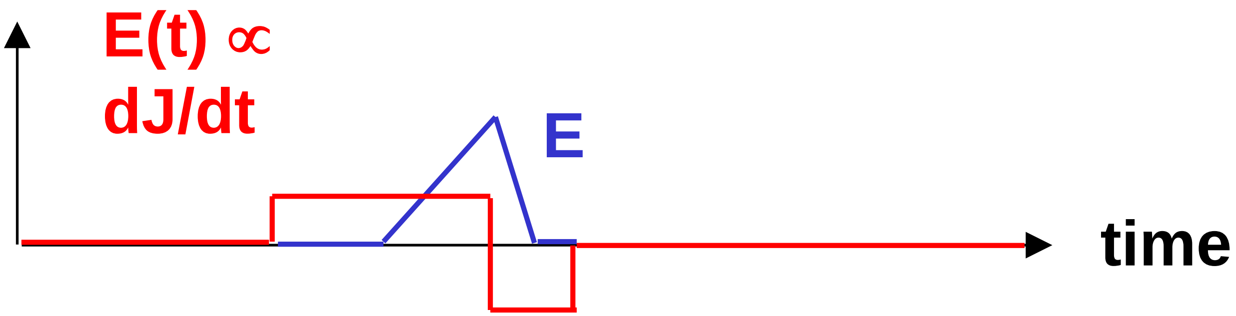


Fig. 4. Current and voltage waveforms given by a textured sample.

Caution : inductive pick-up



$$J(t) \rightarrow B(t) \rightarrow E(t) \propto dJ(t)/dt\ !$$



In practice, compensation circuits are needed (Rogowski coil, dummy loop…)

Outline

- Transport measurements - R(T)
- Transport measurements - E(J)

- Magnetic measurements (general)
- Magnetic measurements - M(H)

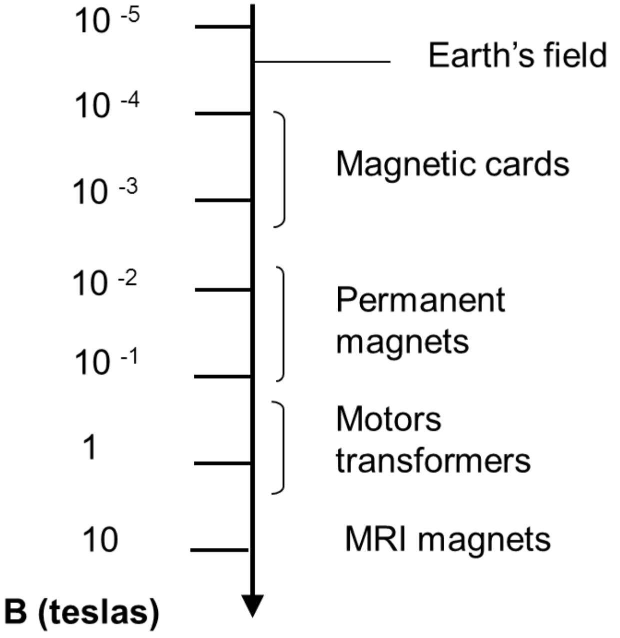
What are we talking about ?

$$\underline{B} = \mu_0 (\underline{H} + \underline{M})$$

H = magnetic field [A / m]

M = magnetization [A / m]

B = magnetic induction [T]



10^{-5}

Earth's field

10^{-4}

Magnetic cards

10^{-3}

10^{-2}

Permanent magnets

10^{-1}

Motors transformers

1

MRI magnets

10

B (teslas)

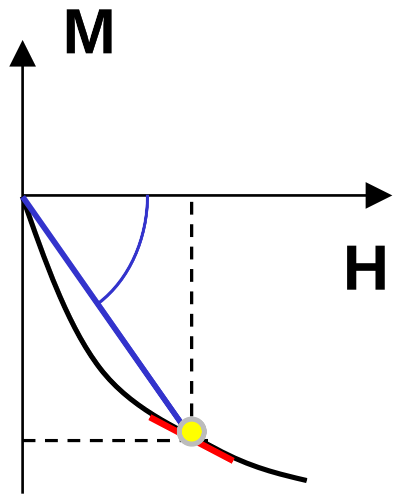
And a little bit more …

m = magnetic moment $[A.m^2]$

M = magnetization $[A / m]$
(= m / V)

χ_{DC} = magnetic susceptibility
(= M / H)
[DC]

χ_{AC} = magnetic susceptibility
(= dM / dH)
[AC]



What do we need to measure ?

M (H)

obtained through
measuring…

m = magnetic moment
$M = m / V$

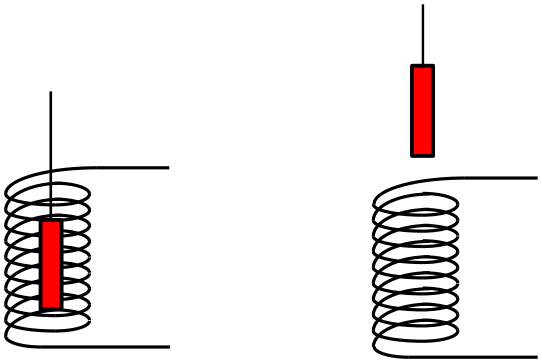
B = magnetic induction
without sample $H = B/\mu_0$

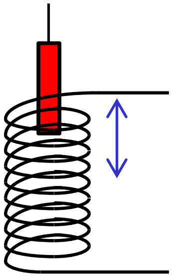
i = magnet current
$H = k.\,i$ (k ~ magnet)

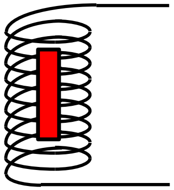
How can we measure ?

A lot of magnetic measurements are carried out using Faraday’s law

$$V = -N\frac{d\Phi}{dt}$$





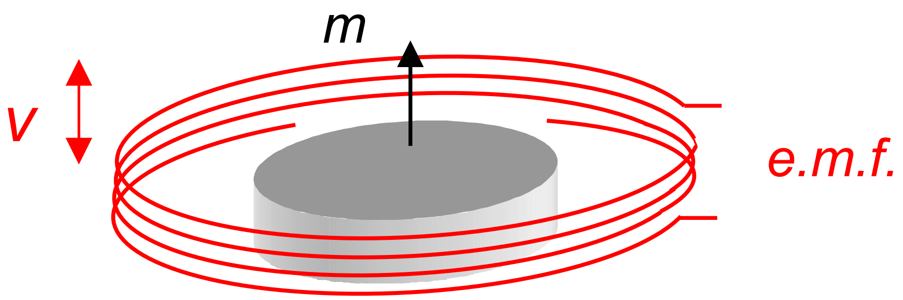


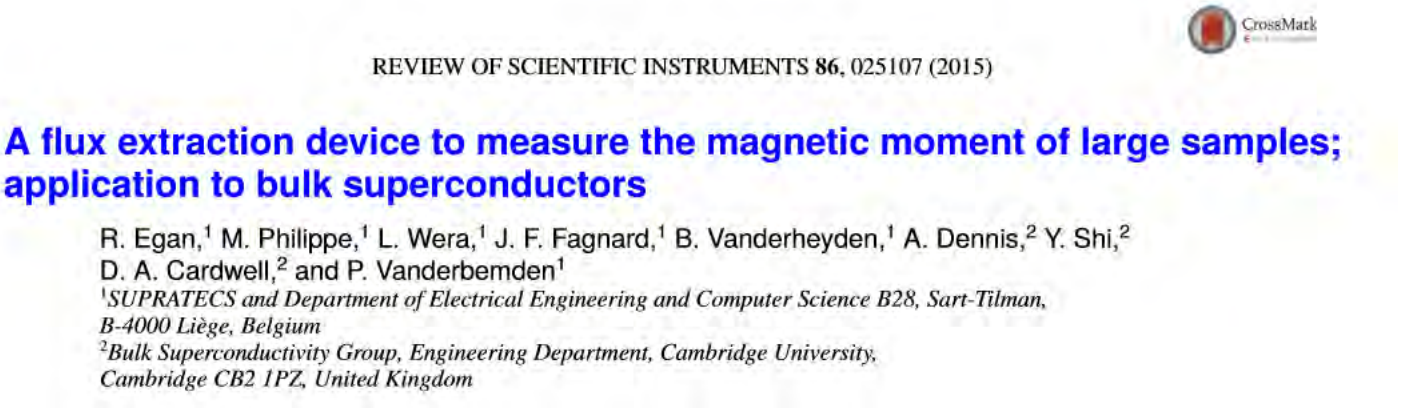
Applications :

- Extraction method (cf. PPMS)
- Vibrating Sample Magnetometer (VSM)
- Measure of a flux variation with analog or digital integration

$H(t) \rightarrow d\Phi(t)/dt \rightarrow \Delta\Phi$

!!! Caution : If one wants probe the magnetic moment, the sensing coil must be much larger than the sample !!!





CrossMark

REVIEW OF SCIENTIFIC INSTRUMENTS **86**, 025107 (2015)

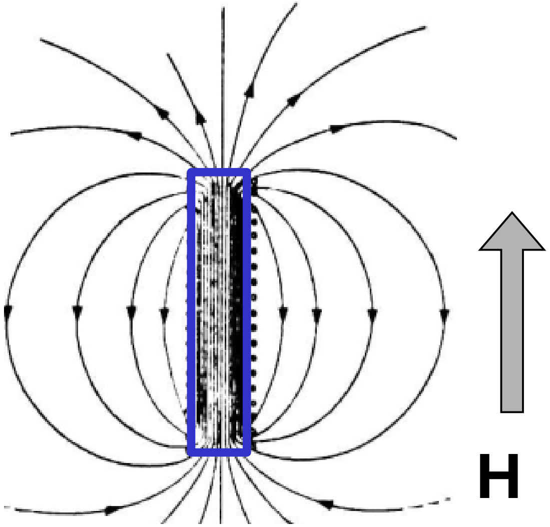
A flux extraction device to measure the magnetic moment of large samples; application to bulk superconductors

R. Egan,[1] M. Philippe,[1] L. Wera,[1] J. F. Fagnard,[1] B. Vanderheyden,[1] A. Dennis,[2] Y. Shi,[2]
D. A. Cardwell,[2] and P. Vanderbemden[1]
[1]*SUPRATECS and Department of Electrical Engineering and Computer Science B28, Sart-Tilman, B-4000 Liège, Belgium*
[2]*Bulk Superconductivity Group, Engineering Department, Cambridge University, Cambridge CB2 1PZ, United Kingdom*

! “Demagnetizing” effects !

A magnetized sample (e.g. $M > 0$) of ***finite size*** creates a field in the ***surrounding space*** and ***within the sample*** itself.

This field – called ***demagnetizing field*** H_D – is always ***opposite in direction to the sample magnetization***.



The ***total*** applied field, H_T, is the sum of the field generated by the magnet H, ***and*** the demagnetizing field H_D. In the simple case $H \,//\, H_D$, one has

$$H_T = H + H_D$$

with

$$H_D = - D\, M$$

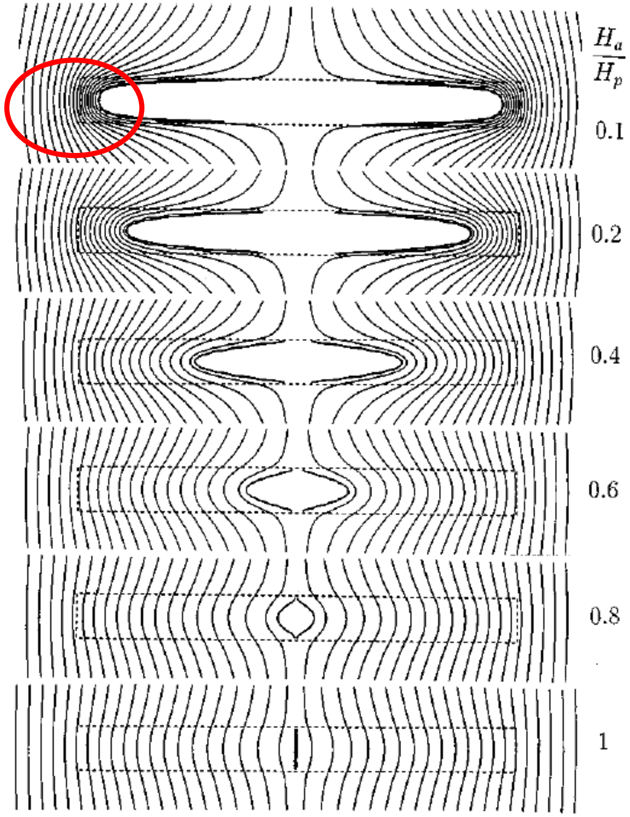
D represents the dimensionless ***demagnetizing factor***

Therefore …

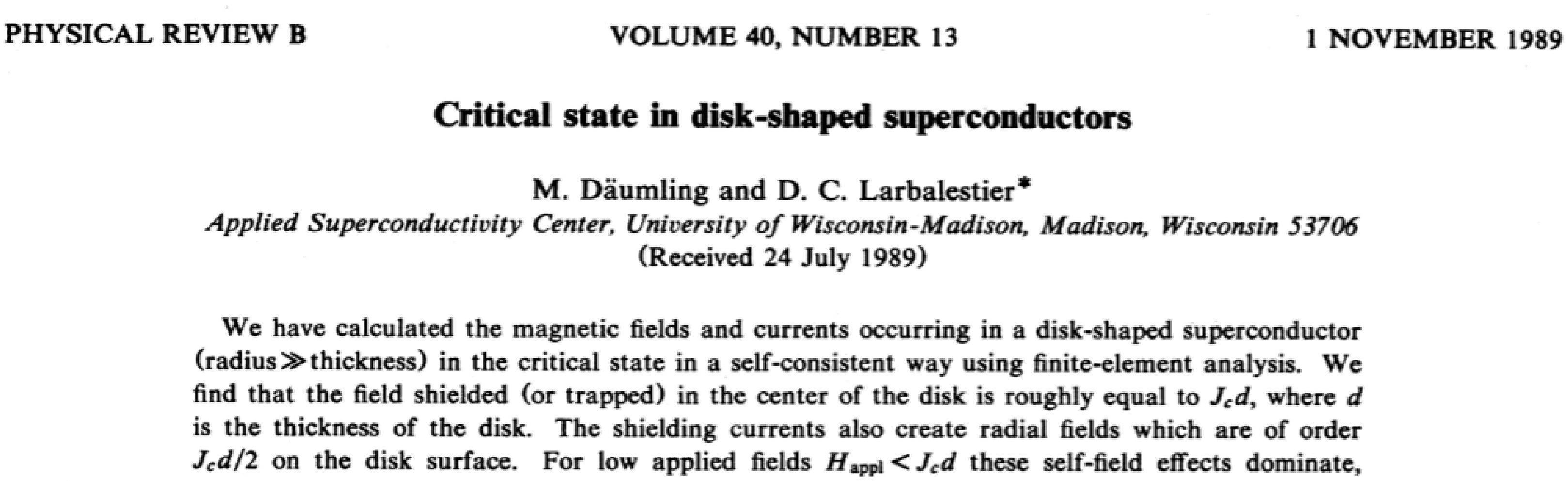
For ferromagnetic materials (M > 0),
H_T is smaller than H ("de-magnetizing")

while for superconductors in the diamagnetic state (M < 0),
H_T is bigger than H ("re-magnetizing" ?).

Superconductor



<u>NB</u> : To understand magnetic flux penetration in Type-II superconductors of finite size, see…



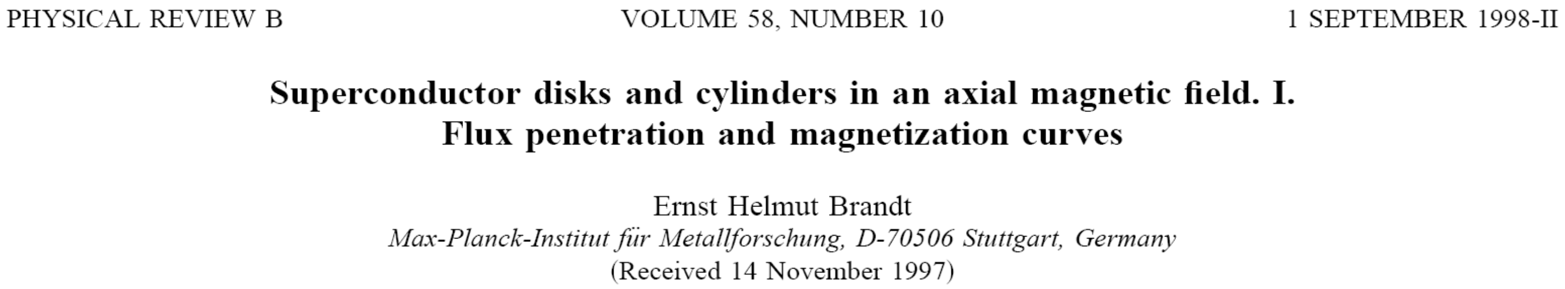
PHYSICAL REVIEW B VOLUME 40, NUMBER 13 1 NOVEMBER 1989

Critical state in disk-shaped superconductors

M. Däumling and D. C. Larbalestier*
Applied Superconductivity Center, University of Wisconsin-Madison, Madison, Wisconsin 53706

(Received 24 July 1989)

We have calculated the magnetic fields and currents occurring in a disk-shaped superconductor (radius $\gg$ thickness) in the critical state in a self-consistent way using finite-element analysis. We find that the field shielded (or trapped) in the center of the disk is roughly equal to $J_c d$, where d is the thickness of the disk. The shielding currents also create radial fields which are of order $J_c d/2$ on the disk surface. For low applied fields $H_{\text{appl}} < J_c d$ these self-field effects dominate,



PHYSICAL REVIEW B VOLUME 58, NUMBER 10 1 SEPTEMBER 1998-II

Superconductor disks and cylinders in an axial magnetic field. I.
Flux penetration and magnetization curves

Ernst Helmut Brandt
Max-Planck-Institut für Metallforschung, D-70506 Stuttgart, Germany

(Received 14 November 1997)

…. as well as all Helmut Brandt's papers ☺

Note however the important distinction :

Demagnetizing effects should always be taken into account when the sample cannot be considered infinitely long

BUT…

the conventional « demagnetzing factor » approach, strictly speaking, is valid for linear materials.

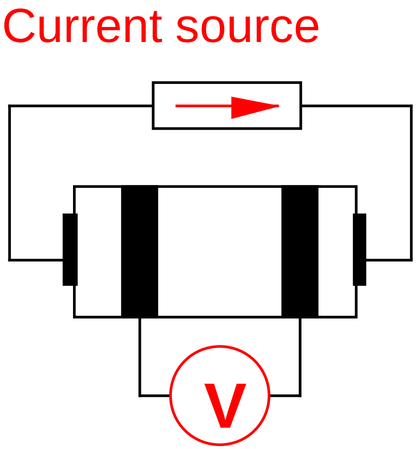
For type-II superconductors, only (semi-) analytical calculations and numerical modelling are appropriate !

Outline

- Transport measurements - R(T)
- Transport measurements - E(J)

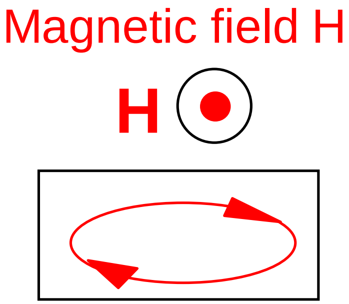
- Magnetic measurements (general)
- Magnetic measurements - M(H)

Transport measurement



Transport current
(applied externally)

Magnetic measurement



Induced current
(by the applied magnetic field)

ADVANTAGE of
magnetic measurements :

No need
of electrical contacts !

DRAWBACK of
magnetic measurements :

Requires a suitable model
(geometry-dependent!)

Bean model : relation $B \leftrightarrow J_c$

Hypotheses :

$H_{C1} \rightarrow 0$ $H_{C2} \rightarrow \infty$ Very strong pinning

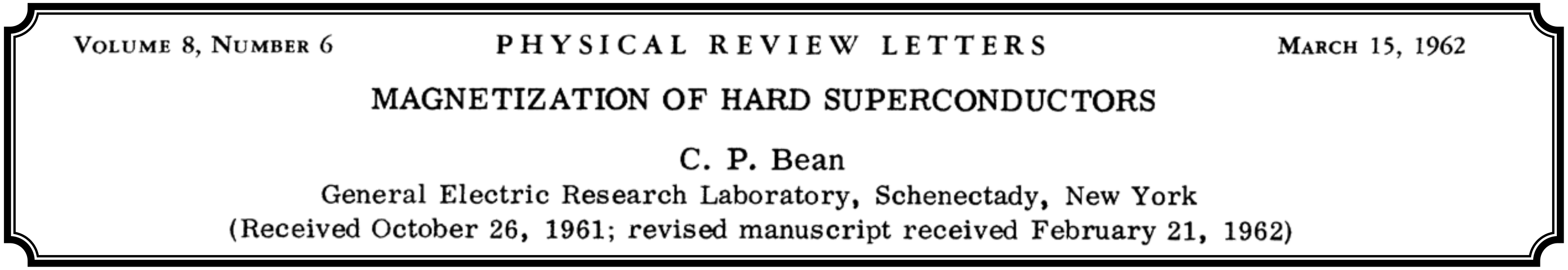
---

Model :

$$\boxed{\text{curl } B = \mu_0 J}$$

$$\boxed{J = +J_c\,,\quad -J_c \quad \text{or} \quad 0}$$

Critical state



VOLUME 8, NUMBER 6 PHYSICAL REVIEW LETTERS MARCH 15, 1962

MAGNETIZATION OF HARD SUPERCONDUCTORS

C. P. Bean

General Electric Research Laboratory, Schenectady, New York

(Received October 26, 1961; revised manuscript received February 21, 1962)

Bean model : relation B $\leftrightarrow$ J_c

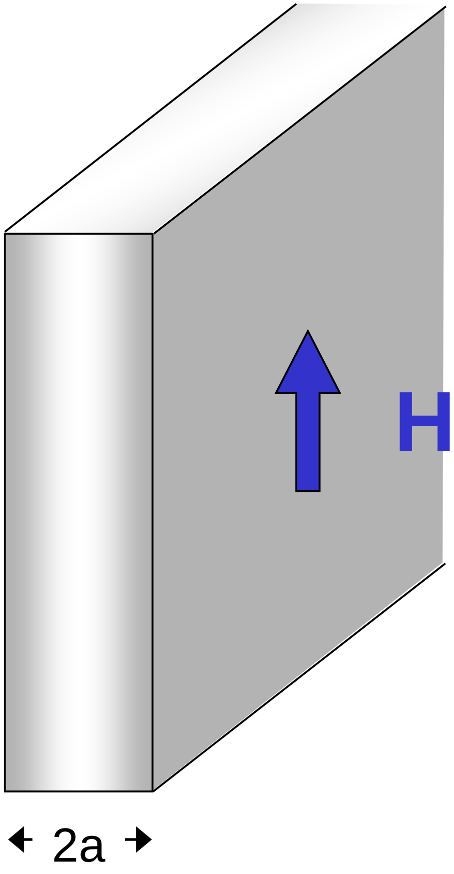
2 additional hypotheses

(1) supercond. ∞ || applied field
(ex. inifinite slab)

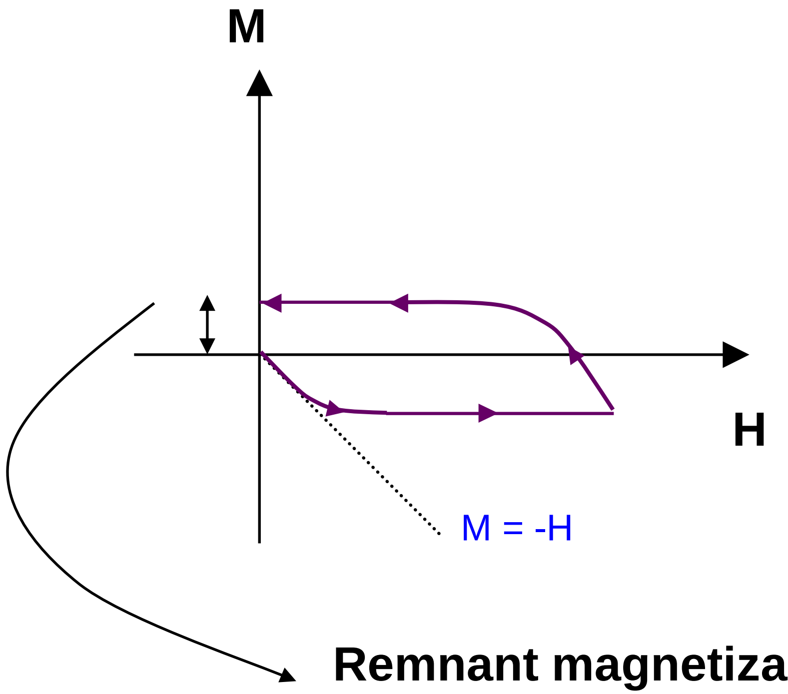
$$\boxed{\mathrm{curl}\, B = \pm \mu_0 J_c \text{ ou } 0}$$

$$\boxed{\left|\frac{\partial B}{\partial y}\right| = \mu_0 J_c \text{ ou } 0}$$

(2) J_c = constant (indep. of B)



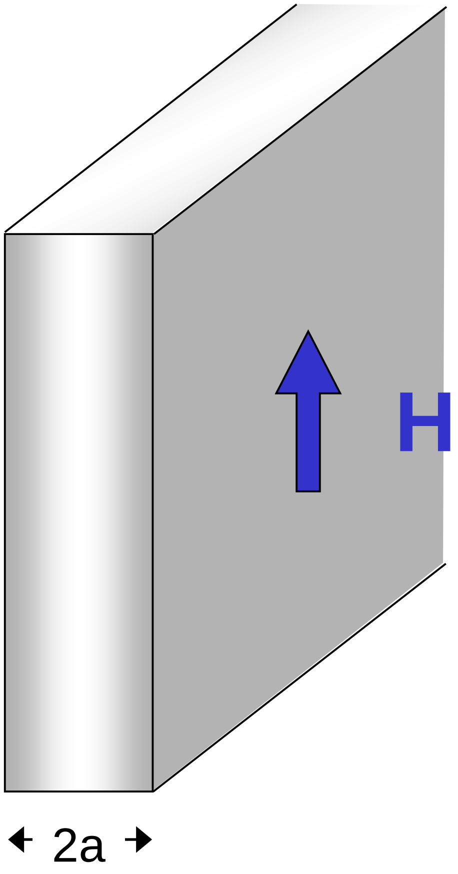
Bean model : relation B ↔ J_c



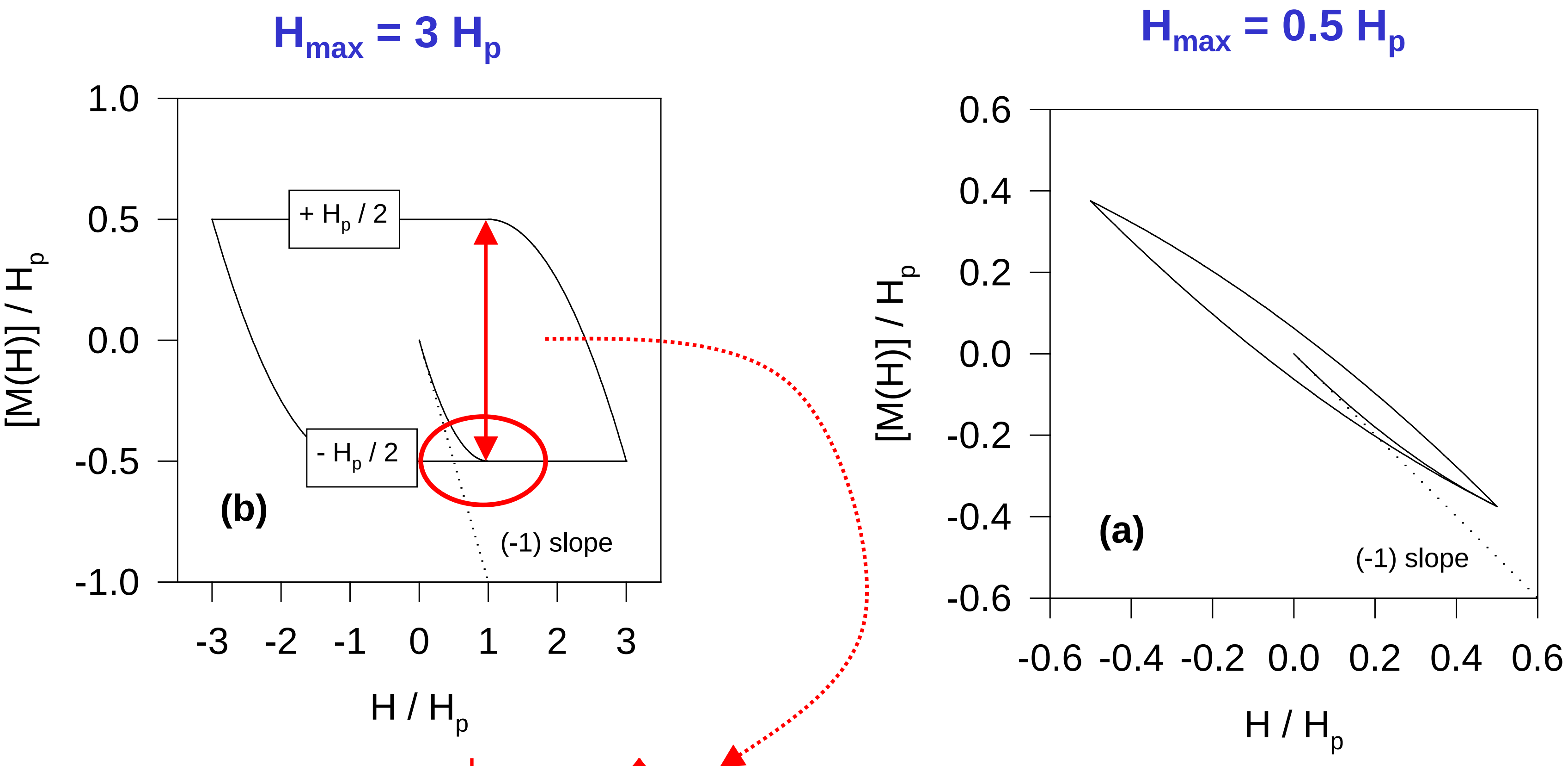
Remnant magnetization

= $(J_c\ a) / 2$

Indirect determination of J_c



Different "M(H)" curves for type II (hard) superconductor as a function of H_{max}

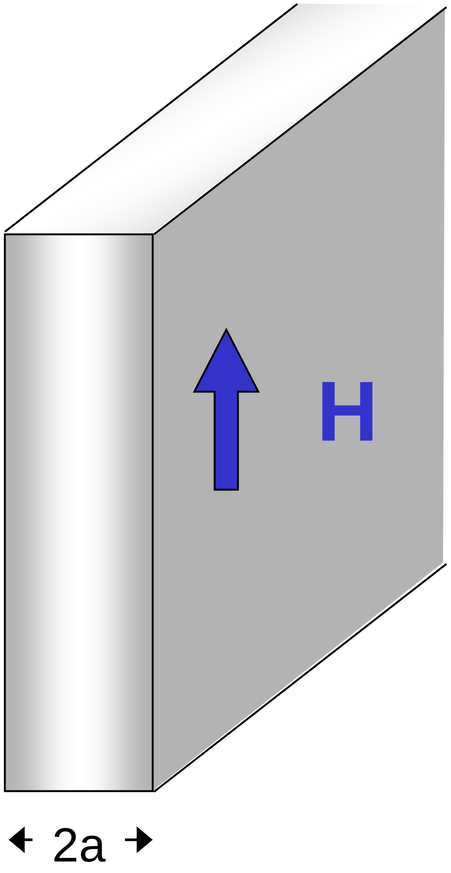


The difference betw. M↓ and M↑ is Hp (= J_c.a) in the case of an infinite slab

BUT… this is only true when the maximum field H_{max} is large enough !

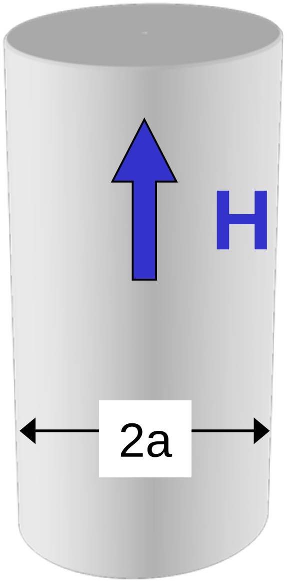
The relation between ΔM and J_c depends on the geometry of the sample

Infinite slab



$$J_c = \frac{\Delta M}{a}$$

Infinite cylinder



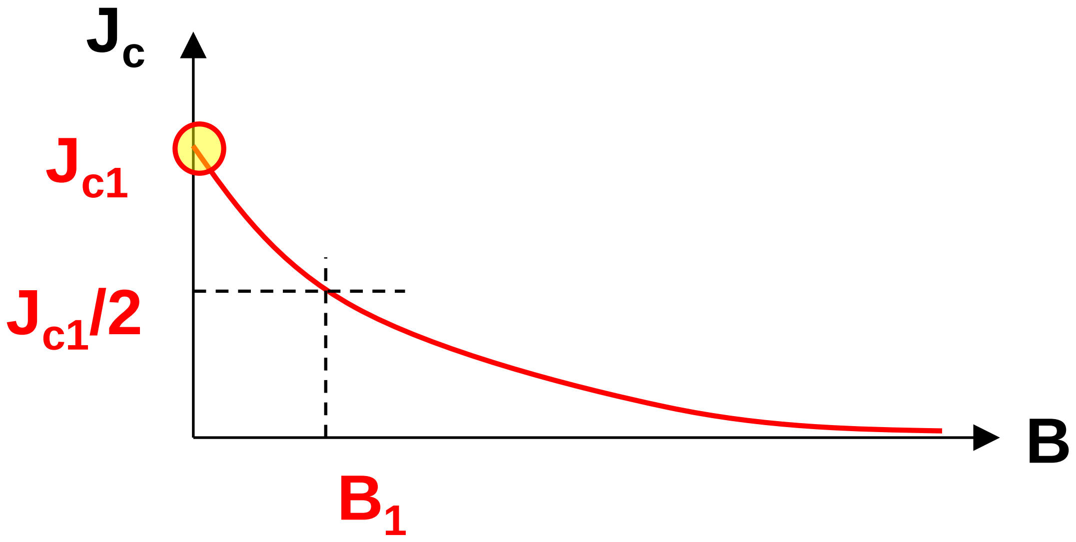
$$J_c = \frac{3\Delta M}{2a}$$

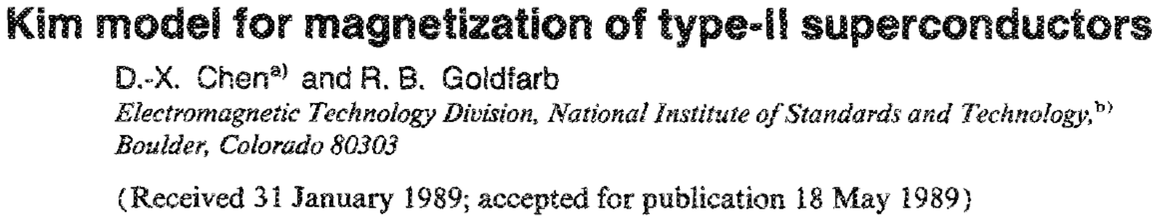
And what happens in the case of $J_c(B)$?

A model of $J_c(B)$ is required !

Ex : Kim model

$$\boxed{J_c = J_{c1}\left(\frac{B_1}{B+B_1}\right)}$$





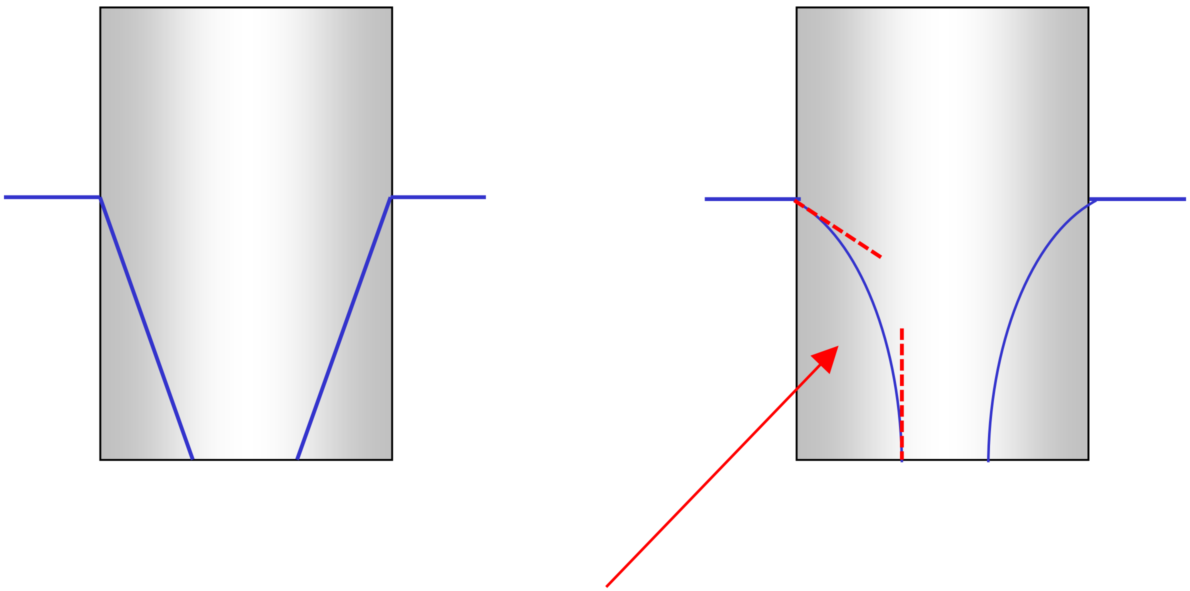
Kim model for magnetization of type-II superconductors

D.-X. Chen[a] and R. B. Goldfarb
Electromagnetic Technology Division, National Institute of Standards and Technology,[b]
Boulder, Colorado 80303

(Received 31 January 1989; accepted for publication 18 May 1989)

J. Appl. Phys. **66** (6) 15 September 1989

Consequences on the magnetic field penetration



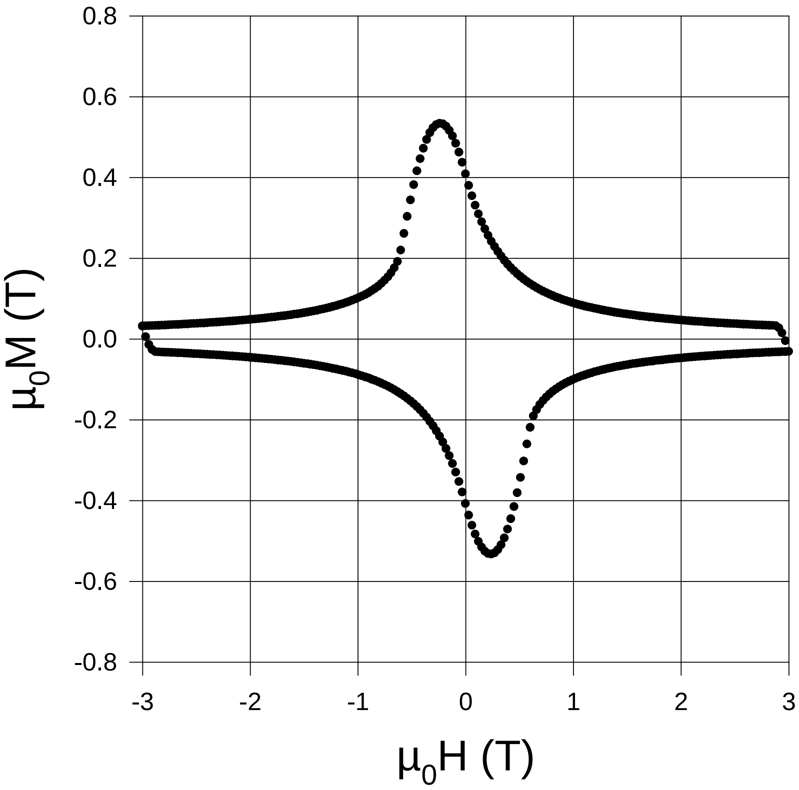
Constant J_c

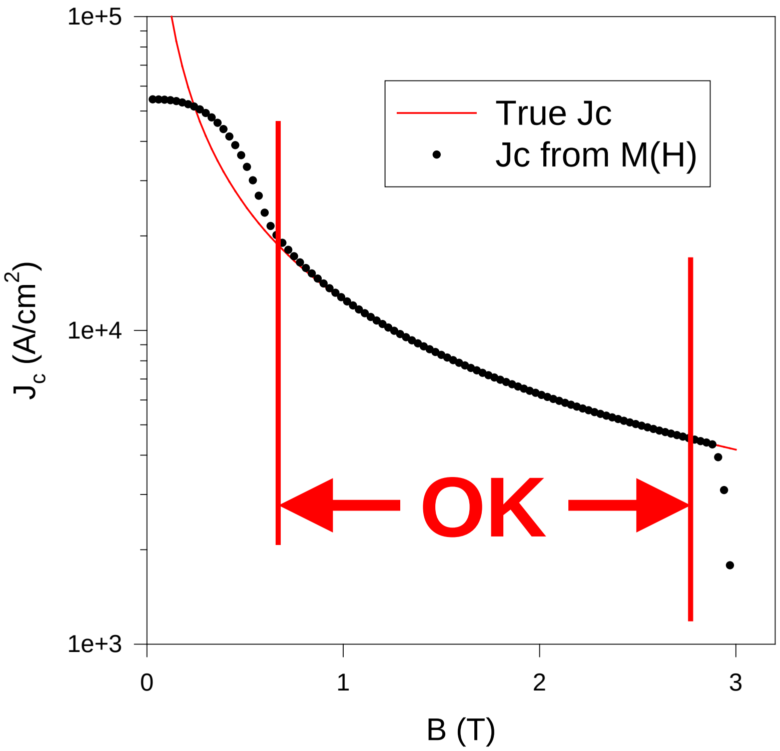
$J_c(B)$

Local slope = J_c

Completely different magnetization curves are expected !

Infinite slab with $J_c \propto (1/B)$

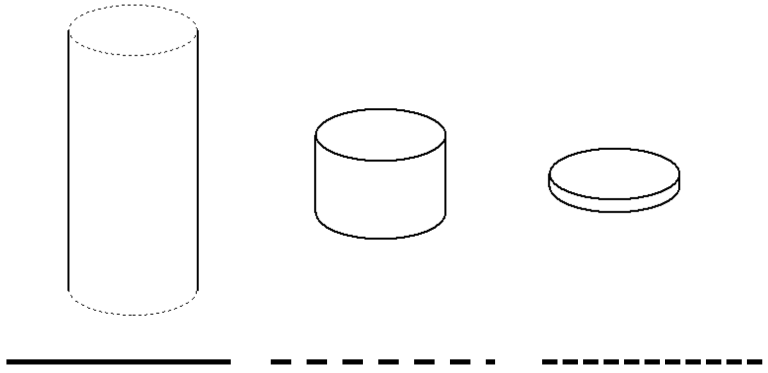


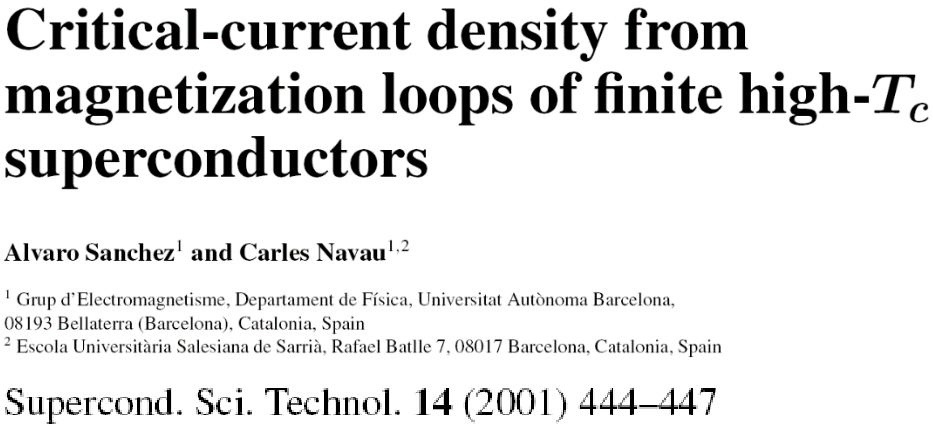


Remarkably, the Jc(B) can often (not always…) be determined from ΔM, provided that the magnetic field range does not extend too close to 0 and to Hmax !

And what happens if the superconductor cannot be assumed to be infinite ?

Modelling needed !



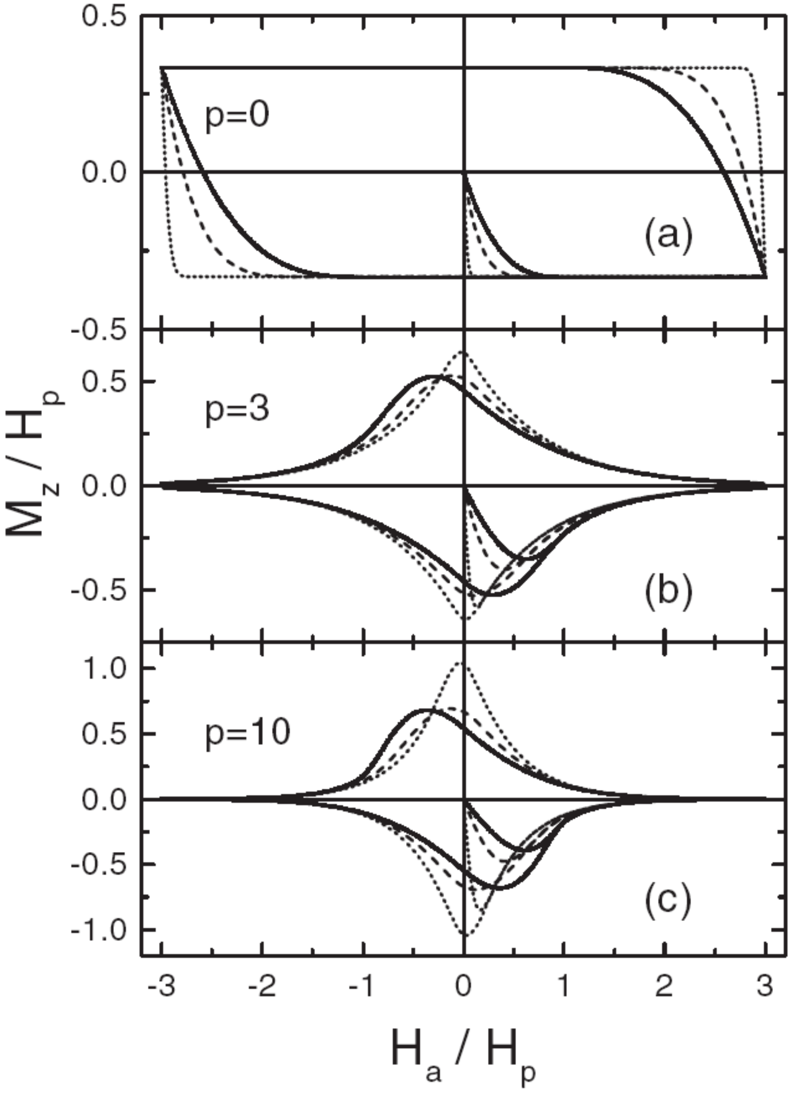


Critical-current density from magnetization loops of finite high-T_c superconductors

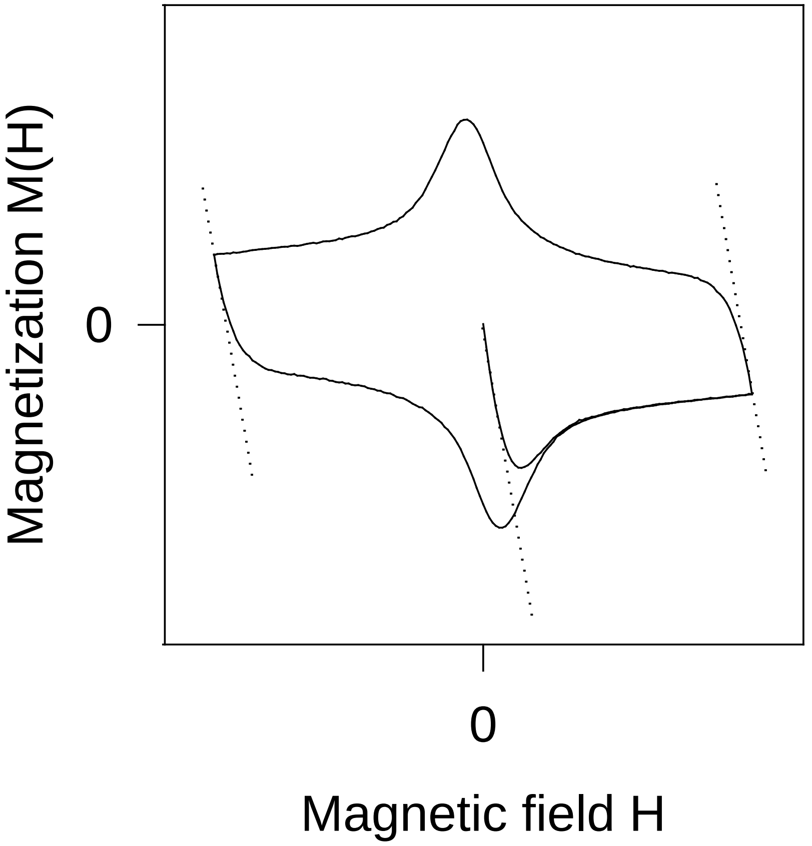
Alvaro Sanchez[1] **and Carles Navau**[1,2]

[1] Grup d'Electromagnetisme, Departament de Física, Universitat Autònoma Barcelona, 08193 Bellaterra (Barcelona), Catalonia, Spain
[2] Escola Universitària Salesiana de Sarrià, Rafael Batlle 7, 08017 Barcelona, Catalonia, Spain

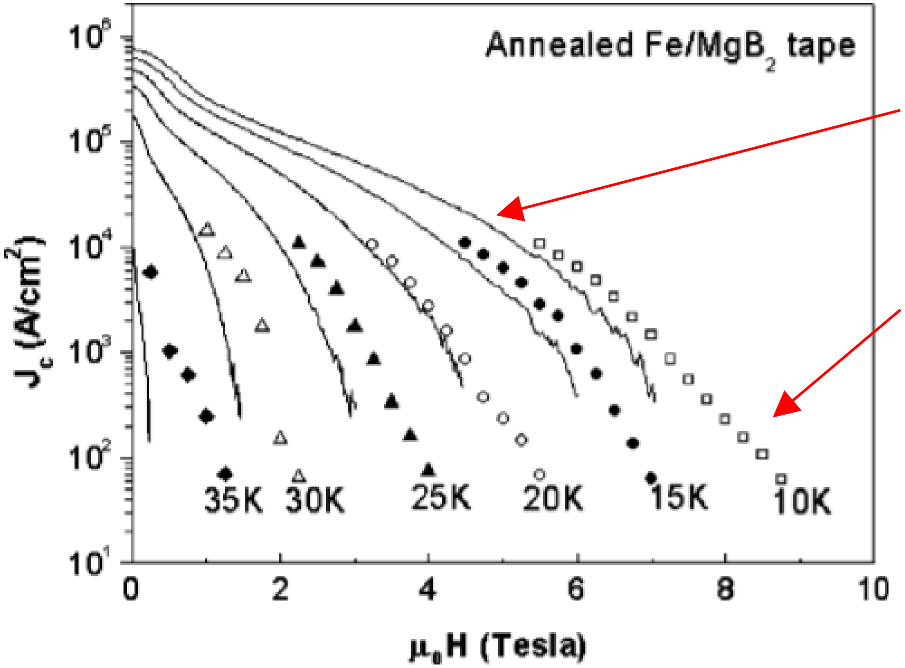
Supercond. Sci. Technol. **14** (2001) 444–447



A typical M(H) curve at “medium” applied fields…



If weak links (granularity) are not a problem, nice agreement between magnetic Jc & transport Jc



MAGNETIC

TRANSPORT

Available online at www.sciencedirect.com

Physica C 385 (2003) 286–305

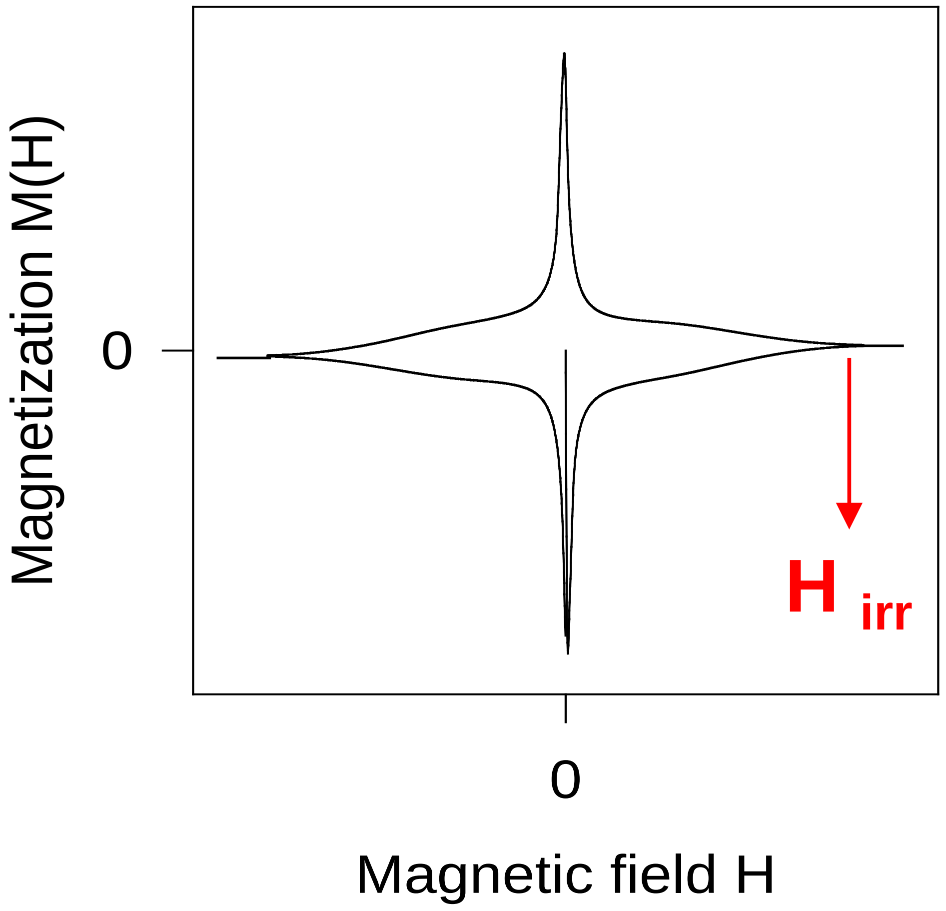
www.elsevier.com/locate/physc

Superconducting properties of MgB$_2$ tapes and wires

R. Flükiger *, H.L. Suo, N. Musolino, C. Beneduce, P. Toulemonde, P. Lezza

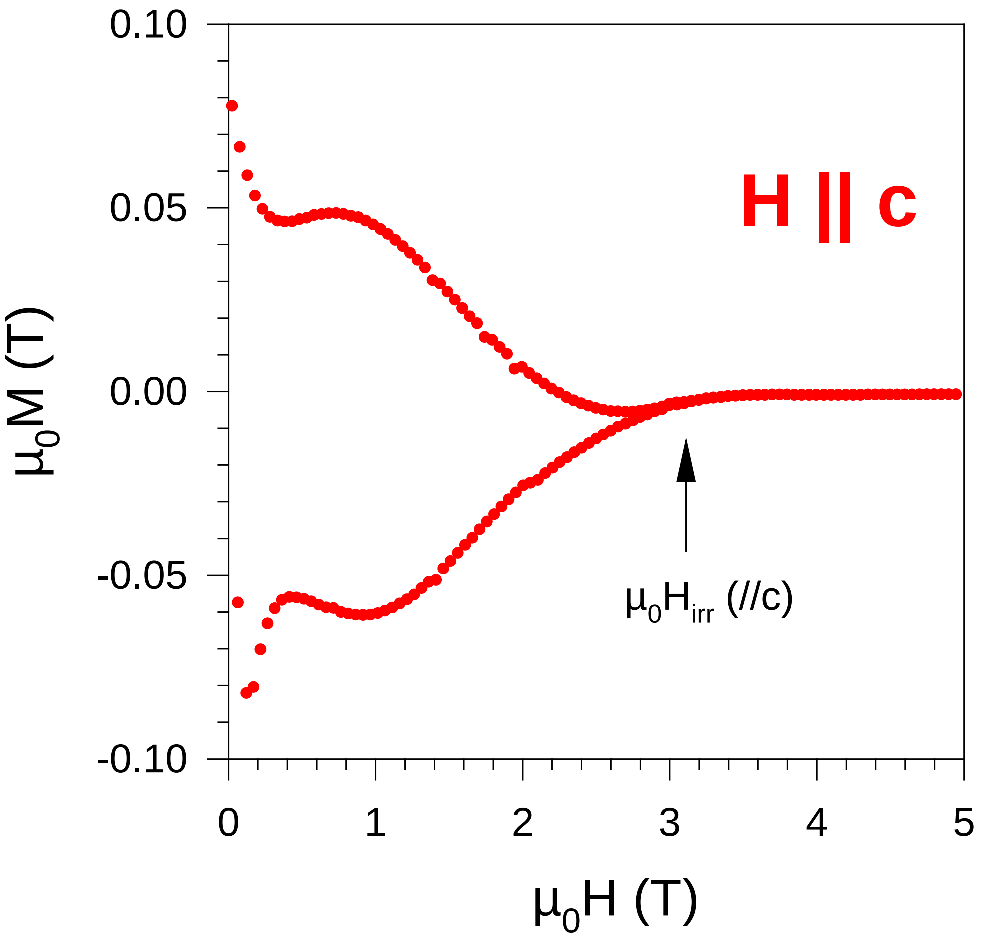
Département de Physique de la Matière Condensée, Université de Genève, 24 Quai Ernest-Ansermet, CH-1211 Genève 4, Switzerland

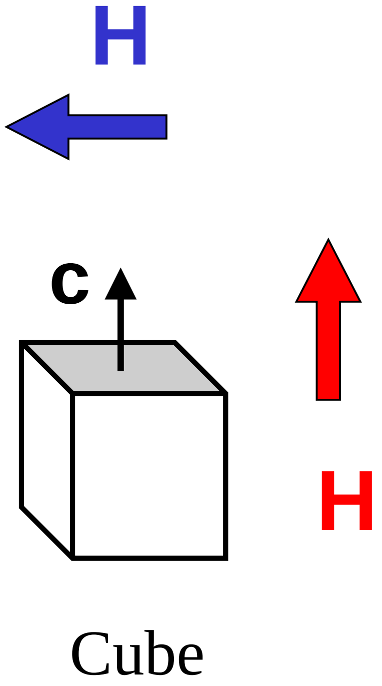
And when the applied field is very large …



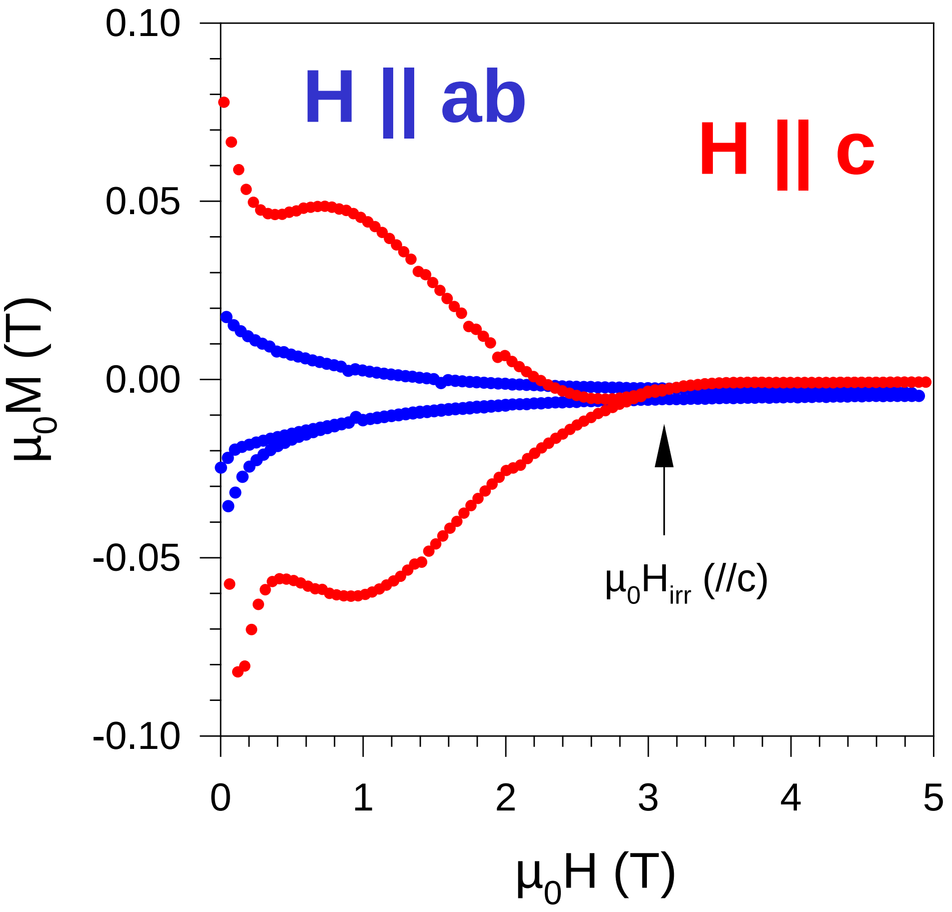
➡ The irreversibility field can be determined from the point where the upper and lower branches of the magnetization loop merge into one

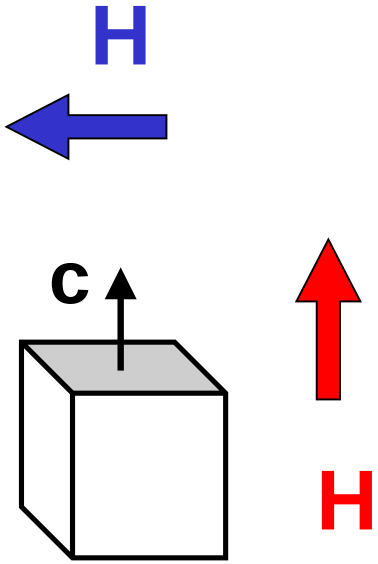
A typical curve for $YBa_2Cu_3O_7$





A typical curve for $YBa_2Cu_3O_7$



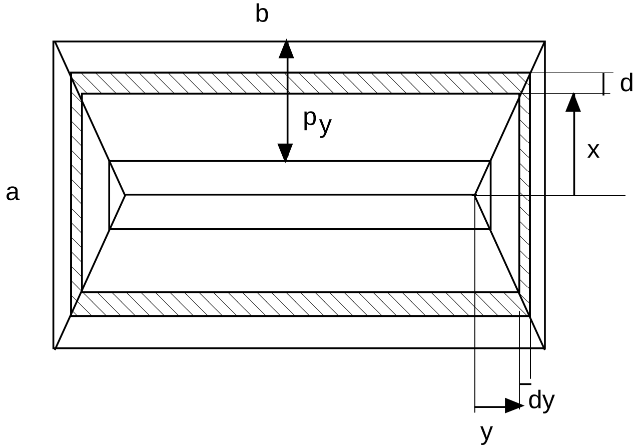


Cube

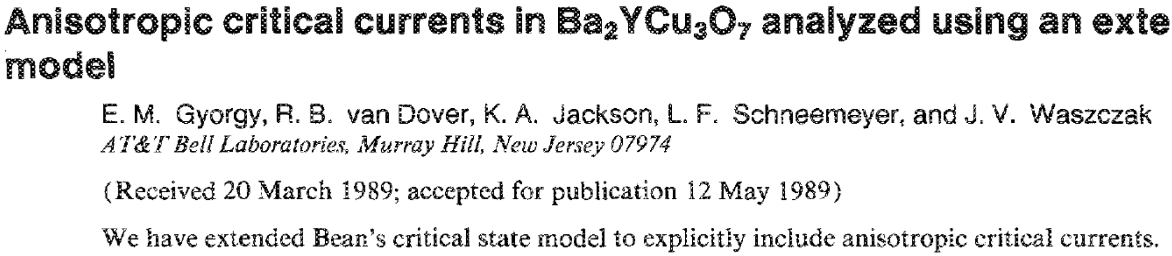
Anisotropy of the current loops should be taken into account to determine the critical current density Jc

Anisotropic Bean model

Analytical calulations can be made in simple geometries (ex. rectangle)



But some results have been published for quite a long time now !



Anisotropic critical currents in $Ba_2YCu_3O_7$ analyzed using an extended Bean model

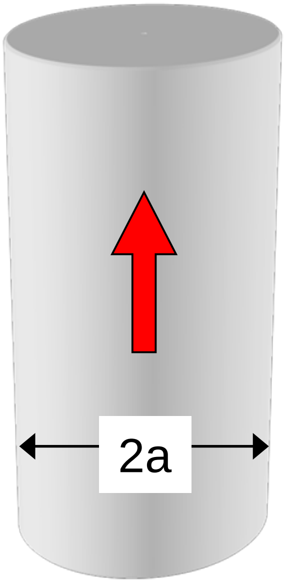
E. M. Gyorgy, R. B. van Dover, K. A. Jackson, L. F. Schneemeyer, and J. V. Waszczak
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(Received 20 March 1989; accepted for publication 12 May 1989)

We have extended Bean's critical state model to explicitly include anisotropic critical currents.

Appl. Phys. Lett. 55 (3),17 July ,1989

And what happens if we consider an E-J curve instead of the Bean model ?



H(t) → B(t)

Do NOT forget Faraday's law

$$\boxed{E = \left(\frac{a}{2}\right)\frac{dB}{dt}}$$

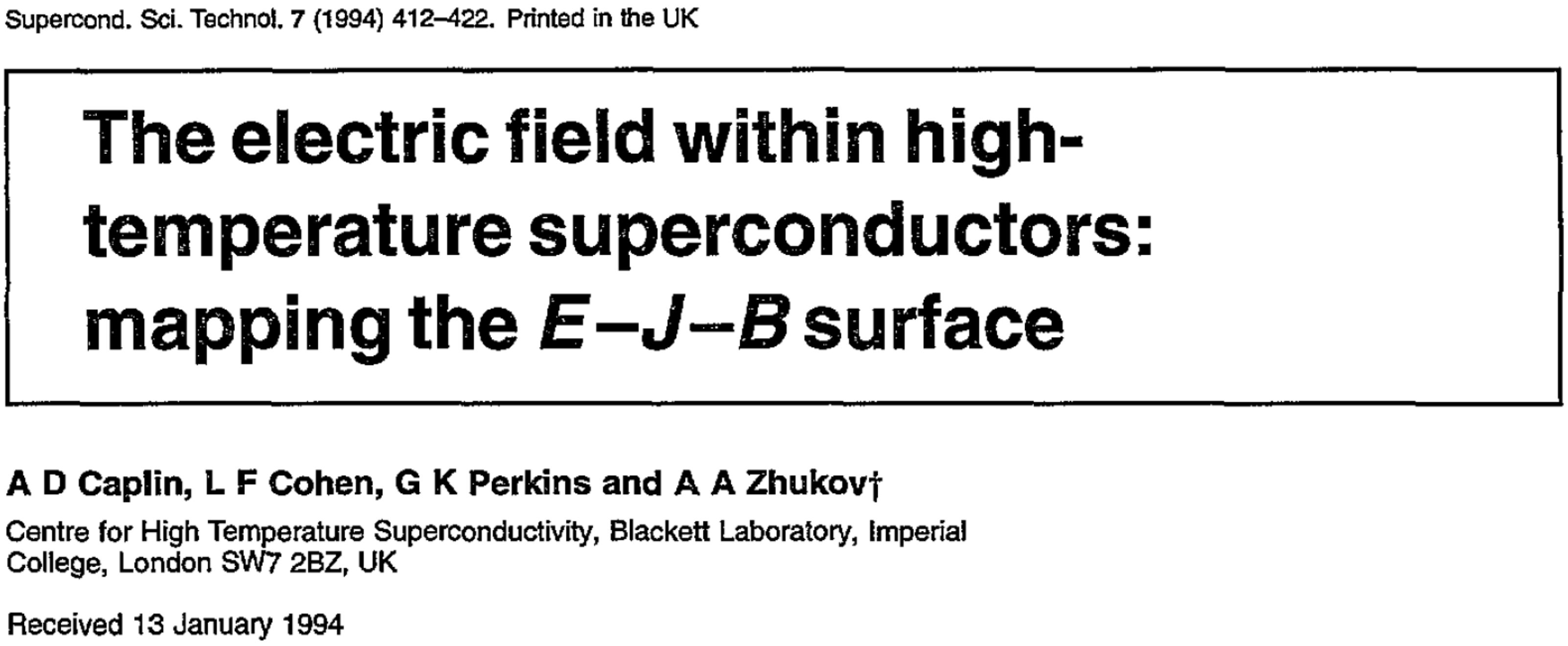
There is always an **electric field** in magnetic experiments !
The amplitude of this field is **much smaller than in transport experiments**

Do not forget to consider these 3 quantities…

Current density : J (A/m²)

Magnetic flux density : B (T)

Electric field : E (V/m)



Supercond. Sci. Technol. 7 (1994) 412–422. Printed in the UK

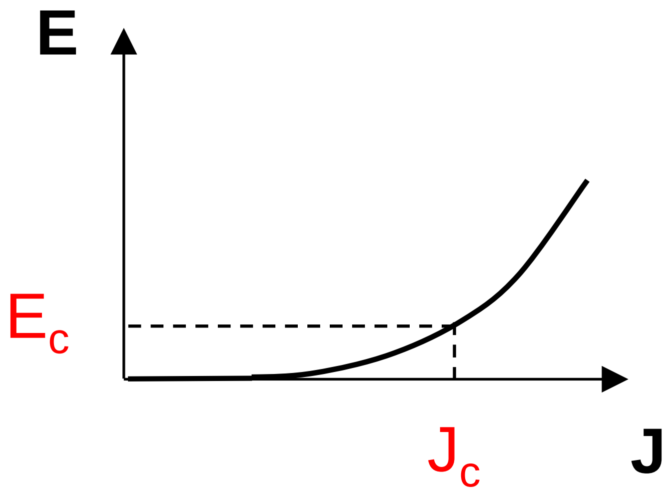
The electric field within high-temperature superconductors: mapping the E–J–B surface

A D Caplin, L F Cohen, G K Perkins and A A Zhukov[†]

Centre for High Temperature Superconductivity, Blackett Laboratory, Imperial
College, London SW7 2BZ, UK

Received 13 January 1994

Consequence …



The amplitude of induced currents increases for large dB/dt !

Always specify dB/dt !

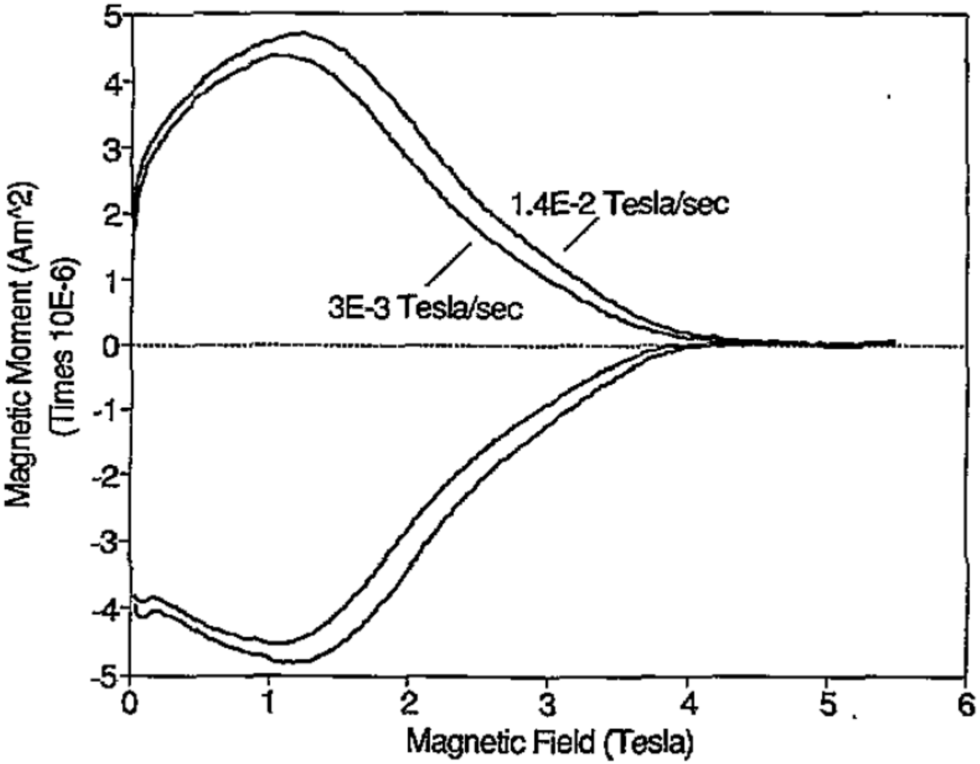
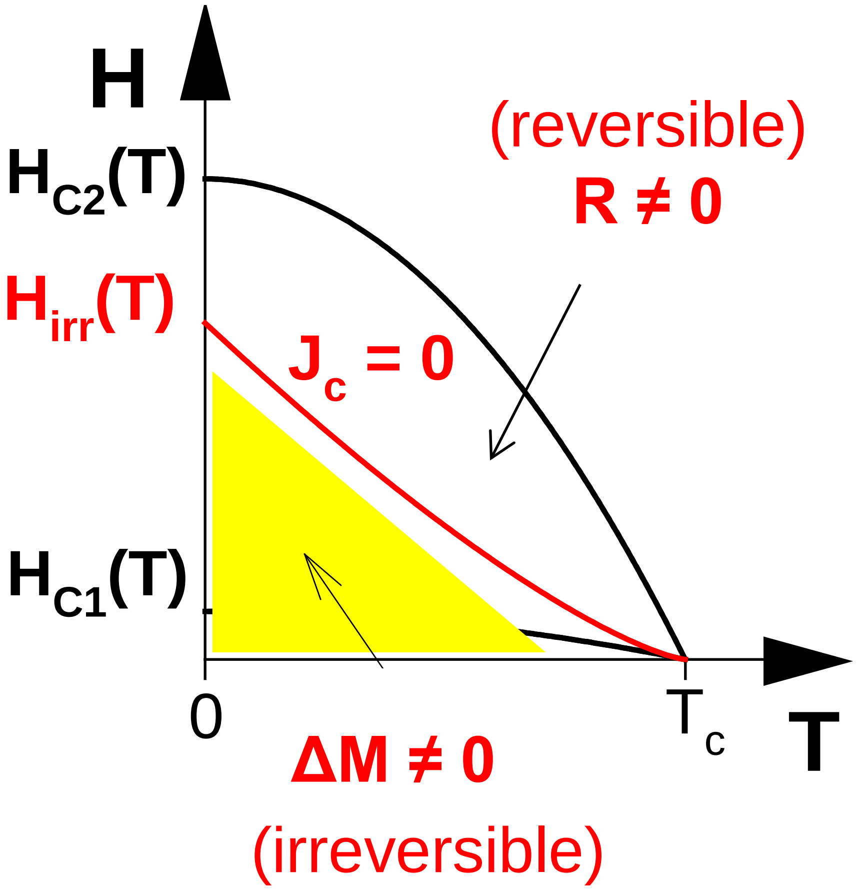
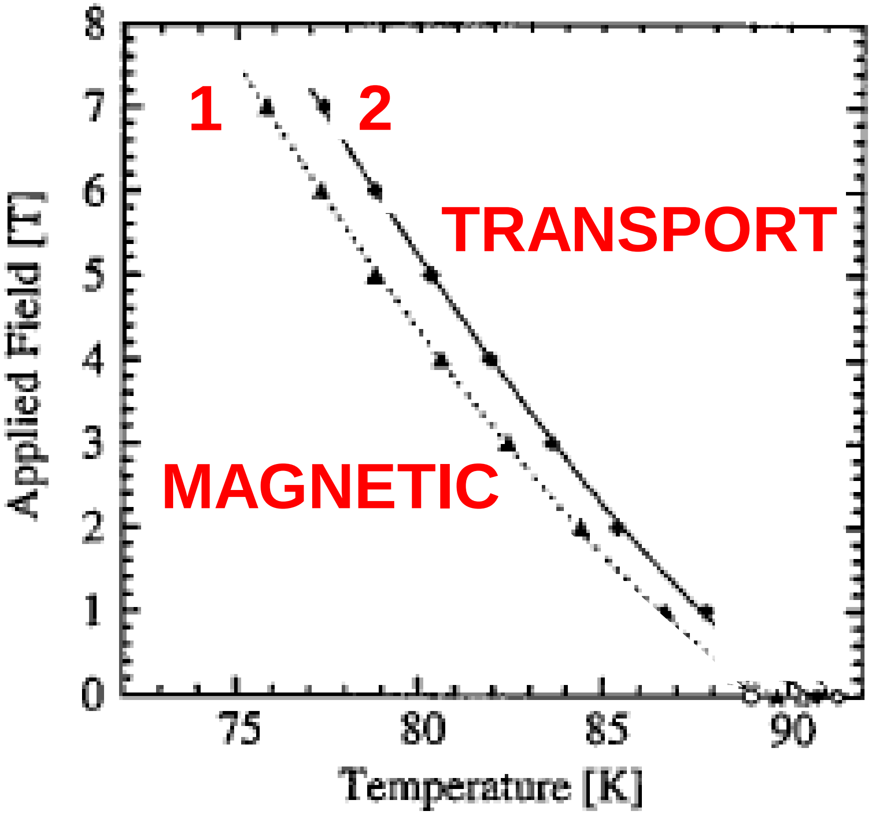


Figure 1. Typical magnetization loops of a high-quality $YBa_2Cu_3O_7$ single crystal at 84 K. Two loops are shown, the outer one having a field sweep rate $\dot{H}_{app}$ of about five times the inner one. H_{app} is parallel to the c-axis. Note the maximum (the 'fishtail' feature) in the magnetic moment at

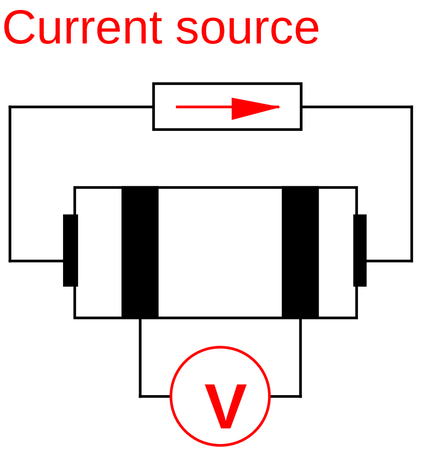
Irreversibility field from TRANSPORT and MAGNETIC



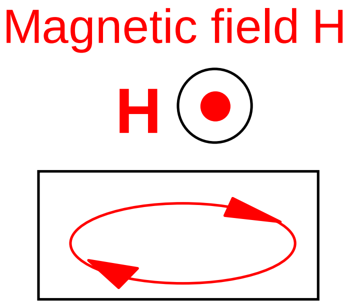


Doyle et al., APL 73, 117 (1998)

Conclusion



Transport current
(applied externally)



Induced current
(by the applied magnetic field)

Both kind of measurements are very useful
and can provide invaluable information on the material properties

BUT … Be always careful when interpreting the results !

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