Diva workshop 2015
Diva in 2 dimensions

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Interpolation 150 years ago...
What is Diva?

**Data Interpolating Variational Analysis**

What is not Diva?

- a plotting tool
- a *black-box*
- a numerical model

- a method to produce gridded fields
- a set of bash scripts and Fortran programs
A little bit of history

Code development (1990-1996)

- Variational Inverse Method (VIM) (Brasseur, 1991, JMS, JGR)
- cross-validation (Brankart and Brasseur, 1996, JAOT)
- error computation (Brankart and Brasseur, 1998, JMS; Rixen et al., 2000, OM)

2D-analysis (2006-2007)

3D-analysis (2007-2008)

4D-analysis (2008-2009)

Web tools:

2011-2012

On-going:

- General: user-driven developments
A little bit of history

Code development (1990-1996)

2D-analysis (2006-2007)

- set of bash scripts (divamesh, divacalc, ...)
- Fortran executables
- parameters optimization tools
- Matlab/Octave scripts for plotting
A little bit of history

Code development (1990-1996)

2D-analysis (2006-2007)

3D-analysis (2007-2008)

- superposition of 2D layers
- automated treatment and optimization
- stability constraint (Ouberdous et al.)
A little bit of history

Code development (1990-1996)

2D-analysis (2006-2007)

3D-analysis (2007-2008)

4D-analysis (2008-2009)

- start from ODV spreadsheet
- detrending (with J. Carstensen, DMU)
- NetCDF 4-D climatology files
A little bit of history

Code development  (1990-1996)

2D-analysis  (2006-2007)
3D-analysis  (2007-2008)
4D-analysis  (2008-2009)

Web tools

- On-line analysis (Barth et al., 2010, Adv. Geosci.)
A little bit of history

<table>
<thead>
<tr>
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Web tools

2011-2012

- multivariate approach
- data transformation tools
- 4-D graphical interface
- implementation of source/decay terms
- advanced error computation *(Troupin et al., 2012, OM)*
A little bit of history

Code development (1990-1996)

2D-analysis (2006-2007)

3D-analysis (2007-2008)

4D-analysis (2008-2009)

Web tools

2011-2012

On-going:

- Modernisation of the code structure
- n-dimensional generalisation
- Spatially correlated observations errors
- Optimized and approximate error calculations (clever poor man)
- Analysis at a specific distance from bottom
- ... ☕️
A little bit of history

Code development (1990-1996)

2D-analysis (2006-2007)

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4D-analysis (2008-2009)

Web tools

2011-2012

On-going:

General: user-driven developments
Diva related tools

**Diva**: base tool (command line), 2D analysis

**Godiva**: automatic repetition of 2D analysis

**Diva-on-web**: 2D analysis with your data on our server

**OceanBrowser**: visualisation tool of 4D NetCDF files

**divand**: multi-dimension analysis (lon, lat, time, depth)

**divaformatlab**: wrapper to use in matlab
DIVA: Data-Interpolating Variational Analysis

Formulation: minimize cost function $J[\varphi]$

$$\min J[\varphi] = \sum_{i=1}^{N} \mu_i \left[ d_i - \varphi(x_i, y_i) \right]^2$$

$$+ \int_D \left( \nabla \nabla \varphi : \nabla \nabla \varphi + \alpha_1 \nabla \varphi \cdot \nabla \varphi + \alpha_0 \varphi^2 \right) \, dD$$
DIVA: Data-Interpolating Variational Analysis

Formulation: minimize cost function $J[\varphi]$

$$\min J[\varphi] = \sum_{i=1}^{N} \mu_i \left[ d_i - \varphi(x_i, y_i) \right]^2$$  \hspace{1cm} \text{data–analysis misfit}$$

$$+ \int_{D} \left( \nabla \nabla \varphi : \nabla \nabla \varphi + \alpha_1 \nabla \varphi \cdot \nabla \varphi + \alpha_0 \varphi^2 \right) \, dD$$  \hspace{1cm} \text{field regularity}$$
Analysis parameters are related to data

Non-dimensional version:

\[ L = \text{length scale} \quad \rightarrow \quad \tilde{\nabla} = L \nabla \quad (1) \]

\[ \rightarrow \quad D = L^2 \tilde{D} \quad (2) \]
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\[ \tilde{J}[\varphi] = \sum_{i=1}^{N} \mu_i L^2 [d_i - \varphi(x_i, y_i)]^2 \]
\[ + \int_{\tilde{D}} \left( \mathbf{\tilde{\nabla}} \mathbf{\tilde{\nabla}} \varphi \cdot \mathbf{\tilde{\nabla}} \mathbf{\tilde{\nabla}} \varphi + \alpha_1 L^2 \mathbf{\tilde{\nabla}} \varphi \cdot \mathbf{\tilde{\nabla}} \varphi + \alpha_0 L^4 \varphi^2 \right) d\tilde{D} \]
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- \( \alpha_0 \rightarrow L \) for which data-analysis misfit \( \simeq \) regularity term: \( \alpha_0 L^4 = 1 \)
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\]

- \( \alpha_0 \rightarrow L \) for which data-analysis misfit \( \simeq \) regularity term: \( \alpha_0 L^4 = 1 \)
- \( \alpha_1 \rightarrow \) influence of gradients: \( \alpha_1 L^2 = 2\xi, \quad \xi = 1 \)
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- \( \alpha_1 \rightarrow \) influence of gradients: \( \alpha_1 L^2 = 2\xi, \quad \xi = 1 \)
- \( \mu_i L^2 \rightarrow \) weight on data:
  \[ \mu_i L^2 = 4\pi \frac{\text{signal}}{\text{noise}_i} \]
Analysis parameters are related to data

Non-dimensional version:

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Coefficients \( \alpha_0, \alpha_1 \) and \( \mu_i \) related to

1. Correlation length \( L \)
2. Signal-to-noise \( \lambda \)
3. Observational noise standard deviation \( \epsilon_i^2 \)
Main analysis parameters

Correlation length $L$:

- Measure of the *influence* of data points
- Estimated by a least-square fit of the covariance function

Signal-to-noise ratio $\lambda$:

- Measure of the *confidence* in data
- Estimated with Generalized Cross Validation techniques
Minimization with a finite-element method

Field regularity → plate bending problem → finite-element solver

Advantages:
- boundaries taken into account
- numerical cost (almost independent on data number)
- no \textit{a posteriori} masking (except if based on error level)
Minimization with a finite-element method

Triangular FE only covers sea: \[ J[\varphi] = \sum_{e=1}^{N_e} J_e(\varphi_e) \] (3)

In each element: \( \varphi_e(r_e) = q_e^T s(r_e) \) with \[
\begin{cases}
  s & \rightarrow \text{shape functions} \\
  q & \rightarrow \text{connectors} \\
  r_e & \rightarrow \text{position}
\end{cases}
\] (4)

(4) in (3) + variational principle

\[ J_e(q_e) = q_e^T K_e q_e - 2q_e^T g_e + \sum_{i=1}^{N_{de}} \mu_i d_i \] (5)

where \[
\begin{cases}
  K_e & \rightarrow \text{local stiffness matrix} \\
  g & \rightarrow \text{vector depending on local data}
\end{cases}
\]
Minimization with a finite-element method

On the whole domain:

\[ J(q) = q^T K q - 2q^T g + \sum_{i=1}^{N_d} \mu_i d_i \]  

(3)

Minimum:

\[ q = K^{-1} g \]  

(4)

\[ q = \begin{bmatrix} K^{-1} \end{bmatrix} g \]  

(5)

Stiffness matrix

Connectors (new unknowns)

Charge vector

Mapping of data on FEM \( \rightarrow \) transfer operator \( T_2 \) \( \rightarrow \) \( g = T_2(r)d \)

Solution at any location \( \rightarrow \) transfer operator \( T_1 \) \( \rightarrow \) \( \varphi(r) = T_1(r)q \)

Results obtained at any location \( \rightarrow \) \( \varphi = T_1(r)K^{-1}T_2(r)d \)
Diva Cocktail Recipe

Ingredients:
- 1 1/2 oz vodka
- 1/2 oz passion-fruit juice
- 1/2 oz lime juice
- 1 tbsp cherry juice
- fill with soda
Diva Cocktail Recipe

Ingredients:
- Smoothness
- Observation constraint
- Behaviour constraint
Want to use Diva?

Playing...

Want to use Diva?

With your own data...

http://gher-diva.phys.ulg.ac.be/web-vis/diva.html or ODV or matlab wrapper
Want to use Diva?

For serious work:

2D version (for production), open source, GPL
nD version (for research), open source, GPL

http://modb.oce.ulg.ac.be/mediawiki/index.php/DIVA
Running **Diva** in 2D: input files

1. `data.dat`: contains the observations

```
36.5500 45.163 17.7138
33.7500 44.167 18.135
32.7500 44.167 18.51
36.2500 43.833 18.5892
33.2500 45.083 18.2326
32.7833 43.917 18.477
32.7500 43.500 18.59
37.2433 44.833 18.1555
36.5000 44.000 18.19
35.8333 43.750 18.62
34.2500 43.832 18.29
35.6500 44.000 18.75
38.0000 44.000 18.155
37.8200 44.368 17.1916
39.0000 42.500 18.23
33.1333 44.433 18.001
33.0500 44.433 18.09
33.2500 44.167 18.231
32.5333 44.833 18.014
38.0167 44.447 18.0568
```
Running Diva in 2D: input files

1. `data.dat`: contains the observations \( x | y | \text{value} \)  
2. `coast.cont`: delimits land and sea (coastline or isobaths)
Running Diva in 2D: input files

1. `data.dat`: contains the observations \( x | y | \text{value} \)
2. `coast.cont`: delimits land and sea (coastline or isobaths)
3. `param.par`: analysis parameters \( L, \lambda, \text{resolution, } \ldots \)

```plaintext
# Lc: correlation length (in units coherent with your data) #
1.5
# icoordchange (=0 if no change of coordinates is to be performed; =1 if positions are in degree #
2
# ispec: output files required #
0
# ireg: mode selected for background field: 0=null guess; 1=mean of data; 2=regression plan if a
2
# xori: x-coordinate of the first grid point of the output#
27
# xori: y-coordinate of the first grid point of the output#
40
# dx: step of output grid#
0.1
# dy: step of output grid#
0.1
# nx: number of grid points in the x-direction#
151
# ny: number of grid points in the y-direction#
76
# valex: exclusion value#
-99
# snr: signal to noise ratio of the whole dataset#
0.5
# varbak: variance of the background field. If zero, no error fields are produced. If one, relat
1.0
```
Workflow in 2D

Select region of study

<table>
<thead>
<tr>
<th></th>
<th>27°E</th>
<th>30°E</th>
<th>33°E</th>
<th>36°E</th>
<th>39°E</th>
<th>42°E</th>
</tr>
</thead>
<tbody>
<tr>
<td>46°N</td>
<td></td>
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<td></td>
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<tr>
<td>44°N</td>
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<tr>
<td>42°N</td>
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<tr>
<td>40°N</td>
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</tbody>
</table>
Workflow in 2D

Extract topography, for example via

Workflow in 2D

Generate contour
Workflow in 2D

Extract data
Workflow in 2D

Evaluate analysis parameters
Workflow in 2D

Create finite-element mesh
Workflow in 2D

Generate analysis
Workflow in 2D

Generate error field
When to use 2D version

- occasional use
- 2D fields like benthic properties
- for implementation of special features by your own (e.g., multiplicative bias correction, special background field creation based on habitats)
- ...

otherwise: use 3D or 4D version directly
Diva in 4 dimensions