6th Diva user workshop

Theory and 2-D version

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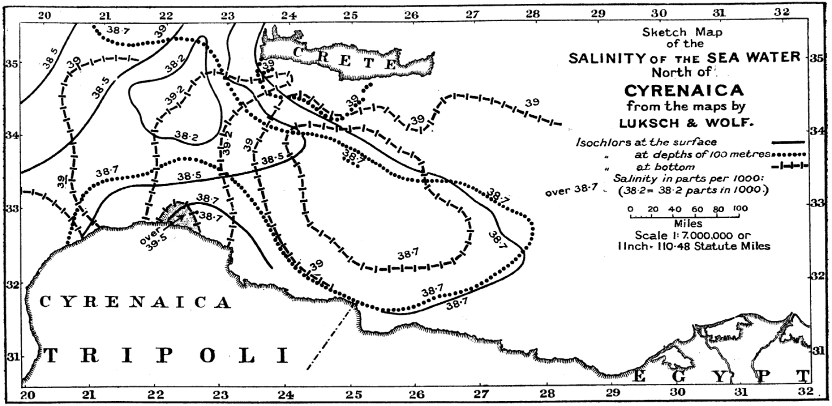
Diva: interpolation (gridding) of in situ data



[J.W. Gregory (1916), Cyrenaica, *The Geography Journal*, **47 (5)**, 321-342]

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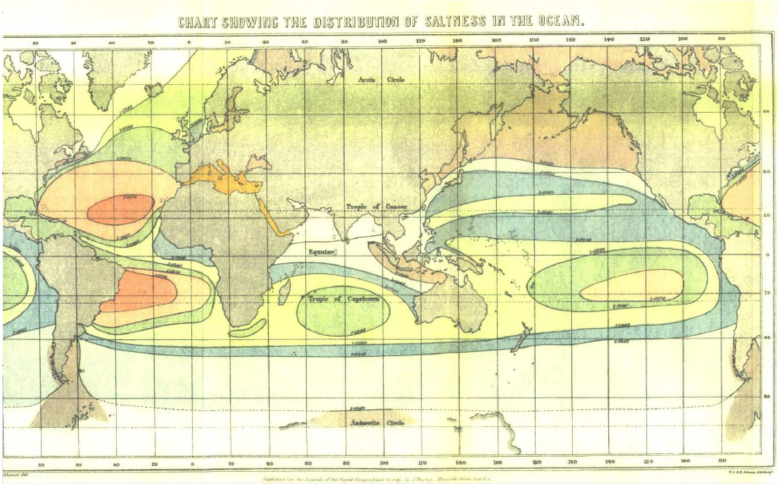


What is Diva?

Data
Interpolating
Variational
Analysis

What is Diva?

- a method to produce gridded field
- a set of scripts and Fortran programs

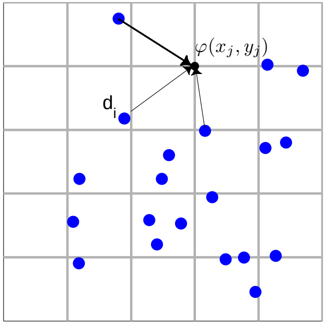
What is not Diva?

- a plotting tool
- a *black-box*
- a numerical model

⚠ gridding and plotting are different tasks

DIVA: Data-Interpolating Variational Analysis

N_d data points d_i • $\rightarrow$ gridded field



Formulation: minimize cost function $J[\varphi]$

$$\min J[\varphi] = \sum_{i=1}^{N} \mu_i \left[d_i - \varphi(x_i, y_i)\right]^2 \qquad \text{data–analysis misfit}$$

$$+ \int_D \left(\nabla\nabla\varphi : \nabla\nabla\varphi + \alpha_1 \nabla\varphi \cdot \nabla\varphi + \alpha_0 \varphi^2\right) \mathrm{d}D \qquad \text{field regularity}$$

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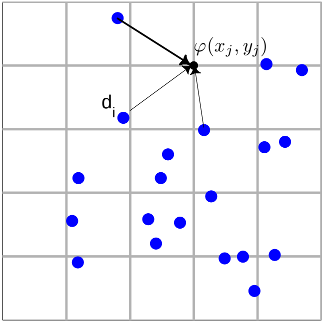

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Four ways to use Diva

1. Command line (batch processing)
2. Ocean Data View
3. Diva-on-web
4. Matlab package

```
divastripped : bash
//////////////////////////////////////////////////
Mesh generation
//////////////////////////////////////////////////

Reading parameters

Copying output files for visualisation
in directory ./output/meshvisu/

---------------
Mesh is created
---------------

//////////////////////////////////////////////////
Going to analyse a field
//////////////////////////////////////////////////

Number of data points: 3

VARBAK: 1.0

Errors will be calculated

Output of results copied in ./output/

Creating netcdf file for field and associated error
in directory ./output/ghertonetcdf/

--------------------
Analysis is finished
--------------------

Check the results in
./ouput/ghertonetcdf/results.nc (netcdf)
[charles@gher13 divastripped]$
```

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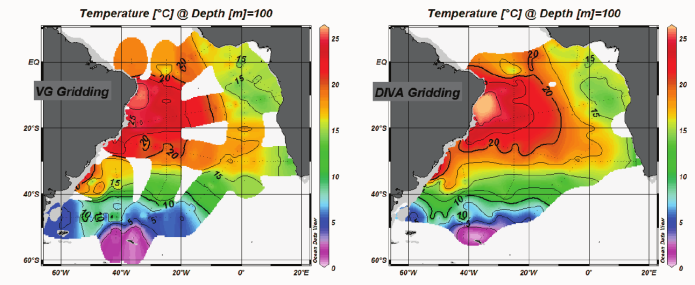

[R. Schlitzer (2009), Ocean Data View User's Guide, Version 4.2]

Four ways to use Diva

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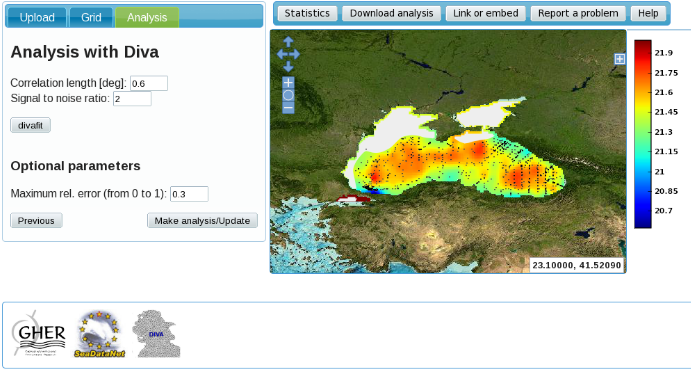
http://gher-diva.phys.ulg.ac.be/web-vis/diva.html



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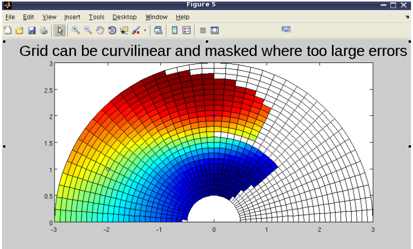
Package available at
http://modb.oce.ulg.ac.be/mediawiki/upload/divaformatlab.zip



Requires executables of Diva (mesh and analysis)

A little bit of history

Code development (1990-1996):

- Variational Inverse Method (VIM) (Brasseur, 1991, JMS, JGR)
- cross-validation (Brankart and Brasseur, 1996, JAOT)
- error computation (Brankart and Brasseur, 1998, JMS; Rixen et al., 2000, OM)

2D-analysis (2006-2007):

- set of bash scripts (`divamesh`, `divacalc`, `...`)
- Fortran executables
- parameters optimization tools
- Matlab/Octave scripts for plotting

3D-analysis (2007-2008):

- superposition of 2D layers
- automated treatment and optimization
- stability constraint (Ouberdous et al.)
- advanced error computation (Troupin et al., 2012, OM)

A little bit of history

4D-analysis (2008-2009):

- start from ODV spreadsheet
- *detrending* (with J. Carstensen, DMU)
- NetCDF 4-D climatology files

Web tools

- On-line analysis (Barth et al., 2010, Adv. Geosci.) *http://gher-diva.phys.ulg.ac.be/web-vis/diva.html*
- Climatology viewer: *http://gher-diva.phys.ulg.ac.be/web-vis/clim.html*

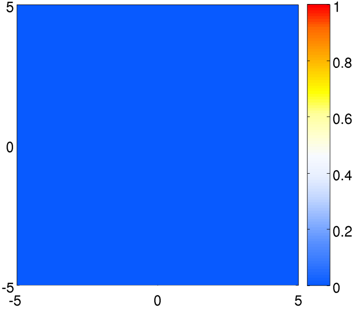
2010 (on-going)

- transition to Fortran95
- multivariate approach
- data transformation tools
- 4-D graphical interface
- implementation of *source/decay* terms
- ...

General: user-driven developments

A simple example with a few points

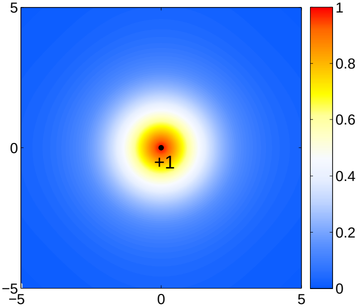
Zero background



- influence around data point
- variable radius of influence
- distance between data points
- separation by obstacles
- confidence in measurements

A simple example with a few points

Square domain, data at center



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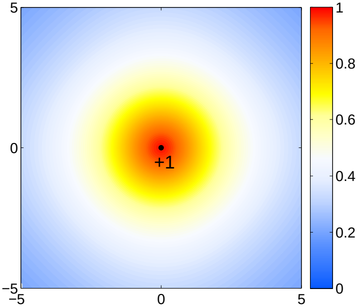

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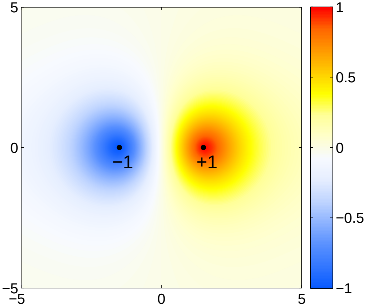
A simple example with a few points

Two symmetric data points



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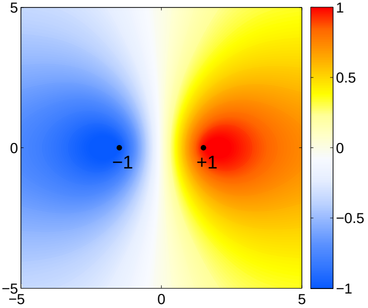

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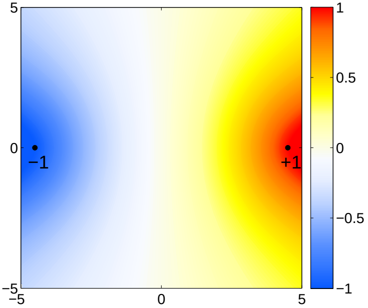

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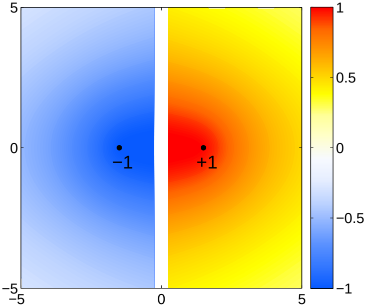
A simple example with a few points

Physical barrier



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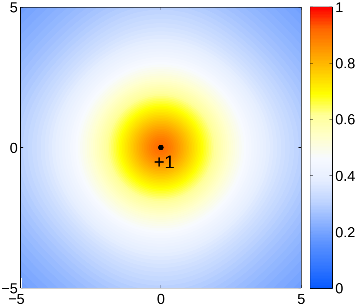
Data at center



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Analysis parameters are related to data

Non-dimensional version:

$$L = \text{length scale} \quad \rightarrow \quad \tilde{\boldsymbol{\nabla}} = L\boldsymbol{\nabla} \tag{1}$$

$$\rightarrow \quad D = L^2 \tilde{D} \tag{2}$$

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- $\alpha_0 \rightarrow L$ for which data-analysis misfit $\simeq$ regularity term: $\alpha_0 L^4 = 1$
- $\alpha_1 \rightarrow$ influence of gradients: $\alpha_1 L^2 = 2\xi, \quad \xi = 1$
- $\mu_i L^2 \rightarrow$ weight on data: $\mu_i L^2 = 4\pi \frac{\text{signal}}{\text{noise}_i}$

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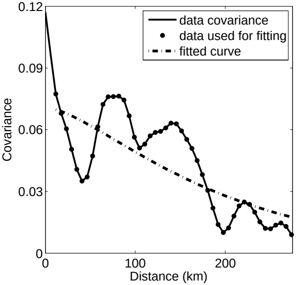
Coefficients α_0, α_1 and μ_i related to

1. Correlation length L
2. Signal-to-noise λ
3. Observational noise standard deviation ϵ_i^2

Main analysis parameters

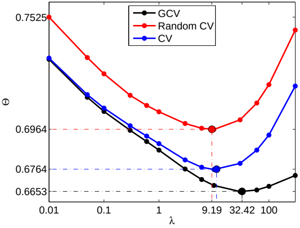
Correlation length L:

- Measure of the *influence* of data points
- Estimated by a least-square fit of the covariance function

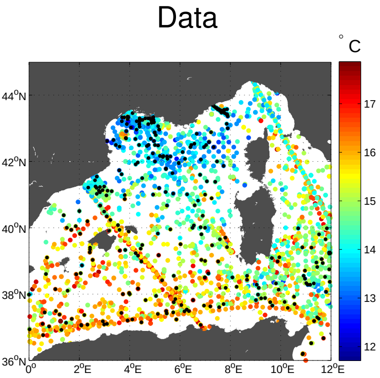


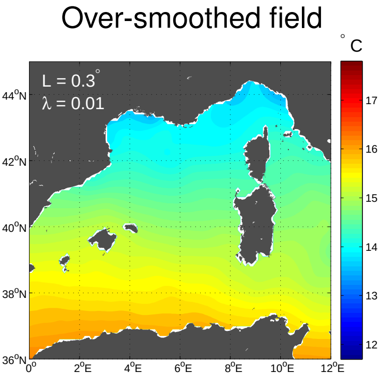
Signal-to-noise ratio λ:

- Measure of the *confidence* in data
- Estimated with Generalized Cross Validation techniques



Parameters: smoothness vs. data influence





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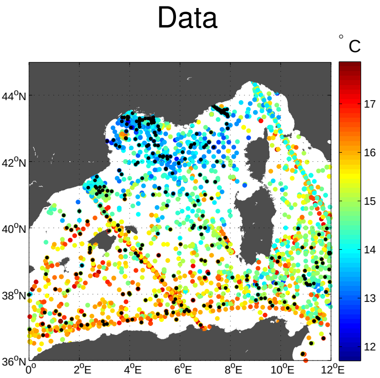

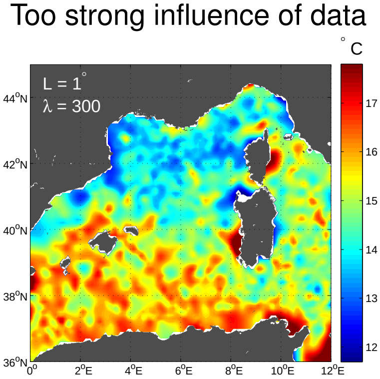

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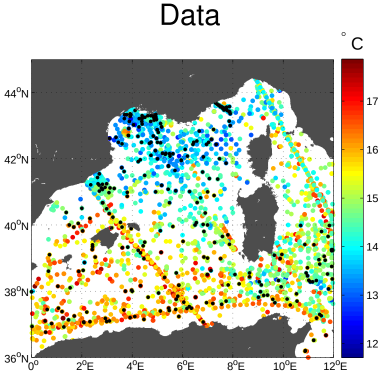
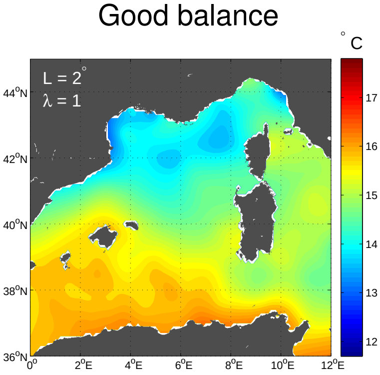
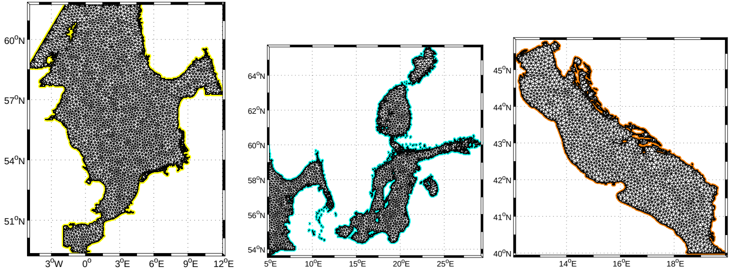




Minimization with a finite-element method

Field regularity → plate bending problem → finite-element solver



Advantages:

- boundaries taken into account
- numerical cost (almost independent on data number)
- no *a posteriori* masking

Minimization with a finite-element method

$$\text{Triangular FE only covers sea:} \qquad J[\varphi] = \sum_{e=1}^{N_e} J_e(\varphi_e) \tag{3}$$

$$\text{In each element: } \varphi_e(\mathbf{r_e}) = \mathbf{q_e}^T \mathbf{s}(\mathbf{r_e}) \quad \text{with} \begin{cases} \mathbf{s} & \rightarrow \text{shape functions} \\ \mathbf{q} & \rightarrow \text{connectors} \\ \mathbf{r_e} & \rightarrow \text{position} \end{cases} \tag{4}$$

(4) in (3) + variational principle

$$J_e(\mathbf{q_e}) = \mathbf{q_e}^T \mathbf{K_e} \mathbf{q_e} - 2\mathbf{q_e}^T \mathbf{g_e} + \sum_{i=1}^{N_{d_e}} \mu_i d_i \tag{5}$$

$$\text{where} \begin{cases} \mathbf{K_e} & \rightarrow \text{local stiffness matrix} \\ \mathbf{g} & \rightarrow \text{vector depending on local data} \end{cases}$$

Minimization with a finite-element method

On the whole domain: $$J(\mathbf{q}) = \mathbf{q}^T\mathbf{K}\mathbf{q} - 2\mathbf{q}^T\mathbf{g} + \sum_{i=1}^{N_d} \mu_i d_i \qquad (3)$$

Minimum: $$\mathbf{q} = \mathbf{K}^{-1}\mathbf{g} \qquad (4)$$

$$\mathbf{q} = \mathbf{K}^{-1}\mathbf{g} \qquad (5)$$

- Stiffness matrix
- Connectors (new unknowns)
- Charge vector

Mapping of data on FEM $\rightarrow$ transfer operator $\mathbf{T_2}$ $\rightarrow$ $\mathbf{g} = \mathbf{T_2}(\mathbf{r})\mathbf{d}$

Solution at any location $\rightarrow$ transfer operator $\mathbf{T_1}$ $\rightarrow$ $\varphi(\mathbf{r}) = \mathbf{T_1}(\mathbf{r})\mathbf{q}$

Results obtained at any location $\rightarrow \varphi = \mathbf{T_1}(\mathbf{r})\mathbf{K}^{-1}\mathbf{T_2}(\mathbf{r})\mathbf{d}$