

Elastic scattering, total cross sections and ρ parameters at the LHC

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November 13, 2017

(mainly) with O.V. Selyugin

Analytic S matrix theory

$$\mathcal{A}(s, t, u)$$

elastic amplitude
dominated at large s
by singularities in the
complex j plane

$$\mathcal{A} = \sum_i C_i s^{\alpha_i(t)} + \int d\omega c(\omega) s^{\alpha(\omega, t)}$$

unknown
functions of t

in the case of
simple poles

simple poles
imply cuts

Knowing the discontinuities
of the ($C=+1$ and $C=-1$) amplitudes
(the imaginary part)
allows one to reconstruct
the whole amplitude

Real part/s

unknown

Imaginary part/s

$$\rho_{\pm} \sigma_{\pm} = \frac{A_{\pm}(s_0, 0)}{2m_p p} + \frac{E}{\pi p} \mathbf{P} \int_{m_p}^{\infty} \left[\frac{\sigma_{\pm}}{E'(E' - E)} - \frac{\sigma_{\mp}}{E'(E' + E)} \right] p' dE'$$

$$+ = pp; - = p\bar{p}, t=0$$

The unitarity of the S matrix
(i.e. $\sigma_{\text{elastic}} < \sigma_{\text{tot}}$)
implies

$$|A(s, b)|^2 \leq 2 \text{Im}(A(s, b))$$

$(A(s, b) = \mathcal{A}(s, b)/s)$

Fourier transform of transverse momentum variable
 \approx partial wave

$$(\ell = b\sqrt{s})$$

This leads to

$$\frac{\sigma_{\text{tot}}}{\log^2 s} \rightarrow \text{constant} \quad \text{as } s \rightarrow \infty$$

2 classes of models

Analytic fits: *many parameters*

- pick a number of known analytic functions
- fit the data
- reduce the number of parameters via some assumption (universality, quark counting, etc.)

Physical models: *many assumptions*

- choose a low-energy parametrisation
- extrapolate it to high energy
- implement unitarity

Analytic parametrisations

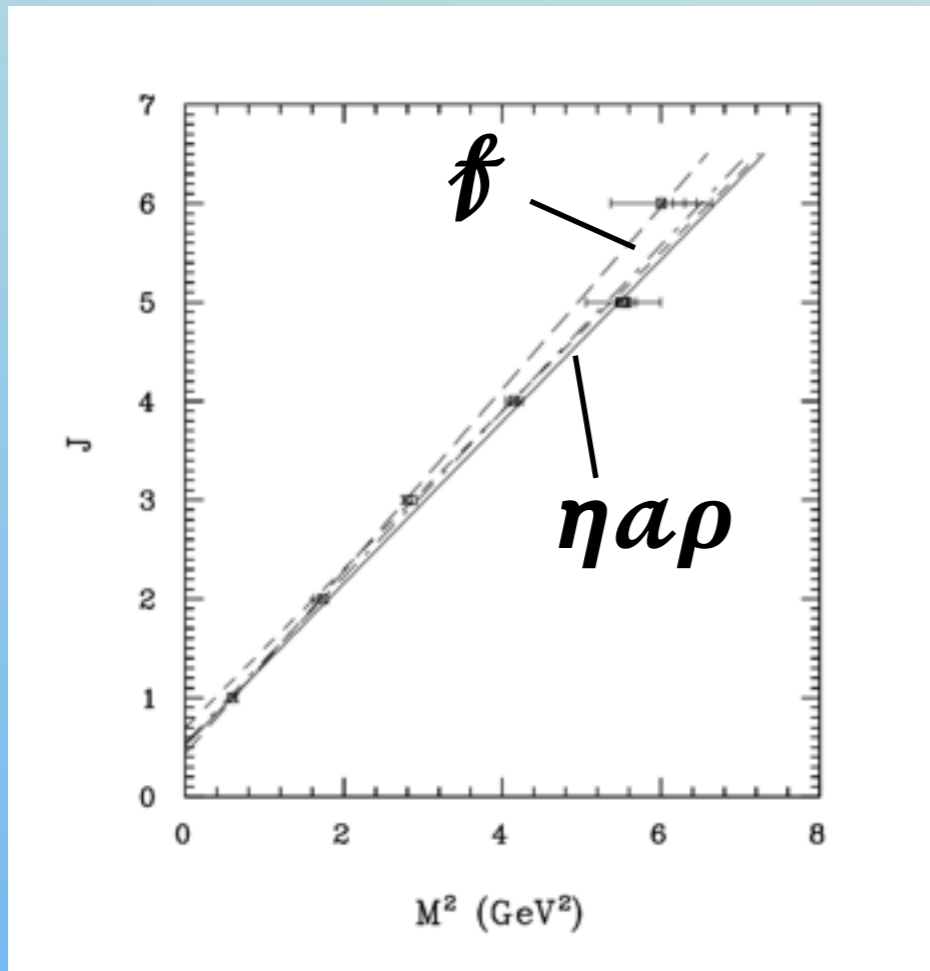
Usually for $t=0$

Start with a cross section parametrisation
(about 6 parameters for pp/pp)

Extend it to the real part using dispersion relations

Use s^α , $\log(s)$, $\log^2(s)$

Physical models

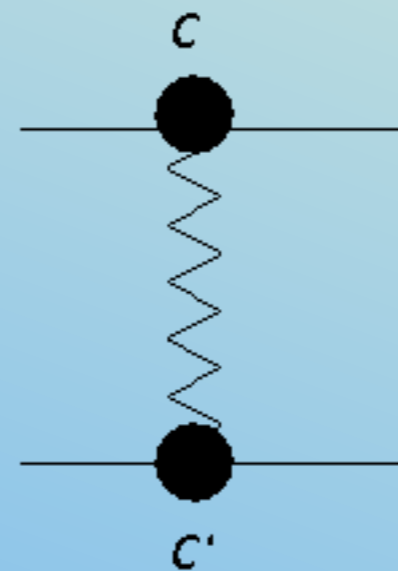


Regge trajectory

$$J = \alpha_0 + \alpha' M^2$$

Elastic amplitude

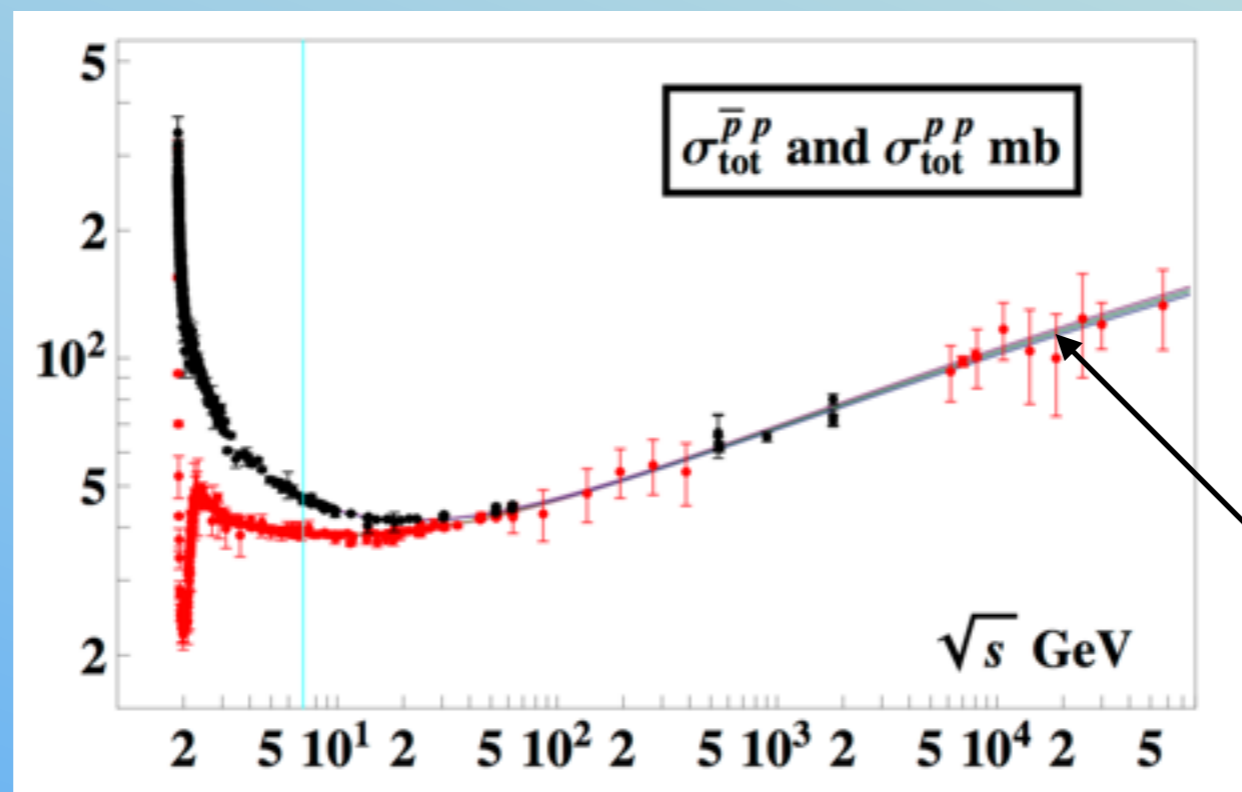
$$\mathcal{A} = c(t)c'(t) (is)^{\alpha_0 + \alpha' t}$$



Pomeron

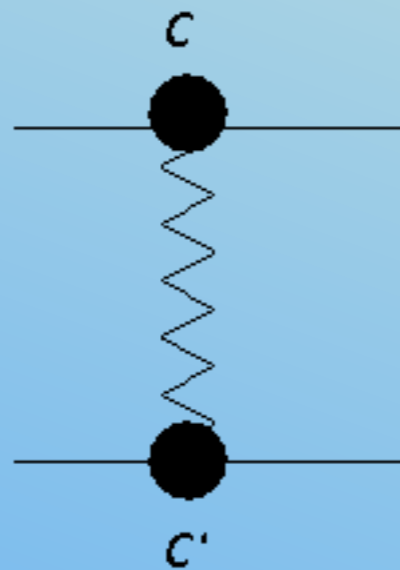
All meson and baryon exchanges lead to falling total cross sections

$$\Im m A/s \propto \sigma_{tot}$$



Pomeron amplitude for $\sqrt{s} < 100 \text{ GeV}$

$$\mathcal{A} = c(t)c'(t) (is)^{\alpha_0 + \alpha' t}$$

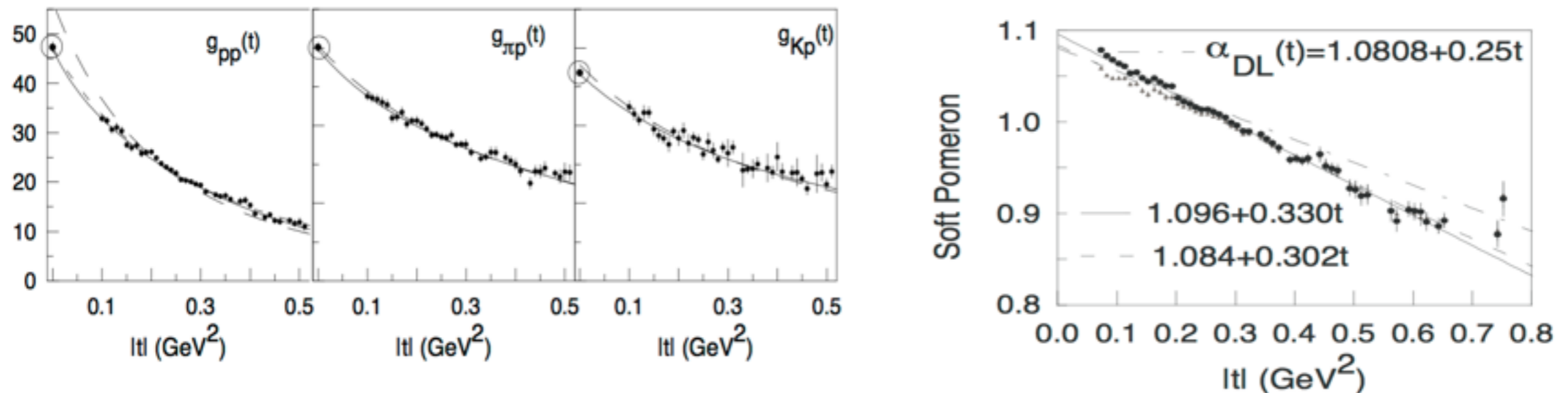


$$\begin{aligned}\alpha_0 &\approx 1.1 \\ \alpha' &\approx 0.3 \text{ GeV}^{-2} \\ c(t) &\approx F_1(t)\end{aligned}$$

Glueball exchange?

Get the trajectories and form factors directly from the data

[JRC, A. Lengyel, E. Martynov, hep-ph/0511073](#)



$$5 \text{ GeV} < \sqrt{s} < 100 \text{ GeV}, 0.1 \text{ GeV}^2 < |t| < 0.5 \text{ GeV}^2$$

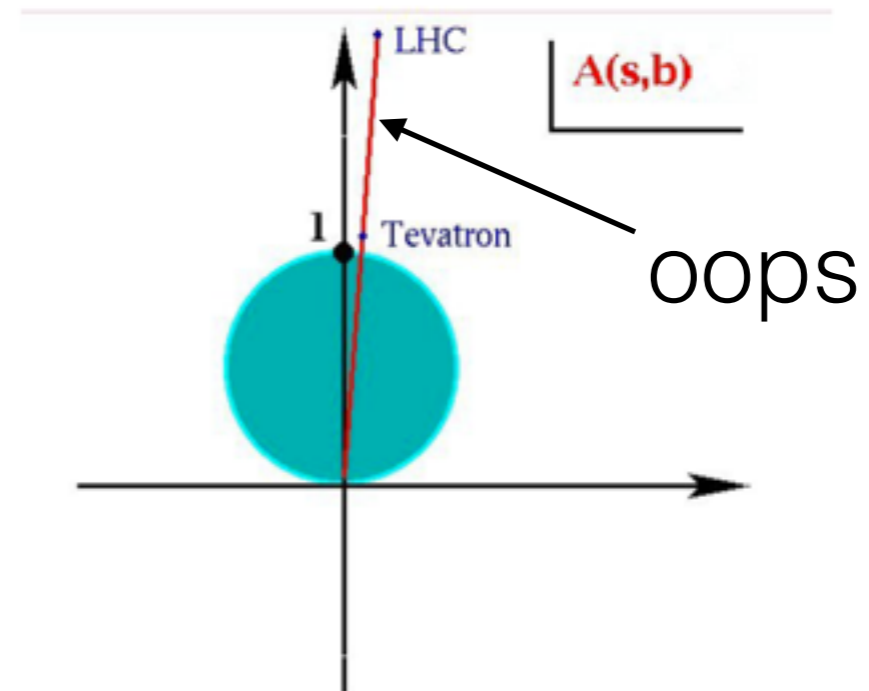
**non exponential
form factors**

linear trajectory

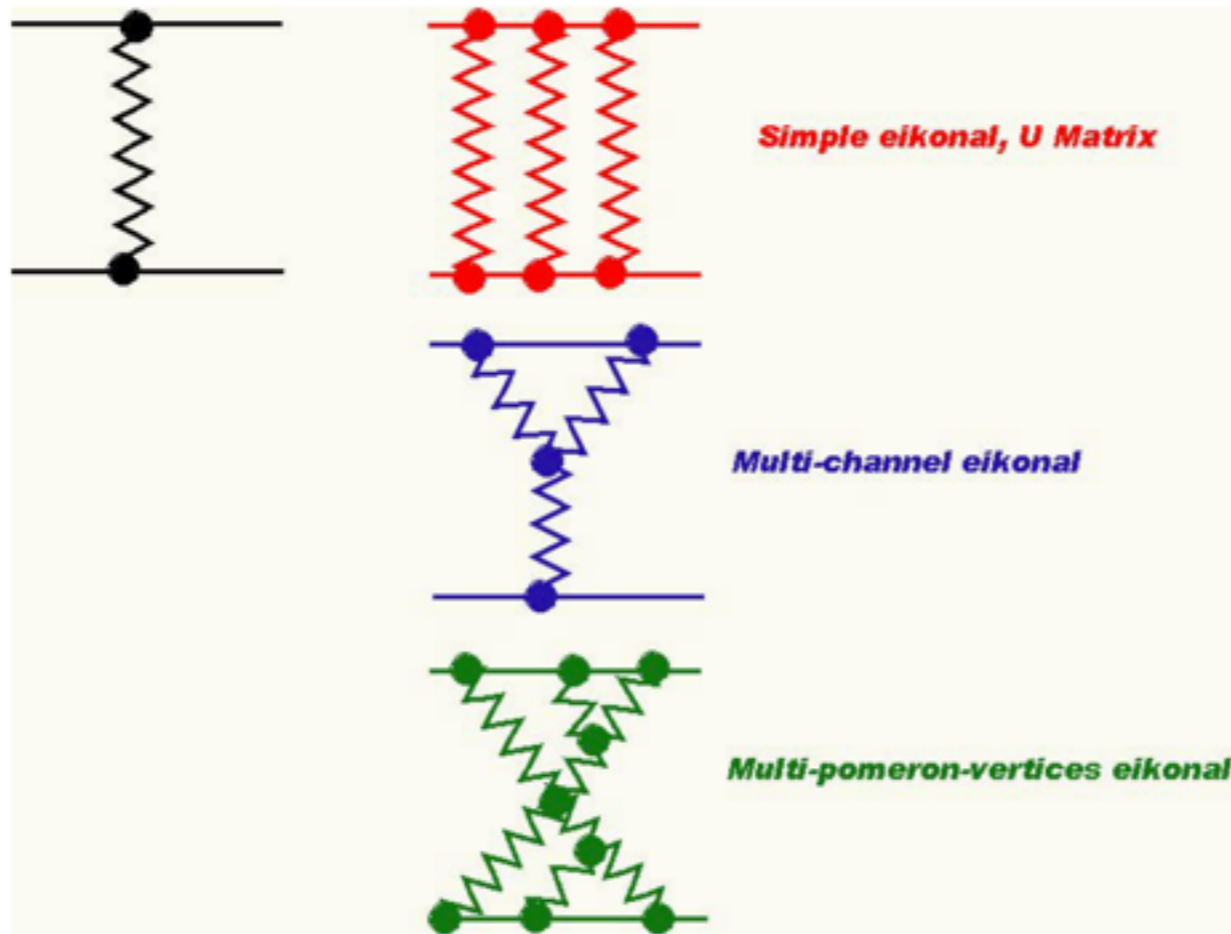
Unitarity

partial wave $A(s, b)$, ($\ell = b\sqrt{s}$)

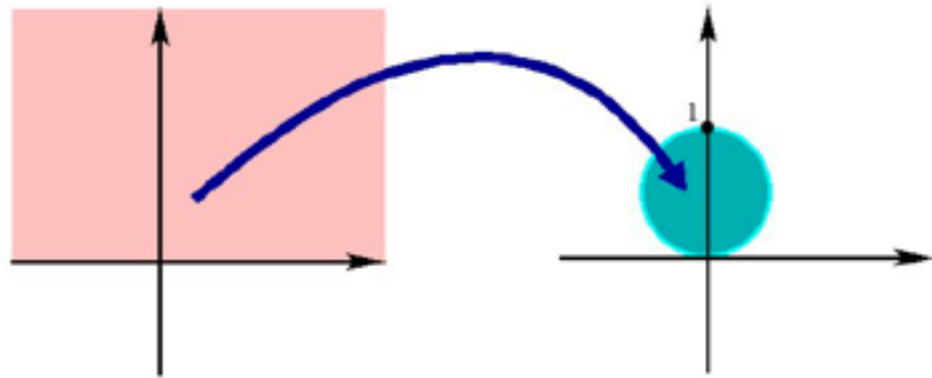
$$|A(s, b)|^2 \leq 2\text{Im}(A(s, b))$$



Unitarisation

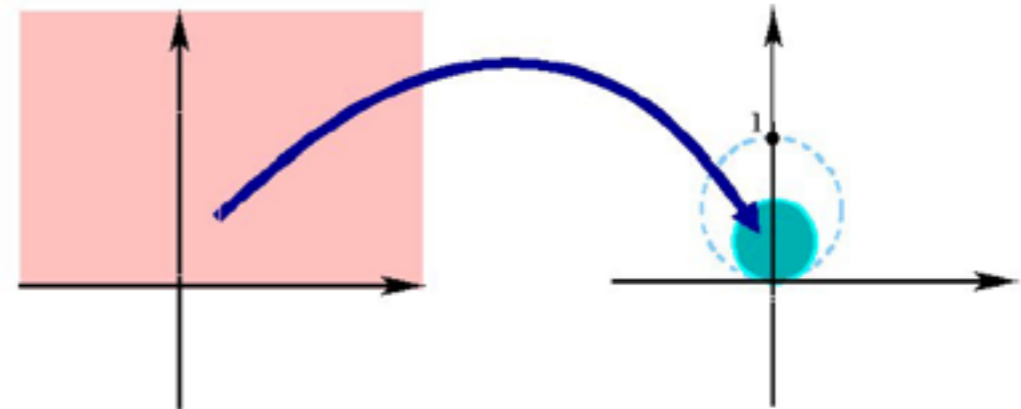


Even in the simplest case we cannot calculate the resummation

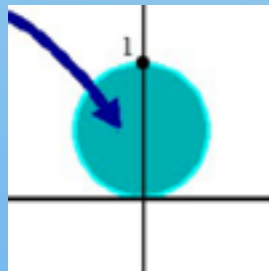
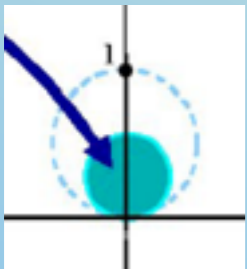


eikonal: $\mathbf{U}(s, b) = i(1 - \exp(iA(s, b)))$
 U-matrix: $\mathbf{U}(s, b) = \frac{A(s, b)}{1 - iA(s, b)/2}$

True unitarisation



eikonal
 $\mathbf{U}(s, b) = \frac{i}{\omega} (1 - \exp(i\omega A(s, b)))$
 or U-matrix
 $\mathbf{U}(s, b) = \frac{A(s, b)}{1 - i\omega A(s, b)/2}$



Total cross sections @ LHC

$$\Im m \mathcal{A}(s, 0) / s \propto \sigma_{tot}$$

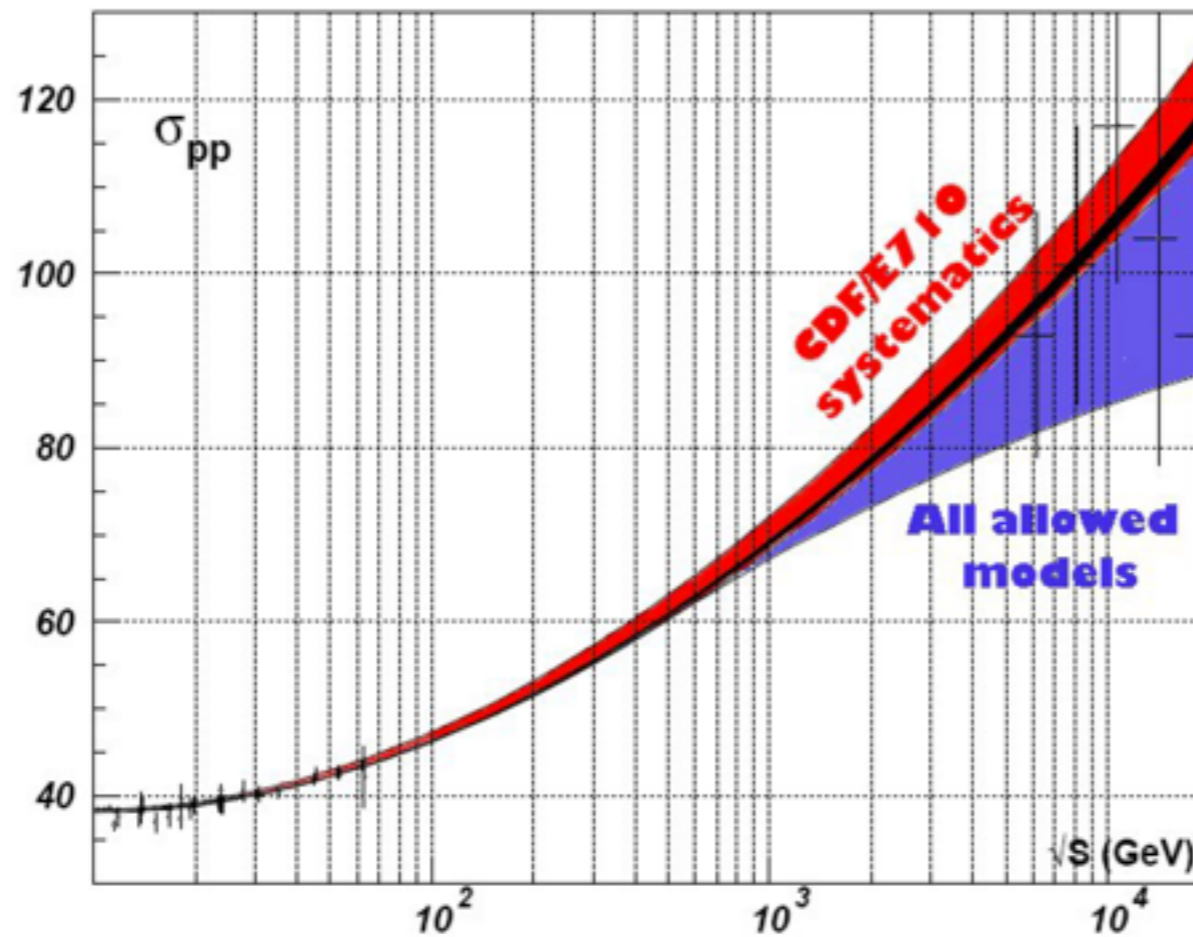
COMPETE fits

qcd.theo.phys.ulg.ac.be/compete

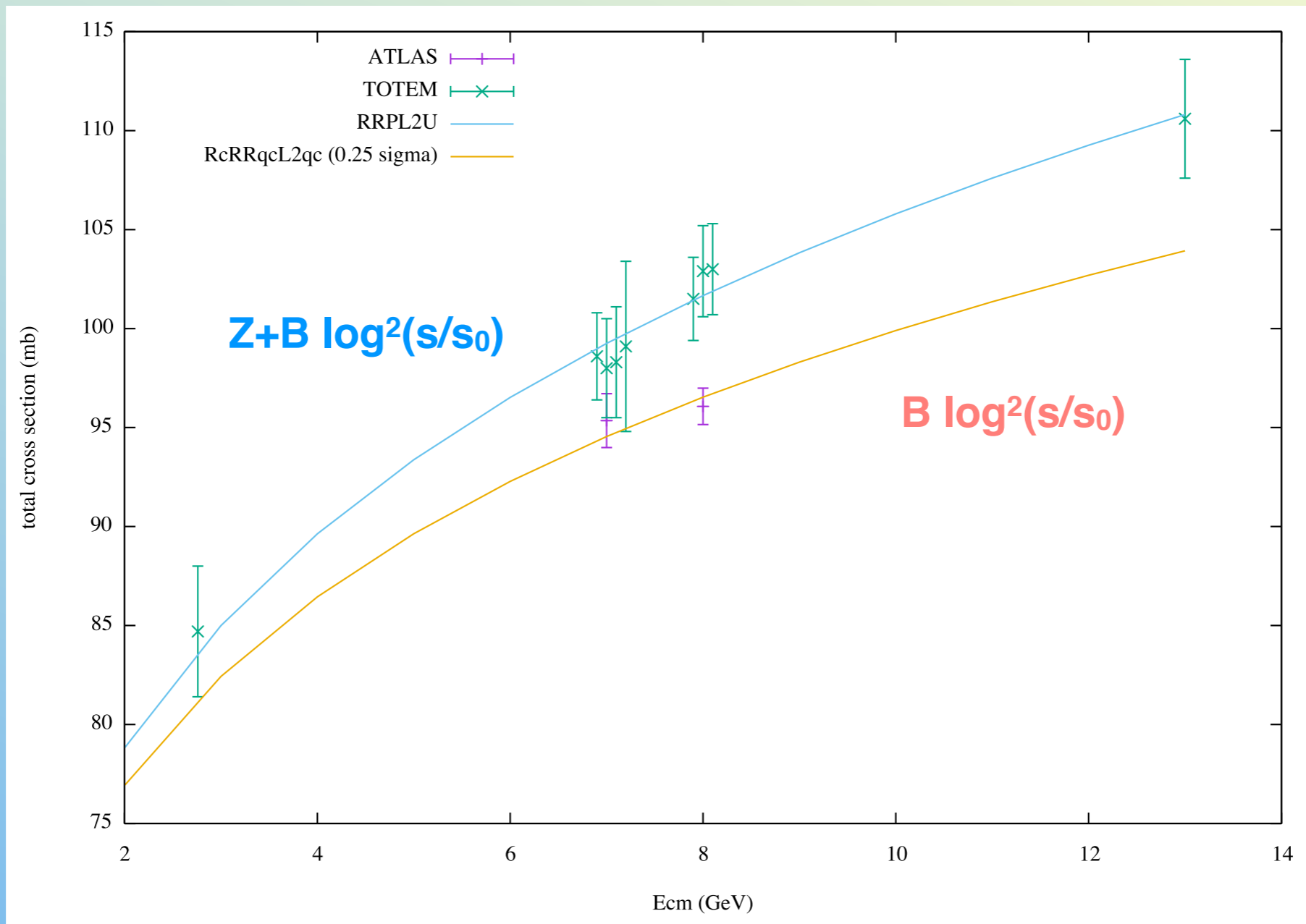
J.R. Cudell, V.V. Ezhela, P. Gauron, K. Kang,⁴ Yu.V. Kuyanov, S.B. Lugovsky, E. Martynov, B. Nicolescu, E.A. Razuvaev, and N.P. Tkachenko, PRL

double poles ($\log(s)$)/ triple poles ($\log^2(s)$)
+ 2 undegenerate lower trajectories

hep-ph/0206172

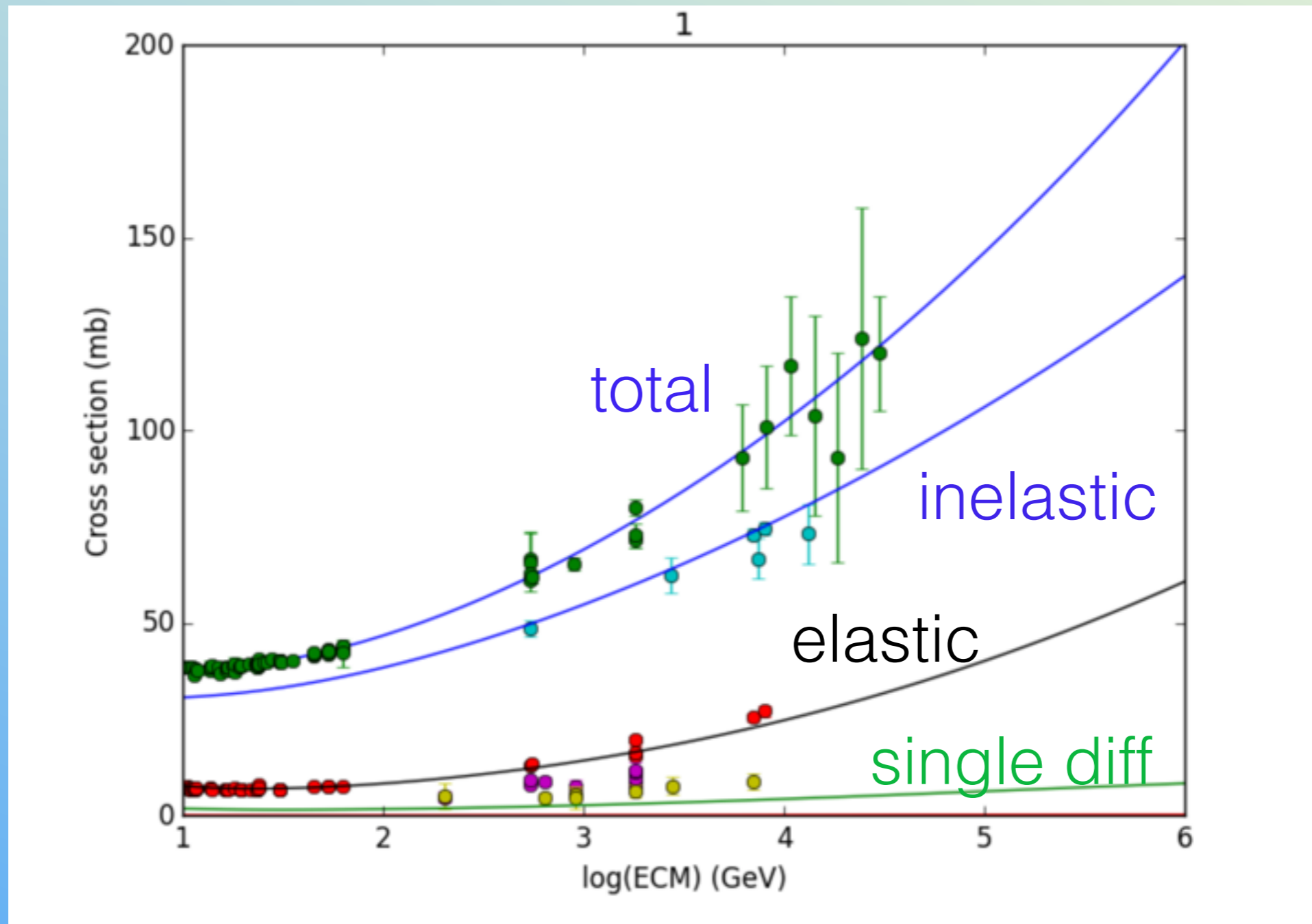


$\sigma_{tot}^{pp}(10 \text{ TeV})$
84 – 112 mb
 $\sigma_{tot}^{pp}(14 \text{ TeV})$
90 – 117 mb



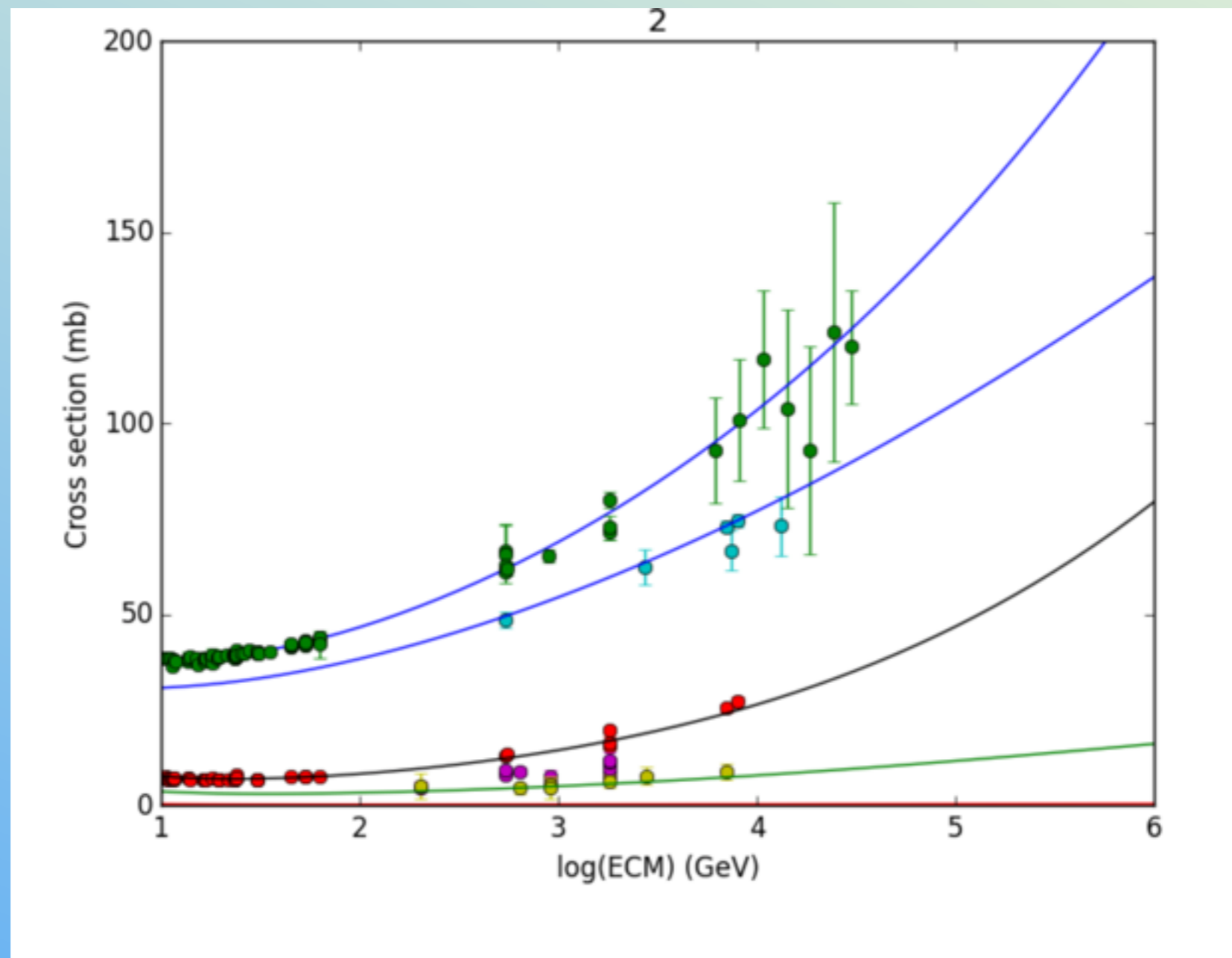
Reminiscent of the TeVatron
problem

Simple eikonal (Black disk)



A Vanthieghem & JRC, <https://matheo.ulg.ac.be/handle/2268.2/1301>

U Matrix (full unitarity)



A Vanthieghem & JRC, <https://matheo.ulg.ac.be/handle/2268.2/1301>

The elastic cross section @ LHC

$$|\mathcal{A}(s, t)|^2 \propto \frac{d\sigma}{dt}$$

Simplest parametrisation

$$\mathcal{A}(s, t) = \frac{ih s}{4\pi} \log^2 \left(\frac{s}{s_0} \right) \left(\frac{s}{s_0} \right)^{\frac{B_1}{2}t + \frac{B_2}{2}t^2} F_1^2(t)$$

+divide each dataset by a factor n_i

	statistical	statistical+ systematic	statistical+ normalisation
χ^2	48337	421	1812
h (GeV^{-2})	0.30	0.30	0.31
B_1 (GeV^{-2})	0.55	0.55	0.58
B_2 (GeV^{-4})	-0.39	-0.39	-0.26
$\sigma_{tot}(7 \text{ TeV})$ (mb)	95.3	95.1	96.8
$\sigma_{tot}(8 \text{ TeV})$ (mb)	98.2	98.0	99.7
TOTEM n_i at 7 TeV			1.03
TOTEM n_i at 8 TeV			1.05,1.06
ATLAS n_i at 7 TeV			0.98
ATLAS n_i at 8 TeV			0.94

More sophisticated: Selyugin's High Energy Generalised Structure (HEGS)

- Eikonal unitarisation
- Vector and tensor form factors from GPD's
- Coulomb region
- $s \leftrightarrow u$ crossing symmetry
- normalisation coefficients for each dataset

$$n_{\text{TOTEM}} \approx 0.9, n_{\text{ATLAS}} \approx 1$$

Small $|t|$ description

[O. Selyugin, arXiv:1410.6305](https://arxiv.org/abs/1410.6305)

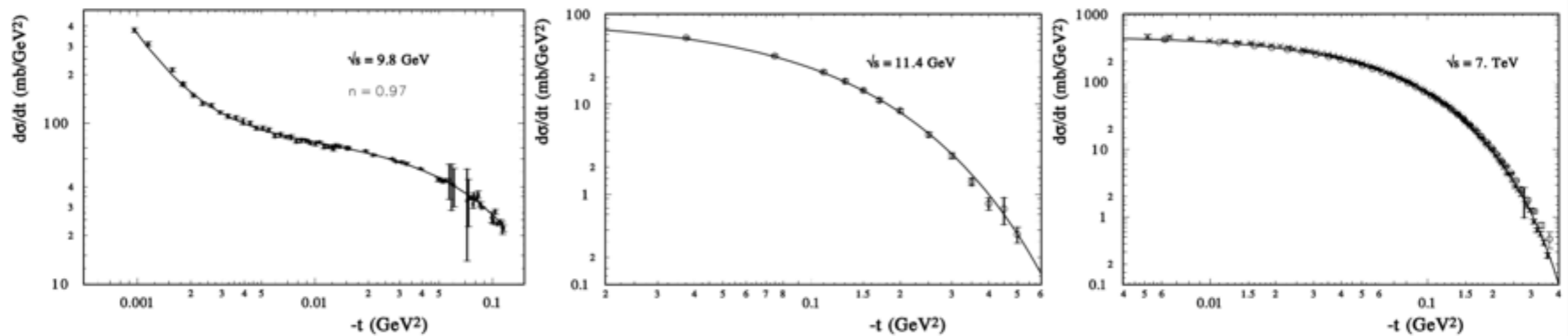
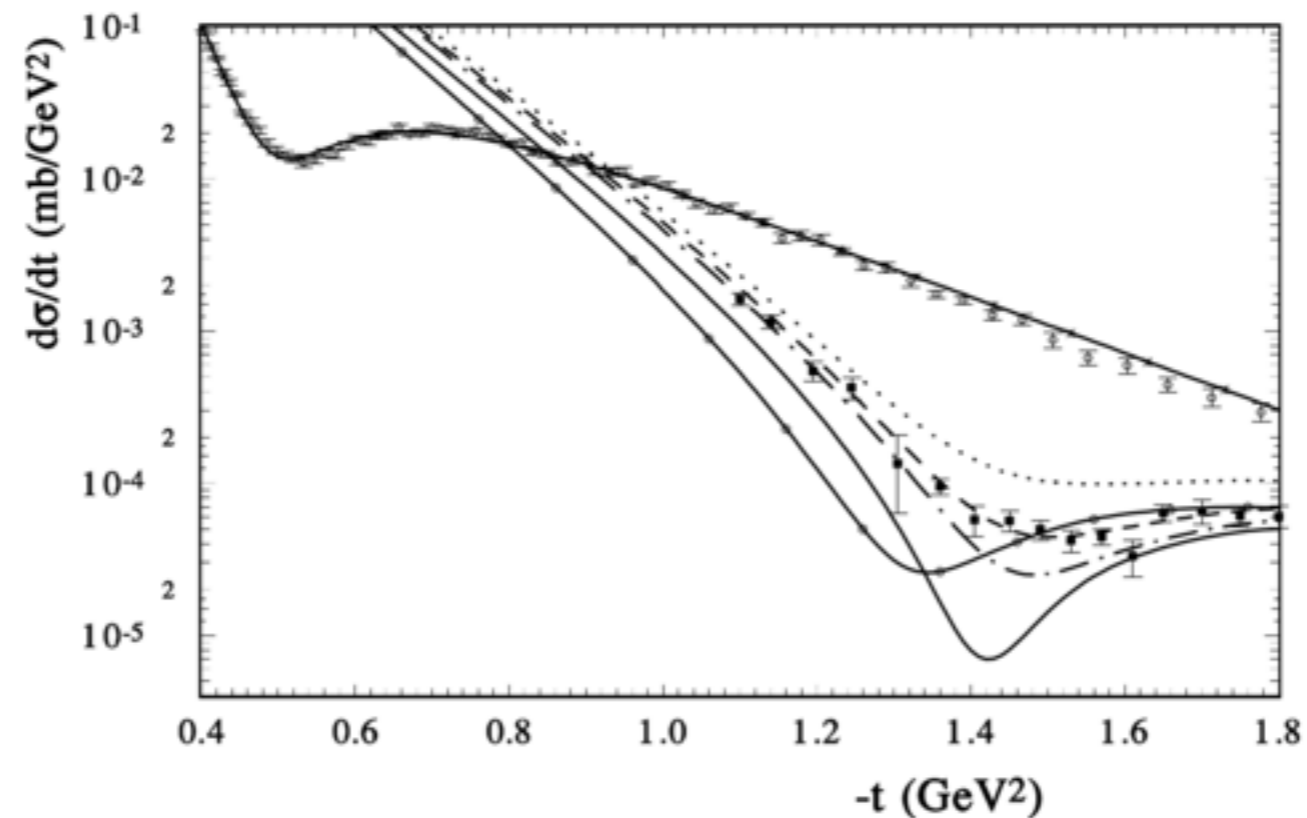


FIGURE 1. $d\sigma/dt$ a) for pp (left panel) at $\sqrt{s} = 9.8$ GeV and b) $p\bar{p}$ (central panel) at $\sqrt{s} = 11.3$ GeV and c) pp (right panel) at $\sqrt{s} = 7$ TeV [cross - TOTEM data [10], circles - ATLAS data [11]].

Large $|t|$ description



[O. Selyugin, arXiv:1609.08847](https://arxiv.org/abs/1609.08847)

Figure 2: The model calculation of the diffraction minimum in $d\sigma/dt$ of pp at $\sqrt{s} = 13.4; 16.8; 19.4; 30.4; 52.8; 7000$ GeV; (lines correspondingly - dots; short dash; dot-dash; solid; solid+circles; solid+ants); the squares - the data at $\sqrt{s} = 16.82$ GeV, and the circles - the data at $\sqrt{s} = 7$ TeV [32].

Small $|t|$

O. Selyugin, arXiv:1505.02426 doi: 10.1103/PhysRevD.91.113003

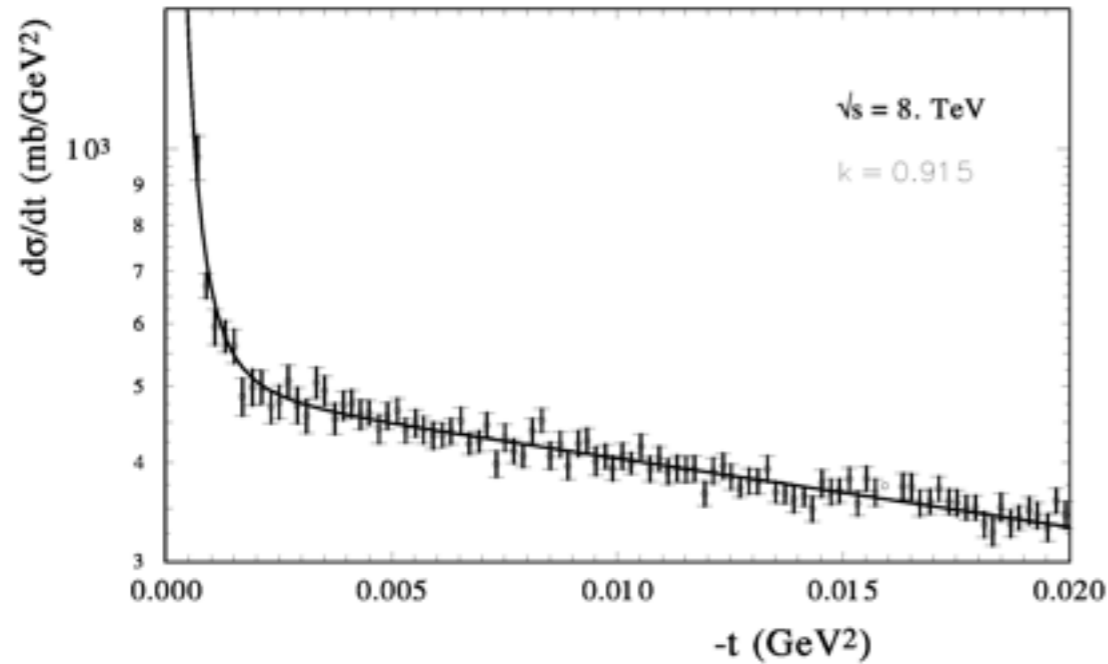


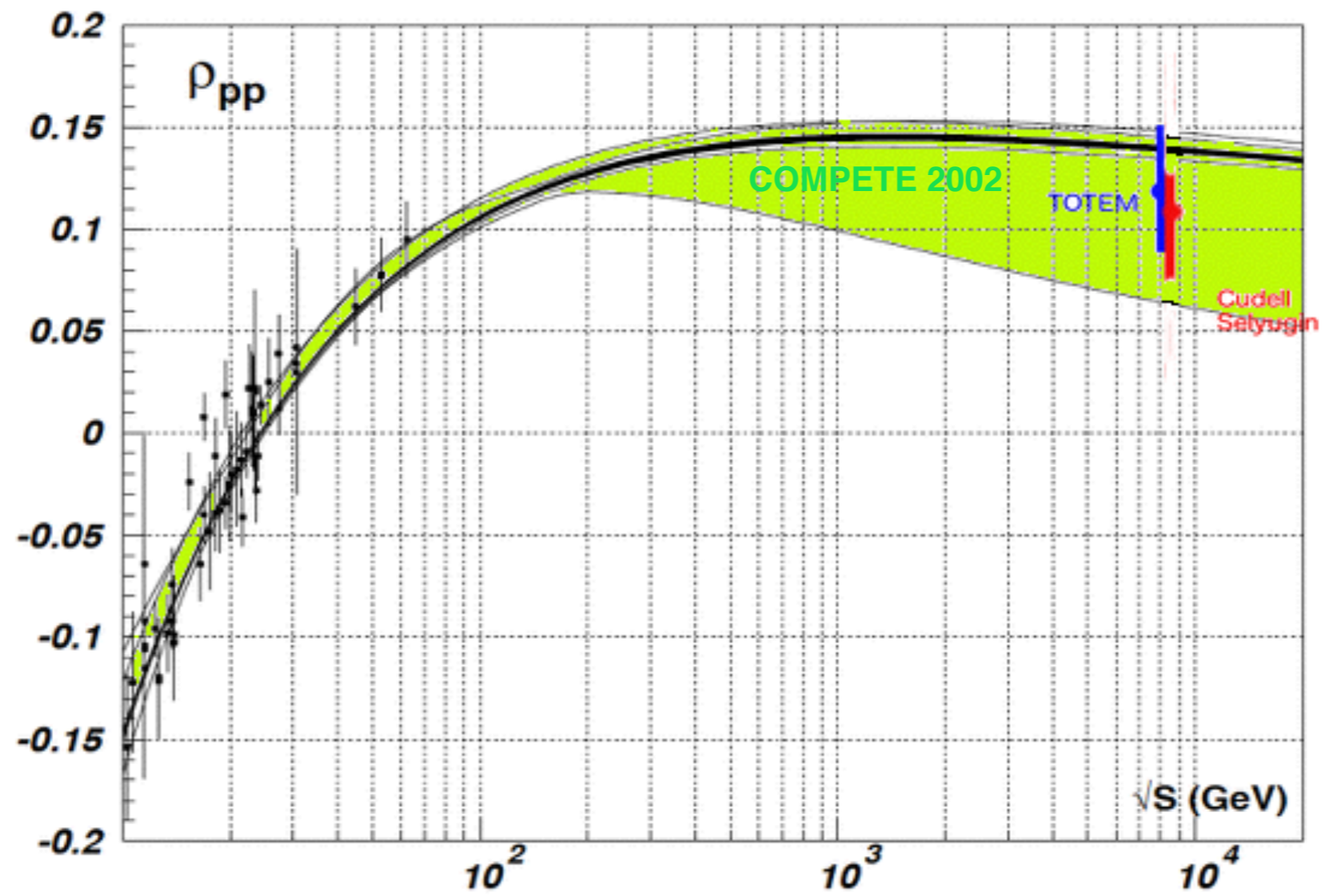
TABLE V. The obtained [85] and predicted sizes of $\sigma_{\text{tot}}(s)$ (in mb) and $\rho(t=0, s)$.

\sqrt{s} , GeV	$\sigma_{\text{tot-exp}}$	σ_{tot}	$\rho(t=0, s)$
19.42	38.98 ± 0.4	39.58 ± 0.8	-0.005 ± 0.0006
22.96	39.42 ± 0.4	39.89 ± 0.8	-0.005 ± 0.0006
52.8	42.85 ± 0.7	43.15 ± 0.5	0.074 ± 0.005
541	62.72 ± 0.2	62.72 ± 0.2	0.128 ± 0.005
1800	77.3 ± 0.38	77.3 ± 0.38	0.127 ± 0.02
7000	98.0 ± 2.6	97.16 ± 0.5	0.121
7000	$96.4 \pm 2.$	97.16 ± 0.5	0.121
8000	$101. \pm 2.1$	99.4 ± 0.5	0.12
14000	$104 \pm 26.$	108.76 ± 0.5	0.1176
30000	$120. \pm 15$	122.7 ± 0.5	0.11
57000	133 ± 23	135.4 ± 0.5	0.11

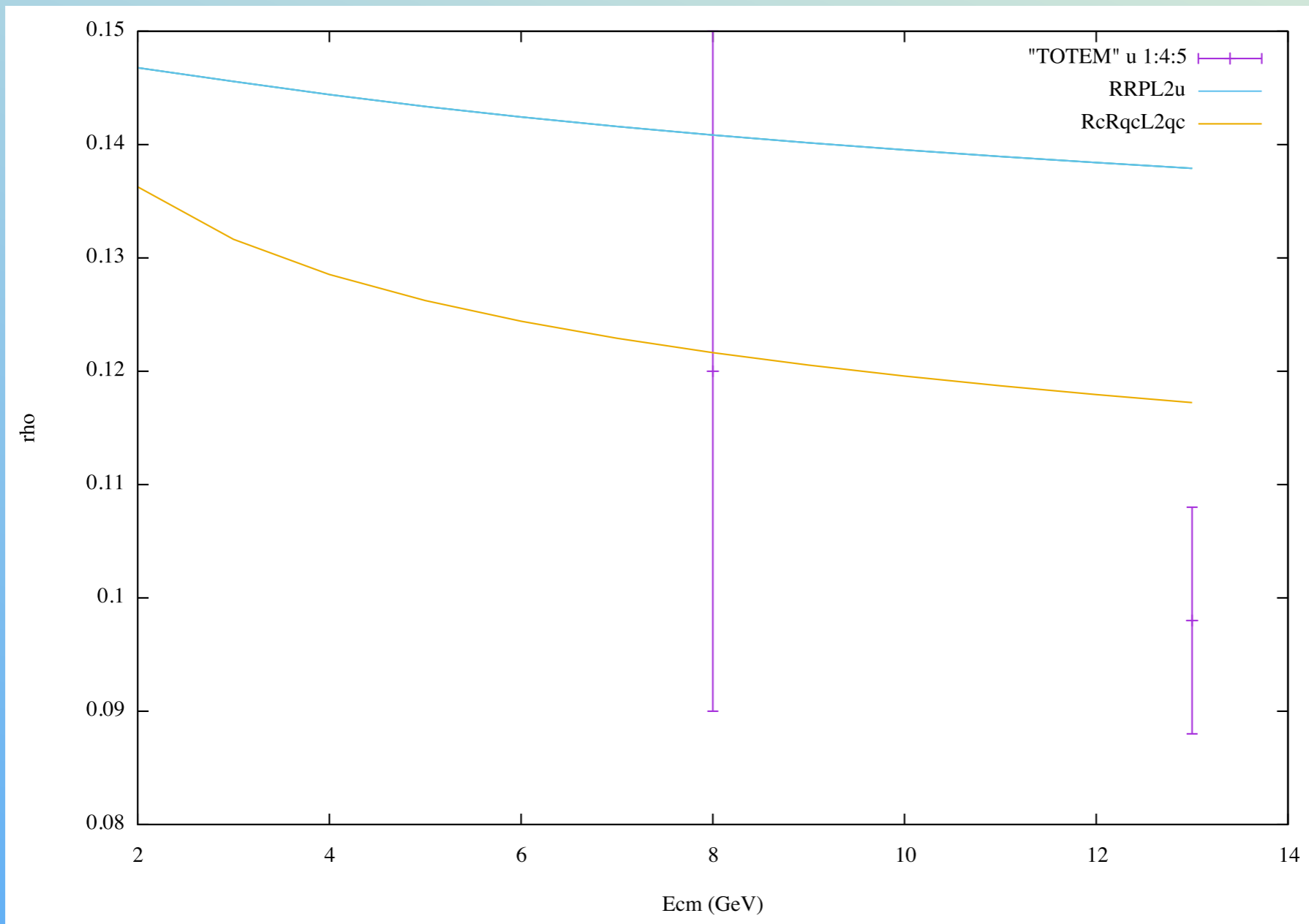
The ρ parameter

$$\frac{\Re \mathcal{A}(s, 0)}{\Im \mathcal{A}(s, 0)} = \rho(s, 0)$$

TOTEM 2015



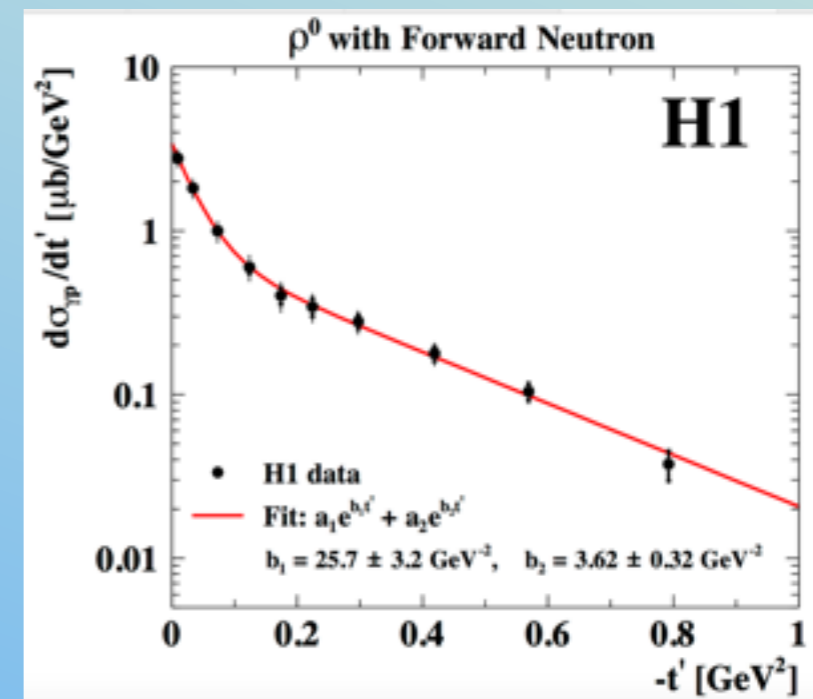
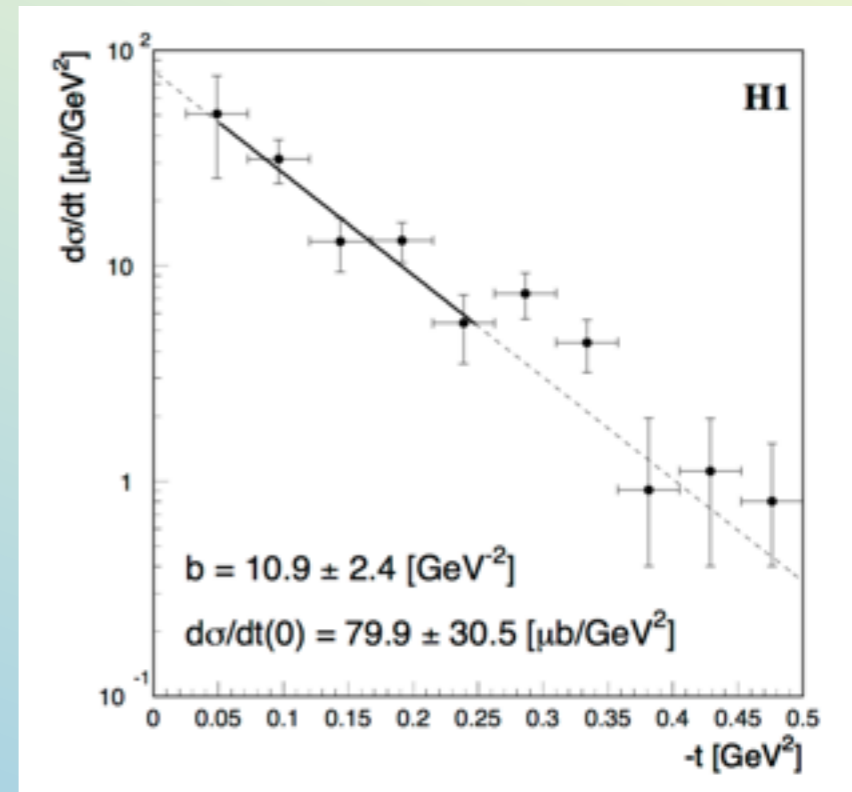
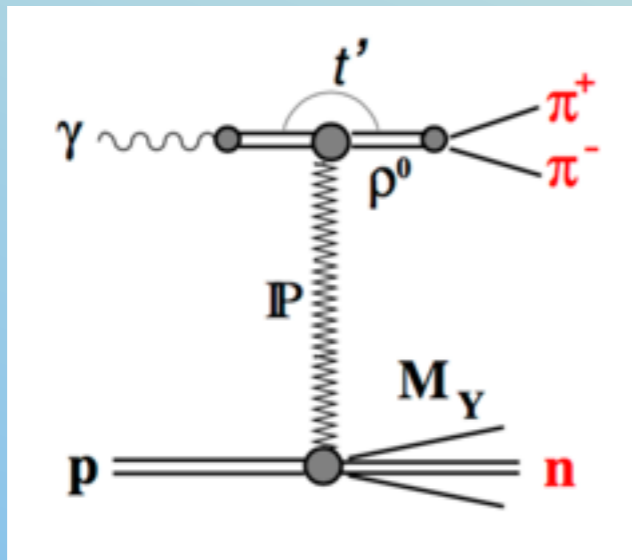
13 TeV?



Conclusions

- What is the source of the disagreement between TOTEM and ATLAS?
 - ▶ Overall normalisation?
 - ▶ Single-diffractive background?
- COMPETE central values are meaningless.
- Models are flexible enough not to favour either dataset.

H1 neutrons



$x_L > 0.95$, $\theta_n < 0.75$ mrad